WEB APPENDIX

The Customer Journey as a Source of Information

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Contents

A Model priors

We detail the specification of the prior distribution for the model parameters.

First, for the population covariance matrix Σ that governs customer heterogeneity in (9), we choose the standard Wishart prior for the precision matrix Σ^{-1},

$$\Sigma^{-1} \sim \text{Wishart}(r_0, R_0).$$

Second, we put priors on the Pitman-Yor process discount and strength parameters, d and a,[16] respectively by

$$\begin{aligned} d &\sim \text{Beta}(\phi_0^d, \phi_1^d) \\ a &\sim \text{Gamma}(\phi_0^a, \phi_1^a). \end{aligned}$$

Third, we put priors on the location parameters θ_c by defining the base distribution of the Pitman-Yor process, F_0. As described in (13), the location parameters are drawn from $\theta_c \sim F_0(\boldsymbol{\phi}_0)$. Following the notation in (10), consider θ^ω and θ^ρ the components of θ that correspond to query parameters $\boldsymbol{\omega}_j$ and click-purchase parameters $\boldsymbol{\rho}_j$, respectively. We define F_0 as a multivariate distribution factorized by each of the components of θ, defined by

$$F_0(\theta|\boldsymbol{\phi}_0) = \left(\prod_{m=1}^{M} F_{0m}^{\omega}(\theta_m^{\omega}|\phi_{0m}) \right) \times \mathcal{N}(\theta^\rho|\mu_0, V_0),$$

where we assume Gaussian priors for the location parameter of click and purchase preferences, and F_{0m}^q is defined accordingly to the support of the parameter that governs the distribution of each query variable m described in (2). That is,

$$F_{0m}^{\omega}(\theta_m^{\omega}|\phi_{0m}) = \begin{cases} \text{Beta}(\phi_{0ma}, \phi_{0mb}) & \text{if } q_{ijm} \text{ is binary} \\ \text{Dirichlet}(\phi_{0m}) & \text{if } q_{ijm} \text{ is categorical} \\ \text{Gamma}(\phi_{0ma}, \phi_{0mb}) & \text{if } q_{ijm} \text{ is continuous positive-valued} \\ \mathcal{N}(\phi_{0m\mu}, \phi_{0m\sigma}) & \text{if } q_{ijm} \text{ is continuous.} \end{cases}$$

[16] We restrict the model to $a > 0$.

Finally, we put mean-zero Gaussian priors on all other parameters in the model including η in (3), and α^0_ℓ, $\boldsymbol{\alpha}^w_\ell$ and $\boldsymbol{\alpha}^\beta_\ell$ in (4)

$$
\begin{aligned}
\eta &\sim \mathcal{N}(0, s^2_\eta)\\
\boldsymbol{\alpha}^0 &\sim \mathcal{N}(\mathbf{0}, S_{\alpha,0})\\
\boldsymbol{\alpha}^w_\ell &\sim \mathcal{N}(\mathbf{0}, S_{\alpha,w}), \forall \ell\\
\boldsymbol{\alpha}^\beta_\ell &\sim \mathcal{N}(\mathbf{0}, S_{\alpha,\beta}), \forall \ell
\end{aligned}
$$

B Blocked-Gibbs sampler algorithm

Our Metropolis-within-Gibbs MCMC sampling algorithm is based on Ishwaran and James (2001) approximation using the stick-breaking representation of the Pitman-Yor (PY) Process, truncating the infinite mixture by setting $V_C = 1$ for a large enough integer C. This approximation allows us to draw context memberships of different journeys in parallel, significantly increasing our sampling scheme's speed. We use adaptive Metropolis-Hastings (M-H) steps to update the PY parameters d and a as these full conditionals do not have a closed form (a has closed form only if $d = 0$). We use Gibbs steps for all other parameters as their full conditionals have closed form. Similarly to the click and purchase components, we use data augmentation for the filter decisions and define $u^f_{ij\ell} = \alpha^0_\ell + \mathbf{w}_{ij\ell}{}' \cdot \boldsymbol{\alpha}^w_\ell + \boldsymbol{\beta}^{x}_{ij}{}' \cdot \boldsymbol{\alpha}^\beta_\ell + \varepsilon^f_{ij\ell}$, such that $\varepsilon^f_{ij\ell} \sim \mathcal{N}(0,1)$ and $f_{ij\ell} = \mathbb{1}(u^f_{ij\ell} > 0)$.

We sequentially update the parameters by,

1. Draw latent click utilities for alternative $k \in \text{Page}_{ijt} \cup \{s\}$ using a truncated Gaussian by,

$$
u^c_{ijtk} \sim \begin{cases}
\text{Truncated-}\mathcal{N}\left(\bar{u}^c_{ijtk}, 1, \text{lower} = -\infty, \text{upper} = 0\right) & \text{if } y^c_{ijt} = e\\
\text{Truncated-}\mathcal{N}\left(\bar{u}^c_{ijtk}, 1, \text{lower} = \max\{u^c_{ijt-k}, 0\}, \text{upper} = \infty\right) & \text{if } y^p_{ij} = k\\
\text{Truncated-}\mathcal{N}\left(\bar{u}^c_{ijtk}, 1, \text{lower} = -\infty, \text{upper} = \max\{u^c_{ijt-k}\}\right) & \text{otherwise,}
\end{cases}
$$

where $\bar{u}^c_{ijtk} = \beta^{0c}_{ij} + \mathbf{x}^c_{ijtk}{}' \cdot \boldsymbol{\beta}^x_{ij} + \text{log-rank}_{ijtk} \cdot \eta$ if $k \in \text{Page}_{ijt}$, and $\bar{u}^c_{ijtk} = \beta^{0s}_{ij}$ if $k = s$.

2. Draw latent purchase utilities by,

$$
u^p_{ijk} \sim \begin{cases}
\text{Truncated-}\mathcal{N}\left(\bar{u}^p_{ijk}, 1, \text{lower} = -\infty, \text{upper} = 0\right) & \text{if } y^p_{ij} = \text{NoPurchase}\\
\text{Truncated-}\mathcal{N}\left(\bar{u}^p_{ijk}, 1, \text{lower} = \max\{u^p_{ij-k}, 0\}, \text{upper} = \infty\right) & \text{if } y^p_{ij} = k\\
\text{Truncated-}\mathcal{N}\left(\bar{u}^p_{ijk}, 1, \text{lower} = -\infty, \text{upper} = \max\{u^p_{ij-k}\}\right) & \text{otherwise,}
\end{cases}
$$

where $\bar{u}^p_{ijk} = \beta^{0p}_{ij} + \mathbf{x}_{ijk}{}' \cdot \boldsymbol{\beta}^x_{ij}$.

3. Draw latent filter utilities by,

$$u^f_{ij\ell} \sim \begin{cases} \text{Truncated-}\mathcal{N}\left(\alpha^0_\ell + \mathbf{w}_{ij\ell}{}' \cdot \boldsymbol{\alpha}^w_\ell + \boldsymbol{\beta}^{x}_{ij}{}' \cdot \boldsymbol{\alpha}^\beta_\ell, 1, \text{lower} = -\infty, \text{upper} = 0\right) & \text{if } f_{ij\ell} = 0 \\ \text{Truncated-}\mathcal{N}\left(\alpha^0_\ell + \mathbf{w}_{ij\ell}{}' \cdot \boldsymbol{\alpha}^w_\ell + \boldsymbol{\beta}^{x}_{ij}{}' \cdot \boldsymbol{\alpha}^\beta_\ell, 1, \text{lower} = 0, \text{upper} = \infty\right) & \text{if } f_{ij\ell} = 1. \end{cases}$$

4. Draw individual-level stable preferences $\boldsymbol{\mu}_i$. We define a vector of click, purchase, and filter latent utilities for journey j,

$$\tilde{u}_{ij} = \begin{bmatrix} \left[u^c_{ijtk} - \text{log-rank}_{ijtk} \cdot \eta\right]_{tk} \\ \left[u^p_{ijk}\right]_k \\ \left[u^f_{ij\ell} - \alpha^0_\ell - \mathbf{w}_{ij\ell}{}' \cdot \boldsymbol{\alpha}^w_\ell\right]_\ell \end{bmatrix}^\top,$$

and $\widetilde{\mathbf{X}}_{ij}$ the corresponding "stacked" matrix of vectors multiplying $\boldsymbol{\beta}_{ij}$ in equations (3), (4), and (6). That is,

$$\widetilde{\mathbf{X}}_{ij} = \begin{bmatrix} \widetilde{\mathbf{X}}^c_{ij} \\ \widetilde{\mathbf{X}}^p_{ij} \\ \widetilde{\mathbf{A}}^f_{ij} \end{bmatrix},$$

where $\widetilde{\mathbf{X}}^c_{ij}$ is the matrix of stacked click covariates. Specifically,

$$\widetilde{\mathbf{X}}^c_{ij} = \begin{bmatrix} \widetilde{\mathbf{X}}^c_{ij1} \\ \vdots \\ \widetilde{\mathbf{X}}^c_{ijt} \\ \vdots \\ \widetilde{\mathbf{X}}^c_{ijT_{ij}} \end{bmatrix} \text{ and } \widetilde{\mathbf{X}}^c_{ijt} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & \mathbf{x}^c_{ijt1}{}' \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & 0 & \mathbf{x}^c_{ijtk}{}' \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & 0 & \mathbf{x}^c_{ijtK_{ijt}}{}' \end{bmatrix}.$$

Similarly, $\widetilde{\mathbf{X}}^p_{ij}$ is the matrix of stacked purchased covariates,

$$\widetilde{\mathbf{X}}^p_{ij} = \begin{bmatrix} 0 & 0 & 1 & \mathbf{x}_{ij1}{}' \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 1 & \mathbf{x}_{ijk}{}' \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 1 & \mathbf{x}_{ijtK_{ij}}{}', \end{bmatrix},$$

and $\widetilde{\mathbf{A}}_{ij}^{f} = \left[\boldsymbol{\alpha}_1^{\beta} \ldots \boldsymbol{\alpha}_\ell^{\beta} \ldots \boldsymbol{\alpha}_{L_{ij}}^{\beta}\right]'$.

The columns of each of these matrices multiply $\boldsymbol{\beta}_{ij} = \left(\beta_{ij}^{0c}, \beta_{ij}^{0s}, \beta_{ij}^{0p}, \boldsymbol{\beta}_{ij}^{x\prime}\right)'$, respectively; which yields the terms in (3), (4), and (6).

