Fiscal Policy Stabilization: Purchases or Transfers? *

Neil R. Mehrotra†

This Draft: July 1, 2013
Original Draft: April 1, 2011

Abstract

Both government purchases and transfers figure prominently as tools for counteracting recessions. However, representative agent models rule out transfers by construction. This paper builds a borrower-lender model with credit spreads and examines both the purchases and transfers multiplier. Under flexible prices, transfers affect output through wealth effects on labor supply and carry small multipliers. Under sticky prices, both multipliers depend on the degree of monetary accommodation. When the zero lower bound is binding, both purchases and transfers carry multipliers above unity, but the transfer multiplier relative to the purchases multiplier is increasing in the debt-elasticity of the credit spread.

Keywords: fiscal policy, transfers, zero lower bound.

JEL Classification: E62

---

*I would like to thank Ricardo Reis and Michael Woodford for helpful discussions and Guido Lorenzoni, Bruce Preston, Guilherme Martins, Stephanie Schmitt-Grohe, and Dmitriy Sergeyev for useful comments. I would also like to thank seminar participants at Columbia University, the EconCon Conference 2011, and the Federal Reserve Board for their helpful suggestions. First draft: April 2011.

†Columbia University, Department of Economics, e-mail: nrm2111@columbia.edu
1 Introduction

The Great Recession has brought renewed attention to the possibility of using fiscal policy to counteract recessions. Between 2007 and 2010, policymakers adopted a series of historically large fiscal interventions in an attempt to raise output, reduce unemployment, and stabilize consumption and investment. In addition to some increases in government purchases, policymakers have also relied heavily on transfers of various forms - to individuals, institutions, and state and local governments - as instruments of fiscal policy. Table 1 provides the Congressional Budget Office breakdown of the various components of the Recovery Act and estimates for the associated policy multiplier. Transfers account for more than half of the expenditures in the Recovery Act.

Table 1: Outlays and Estimated Policy Multipliers for American Recovery and Reinvestment Act

<table>
<thead>
<tr>
<th>Category</th>
<th>Estimated Multiplier (High)</th>
<th>Estimated Multiplier (Low)</th>
<th>Outlays</th>
</tr>
</thead>
<tbody>
<tr>
<td>Purchases of goods and services by the federal government</td>
<td>2.5</td>
<td>1.0</td>
<td>$88 bn</td>
</tr>
<tr>
<td>Transfers to state and local governments for infrastructure</td>
<td>2.5</td>
<td>1.0</td>
<td>$44 bn</td>
</tr>
<tr>
<td>Transfers to state and local governments not for infrastructure</td>
<td>1.9</td>
<td>0.7</td>
<td>$215 bn</td>
</tr>
<tr>
<td>Transfers to persons</td>
<td>2.2</td>
<td>0.8</td>
<td>$100 bn</td>
</tr>
<tr>
<td>One-time Social Security payments</td>
<td>1.2</td>
<td>0.2</td>
<td>$18 bn</td>
</tr>
<tr>
<td>Two-year tax cuts for lower and middle income persons</td>
<td>1.7</td>
<td>0.5</td>
<td>$168 bn</td>
</tr>
<tr>
<td>One-year tax cuts for higher income persons (AMT fix)</td>
<td>0.5</td>
<td>0.1</td>
<td>$70 bn</td>
</tr>
</tbody>
</table>

In contrast to government purchases, the effectiveness of transfers as a instrument of stabilization has only recently garnered attention in the literature. Empirical work by Johnson, Parker and Souleles (2006) demonstrate that an economically significant portion of tax rebates (intended as stimulus) are spent. The authors track changes in consumption in the Consumer Expenditures Survey and use the timing of rebates as a source of exogenous variation. Agarwal, Liu and Souleles (2007) provide additional evidence of sizable consumption effects by examining spending and saving behavior of households using credit card data. This literature finds an economically significant and persistent response of household consumption to rebates. Recent work by Oh and Reis (2012) and Giambattista and Pennings (2012) have emphasized the important role of transfers in recent stimulus programs and have posited models to determine the effect of these programs. Similarly, work by Kaplan and Violante (2011) and Bilbiie, Monacelli and Perotti (2012) have further examined the channels by which transfers effect aggregate output, employment and consumption.

In this paper, I examine the role of transfers as an instrument of fiscal policy with an emphasis on purchases and transfers as alternative policies. To provide a role for transfers, the model features patient and impatient households along with a credit spread which generates borrowing and lending.
in steady state. The model allows for flexible or sticky prices to determine how the conclusions of the representative agent RBC and New Keynesian models carry over to a multiagent setting. Additionally, the model demonstrates how a broad class of deficit-financed government expenditures can be represented as some combination of government purchases and transfers.