We further define $\widetilde{\mathbf{X}}_i$ as

$$\widetilde{\mathbf{X}}_i = \begin{bmatrix} \widetilde{\mathbf{X}}_{i1} \\ \vdots \\ \widetilde{\mathbf{X}}_{ij} \\ \vdots \\ \widetilde{\mathbf{X}}_{iJ_i} \end{bmatrix},$$

and $\widetilde{\mathbf{u}}_i$ as

$$\widetilde{\mathbf{u}}_i = \begin{bmatrix} \widetilde{u}_{i1} - \widetilde{\mathbf{X}}_{i1} \cdot \boldsymbol{\rho}_1 \\ \vdots \\ \widetilde{u}_{ij} - \widetilde{\mathbf{X}}_{ij} \cdot \boldsymbol{\rho}_j \\ \vdots \\ \widetilde{u}_{iJ_i} - \widetilde{\mathbf{X}}_{iJ_i} \cdot \boldsymbol{\rho}_{J_i} \end{bmatrix}.$$

Finally, we draw $\boldsymbol{\mu}_i \sim \mathcal{N}(\widetilde{\mu}_i, \widetilde{S}_i)$ where

$$\begin{aligned} \widetilde{S}_i^{-1} &= \Sigma^{-1} + \widetilde{\mathbf{X}}_i' \widetilde{\mathbf{X}}_i \\ \widetilde{\mu}_i &= \widetilde{S}_i \left(\Sigma^{-1} \cdot \mathbf{0} + \widetilde{\mathbf{X}}_i' \widetilde{\mathbf{u}}_i\right). \end{aligned}$$

5. Draw context membership z_j as follows

$$p(z_j = c | \cdot) = \frac{\pi_c \mathcal{P}_{jc}}{\sum_{c'=1}^{C} \pi_{c'} \mathcal{P}_{jc'}},$$

where $\mathcal{P}_{jc} = \left(\prod_{m=1}^{M} p(q_{ijm} | \theta_{cm}^{\omega})\right) \cdot p\left(\widetilde{u}_{ij} - \widetilde{\mathbf{X}}_{ij} \boldsymbol{\mu}_i | \widetilde{\mathbf{X}}_{ij} \theta_j^{\rho}, 1\right)$, with $p(q_{ijm} | \theta_{cm}^{\omega})$ denoting the pdf of query variables as defined in (2), and $p\left(\widetilde{u}_{ij} - \widetilde{\mathbf{X}}_{ij} \cdot \boldsymbol{\mu}_i | \widetilde{\mathbf{X}}_{ij} \cdot \theta_j^{\rho}, 1\right)$ denoting the product of elementwise normal pdf evaluated at each components of $\widetilde{u}_{ij} - \widetilde{\mathbf{X}}_{ij} \cdot \boldsymbol{\mu}_i$ with mean $\widetilde{\mathbf{X}}_{ij} \cdot \theta_j^{\rho}$ and variance 1.

6. Draw the query components of context location parameters θ_c^ω for each context c. We denote $\mathcal{J}(c)$ the set of journeys j such that $z_j = c$, and $n_c = |\mathcal{J}(c)|$ the number of journeys in that set. For each query variable m, we draw θ_{cm}^ω depending on the type of query variable modeled in (2). Specifically,

$$\theta_{cm}^\omega \sim \begin{cases} \text{Beta}\left(\phi_{0ma} + \sum\limits_{j \in \mathcal{J}(c)} q_{ijm}, \phi_{0mb} + n_c - \sum_{j \in \mathcal{J}(c)} q_{ijm}\right) & \text{if } q_{ijm} \text{ is binary} \\ \text{Dirichlet}\left(\phi_{0m} + [nq_{cm1}, \ldots, nq_{cmN_m}]^\top\right) & \text{if } q_{ijm} \text{ is categorical} \\ \text{Gamma}\left(\phi_{0ma} + n_c, \phi_{0mb} + \sum\limits_{j \in \mathcal{J}(c)} q_{ijm}\right) & \text{if } q_{ijm} \text{ is continuous positive-valued} \\ \mathcal{N}(\tilde{\mu}_{cm}, \tilde{s}_{cm}) & \text{if } q_{ijm} \text{ is continuous,} \end{cases}$$

where $nq_{cmn} = \sum_{j \in \mathcal{J}(c)} \mathbb{1}(q_{ijm} = n)$, $\tilde{s}_{cm}^{-1} = \left[\phi_{0m\sigma}^{-1} + \sigma_m^{-2}\right]$ and $\tilde{\mu}_{cm} = \tilde{s}_{cm} \sum_{j \in \mathcal{J}(c)} q_{ijm}$.

7. Draw the click-purchase context location parameters θ^ρ. We define $\bar{\mathbf{X}}_c$ and $\bar{\mathbf{u}}_c$ as

$$\bar{\mathbf{X}}_c = \left[\left[\tilde{\mathbf{X}}_{i(j)j}\right]_{j \in \mathcal{J}(c)}\right], \text{ and } \qquad \bar{\mathbf{u}}_c = \left[\left[\tilde{u}_{i(j)j} - \tilde{\mathbf{X}}_{i(j)j} \cdot \boldsymbol{\mu}_{i(j)}\right]_{j \in \mathcal{J}(c)}\right],$$

where $i(j)$ denotes the customer to whom journey j belongs to.

We draw $\theta_c^\rho \sim \mathcal{N}(\bar{\mu}_c, \bar{S}_c)$, where

$$\begin{aligned} \bar{S}_c^{-1} &= V_0^{-1} + \bar{\mathbf{X}}_c' \bar{\mathbf{X}}_c \\ \bar{\mu}_c &= \bar{S}_c \left(V_0^{-1} \cdot \mu_0 + \bar{\mathbf{X}}_c' \bar{\mathbf{u}}_c\right). \end{aligned}$$

8. Draw ranking effect η. Defining $\mathbf{r}$ as the vector of all log-rank$_{ijtk}$ values, and the vector of differences in click utilities $\mathbf{u}^{\mathrm{r}} = \left[\{u_{ijtk}^c - \beta_{ij}^{0c} + \mathbf{x}_{ijtk}^{c}{}' \cdot \boldsymbol{\beta}_{ij}^x\}_{ijtk}\right]$, we draw η by

$$\eta \sim \mathcal{N}(\bar{\mu}_\eta, \bar{s}_\eta^2),$$

where

$$\begin{aligned} \overline{s_\eta}^{-1} &= s_\eta^{-1} + \mathbf{r}'\mathbf{r} \\ \bar{\mu}_\eta &= \bar{s}_\eta \left(s_\eta^{-1} \cdot 0 + \mathbf{r}' \mathbf{u}^{\mathrm{r}}\right). \end{aligned}$$

9. Draw $\boldsymbol{\alpha}^0$. We define $\tilde{\mathbf{u}}_0^f$ as the vector of residual filter utilities where each component is an observation (i,j,ℓ) defined as

$$\tilde{u}_{0ij\ell}^f = u_{ij\ell}^f - \mathbf{w}_{ij\ell}{}' \cdot \boldsymbol{\alpha}_\ell^w - \boldsymbol{\beta}_{ij}^{x}{}' \cdot \boldsymbol{\alpha}_\ell^\beta.$$

We also define a binary matrix that multiplies the vector of intercepts $\boldsymbol{\alpha}^0$ to yield the respective level for each observation. In other words, this matrix encodes in binary variables to which level ℓ the observation (row) belongs, such that the entry in each row that represents the observation (i,j,ℓ) takes the value one for column ℓ, and zero for all others. Consequently, we draw $\boldsymbol{\alpha}^0$ by

$$\boldsymbol{\alpha}^0 \sim \mathcal{N}(\bar{\boldsymbol{\mu}}_{\alpha,0}, \bar{S}_{\alpha,0}),$$

where

$$\begin{aligned}\bar{S}_{\alpha,0}^{-1} &= S_{\alpha,0}^{-1} + \tilde{\mathbf{w}}_0' \tilde{\mathbf{w}}_0 \\ \bar{\boldsymbol{\mu}}_{\alpha,0} &= \bar{S}_{\alpha,0}\left(S_{\alpha,0}^{-1} \cdot 0 + \tilde{\mathbf{w}}_0' \tilde{\mathbf{u}}_0^f\right).\end{aligned}$$

10. Draw $\boldsymbol{\alpha}_\ell^w$. We define $\tilde{\mathbf{u}}_{w,\ell}^f = \left[\{u_{ij\ell}^f - \alpha_\ell^0 - \boldsymbol{\beta}_{ij}^{x}{}' \cdot \boldsymbol{\alpha}_\ell^\beta\}_{ij}\right]$ as the vector of residual filter utilities, and $\mathbf{W}_\ell = \left[\{\mathbf{w}_{ij\ell}'\}_{ij}\right]$ the matrix of filter controls for level ℓ, and draw $\boldsymbol{\alpha}_\ell^w$ by

$$\boldsymbol{\alpha}_\ell^w \sim \mathcal{N}(\bar{\boldsymbol{\mu}}_\ell^w, \bar{S}_{\alpha,w,\ell}),$$

where

$$\begin{aligned}\bar{S}_{\alpha,w,\ell}^{-1} &= S_{\alpha,w}^{-1} + \mathbf{W}_\ell' \mathbf{W}_\ell \\ \bar{\boldsymbol{\mu}}_\ell^w &= \bar{S}_{\alpha,w,\ell}\left(S_{\alpha,w}^{-1} \cdot 0 + \mathbf{W}_\ell' \tilde{\mathbf{u}}_{w,\ell}^f\right).\end{aligned}$$

11. Draw $\boldsymbol{\alpha}_\ell^\beta$. We define $\tilde{\mathbf{u}}_{w,\ell}^b = \left[\{u_{ij\ell}^f - \alpha_\ell^0 - \mathbf{w}_{ij\ell}' \cdot \boldsymbol{\alpha}_\ell^w\}_{ij}\right]$ as the vector of residual filter utilities, and $\mathbf{B}_\ell = \left[\{\boldsymbol{\beta}_{ij}'\}_{ij}\right]$ the matrix of preferences, where each row of the matrix contains the vector of preferences corresponding to the respective row in $\tilde{\mathbf{u}}_{w,\ell}^b$. Draw $\boldsymbol{\alpha}_\ell^\beta$ by

$$\boldsymbol{\alpha}_\ell^\beta \sim \mathcal{N}(\bar{\boldsymbol{\mu}}_\ell^\beta, \bar{S}_{\alpha,\beta,\ell}),$$

where

$$\bar{S}^{-1}_{\alpha,\beta,\ell} = S^{-1}_{\alpha,\beta} + \mathbf{B}'_\ell \mathbf{B}_\ell$$
$$\bar{\boldsymbol{\mu}}^{\beta}_\ell = \bar{S}_{\alpha,\beta,\ell}\big(S^{-1}_{\alpha,\beta}\cdot 0 + \mathbf{B}'_\ell \tilde{\mathbf{u}}^b_{w,\ell}\big).$$

12. (M-H step) Draw a proposal $a^{\text{prop}} \sim p_{a-\text{prop}}(\cdot|a)$. Update $a = a^{\text{prop}}$ with probability

$$\alpha(a,a^{\text{prop}}) = \min\left\{1, \frac{\text{Gamma}(a^{\text{prop}}|\phi^a_0,\phi^a_1)}{\text{Gamma}(a|\phi^a_0,\phi^a_1)} \cdot \frac{\prod_{c=1}^{C-1}\text{Beta}(V_c|1-d, a^{\text{prop}}+c\cdot d)}{\prod_{c=1}^{C-1}\text{Beta}(V_c|1-d, a+c\cdot d)} \cdot \frac{p_{a-\text{prop}}(a|a^{\text{prop}})}{p_{a-\text{prop}}(a^{\text{prop}}|a)}\right\}.$$

We use a log-normal $p_{a-\text{prop}}(\cdot|a) = \log\mathcal{N}(\log(a), \tau^2_n)$, where we use a vanishing adaptation procedure (Atchadé and Rosenthal, 2005) to adapt the proposal step size to target an acceptance rate of 0.44 (Gelman et al., 1995) through

$$\tau^2_n = \begin{cases} \tau^2_0 & n \leqslant 200, \\ |\tau^2_{n-1} + \frac{\epsilon}{n}(ap_n - 0.44)| & n > 200, \end{cases}$$

where ap_n is the empirical acceptance rate up to iteration n. Note the proposal distribution is not symmetric and yields a ratio $\frac{p_{a-\text{prop}}(a|a^{\text{prop}})}{p_{a-\text{prop}}(a^{\text{prop}}|a)} = \frac{a^{\text{prop}}}{a}$.

13. (M-H step) Draw a proposal $d^{\text{prop}} \sim p_{d-\text{prop}}(\cdot|d)$. Update $d = d^{\text{prop}}$ with probability

$$\alpha(d,d^{\text{prop}}) = \min\left\{1, \frac{\text{Beta}\big(d^{\text{prop}}|\phi^d_0,\phi^d_1\big)}{\text{Beta}\big(d|\phi^d_0,\phi^d_1\big)} \cdot \frac{\prod_{c=1}^{C-1}\text{Beta}(V_c|1-d^{\text{prop}}, a+c\cdot d^{\text{prop}})}{\prod_{c=1}^{C-1}\text{Beta}(V_c|1-d, a+c\cdot d)} \cdot \frac{1/p_{d-\text{prop}}(d^{\text{prop}}|d)}{1/p_{d-\text{prop}}(d|d^{\text{prop}})}\right\}.$$

We use a logit-normal proposal distribution $p_{d-\text{prop}}(\cdot|d) = \text{logit}-\mathcal{N}(\text{logit}(d), s^2_n)$, where the logit function is defined by $\text{logit}(d) = \log\big(\frac{d}{1-d}\big)$, and the logit-normal pdf is defined by $\text{logit}-\mathcal{N}(x|\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}}\frac{1}{x(1-x)}\exp\Big\{-\frac{(\text{logit}(x)-\mu)^2}{2\sigma^2}\Big\}$. We adapt s^2_n analogously to τ^2_n in the previous step.

14. Draw context probabilities π_c, by drawing the stick parameters V_c from

$$V_c \sim \text{Beta}\left(1-d+n_c, a+c\cdot d+\sum_{c'=c+1}^{C} n_{c'}\right),$$

and compute π_c according to (14).

15. Draw population covariance matrix Σ, by

$$\Sigma^{-1} \sim \text{Wishart}(r_1, R_1),$$

where

$$\begin{aligned} r_1 &= r_0 + I \\ {R_1}^{-1} &= {R_0}^{-1} + \sum_i \boldsymbol{\mu}_i \cdot {\boldsymbol{\mu}_i}' \end{aligned}$$

C Posterior distribution of holdout journey preferences

We outline the procedure to update the posterior distribution of preferences for holdout journeys, given data on the focal journey and past journeys. (This corresponds to the right-hand side of (16)). There are several relevant considerations for such a procedure.

First, we leverage the Pitman-Yor process when making inferences on new journeys, by allowing for a previously unobserved context to be discovered in this focal journey. Second, as the posterior of global parameters is obtained with a large number of journeys in the training sample, we approximate the posterior of these parameters given all training data plus focal journey j, by the posterior without focal journey j. That is, the inference on global parameters remains largely unchanged by the addition of a single journey (except for discovering a context that has not been observed before, as mentioned above). This assumption allows us to maintain computational efficiency by not re-estimating the whole model when updating the inference on current journey preferences as new data arrives. Third, as commented in Section 3.4, the context of past journeys is conditionally dependent on the focal journey given past and current journey data, because stable preferences and contexts both jointly determine the outcomes in both journeys. Therefore, in the process of drawing preferences for new journeys, we update the inferences for past journeys of the focal customer as well.

For each customer i, we denote the focal (holdout) journey by j, with j' referring to journeys different from the focal one. The set of past journeys (not including j) is denoted by $\mathcal{J}(i)$, the vector of contexts of all past journeys by $\mathbf{z}_{i,-j} = \{z_{ij'}\}_{j' \in \mathcal{J}(i)}$, the entire journey data for a journey j' by $\mathcal{H}_{i,j'} = \{\mathbf{q}_{ij'}, y^c_{ij'1:T_{j'}}, f_{ij'1:L}, y^p_{ij'}\}$, the collection of past journey data by $\mathcal{H}_i = \bigcup_{j' \in \mathcal{J}(i)} \mathcal{H}_{i,j'}$, the set of global parameters by Φ,[17] and all training data by $\mathcal{D}$.

We update the posterior of preferences for focal journey j, $\boldsymbol{\beta}_{ij}$, by

$$\begin{aligned} p(\boldsymbol{\beta}_{ij} | \mathbf{q}_{ij}, y^c_{ij1:t}, \mathcal{L}_{ijt}, \mathcal{H}_i, \mathcal{D}) &= \int p(\boldsymbol{\beta}_{ij} | \mathbf{q}_{ij}, y^c_{ij1:t}, \mathcal{L}_{ijt}, \mathcal{H}_i, \Phi) \cdot p(\Phi | \mathcal{D}, \mathbf{q}_{ij}, y^c_{ij1:t}, \mathcal{L}_{ijt}) d\Phi \\ &\approx \int p(\boldsymbol{\beta}_{ij} | \mathbf{q}_{ij}, y^c_{ij1:t}, \mathcal{L}_{ijt}, \mathcal{H}_i, \Phi) \cdot p(\Phi | \mathcal{D}) d\Phi, \end{aligned} \tag{19}$$

where $p(\Phi|\mathcal{D})$ is the posterior distribution of the global parameters given the training data. We expand the left term in (19), by drawing customer stable preferences, context-specific

[17] Note that the global parameters are $\Phi = \left[\Sigma, \eta, a, d, \{\pi_c, \theta_c\}_{c=1}^C, \{\alpha^0_\ell, \boldsymbol{\alpha}^w_\ell, \boldsymbol{\alpha}^\beta_\ell\}_{\ell \in \{1,\dots,L\}}\right]$.

parameters, focal context membership, and past journeys contexts and marginalizing them,

$$p(\boldsymbol{\beta}_{ij}|\mathbf{q}_{ij},y^c_{ij1:t},\mathcal{L}_{ijt},\mathcal{H}_i,\Phi)= \int p(\boldsymbol{\beta}_{ij},\boldsymbol{\mu}_i,\boldsymbol{\gamma}_j,\boldsymbol{\gamma}_{-j},z_{ij},\mathbf{z}_{i,-j}|\mathbf{q}_{ij},y^c_{ij1:t},\mathcal{L}_{ijt},\mathcal{H}_i,\Phi)\cdot d\boldsymbol{\mu}_i\cdot d\boldsymbol{\gamma}_j\cdot d\boldsymbol{\gamma}_{-j}\cdot dz_{ij}\cdot d\mathbf{z}_{i,-j}, \quad (20)$$

where $\boldsymbol{\gamma}_{-j}=\{\boldsymbol{\gamma}_{j'}\}_{j'\in\mathcal{J}(i)}$. Finally, noting that $\boldsymbol{\gamma}_j=\begin{pmatrix}\boldsymbol{\omega}_j\\ \boldsymbol{\rho}_j\end{pmatrix}$ we can write this posterior as being proportional to the joint

$$\begin{aligned} &p(\boldsymbol{\beta}_{ij},\boldsymbol{\mu}_i,\boldsymbol{\gamma}_j,\boldsymbol{\gamma}_{-j},z_{ij},\mathbf{z}_{i,-j}|\mathbf{q}_{ij},y^c_{ij1:t},\mathcal{L}_{ijt},\mathcal{H}_i,\Phi)\\ &\quad\propto\ p(\boldsymbol{\beta}_{ij},\boldsymbol{\mu}_i,\boldsymbol{\gamma}_j,\boldsymbol{\gamma}_{-j},z_{ij},\mathbf{z}_{i,-j},\mathbf{q}_{ij},y^c_{ij1:t},\mathcal{L}_{ijt},\mathcal{H}_i|,\Phi)\\ &\quad=\ p(\mathbf{q}_{ij}|\boldsymbol{\omega}_j)\cdot p(y^c_{ij1:t}|\boldsymbol{\mu}_i,\boldsymbol{\rho}_j,\eta)\cdot p(\mathcal{L}_{ijt}|\boldsymbol{\mu}_i,\boldsymbol{\rho}_j,\boldsymbol{\alpha}^0,\boldsymbol{\alpha}^w,\boldsymbol{\alpha}^\beta)\cdot\mathbf{1}\{\boldsymbol{\beta}_{ij}=\boldsymbol{\mu}_i+\boldsymbol{\rho}_j\}\\ &\qquad\cdot\prod_{j'\in\mathcal{J}(i)}p(\mathcal{H}_{ij'}|\boldsymbol{\mu}_i,\boldsymbol{\gamma}_{-j},\Phi)\\ &\qquad\cdot p(\boldsymbol{\mu}_i|\Sigma)\cdot p\Big(z_{ij},\boldsymbol{\gamma}_j|a,d,\{\pi_c,\theta_c\}_{c=1}^{\widetilde{C}}\Big)\cdot\prod_{j'\in\mathcal{J}(i)}p\Big(z_{ij'},\boldsymbol{\gamma}_{j'}|a,d,\{\pi_c,\theta_c\}_{c=1}^{\widetilde{C}}\Big), \end{aligned} \quad (21)$$

where $\widetilde{C}$ is the number of contexts (which is a latent variable, and thus, it is drawn from the posterior $p(\Phi|\mathcal{D})$).