My analysis reveals that several insights from the representative agent setting carry over to a multiagent setting with credit spreads. Under flexible prices, fiscal policy only affects output and employment through a wealth effect on labor supply. If preferences or the structure of labor markets eliminate wealth effects on labor supply, neither purchases nor transfers will have any effect on output or employment. However, even in the presence of wealth effects, the deviations from the representative agent benchmark are small for plausible calibrations. The government purchases multiplier on output is positive and driven by the negative wealth effect on labor supply, while the transfers multiplier is close to zero as wealth effects lead to offsetting movements in hours worked by the households that provide and receive the transfer. A sensitivity analysis reveals that the sensitivity of the credit spread to outstanding debt or borrower income does not affect these results.

Under sticky prices, fiscal policy now has both a supply effect (via wealth effect on labor supply) and a demand effect (via countercyclical markups). In the absence of wealth effects, a Phillips curve can be derived in terms of output and inflation. So long as the instrument of monetary policy is not constrained, the central bank may implement any combination of output and inflation irrespective of the stance of fiscal policy. In this sense, fiscal policy is irrelevant for determining aggregate output or inflation as monetary policy is free to undo any effect of fiscal policy. More generally, the tradeoff between purchases and transfers will depend on the monetary policy rule. In the presence of wealth effects, purchases or transfers may lower wages and shift the Phillips curve. Under a Taylor rule and a standard calibration, transfers continue to have small effects on output and employment relative to purchases. The primacy of monetary policy in determining the effect of fiscal policy is analogous to the conclusions of Woodford (2010) and Curdia and Woodford (2010). The presence of a credit spread alters the implementation of monetary policy (rule) but leave the feasible set (Phillips curve) unchanged.

When the instrument of monetary policy is constrained by, for example, the zero lower bound on the nominal interest rate, the choice between purchases and transfers once again becomes relevant and monetary policy cannot substitute for fiscal policy. Moreover, the behavior of the credit spread and its dependence on endogenous variables such as aggregate borrowing and income will determine
the relative merits of purchases versus transfers. In the model, an exogenous shock to the credit spread causes the zero lower bound to bind. Under the calibration considered, purchases act more directly to increase output and inflation while transfers allow for a faster reduction in private sector debt. Unlike representative agent models of the ZLB, fiscal interventions in this setting hasten the escape from the zero lower bound due to the endogenous effect of debt reduction on credit spreads, and consumption multipliers for each policy are typically positive. A credit spread that is more elastic to changes in private sector debt favors transfers, while a spread that is more elastic to borrower income favors purchases.

The paper is organized as follows: Section 2 briefly summarizes related literature on fiscal policy in a non-representative agent setting and its role in stabilizing business cycles. Section 3 presents the model and introduces credit spreads and fiscal policy. Section 4 compares purchases and transfers in the case of no wealth effects on labor supply. Alternatively, Section 5 considers purchases and transfers in the presence of wealth effects. Section 6 examines the effect of purchases and transfers at the zero lower bound and Section 7 concludes.

2 Related Literature

The model of patient and impatient agents draws on the borrower-saver model used in Campbell and Hercowitz (2005), Iacoviello (2005), and Monacelli (2009) where different rates of time preference among households allow for borrowing and lending in steady state. Differing rates of time preference are a staple in financial accelerator models such as Bernanke, Gertler and Gilchrist (1999), but these models typically go further and link the discount rate to the structure of production. The structure of model considered here closely relates to the model used by Bilbiie, Monacelli and Perotti (2012) which focuses on the aggregate effects of income redistribution. My work differs in considering the role of credit spreads on the choice of purchases and transfers and the analysis of alternative fiscal instruments at the zero lower bound. Also, like Eggertsson and Krugman (2010), I also analyze fiscal policy in a two-agent setting where an exogenous debt shock causes the zero lower bound to bind and borrowers reduce consumption as debt is repaid. However, my model differs in considering deficit-financed fiscal policy, credit spreads that are partly determined endogenously, and analyzing the importance of wealth effects on labor supply both at the zero lower bound and away from the zero lower bound.