We update those parameters using steps 1, 2, 3, and 4 exactly as shown in Web Appendix B, and we adapt steps 5, 6, and 7 to allow for previously unobserved contexts to be drawn by:

5*. Draw context membership $z_j\in\{1...\widetilde{C}\}\cup\{\widetilde{C}+1\}$ as follows

$$p(z_j=c|\cdot)\propto \begin{cases}(n_c-d)\cdot\mathcal{P}_{jc}, & \text{if } c\leqslant\widetilde{C}\\ (a+d\cdot\widetilde{C})\cdot\mathcal{P}^*_j & \text{if } c=\widetilde{C}+1\end{cases}$$

where $\mathcal{P}_{jc}$ as defined in step 5 in Web Appendix B, and $\mathcal{P}^*_j=\Big(\prod_{m=1}^M p(q_{ijm}|\phi_{0m})\Big)\cdot p\Big(\widetilde{u}_{ij}-\widetilde{\mathbf{X}}_{ij}\boldsymbol{\mu}_i|\widetilde{\mathbf{X}}_{ij},\mu_0,V_0\Big)$ the product of posterior predictive likelihoods[18] such that

$$p(q_{ijm}|\phi_{0m})=\int p(q_{ijm}|\theta^w_m)\cdot p(\theta^w_m|\phi_{0m})d\theta^w_m$$
$$p\Big(\widetilde{u}_{ij}-\widetilde{\mathbf{X}}_{ij}\boldsymbol{\mu}_i|\widetilde{\mathbf{X}}_{ij},\mu_0,V_0\Big)=\int p\Big(\widetilde{u}_{ij}-\widetilde{\mathbf{X}}_{ij}\boldsymbol{\mu}_i|\widetilde{\mathbf{X}}_{ij}\theta^\rho_j\Big)\mathcal{N}(\theta^\rho|\mu,V_0)d\theta^\rho.$$

[18] As all prior-likelihood pairs are conditionally conjugate, these posterior predictive likelihoods have closed form.

6*. If $z_{ij} = \widetilde{C} + 1$, then draw the query components of context location parameters $\theta^{\omega}_{\widetilde{C}+1}$ following step 6 in Web Appendix B.

7*. If $z_{ij} = \widetilde{C} + 1$, then draw the click-purchase context location parameters $\theta^{\rho}_{\widetilde{C}+1}$ following step 7 in Web Appendix B.

$**$: If $z_{ij} = \widetilde{C} + 1$, then update $\widetilde{C} = \widetilde{C} + 1$, and repeat the same steps for all $j' \in \mathcal{J}(c)$.

D Algorithm for computing purchase probabilities

Algorithm 1 Computing purchase probabilities

Input

A vector of preferences $\boldsymbol{\beta}_{ij}$

A set of products with at least one click $\mathcal{C}_{ij}^{obs} = \{k \mid \exists t' \leqslant t, y_{ijt'}^c = k\}$

Number of samples S for the Monte Carlo approximation

Trained predictor function $\hat{g}_{\mathcal{C}}(\mathbf{x}, \boldsymbol{\beta})$

Output

$p(y_{ij}^p | y_{ij1:t}^c, \boldsymbol{\beta}_{ij})$

Procedure

for all $s \leftarrow 1 : S$ **do**

  Initialize consideration set $\mathcal{C}_{ij} \leftarrow \mathcal{C}_{ij}^{obs}$

  for all $k \notin \mathcal{C}_{ij}^{obs}$ **do**

    Draw $u \sim U(0,1)$

    if $u \leqslant \hat{g}_{\mathcal{C}}(\mathbf{x}_{ijk}, \boldsymbol{\beta}_{ij})$ **then**

      $\mathcal{C}_{ij} \leftarrow \mathcal{C}_{ij} \cup \{k\}$

    end if

  end for

  Compute $p_s = p(y_{ij}^p | \mathcal{C}_{ij}, \boldsymbol{\beta}_{ij})$ using GLK simulator and Equation (6)

end for

Return $p(y_{ij}^p | y_{ij1:t}^c, \boldsymbol{\beta}_{ij}) \approx \frac{1}{S} \sum_{s=1}^{S} p_s$

E Details on the computation of consideration probabilities (XGBoost)

We estimate tree-based classifiers (XGBoost and Random Forest) to predict consideration in hold out journeys. We train such models using the data from the training sample (including clicks as dependent variable and the product attributes as features) as well as draws from the posterior distribution of the vector of preferences (which are included as additional features in our classifier).

In our empirical application, consideration is operationalized slightly differently for the two types of flights one-way and roundtrip. For one-way itineraries, the details page is shown after a single click on a oneway results page; moment at which we assume the flight is being considered. For roundtrip itineraries, on the other hand, the customer must click on the outbound component of the flight (on an outbound results page), *and* on the inbound (return) component of the flight in order to see the details page and for the product to be considered. Accordingly, we train three different models, each aiming at a different prediction task: One that predicts consideration for oneway flights ($\hat{g}_{ow}$), another one that predicts whether the outbound component of a roundtrip flight is considered ($\hat{g}_{out}$), and another one that predicts, conditional on the outbound component being considered, whether the inbound component is also considered ($\hat{g}_{in}$).

Following, (18), we compute the consideration probabilities given whether the customer has clicked on the itinerary, or a portion of the itinerary. That is,

$$p(k \in \mathcal{C}_{ij}|\text{OneWay}, y^c_{ij1:t}, \boldsymbol{\beta}_{ij}) \approx \begin{cases} 1 & \text{if flight was clicked on before} \\ \hat{g}_{ow}(\mathbf{x}_{ijk}, \beta_{ij}) & \text{if flight has not been clicked on yet,} \end{cases}$$

$$p(k \in \mathcal{C}_{ij}|\text{Roundtrip}, y^c_{ij1:t}, \boldsymbol{\beta}_{ij}) \approx \begin{cases} 1 & \text{if both legs were clicked on} \\ 1 \cdot \hat{g}_{in}(\mathbf{x}_{ijk}, \beta_{ij}) & \text{if only outbound leg was clicked on} \\ \hat{g}_{out}(\mathbf{x}_{ijk}, \beta_{ij}) \cdot \hat{g}_{in}(\mathbf{x}_{ijk}, \beta_{ij}) & \text{if no leg has been clicked on.} \end{cases}$$

Because the parameters $\boldsymbol{\beta}_{ij}$ are estimated in a Bayesian manner (i.e., we don't have a point estimate but a posterior distribution), we draw a sample of 50 draws from the posterior distribution of $\boldsymbol{\beta}_{ij}$ when training the consideration of each journey. Specifically, for each product k in a journey, we create 50 observations, each with a feature vector concatenating the vector of product attributes, $\mathbf{x}_{ijk}$, and the drawn preferences $\widetilde{\boldsymbol{\beta}}_{ijd}$. We sample 1,000,000 observations ($\sim$1% of total) to train the classifiers. We use oneway observations to train $\hat{g}_{ow}$; and roundtrip observations to train $\hat{g}_{out}$. To train $\hat{g}_{in}$, we only use roundtrip observations such that the outbound leg of the corresponding itinerary was clicked on.[19] We use as binary outcomes whether the corresponding product of each observation was clicked on during the journey,[20] and the corresponding cross-entropy loss (i.e., binary logistic) to train the models.

We use a 80%-20% training/test split, and ten-fold cross-validation on the training sample over a grid to tune the hyperparameters of each classifier (e.g., the learning rate and the maximum depth of the trees for the XGBoost). Table E.2 shows the performance of both the XGBoost and the Random Forest on each prediction task. Because the XGBoost overall accuracy metrics (F1 and AUC) are superior in all tasks, we use the results of the XGBoost when augmenting consideration sets.

	Consideration				
Model	Balanced accuracy	Precision	Recall	F1	AUC
Oneway ($\hat{g}_{ow}$)					
XGBoost	0.2287	0.3893	0.0680	0.1158	0.9064
Random Forest	0.3403	0.6486	0.0320	0.0610	0.6898
Outbound ($\hat{g}_{out}$)					
XGBoost	0.9598	0.9406	0.9789	0.9594	0.9958
Random Forest	0.8027	0.8304	0.7749	0.8017	0.9593
Inbound ($\hat{g}_{in}$)					
XGBoost	0.3488	0.5482	0.1494	0.2348	0.9233
Random Forest	0.3928	0.6879	0.0977	0.1711	0.7737

Table E.2: Performance of XGBoost consideration predictors.

[19] Arguably, there could be selection bias affecting our sample, as we would make predictions for those not clicked on yet based only on those clicked on the outbound leg. However, we argue that this approach is the most sensible given the task at hand. First, any potential selection bias should hurt the out-of-sample performance, and, thus, be captured by the out-of-sample performance of the predictions of the whole model. Second, those predictions should only be relevant for products that their outbound leg was clicked on, or that the outbound model predicts will be clicked on. Therefore, even if predictions are off for products that are unlikely to be clicked on, these are captured already by $\hat{g}_{out}$.

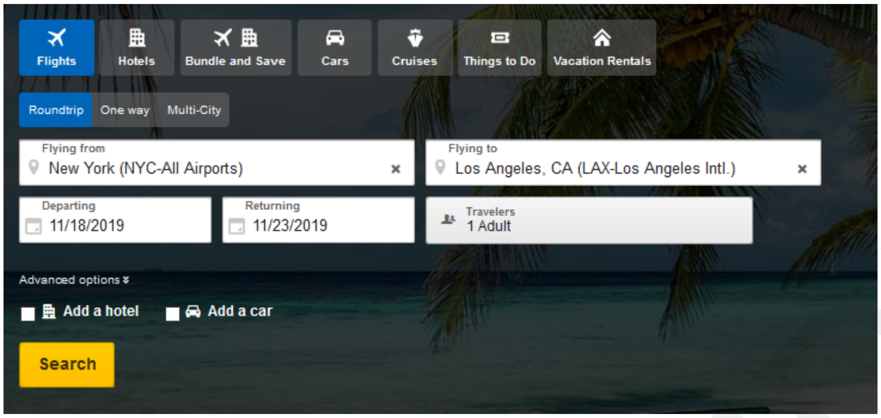
[20] For the outbound leg model, we use as an outcome whether that product has the same exact outbound leg as any product that was clicked on during the journey, that is, if an outbound leg is clicked on within a results page, all returning flights displayed on the next page (which share the same already-clicked outbound leg) are defined as positive labels for the predictive model.