1The Appendix relates the credit spread model considered here to models with rule-of-thumb households, models with borrowing constraints, and overlapping generations models.
The effect of fiscal policy has also been examined in models with rule-of-thumb agents - agents who do not participate in financial markets and simply consume their income each period. Mankiw (2000) analyzes the effects of changes in taxation in a savers-spenders framework, noting that such a model provides a justification for temporary reductions in taxes as stimulus. Gali, Lopez-Salido and Valles (2007) examine the effect of rule-of-thumb consumers on the government purchases multiplier, finding that the presence of these agents can boost the multiplier above one. However, the effect of nominal rigidities and labor market frictions in their model have substantial effects on the government purchases multiplier even in the absence of rule-of-thumb consumers. In a model with rule-of-thumb agents, Giambattista and Pennings (2012) also compare the transfers multiplier to the government purchases multiplier finding cases in which the former can exceed the later. In contrast to a rule-of-thumb model, my model allows for intertemporal optimization on the part of both households and better fits the empirical evidence on tax rebates by allowing for a persistent response to temporary tax rebates as documented in Agarwal, Liu and Souleles (2007).

My work also relates to a literature on the effects of the public debt and transfers in settings with credit frictions such as borrowing constraints and incomplete markets. Aiyagari and McGrattan (1998) examine the optimal level of public debt in a heterogenous agent model with idiosyncratic earnings risk and capital as the variable factor of production. A higher level of public debt can increase welfare by easing liquidity constraints but lowers output by reducing precautionary saving and decreasing capital. Woodford (1990) presents a stylized overlapping generations model with capital to illustrated that increases in the public debt can both increase welfare and increase output, countering the view that high levels of debt must necessarily crowd out investment. This paper differs from this literature by considering an aggregate demand channel for changes in the public debt and focusing on short-term rather than long-run effects of fiscal policy.

A small literature has studied the conduct of fiscal policy for stabilization purposes in heterogeneous agent models with incomplete markets. Heathcote (2005) considers the short-run effect of tax cuts in a model with idiosyncratic income risk, but where both hours and capital are variable factors of production. He finds that a tax rebate has a multiplier of 0.15, and somewhat higher multipliers when considering reductions in distortionary taxes. His work does not consider the aggregate demand effect of alternative fiscal policies. Moreover, the output effect comes from investment rather than hours since he assumes GHH preferences and no wealth effects on labor supply. Similarly, Oh and Reis (2012) consider the effect of targeted transfers as fiscal stimulus and find very low transfer multipliers. The increase in hours worked by households that experience a negative wealth
shock does not offset the decrease in hours worked by households that receive transfers. The model considered here differs by treating only hours as a variable factor, considering sticky prices as the nominal rigidity, and using credit spreads as opposed to borrowing constraints to allow for financial intermediation.

3 Model

The model consists of two types of household, monopolistically competitive firms, a monetary authority that sets the deposit rate as its policy instrument, and a fiscal authority. The two-agent model facilitates the introduction of sticky prices and monetary policy to examine aggregate demand effects, and allows for the use of log-linearization to understand the key mechanisms at work. To generate borrowing and lending in steady state, the lender and borrower household are assumed to differ in their rates of time preference. An equilibrium credit spread is introduced to ensure that both agent’s Euler equations are satisfied in steady state.

3.1 Households

A measure $1-\eta$ of patient household chooses consumption and real savings to maximize discounted expected utility:

$$\max_{\{C^s_t, N^s_t, D_t\}} E \sum_{t=0}^{\infty} \beta^t U (C^s_t, N^s_t)$$

subject to $C^s_t = W_t N^s_t + \frac{1+i_{t-1}^d}{\Pi_t} D_{t-1} - D_t + \Pi_t^f - T_t$

where $D_t$ is real savings of the patient household and $\Pi_t^f$ are any profits from the real or financial sectors\(^2\). The government may collect non-distortionary lump sum taxes $T_t$ that are levied uniformly across households. The period utility function $U (C, N)$ is twice continuously differentiable, increasing, and concave in consumption: $U_c (C, N) > 0$, $U_{cc} (C, N) < 0$ and decreasing and convex in hours: $U_h (C, N) < 0$, $U_{hh} (C, N) < 0$. While patient households could choose to borrow, for sufficiently small shocks, the interest rate on borrowings would be too high and the patient household only saves.

\(^2\)If equity in the firms and intermediaries were traded and short-selling ruled out, the patient household would accumulate all shares in steady state. For sufficiently small shocks, the assumption that patient households own all shares would continue to hold in the stochastic economy.
A measure \( \eta \) of impatient household chooses consumption and real borrowings to maximize discounted expected utility:

\[
\max\{C^b_t, N^b_t, B_t\} \quad E_0 \sum_{t=0}^{\infty} \gamma^t U\left(C^b_t, N^b_t\right)
\]

subject to \( C^b_t = W_t N^b_t + B_t - \frac{1 + \tau^b_t}{\Pi_t} B_{t-1} - T_t \)

where \( B_t \) is the real borrowings of the impatient household. The impatient household’s discount rate \( \gamma < \beta \) ensures that the household chooses not to save and to only borrow in the neighborhood of the steady state. The impatient household’s optimality conditions are analogous to those of the patient household and standard:

\[
\lambda^i_t = U^i_t \left(C^i_t, N^i_t\right) \quad (1)
\]

\[
\lambda^i_t W_t = -U^h_t \left(C^i_t, N^i_t\right) \quad (2)
\]

\[
\lambda^s_t = \beta E_{t+1} \lambda^s_{t+1} \frac{1 + \tau^d_t}{\Pi_{t+1}} \quad (3)
\]

\[
\lambda^b_t = \gamma E_{t+1} \lambda^b_{t+1} \frac{1 + \tau^b_t}{\Pi_{t+1}} \quad (4)
\]

for \( i \in \{s, b\} \) in equations (1) and (2). The difference between the borrowing rate and the deposit rate allows both agents Euler equations to be satisfied in the non-stochastic steady state, with the interest rates determined by the patient and impatient household’s discount rates.

Aggregate consumption \( C_t \) and labor supply \( N^\text{sup}_t \) are simply the weighted sum of each household’s consumption and labor supply:

\[
C_t = \eta C^b_t + (1 - \eta) C^s_t \quad (5)
\]

\[
N^\text{sup}_t = \eta N^b_t + (1 - \eta) N^s_t \quad (6)
\]

As my analysis demonstrates, wealth effects play a critical role in determining the effect of fiscal policy on output, employment and consumption.

**Definition 1.** Wealth effects are absent from household labor supply if the household’s labor supply has the following representation:

\[
W_t = v_i (N^i_t)
\]

for some function \( v_i \) that is increasing.
Wealth effects on labor supply are eliminated under the preference specification considered by Greenwood, Hercowitz and Huffman (1988):

\[
U(C, N) = \frac{\left(C - \gamma N^{1 + \frac{1}{\varphi}}\right)^{1-\sigma}}{1 - \sigma}
\]

where \(\varphi\) is the Frisch elasticity of labor supply. Under GHH preferences, labor supply takes the form shown in the definition:

\[
W_t = \gamma \left(1 + \frac{1}{\varphi}\right) \left(N_t^i\right)^{1/\varphi}
\]

Aside from GHH preferences, wealth effects on labor supply would also be absent in a model with labor market rigidities. Under a rigid real wage, the labor supply relation no longer holds for each household:

\[
W_t > -\frac{U_{ib} \left(C_i^t, N_i^t\right)}{U_c \left(C_i^t, N_i^t\right)}
\]

for \(i \in \{s, b\}\). In a model where wages remained constant - the case of perfect wage rigidity considered by (Blanchard and Gali, 2010) and Shimer (2012) - fiscal multipliers are determined exclusively by firm’s labor demand condition. Under wage rigidity, household’s labor supply can be represented (locally) by a constant function \(v_i \left(N_i^t\right) = c = \bar{W}\) satisfying the definition of no wealth effects.

To obtain an aggregate labor supply curve and an aggregate IS curve, I must log-linearize the household’s labor supply and Euler equations. In the general case with wealth effects, labor supply is a function of the aggregate wage and the household’s consumption:

\[
w_t = \frac{1}{\varphi_i} n_i^t + \frac{1}{\sigma_i} c_i^t
\]

for \(i \in \{s, b\}\) where the lower case variables represent log deviations from steady state, \(\varphi_i\) is the household’s Frisch elasticity and \(\sigma_i\) is the household’s intertemporal elasticity of substitution. Solving for each agent’s labor supply \(n_i^t\), aggregate labor supply is the weighted sum of each agent’s log-linearized labor supply (where the weight is the steady state share of employment for each household). Similarly, an aggregate IS equation can be obtained by a weighted sum of each agent’s log-linearized Euler equation:

\[
w_t = \frac{1}{\varphi} n_t + l_b \frac{\varphi_b}{\varphi \sigma_b} c_t^b + (1 - l_b) \frac{\varphi_s}{\varphi \sigma_s} c_t^s
\]

\[
c_t = E_t c_{t+1} + s_b \sigma_b i_t^b + (1 - s_b) \sigma_s i_t^s - \bar{\sigma} E_t \pi_{t+1}
\]
with \( l_b = \frac{\eta N_b}{\overline{N}} \) and \( s_b = \frac{\eta C_b}{\overline{C}} \). The parameters \( \bar{\varphi} = l_b \varphi_b + (1 - l_b) \varphi_s \) and \( \bar{\sigma} = s_b \sigma_b + (1 - s_b) \sigma_s \) are the appropriate weighted aggregate Frisch elasticity and aggregate intertemporal elasticity of substitution respectively. Relative to a standard representative household model, the labor supply curve depends on the distribution of consumption (as opposed to just the level of consumption) and the IS curve depends on the real borrowing rate (in addition to the real deposit rate).