F Empirical application: Additional figures and summary statistics

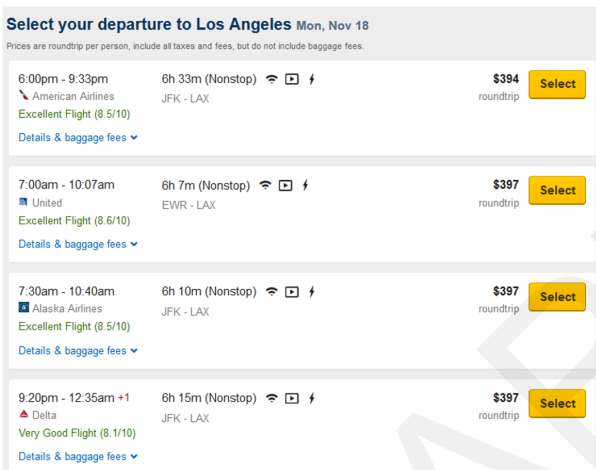
F.1 Example of purchase journey steps

F.2 Product attributes

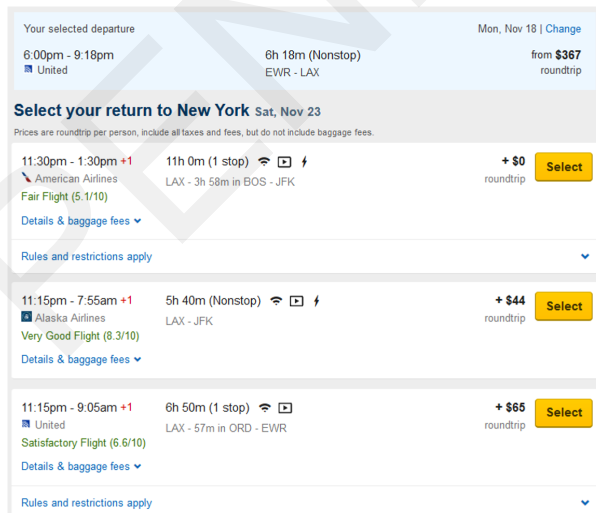
(a) Example of query page



(b) Example of outbound page results



(c) Example of inbound page results



(d) Example of flight details results

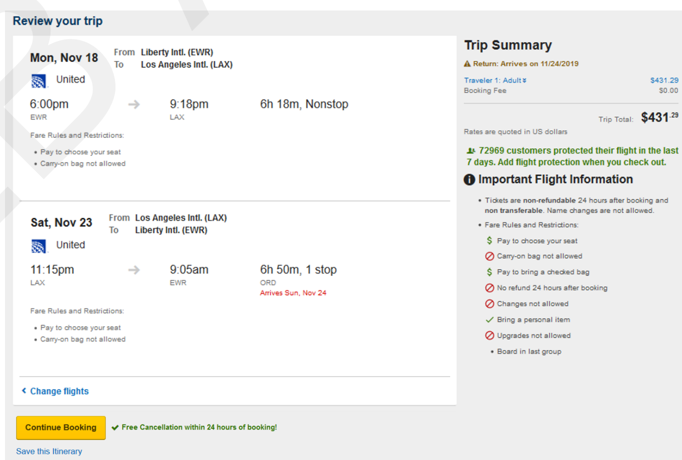


Figure F.2: Purchase journey steps

Product attribute	Mean	SD	Quantiles 5%	50%	95%
Product level attributes					
Price	1,547	3,269	196	751	5,320
Cheapest price per journey	698	1,526	98	401	2,117
Outbound level attributes					
Length of trip (hours)	11.28	8.49	2.05	8.42	28.60
Shortest length of trip per journey (hours)	5.86	5.05	1.25	4.07	17.08
Number of stops: Non stop	0.20	.	0	0	1
Number of stops: One stop	0.59	.	0	1	1
Number of stops: 2+ stops	0.21	.	0	0	1
Alliance: Alaska Airlines	0.04	.	0	0	0
Alliance: Frontier	0.01	.	0	0	0
Alliance: JetBlue	0.03	.	0	0	0
Alliance: Multiple alliances	0.07	.	0	0	1
Alliance: Other – No alliance	0.07	.	0	0	1
Alliance: OneWorld (American)	0.27	.	0	0	1
Alliance: Skyteam (Delta)	0.27	.	0	0	1
Alliance: Spirit	0.02	.	0	0	0
Alliance: Star Alliance (United)	0.23	.	0	0	1
Dep. time: Early morning (0:00am - 4:59am)	0.04	.	0	0	0
Dep. time: Morning (5:00am – 11:59am)	0.47	.	0	0	1
Dep. time: Afternoon (12:00pm - 5:59pm)	0.31	.	0	0	1
Dep. time: Evening (6:00pm - 11:59pm)	0.18	.	0	0	1
Arr. time: Early morning (0:00am - 4:59am)	0.05	.	0	0	0
Arr. time: Morning (5:00am – 11:59am)	0.24	.	0	0	1
Arr. time: Afternoon (12:00pm - 5:59pm)	0.34	.	0	0	1
Arr. time: Evening (6:00pm - 11:59pm)	0.37	.	0	0	1
Inbound level attributes					
Length of trip (hours)	11.08	9.02	1.83	7.92	29.50
Shortest length of trip per journey (hours)	6.17	5.31	1.25	4.27	17.75
Number of stops: Non stop	0.19	.	0	0	1
Number of stops: One stop	0.70	.	0	1	1
Number of stops: 2+ stops	0.11	.	0	0	1
Alliance: Alaska Airlines	0.02	.	0	0	0
Alliance: Frontier	0.02	.	0	0	0
Alliance: JetBlue	0.02	.	0	0	0
Alliance: Multiple alliances	0.02	.	0	0	0
Alliance: Other – No alliance	0.07	.	0	0	1
Alliance: OneWorld (American)	0.51	.	0	1	1
Alliance: Skyteam (Delta)	0.13	.	0	0	1
Alliance: Spirit	0.05	.	0	0	1
Alliance: Star Alliance (United)	0.15	.	0	0	1
Dep. time: Early morning (0:00am - 4:59am)	0.03	.	0	0	0
Dep. time: Morning (5:00am – 11:59am)	0.65	.	0	1	1
Dep. time: Afternoon (12:00pm - 5:59pm)	0.18	.	0	0	1
Dep. time: Evening (6:00pm - 11:59pm)	0.14	.	0	0	1
Arr. time: Early morning (0:00am - 4:59am)	0.04	.	0	0	0
Arr. time: Morning (5:00am – 11:59am)	0.55	.	0	1	1
Arr. time: Afternoon (12:00pm - 5:59pm)	0.19	.	0	0	1
Arr. time: Evening (6:00pm - 11:59pm)	0.23	.	0	0	1

Table F.3: Summary statistics of product attributes in page results

F.3 Filters construction and summary statistics

As mentioned in the main manuscript, the focal company did not collect the action of "filtering" directly. Rather, we infer such a behavior from the flight results we observe in the data. Specifically, we construct filter data conservatively in the following manner: (1) We infer that a filter was applied if all product results on a page have the same level on a product attribute (e.g., non-stop) and this does not occur in the first page of results.[21] (2) We allow multiple filters on a page as long as they belong to different attributes (e.g., American Airlines and non-stop).

Similar to the click and purchase data, airline data in filters is equally sparse, so we aggregate them into filters at the alliance level. That said, we still infer whether a filter was applied on a page using the airline data, as customers could only apply filters at the airline level and not at the alliance level during the observation window. For example, if a page contains results from multiple OneWorld airlines (e.g., American Airlines and British Airways results), we do not define those results as resulting from a filter, as the platform did not allow customers to filter specifically on alliances. However, we define a filter on the OneWorld alliance if all flights belong to a single airline that belongs to the OneWorld alliance (e.g., all flights American Airlines or all flights British Airways).

Table F.4 shows, per attribute and level, the percentage of first-party journeys where a filter was applied.

[21] Because the website does not filter by default, a constant attribute on the first page reflects limited supply, not a filtering constraint.

Attribute	**Level**	**Proportion journeys filtered**	
		Mean	s.e.
Alliance	OneWorld	0.020	0.001
	Skyteam	0.016	0.001
	Star Alliance	0.017	0.001
	Alaska Airlines	0.003	0.000
	Frontier	0.001	0.000
	JetBlue	0.006	0.000
	Spirit	0.001	0.000
	OTHER_NO_ALLIANCE	0.008	0.001
Stops	Non-stop	0.138	0.002
	One stop	0.038	0.001
Departure time	Early morning (0:00am - 4:59am)	0.004	0.000
	Morning (5:00am - 11:59am)	0.032	0.001
	Afternoon (12:00pm - 5:59pm)	0.027	0.001
	Evening (6:00pm - 11:59pm)	0.028	0.001
Arrival time	Early morning (0:00am - 4:59am)	0.002	0.000
	Morning (5:00am - 11:59am)	0.018	0.001
	Afternoon (12:00pm - 5:59pm)	0.019	0.001
	Evening (6:00pm - 11:59pm)	0.021	0.001

Table F.4: Percentage of journeys with filters in attributes.