### 3.2 Credit Spreads

The credit spread - the difference between the borrowing rate and deposit rate - is treated as a reduced form equation:

\[
\frac{1 + i_b}{1 + i_d} = 1 + \omega_t = E_t \Gamma \left( B_t, W_{t+1} N_{t+1}, Z_t \right)
\]  

(9)

The function \( \Gamma \) is assumed to be weakly increasing in its first and last arguments and weakly decreasing in its middle argument. The assumption that the spread is increasing with the level of household debt \( B_t \) is needed to ensure determinacy of the rational expectations equilibrium and is analogous to the stationarity conditions needed in small open economy models\(^3\). The effect of expected borrower income, \( W_{t+1} N_{t+1} \) on credit spreads is consistent with the observed counter-cyclicality of credit spreads and the fact that spreads lead the business cycle. The dependence of the spread on borrower income would emerge in a model where lending is subject to adverse selection or limited commitment.

The shock \( Z_t \) is an exogenous financial shock that can increase spreads. The financial shock may be interpreted as either a shock to the supply or demand side of the credit market. On the supply side, if financial intermediaries’ capacity to raise funds is constrained by their own net worth, a depletion of equity due to an unexpected loss on the asset side of the balance sheet will cause an increase in borrowing rates. Alternatively, on the demand side, a shock to borrower collateral can likewise make borrowers less creditworthy thereby raising spreads. In particular, in a model with housing as collateral, a shock to house prices would reduce the value of collateral and raise credit spreads for the borrower household.

The log-linearized credit spread can be summarized by two parameters: the elasticity of the spread to private borrowings and the elasticity of the spread to borrower income with \( \chi_b > 0 \) and \( \chi_n \geq 0 \):

\[
\omega_t = \chi_b b_t - \chi_n E_t \left( w_{t+1} + n_{t+1} \right) + z_t
\]

\(^3\)See discussion in Schmitt-Grohé and Uribe (2003).
The elasticity on debt strictly exceeds zero to ensure stationarity. The credit spread may rise due to an exogenous increase in $z_t$ or may rise due to some other shock that drives up the level of debt or decreases borrower’s household income. The log-linearized credit spread is flexible enough to incorporate the type of interest rate spreads seen in a broad class of models. When $\chi_n = 0$, the model exhibits a debt elastic spread as in standard small open economy models. When $\chi_b = \chi_n > 0$, the credit spread varies with the leverage of the borrower household. The canonical financial accelerator model of Bernanke, Gertler and Gilchrist (1999) features a leverage elastic spread. Finally, when $\chi_n > \chi_b > 0$, the credit spread may be described as income elastic strengthening comovement with the business cycle. Variations in these parameters will be used to determine the effect of credit spreads on the choice among fiscal instruments.

### 3.3 Fiscal and Monetary Policy

The instruments of fiscal policy consist of a set of uniform nondistortionary taxes, government consumption, and tax rebates. The fiscal authority may also run a budget deficit subject to a fiscal rule that ensures that the debt returns to its steady state level and subject to an intertemporal solvency condition:

\begin{align}
G_t &= B_t^g - \frac{1 + i_{t-1}^d}{\Pi_t} B_{t-1}^g + T_t \\
T_t &= \phi_b (B_{t-1}^g - B_T^g) - reb_t \\
0 &= \lim_{T \to \infty} E_T \frac{P_t}{P_T} \frac{B_T^g}{\prod_{t} (1 + i_{t-1}^d)}
\end{align}

where $reb_t$ is a lump sum tax rebate delivered to all households. The instruments of fiscal policy are government purchases $G_t$ and a reduction in lump sum taxes $reb_t$. The government’s cost of funds is the policy rate $i_t^d$, not the borrowing rate $i_t^d$. This assumption best fits larger economies like the United States where the government controls the currency. For small open economies and countries in a currency union (such as the Eurozone), the rate at which the government borrows may carry a premium to the policy rate.

The monetary authority is assumed to set a rule for monetary policy so long as its instrument of policy, the deposit rate $i_t^d$, is not constrained by the zero lower bound. I will consider when
monetary policy follows a standard Taylor rule or pursues perfect inflation stabilization:

$$\left( \frac{r^d_t}{\bar{r}_d} \right) = (\Pi_t)^{\phi_x} \left( \frac{Y_t}{Y_t^{n}} \right)^{\phi_y}$$ (13)

$$\Pi_t = 1$$ (14)

When monetary policy is constrained by the zero lower bound, I assume that the deposit rate is set at zero or inflation is perfectly stabilized.

### 3.4 Firms

Monopolistically competitive firms set prices periodically and hire labor in each period to produce a differentiated good. Cost minimization for firms and production function play a key role in examining the effects of various fiscal policy shocks and are given below:

$$MC_t = \frac{W_t N_t}{\alpha Y_t}$$ (15)

$$Y_t = N_t^\alpha$$ (16)

where $\alpha$ is the labor share, $N_t$ is labor demand and $MC_t$ is the firm’s marginal cost which varies over time depending on the rate of inflation and the stance of monetary policy.

Prices are reset via Calvo price setting where $\theta$ is the likelihood of firm resetting its prices in the current period. When $\theta = 1$, prices are set each period and monopolistically competitive firms set prices as a fixed markup over marginal costs:

$$\frac{P_{it}}{P_t} = \frac{\nu}{\nu - 1} MC_t$$

where $\nu$ is the elasticity of substitution among final goods in the Dixit-Stiglitz aggregator. If the initial price level is unity, then prices will be normalized to unity, and marginal costs will be fixed at all periods $MC_t = MC = 1/\mu_p$. When $\theta < 1$, firms will set prices on the basis of future expected marginal costs. The firms pricing problem and the behavior of the price level are summarized by
the following dynamic equations:

\[ F_t = \mu_p \lambda_t^s MC_t Y_t + \theta \beta E_t \Pi_{t+1}^\nu F_{t+1} \]
\[ K_t = \lambda_t^s Y_t + \theta \beta E_t \Pi_{t+1}^{\nu-1} K_{t+1} \]
\[ 1 = \theta \Pi_t^{\nu-1} + (1 - \theta) \left( \frac{K_t}{F_t} \right)^{\nu-1} \]

Firms are owned by the saver households and therefore future marginal costs are discounted by the saver household’s stochastic discount factor.

When prices are flexible, marginal costs are fixed and, to a log-linear approximation, \( mc_t = 0 \). When prices are sticky, a log-linear approximation to the firm’s pricing problem around a zero inflation steady state implies the standard New Keynesian Phillips curve:

\[ \pi_t = \kappa mc_t + \beta E_t \pi_{t+1} \]

where \( \kappa = \frac{(1-\theta)(1-\theta \beta)}{\theta} \).

### 3.5 Equilibrium

Asset market clearing requires that real saving equals real borrowing:

\[ \eta B_t + B_t^g = (1 - \eta) D_t \]

Combining the household’s budget constraints and the government’s budget constraint and firm profits implies an aggregate resource constraint of the form:

\[ Y_t = C_t + G_t \tag{17} \]

Labor market clearing requires:

\[ N_t = N_t^{supply} = \eta N_t^b + (1 - \eta) N_t^s \tag{18} \]

**Definition 2.** An equilibrium is a set of allocations \( \{ Y_t, N_t, C_t^s, C_t^b, N_t^s, N_t^b, \lambda_t^s, \lambda_t^b, B_t, F_t, K_t \} \), a price process for \( \{ W_t, \Pi_t, i^d_t, i^b_t, MC_t \} \), a fiscal policy \( \{ B_t^g, T_t, G_t, reb_t \} \), and initial values for private debt \( B_0 \) and public debt \( B_0^g \) that jointly satisfy the equilibrium conditions listed in the
Appendix.

The fiscal policy considered consists of government purchases and tax rebates, as opposed to transfers. However, deficit-financing of these policies is equivalent to a transfer from saver to borrower households and back again.

**Proposition 1.** Consider an equilibrium under a deficit financed fiscal policy \( \{B_t^g, T_t, G_t, reb_t\} \). There exists a set of household-specific taxes \( T^b_t \) and \( T^s_t \) that implement the same equilibrium and satisfies a balanced budget: 
\[
G_t = \eta T^b_t + (1 - \eta) T^s_t 
\]