G Additional results

G.1 Context-specific parameter estimates

Parameter	Context 1	2	3	4	5	6	7	8	9	10	11
Query											
Is it roundtrip? Yes	0.24	0.91	0.90	0.99	0.97	0.93	0.95	0.99	0.20	0.96	0.97
Is it domestic? (within EU is domestic) Yes	1.00	1.00	1.00	0.00	0.02	1.00	0.00	1.00	0.08	0.00	0.03
Flying from international airport? Yes	0.55	0.74	0.48	0.87	0.74	0.50	0.94	0.62	0.90	0.90	0.90
Market: US Domestic	0.98	0.95	0.97	0.00	0.00	0.97	0.00	0.93	0.00	0.00	0.00
Market: US Overseas	0.00	0.00	0.00	0.83	0.01	0.00	0.72	0.00	0.00	0.78	0.01
Market: Non-US across continent	0.00	0.00	0.00	0.15	0.05	0.00	0.26	0.00	0.09	0.20	0.08
Market: Non-US within continent	0.00	0.02	0.00	0.01	0.07	0.00	0.01	0.04	0.19	0.01	0.10
Market: US North America	0.02	0.02	0.03	0.00	0.88	0.02	0.00	0.03	0.72	0.00	0.81
Type of location searched: Airport	0.92	0.89	0.93	0.88	0.90	0.88	0.89	0.91	0.90	0.85	0.81
Type of location searched: Both	0.02	0.01	0.02	0.02	0.06	0.04	0.04	0.02	0.07	0.04	0.07
Type of location searched: City	0.05	0.10	0.05	0.10	0.04	0.08	0.07	0.07	0.03	0.12	0.12
Trip distance (1000s kms)	1.87	2.53	1.84	9.72	2.71	0.80	9.71	2.36	2.53	8.96	2.77
More than one adult? Yes	0.17	0.39	0.24	0.28	0.46	0.22	0.18	0.50	0.26	0.26	0.47
Traveling with kids? Yes	0.04	0.13	0.06	0.03	0.17	0.05	0.11	0.17	0.10	0.07	0.12
Is it summer season? Yes	0.43	0.39	0.42	0.00	0.26	0.36	0.21	0.00	0.62	0.98	0.21
Holiday season? Yes	0.00	0.00	0.01	0.06	0.07	0.01	0.00	0.32	0.00	0.00	0.09
Does stay include a weekend? Yes	0.15	0.89	0.99	1.00	0.97	0.83	1.00	0.99	0.25	1.00	0.90
Length of stay (only RT) (days)	2.30	5.25	5.63	14.77	9.83	3.88	45.67	6.16	2.81	13.16	7.40
Searching on weekend? Yes	0.19	0.21	0.21	0.25	0.24	0.19	0.26	0.17	0.24	0.27	0.20
Searching during work hours? Yes	0.52	0.52	0.52	0.51	0.55	0.54	0.37	0.59	0.42	0.43	0.60
Time in advance to buy (days)	23.95	58.21	41.18	107.77	81.79	37.34	53.67	111.04	29.07	39.38	92.67
Preferences											
Intercept Search: OW Search	-0.02	-0.28	0.01	-0.14	0.04	-0.19	0.06	0.09	-0.06	-0.18	-0.16
Intercept Search: RT Outbound	-0.22	-0.51	-0.13	-0.70	-0.13	-0.49	-0.10	-0.09	-0.24	-0.41	-0.45
Intercept Search: RT Inbound	-0.03	-0.11	-0.26	-0.18	-0.01	-0.13	-0.11	-0.15	-0.06	-0.10	-0.04
Intercept Click: OW Search	-0.81	-0.37	-0.61	0.04	0.08	-0.09	-0.08	-0.09	-0.36	-0.02	-0.03
Intercept Click: RT Outbound	-0.28	-0.17	-0.18	-0.14	-0.18	-0.43	-0.31	-0.61	-0.04	-0.21	-0.43
Intercept Click: RT Inbound	0.05	-0.18	0.19	0.28	0.35	-0.15	0.43	0.08	-0.02	0.15	-0.13
Price	-0.19	0.02	-0.37	-0.19	-0.18	0.15	-0.34	-0.15	-0.03	-0.06	0.32
Length of trip (hours)	-0.59	-0.54	-0.90	-0.59	-0.85	-0.20	-0.69	-0.46	-0.49	-0.21	-0.09
Number of stops: Non stop	0.11	0.43	0.60	0.31	0.60	0.14	0.35	0.18	-0.02	0.22	0.03
Number of stops: 2+ stops	-0.33	-0.15	-0.28	-0.46	-0.12	-0.06	-0.40	-0.10	-0.10	-0.25	-0.05
Alliance: Skyteam (Delta)	-0.02	-0.12	-0.18	-0.09	-0.14	-0.09	-0.01	-0.06	-0.12	0.03	-0.10
Alliance: Star Alliance (United)	-0.10	-0.19	-0.21	0.14	0.12	-0.16	-0.06	-0.09	0.02	-0.06	-0.06
Alliance: Alaska Airlines	-0.06	-0.08	0.01	0.02	-0.10	-0.01	0.13	-0.01	-0.07	-0.01	-0.04
Alliance: Spirit	-0.21	-0.02	-0.21	0.01	0.00	0.04	-0.06	-0.05	-0.01	0.00	0.01
Alliance: JetBlue	-0.02	0.23	-0.01	0.00	0.08	0.13	-0.13	-0.03	0.04	0.04	0.02
Alliance: Frontier	-0.11	0.06	-0.03	-0.03	0.00	0.02	-0.02	0.01	0.06	0.01	0.00
Alliance: Other – No alliance	-0.09	-0.05	-0.04	0.05	0.02	-0.03	0.12	0.12	-0.14	-0.06	-0.04
Alliance: Multiple alliances	-0.13	-0.07	-0.10	0.02	-0.08	-0.02	-0.09	-0.04	-0.09	0.00	0.03
Outbound dep. time: Early morning (0:00am - 4:59am)	-0.16	-0.01	0.07	0.05	0.11	0.01	-0.08	0.01	-0.03	-0.01	-0.02
Outbound dep. time: Afternoon (12:00pm - 5:59pm)	-0.08	-0.26	-0.09	0.03	0.01	-0.03	-0.14	-0.06	-0.04	-0.10	-0.07
Outbound dep. time: Evening (6:00pm - 11:59pm)	-0.27	-0.21	-0.07	-0.07	-0.10	-0.04	-0.11	-0.01	-0.03	-0.07	-0.07
Outbound arr. time: Early morning (0:00am - 4:59am)	-0.11	-0.14	-0.20	0.00	-0.05	-0.04	-0.11	-0.05	-0.03	-0.05	-0.02
Outbound arr. time: Afternoon (12:00pm - 5:59pm)	0.10	0.13	0.19	-0.11	0.04	-0.11	-0.05	-0.02	-0.09	-0.12	-0.08
Outbound arr. time: Evening (6:00pm - 11:59pm)	-0.01	-0.17	0.12	-0.08	-0.12	0.01	-0.14	-0.14	-0.07	-0.01	-0.06
Inbound dep. time: Early morning (0:00am - 4:59am)	-0.08	0.11	0.00	0.10	-0.01	0.12	-0.07	-0.15	-0.02	0.11	0.10
Inbound dep. time: Afternoon (12:00pm - 5:59pm)	0.10	0.23	0.35	0.05	0.15	-0.01	0.07	0.09	-0.01	0.00	-0.03
Inbound dep. time: Evening (6:00pm - 11:59pm)	0.06	-0.03	0.06	0.05	-0.04	-0.01	0.12	-0.01	0.01	0.03	-0.01
Inbound arr. time: Early morning (0:00am - 4:59am)	-0.11	0.05	0.00	0.02	0.05	0.07	-0.13	-0.07	0.00	0.11	0.08
Inbound arr. time: Afternoon (12:00pm - 5:59pm)	0.01	0.03	0.14	-0.04	0.09	0.00	0.09	0.03	-0.05	-0.02	-0.06
Inbound arr. time: Evening (6:00pm - 11:59pm)	0.20	0.21	0.44	-0.01	0.17	-0.01	-0.05	0.09	0.00	0.02	-0.04

Table G.5: Posterior mean of location click and purchase parameters. Contexts 1-11