**Proof.** Since the saver household purchases the issuance of government debt, the saver’s budget constraint may be expressed using the asset market clearing condition and substituting out for taxes using the government’s budget constraint (10):
\[
C_t^s + \frac{1}{1 - \eta} (\eta B_t + B^g_t) = W_t N_t^b + \Pi_t^f + \frac{1 + i^d_{t-1}}{\Pi_t} \frac{1}{1 - \eta} \left( \eta B_{t-1} + B^g_{t-1} \right) \\
+ B^g_t - \frac{1 + i^d_{t-1}}{\Pi_t} B^g_{t-1} - G_t
\]
Rearranging, we may define a saver specific tax \( T^s_t \):
\[
C_t^s + \frac{\eta}{1 - \eta} B^g_t = W_t N_t^b + \Pi_t^f + \frac{1 + i^d_{t-1}}{\Pi_t} \frac{\eta}{1 - \eta} B_{t-1} - T^s_t \\
T^s_t = \frac{\eta}{1 - \eta} \left( B^g_t - \frac{1 + i^d_{t-1}}{\Pi_t} B^g_{t-1} \right) + G_t
\]
For the borrower household, we may define the borrower specific tax \( T^b_t \):
\[
G_t = \left( B^g_t - \frac{1 + i^d_{t-1}}{\Pi_t} B^g_{t-1} \right)
\]
It is readily verified that the household specific taxes satisfy the balanced budget constraint.

The proposition illustrates an equivalence relation between deficit-financing and transfers between agents. As the budget deficit increases, taxes fall for the borrower household and rise for the saver. A tax rebate represents a pure transfer from savers to borrowers despite the fact that both households receive the tax rebate. A deficit financed increase in purchases represents a combination of both transfers and purchases.

However, the transfer cannot be one way. As the debt is stabilized or decreased, the transfer reverses - borrowers make a transfer back to savers. Thus, in general, the converse of the proposition will not hold. A fiscal authority that can levy household specific taxes can implement a richer set of policies than a fiscal authority constrained to uniform taxation and deficit financing. For example,
a one-way transfer cannot be implemented as a deficit-financed rebate. Moreover, the capacity of the fiscal authority to engineer large transfers depends on the initial level of debt - with high levels of public debt, an increase in transfers requires an increase to higher debt levels where the overall transfer will be blunted by the size of interest payments.

4 Case of No Wealth Effects on Labor Supply

In this section, I examine the effect of purchases and transfers in a setting where household preferences or the structure of labor markets eliminate wealth effects on labor supply. The absence of wealth effects eliminates any effect of fiscal policy on aggregate supply. With prices set freely each period, firms’ incentives to hire labor are unchanged because neither its marginal costs nor its production technology are affected by the change in fiscal policy. When prices are changed only periodically, changes in fiscal policy will have an effect on aggregate demand. When prices are fixed, producers must meet demand at posted prices raising marginal costs. However, the monetary authority is always free to tighten interest rates and dampen demand so long as it is not constrained by the zero lower bound.

4.1 Flexible Prices

When producers are free to set prices each period, prices are a constant markup over marginal costs. Since price is normalized to unity, marginal costs are constant: $\overline{MC} = \frac{1}{\mu_p}$ in all periods.

**Proposition 2.** In the absence of wealth effects on labor supply and if $\theta = 1$, then output and employment are determined independently of fiscal policy

**Proof.** For each household, labor supply is determined by (2):

$$W_t = v_i \left( N_t^i \right)$$

for $i \in \{ s, b \}$. Under the assumptions in Section 3.1, the function $v$ is strictly increasing. Therefore, its inverse exists and combining the labor supply equation with labor market clearing:

$$N_t = \eta v^{-1}_b (W_t) + (1 - \eta) v^{-1}_s (W_t)$$

Using the firm’s production function (11) and labor demand condition (10), wages can be expressed
in terms of employment:

\[ W_t = \alpha MCN_t^{\alpha-1} \]

Replacing wages, aggregate employment is determined independent of fiscal policy. The production function implies that output is also determined independent of fiscal policy. □

Importantly, the irrelevance of fiscal policy holds irrespective of any of the properties of the credit spread, and would continue to obtain in a model with other types of financial frictions (such as borrowing constraints) or a larger number of agents so long as the labor supply relation holds for each agent. Using the economy’s resource constraint (17), it follows that a tax rebate or transfer has no affect on aggregate consumption while an increase in government purchases is offset by an equivalent decrease in consumption. Significantly, the insights of the representative agent model are unchanged in the multiple agent setting.

4.2 Sticky Prices

Under sticky prices, marginal costs are no longer constant and fiscal shocks will affect output and employment through the aggregate demand channel. However, monetary policy can also affect output and employment via the aggregate demand channel, and, since the feasible set of combinations of output and inflation are unchanged by the presence of credit spreads, monetary policy and fiscal policy are redundant.