Parameter	Context 12	13	14	15	16	17	18	19	20	21	22
Query											
Is it roundtrip? Yes	0.09	0.16	0.96	0.31	0.37	0.27	0.13	0.17	0.09	0.05	0.92
Is it domestic? (within EU is domestic) Yes	0.00	0.94	0.19	1.00	0.97	0.00	0.00	0.18	0.06	1.00	0.71
Flying from international airport? Yes	0.90	1.00	0.99	0.44	1.00	0.94	0.89	0.97	0.88	0.52	1.00
Market: US Domestic	0.00	0.05	0.00	0.98	0.15	0.00	0.00	0.00	0.00	0.96	0.01
Market: US Overseas	0.87	0.00	0.00	0.00	0.00	0.66	0.79	0.01	0.00	0.00	0.00
Market: Non-US across continent	0.11	0.00	0.06	0.00	0.00	0.30	0.19	0.15	0.06	0.00	0.01
Market: Non-US within continent	0.01	0.93	0.29	0.01	0.74	0.04	0.01	0.43	0.36	0.02	0.97
Market: US North America	0.00	0.02	0.65	0.01	0.11	0.00	0.00	0.41	0.58	0.02	0.01
Type of location searched: Airport	0.84	0.76	0.83	0.90	0.76	0.85	0.88	0.87	0.79	0.91	0.84
Type of location searched: Both	0.01	0.10	0.08	0.04	0.03	0.04	0.03	0.09	0.16	0.06	0.13
Type of location searched: City	0.15	0.14	0.09	0.05	0.22	0.11	0.10	0.04	0.06	0.03	0.03
Trip distance (1000s kms)	9.81	1.16	2.41	0.50	0.59	8.70	9.84	2.37	2.72	2.39	1.37
More than one adult? Yes	0.19	0.40	0.08	0.13	0.48	0.17	0.29	0.03	0.42	0.45	0.41
Traveling with kids? Yes	0.06	0.09	0.01	0.03	0.13	0.05	0.02	0.01	0.07	0.11	0.12
Is it summer season? Yes	0.57	0.56	0.59	0.43	0.37	0.40	0.01	0.10	0.03	0.02	0.22
Holiday season? Yes	0.00	0.01	0.00	0.00	0.02	0.00	0.15	0.00	0.19	0.37	0.06
Does stay include a weekend? Yes	0.30	0.25	0.99	0.16	0.41	0.15	0.11	0.23	0.24	0.25	0.90
Length of stay (only RT) (days)	6.53	2.92	8.55	1.56	3.92	3.09	3.40	3.17	4.03	4.21	6.96
Searching on weekend? Yes	0.18	0.29	0.14	0.22	0.28	0.26	0.17	0.12	0.22	0.18	0.27
Searching during work hours? Yes	0.34	0.29	0.47	0.58	0.31	0.41	0.48	0.41	0.30	0.50	0.29
Time in advance to buy (days)	28.34	30.26	25.97	14.45	60.45	46.61	121.05	8.12	109.18	116.00	73.98
Preferences											
Intercept Search: OW Search	-0.30	0.12	0.19	-0.16	-0.15	-0.31	-0.02	-0.15	0.14	-0.01	-0.12
Intercept Search: RT Outbound	0.01	-0.04	-0.17	-0.08	-0.23	-0.30	0.03	-0.22	0.09	-0.02	-0.34
Intercept Search: RT Inbound	0.17	-0.08	-0.05	0.02	0.01	-0.03	-0.02	0.01	0.02	-0.01	0.01
Intercept Click: OW Search	-0.31	-0.34	-0.06	-0.46	-0.35	-0.34	-0.28	-0.14	-0.29	-0.32	-0.01
Intercept Click: RT Outbound	0.06	-0.06	-0.52	-0.19	-0.16	-0.24	-0.20	0.02	-0.04	0.02	-0.23
Intercept Click: RT Inbound	0.01	0.00	0.02	0.16	-0.04	-0.04	0.09	-0.03	0.03	-0.04	0.02
Price	-0.38	-0.11	-0.08	-0.04	0.17	0.13	-0.14	-0.15	-0.13	-0.15	0.13
Length of trip (hours)	-0.45	-0.53	-0.33	-0.23	-0.22	-0.08	-0.35	-0.35	-0.38	-0.29	-0.15
Number of stops: Non stop	0.02	0.05	-0.01	-0.05	-0.11	-0.06	-0.04	0.18	-0.06	-0.13	0.06
Number of stops: 2+ stops	-0.15	-0.09	-0.09	-0.09	-0.03	-0.12	-0.30	-0.11	-0.01	-0.07	-0.03
Alliance: Skyteam (Delta)	-0.16	-0.05	-0.07	-0.04	-0.16	-0.12	-0.09	-0.05	-0.08	-0.07	-0.03
Alliance: Star Alliance (United)	-0.07	0.02	-0.07	-0.16	-0.13	-0.18	-0.09	-0.01	-0.03	-0.01	-0.04
Alliance: Alaska Airlines	0.02	-0.03	0.04	-0.07	-0.03	-0.02	0.03	-0.05	0.02	-0.06	-0.03
Alliance: Spirit	0.00	-0.03	-0.02	0.00	0.00	0.02	-0.02	0.04	-0.04	0.02	0.02
Alliance: JetBlue	0.10	-0.03	-0.02	-0.01	-0.02	0.02	-0.10	0.12	-0.08	0.03	0.06
Alliance: Frontier	0.06	-0.04	-0.01	-0.02	-0.04	0.00	-0.01	0.03	0.01	-0.03	0.00
Alliance: Other – No alliance	0.10	-0.13	-0.01	0.02	-0.11	-0.03	-0.05	-0.08	-0.01	-0.03	-0.03
Alliance: Multiple alliances	-0.09	0.12	-0.07	0.02	-0.01	0.02	-0.12	0.04	-0.01	-0.03	0.00
Outbound dep. time: Early morning (0:00am - 4:59am)	-0.02	0.02	-0.04	-0.02	0.07	0.04	-0.12	-0.01	-0.10	0.00	-0.02
Outbound dep. time: Afternoon (12:00pm - 5:59pm)	-0.04	0.00	-0.10	-0.14	-0.08	-0.10	-0.12	-0.09	-0.08	-0.11	-0.01
Outbound dep. time: Evening (6:00pm - 11:59pm)	-0.06	-0.17	0.00	-0.16	-0.19	-0.17	-0.06	-0.07	-0.04	-0.12	-0.04
Outbound arr. time: Early morning (0:00am - 4:59am)	-0.13	-0.09	-0.06	0.00	-0.04	-0.05	-0.06	-0.01	-0.08	-0.10	0.00
Outbound arr. time: Afternoon (12:00pm - 5:59pm)	-0.03	0.04	-0.05	-0.23	-0.04	-0.10	-0.26	-0.08	-0.14	-0.01	0.00
Outbound arr. time: Evening (6:00pm - 11:59pm)	-0.13	-0.14	-0.18	-0.19	-0.22	-0.12	-0.18	0.05	-0.10	-0.09	-0.02
Inbound dep. time: Early morning (0:00am - 4:59am)	0.15	-0.04	-0.07	-0.06	0.00	0.07	-0.09	0.12	-0.04	0.05	0.06
Inbound dep. time: Afternoon (12:00pm - 5:59pm)	-0.02	0.06	-0.06	0.08	-0.02	-0.02	0.01	0.00	0.02	-0.01	0.03
Inbound dep. time: Evening (6:00pm - 11:59pm)	-0.04	0.02	0.03	-0.01	-0.01	-0.01	0.04	-0.03	0.03	-0.02	-0.01
Inbound arr. time: Early morning (0:00am - 4:59am)	0.12	0.04	-0.03	0.00	0.00	0.09	-0.08	0.10	-0.03	0.01	0.02
Inbound arr. time: Afternoon (12:00pm - 5:59pm)	0.00	-0.01	0.00	0.00	0.00	-0.02	0.01	-0.01	0.04	-0.04	0.01
Inbound arr. time: Evening (6:00pm - 11:59pm)	0.00	0.04	0.02	0.10	0.00	0.00	0.00	0.00	0.00	-0.02	-0.02

Table G.6: Posterior mean of location click and purchase parameters. Contexts 12-22

G.2 Relative differences across contexts (all variables)

We normalize the location parameters to account for how they vary across contexts and how much uncertainty their posterior has. First, for each context c, we compute the posterior mean of each location parameter θ_c. Second, we compare these location parameters with the population mean level of those same parameters, but now we include query parameters as well. We subtract these two to measure whether contexts are *above or below average* on each of the query parameters and click and purchase preferences. Finally, we normalize these differences by dividing by the square root of the posterior variance across journeys. This variance is composed by two terms (similar to ANOVA): (1) the within-context posterior variance of each θ_c, which measures the posterior uncertainty of each location parameter θ_c; and (2) the across-context variance of all θ_c with respect to the population mean, which captures how much variance is explained by the differences between contexts. By normalizing the location parameters, we can now compare contexts with respect to whether they score higher or lower than average on each of the query parameters and preferences.

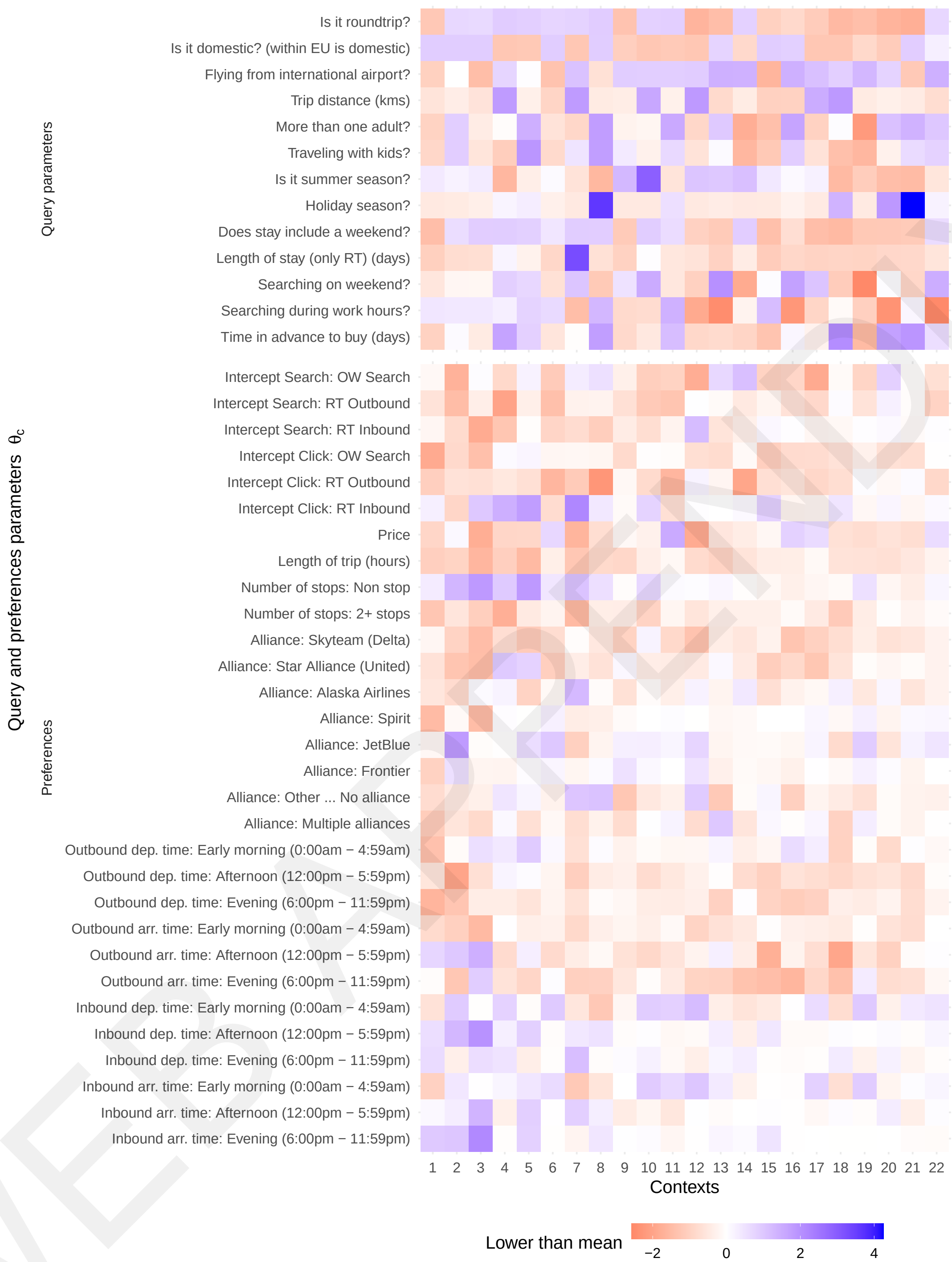


Figure G.3: Posterior mean of all context location parameters θ_c, relative to the average in the population. The top figure shows how each context deviates from the average with respect to the query variables. The bottom figure shows deviations with respect to the preference parameters. Blue (red) boxes mean positive (negative) deviation from the average in the population.

G.3 Benchmark models

We describe in detail how the benchmark models are trained and how the binary prediction scores are normalized per journey. As both benchmark models are built for binary classification tasks (or multi-class classification tasks with a fixed set of classes across observations), we create a series of binary classifications and use normalization to convert these to a multinomial choice task (or varying choice sizes, depending on the consideration set of each journey).

Consider customer i, in journey j and the set $\mathcal{K}_j$ that contains the products customer i can buy in journey j (we also include $k=0$, an additional "no-purchase product" in this set). We assemble the set of all observations $\mathcal{O}=\{(i,j,k)|\, i=1,...,I,\, j=1...,J_i,\, k\in\mathcal{K}_j\}$, where each observation ("row" in our dataset) represents a product in a journey. We create a single training dataset using the clickstream data of the entire journey of each customer in the training data to estimate the benchmark models, which mimic the information seen by the proposed model.

To compute predictions in the test set (i.e., in journeys that have not been observed yet), we create a dataset that changes as information comes in. When making predictions after 5 steps, we use all the information in the journey available within the first 5 steps of the journey. To avoid selection bias and to be able to compare quantities across the different stages of the journey, we hold constant the set of journeys across the two test conditions: after query and after 5 steps (columns of Table 4). Specifically, for journeys shorter than 5 steps, we use the entire journey when making 5-step predictions.

For each observation, we create the binary outcome Y_{ijk}, which equals one if customer i purchased product k during journey j, and zero otherwise ($Y_{ij0}=1$ if the customer ends the journey without a purchase); and a set of features X_{ijk} ("columns" in our dataset) that contain the information for each customer, journey, and product. Specifically, we include five types of features in X_{ijk}:

(1) the set of query variables for journey j (same as those in the query model),

(2) summary statistics of the attributes of all the products shown in the first page of journey j (same as those in the main model),

(3) the clicks and filters (during the focal journey) up to the moment when the prediction is made (capturing what the customer has been clicking so far),

(4) the queries, product attributes, clicks, filters, and purchases from past journeys (capturing the customer's past behavior), and

(5) the attributes of product k.

We now provide details about each of these sets of variables:

(1) We use the same set of query variables as in the main model. We encode all categorical variables as binary (leaving one level out to avoid multicollinearity).

(2) We use the same set of attributes as in the main model, with the exception that we encode categorical variables in full one-hot encoding, such that each level in a categorical variable has a corresponding binary feature. We summarize these features across all products shown on the first page of journey j, and compute the average, minimum, and maximum shown on the first page.

(3) We categorize 'clicks' in two primary ways. Firstly, at the product level, we represent using a binary feature whether the focal product k has been selected or not. In the training data, clicks throughout the entire journey are used to formulate this binary feature since the model undergoes a one-time training. In the test data, this feature is set to one if product k has been clicked on by that point in the journey. If the product remains unclicked, this feature corresponds to the percentage of products clicked in the training data; essentially, in the absence of the feature, we resort to the mean value from the training data.

    Secondly, at the journey level, we aggregate the features of all clicked products within the focal journey, utilizing averages for continuous data and counts for binary data. Mirroring the process mentioned earlier, the training data summary is computed at the end of the journey, whereas the test data incorporates information accessible up to that specific step. Furthermore, we document the total count of clicked products within the focal journey. Filters are integrated in a similar fashion.

(4) We compute the average of variables (1) – (3) plus the attributes of purchased products and the number of past purchases, across all past journeys of customer i. In the training data, for focal journey j, these summaries are computed across journeys 1 through $j-1$; whereas in the test data, we use the summarized across all journeys of customer i in the training data. We also include the number of past journeys (such that a non-linear model can recreate counts).

(5) We include the features of product k as done in the proposed model, and we use a binary feature to distinguish between actual products and the No-Purchase product.

In sum, we generated a training dataset of 258,588 observations and 454 features. We train both binary classifiers, Random Forest (RF) and XGBoost, using a cross-entropy loss (i.e., binary logistic). For the RF, we use honest splitting estimation, where the sample is split in two: one to construct the trees and another to evaluate the predictions. We use a sample fraction of 0.5, a number of variable tries per split of 41 ($\sqrt{\#\text{features}}+20$), an honesty fraction of 0.5, and 2000 trees. For XGBoost, we use 100 rounds with a learning rate of 1 and a maximum depth of trees of 4.

After the models are trained, we compute predictions on the test data, $\widehat{pY}_{ijk}$, in multiple steps. First, we normalize the predictive scores from the benchmarks per journey, such that they sum to one by

$$\widehat{pY}_{ijk}^{\text{norm}} = \frac{\widehat{pY}_{ijk}}{\sum\limits_{k' \in \mathcal{K}(j)} \widehat{pY}_{ijk'}},$$

as these binary predictions are generated independently for all observations. Note that this normalization is not needed for the proposed model as the model provides a probability measure directly. The next steps apply to both benchmark models and our proposed model.

Second, for the incidence predictive task, we label a journey as a purchase if the normalized score for the no-purchase product is lower than 0.5, that is,

$$\widehat{Y}_{ij}^{\text{incidence}} = \mathbf{1}\left\{\widehat{pY}_{ij0}^{\text{norm}} \leqslant 0.5\right\}.$$

We compute balanced accuracy, precision, and recall from these predicted labels.

Third, for the product choice given purchase predictive task, we first compute choice given purchase scores per product by

$$\widehat{pY}_{ijk}^{\text{choice}} = \frac{\widehat{pY}_{ijk}^{\text{norm}}}{\sum\limits_{k' \in \mathcal{K}(j): k \neq 0} \widehat{pY}_{ijk'}^{\text{norm}}},$$

and label the predicted chosen alternative as the product with the maximum score per journey

$$\widehat{Y}_{ij}^{\text{choice}} = \underset{k \in \mathcal{K}(j)}{\operatorname{argmax}} \left\{\widehat{pY}_{ijk}^{\text{choice}}\right\}.$$

We use the predicted labels $\widehat{Y}_{ij}^{\text{choice}}$ to compute hitrate (percentage of journeys where predicted choice equals actual chosen product). In order to provide information on how the model predicts at the product level (what the models were trained for), we use $\widehat{pY}_{ijk}^{\text{choice}}$ to compute balanced accuracy by labeling as one the product with the highest score and computing the confusion matrix using the data at the journey-product level. Note that in such case, precision, recall, and balanced accuracy are all equal, as there is only one chosen product per journey (actual), and only one product is predicted to be chosen.

G.4 Additional predictive validity results

G.4.1 Prediction of proposed vs. benchmark models with 2 clicks

	Incidence	
Model	After query	After 2 steps
Balanced accuracy		
Proposed model	0.62	0.64
Random forest	0.60	0.67
XGBoost	0.50	0.53
Precision		
Proposed model	0.21	0.22
Random forest	0.28	0.31
XGBoost	.	0.60
Recall		
Proposed model	0.83	0.87
Random forest	0.40	0.57
XGBoost	0.00	0.06

(a) Purchase incidence of proposed vs. benchmark models.

	Product choice given purchase	
Model	After query	After 2 steps
Hitrate		
Proposed model	0.16	0.27
Random forest	0.16	0.18
XGBoost	0.03	0.27
Balanced accuracy		
Proposed model	0.58	0.63
Random forest	0.58	0.59
XGBoost	0.51	0.63

(b) Choice given purchase of proposed vs. benchmark models.

Table G.7: Prediction of proposed vs. benchmark models after query and 2 clicks.

G.4.2 Additional prediction measures

We complement the results presented in Table 4 of the main manuscript by adding the F1 measure (harmonic mean of precision and recall) and the Jaccard index (ratio of true positives over the union of true and predicted positive outcomes). We also report precision and recall for choice. We remark that in the choice given purchase prediction task, precision and recall are equal since there is a single positive label per journey (for both true and predicted labels, as there is one product purchased per journey, and only one product is predicted to be chosen per journey). Consequently, both balanced accuracy and the F1 measure are equal to precision and recall.

	Incidence	
Model	After query	After 5 steps
Balanced accuracy		
Proposed model	0.62	0.65
Random forest	0.60	0.70
XGBoost	0.50	0.59
F1		
Proposed model	0.34	0.35
Random forest	0.33	0.44
XGBoost	.	0.30
Jaccard index		
Proposed model	0.20	0.21
Random forest	0.19	0.28
XGBoost	0.00	0.18
Precision		
Proposed model	0.21	0.22
Random forest	0.28	0.33
XGBoost	.	0.60
Recall		
Proposed model	0.83	0.91
Random forest	0.40	0.67
XGBoost	0.00	0.20

(a) Purchase incidence of proposed vs. benchmark models. (Extended set of measures of fit.)

	Product choice given purchase	
Model	After query	After 5 steps
Hitrate		
Proposed model	0.16	0.62
Random forest	0.16	0.19
XGBoost	0.03	0.62
Balanced accuracy		
Proposed model	0.58	0.81
Random forest	0.58	0.59
XGBoost	0.51	0.81
F1		
Proposed model	0.16	0.62
Random forest	0.16	0.19
XGBoost	0.03	0.62
Jaccard index		
Proposed model	0.09	0.45
Random forest	0.09	0.11
XGBoost	0.02	0.45
Precision / recall		
Proposed model	0.16	0.62
Random forest	0.16	0.19
XGBoost	0.03	0.62

(b) Choice given purchase of proposed vs. benchmark models.

Table G.8: Prediction of proposed vs. benchmark models.

G.5 Details on predictions for computing the value of first-party data.

Similarly to the analysis presented in Section 5.4, we compute the choice probabilities for all models at each stage of the journey. We consistently employ all journeys across all stages, ensuring a constant set of journeys when conducting comparisons throughout the journey. For instance, when forecasting after 5 steps (or 2 steps), we consider the initial five (or two) steps for journeys with at least 5 (or 2) steps, while accommodating all available steps for journeys shorter than 5 (or 2) steps. This methodology enables us to assess the journey's value with a conservative lens, as performance on the held-out set would notably enhance if we were to observe a uniform 5 steps across all journeys.

We compute hitrates at the product level, analogous to the approach described in Section 5.4. When exploring the ability of the model to predict what attributes the customer will choose, we compute the probabilities of choosing each level by aggregating the choice probabilities across all products with such a level. For categorical variables, we compute hitrates, and for continuous variables, we utilize the Root Mean Square Error (RMSE).

For example, let us consider a categorical attribute such as number of stops. For each level — Non-stop, One stop, and 2+ stops — we compute the probability that a customer, conditional on making a purchase, will opt for a specific stop level. This is done by aggregating the choice probabilities associated with the 'stop' attribute. For instance, the probability that a customer will select a non-stop flight corresponds to the cumulative choice probabilities of all non-stop flights. Subsequently, the predicted number of stops is identified as the level with the highest choice probability. We then contrast these predicted labels with the actual labels to compute hit rates, which represent the proportion of journeys where we accurately predict the number of stops for the chosen flight. A similar methodology is applied when considering airline alliances, which is also categorical.

For a continuous attribute such as price, we first calculate the square errors between the price of each alternative and the price of the purchased alternative, and then compute the weighted average of those square errors (by journey) using the purchase probabilities as weights, by

$$MSE_{ij}^{\text{Price}} = \sum_{k \in \mathcal{K}(j)} p(y_{ij}^{p} = k | \text{Data}_{ijt}) \cdot (\text{Price}_{ijk} - \text{Price}_{ijk*})^2,$$

where $p(y_{ij}^{p} = k | \text{Data}_{ijt})$ are the purchase probabilities and k^* is the true purchased alternative. First, note that the square errors are independent from the predictions, but the weighted average is not. Second, note that if the model predicts with probability one on alternatives with the same price as the purchased one, then this expectation is zero. Finally, we average those expected square errors and compute the square root

$$RMSE^{\text{Price}} = \sqrt{\frac{1}{J^{oos}} \sum_{ij} MSE_{ij}^{\text{Price}}},$$

where J^{oos} is the number of heldout journeys. We follow the same procedure for the length of the trip. We compute these scores on the normalized prices and lengths to weigh all journeys equally and to avoid searches with more expensive and longer destinations to dominate the score.