To show that the Phillips curve is unchanged, I use a log-linear approximation to the equilibrium conditions to obtain the output inflation tradeoff. Under GHH preferences, the household’s log-linearized labor supply conditions imply:

\[ w_t = \frac{1}{\nu} n_t^i \]

for \( i \epsilon \{s, b\} \). Aggregating using a log-linearized version of (18) and eliminating \( w_t \) using (15):

\[ mc_t = n_t - y_t + \frac{1}{\nu} n_t \]

Eliminating \( n_t \) using the log-linearized production function (16) and using the equation for \( mc_t \), an expectations-augmented Phillips curve is obtained:

\[ \pi_t = \frac{\kappa}{\alpha} \left( 1 - \alpha + \frac{1}{\nu} \right) y_t + \beta E_t \pi_{t+1} \]
The case of wage rigidity is simply the case of $\nu \to \infty$:

$$\pi_t = \frac{K}{\alpha}(1 - \alpha) y_t + \beta E_t \pi_{t+1}$$

If monetary policy seeks to stabilize some combination of output and inflation, the targeting rule for optimal monetary policy will be unaffected by the presence of credit spreads or their variability. Formally, if the central bank chooses a path of $\pi_t, y_t$ to minimize a loss function of the form:

$$L = E_0 \sum_{t=0}^{\infty} \beta^t \left( \pi_t^2 + \lambda y_t^2 \right)$$

subject to the Phillips curve given above, then the target criterion is the standard one

$$\pi_t + \frac{\lambda}{\vartheta} (y_t - y_{t-1}) = 0$$

where $\vartheta$ is the slope of the Phillips curve. Though the loss function here does not follow from a second-order approximation of average utility in a multiple household economy, it is sensible to assume that the central bank will be primarily concerned with maintaining aggregate output rather than distributional considerations. The primacy of monetary policy in determining the effect of fiscal shocks is similar to the conclusions reached in Woodford (2010). He showed that the government purchases multiplier could be larger or smaller than the neoclassical multiplier depending on how aggressively monetary policy responds to inflation.

While, the inflation/output tradeoff is unchanged by credit spreads, the implementation of monetary policy will be affected. This result is analogous to the results presented in Curdia and Woodford (2010) who show that the presence of financial intermediation does not affect the targeting rule for optimal monetary policy but may affect the implementation of optimal monetary policy. In general, setting the correct policy rate $i_t^d$ to implement optimal policy will require the monetary authority to take into account changes in the credit spread. A log-linear approximation to the household’s Euler equations (1) and (3) - (4) and a log-linear approximation to the resource constraint (17) can be combined to derive an aggregate IS equation:

$$i_t^d = E_t \pi_{t+1} - \frac{1}{s\sigma} (y_t - g_t - E_t (y_{t+1} - g_{t+1})) - \frac{s\sigma_b}{\sigma} \omega_t$$

where $\omega_t$ is the credit spread, $\sigma_b$ is the borrower household’s intertemporal elasticity of substitu-
inition, $\tilde{\sigma}$ is a weighted average of households intertemporal elasticity of substitution, $s_b$ is the share of borrower's consumption in total consumption in steady state, and $s_c$ is the share of private consumption in total output in steady state. Fiscal policy will directly affect the determination of interest rates through government purchases and also affect interest rates via the spread. So long as the zero lower bound on nominal interest rates is not binding, there exists a path of interest rates consistent with the target path of output and inflation set by the monetary authority. Any changes in fiscal policy can be accommodated by suitable adjustment of the interest rate. Since a path of output implies a path of employment, monetary policy and fiscal policy are redundant in determining those quantities when the zero lower bound is not binding. Importantly, monetary policy and fiscal policy cannot achieve the same equilibrium allocations and are not equivalent in terms of the distribution of consumption. Fiscal policy may still play a role in targeting some distribution of consumption or level of private debt.

5 Case of Wealth Effects on Labor Supply

In this section, I consider the more conventional case of government purchases and transfers in the presence of wealth effects on labor supply. The canonical RBC and New Keynesian models typically feature wealth effects ensuring both an aggregate supply and an aggregate demand channel for fiscal policy. While the conclusions in this section are not as strong as the case with no wealth effects, the insights from the special case of no wealth effects largely carry over in the calibrated examples considered in this section.

5.1 Representative Agent Benchmark

To allow for wealth effects on labor supply, I consider standard preferences where the level of consumption affects agent’s labor supply. To a log linear approximation, each agent’s labor supply condition relates the wage to hours worked and consumption:

$$w_t = \frac{1}{\varphi_i} n_t^i + \frac{1}{\sigma_i} c_t^i$$

The labor supply approximation given above holds irrespective of whether utility is separable in consumption and hours. To examine how credit spreads affect fiscal multipliers, it is useful to derive a representative agent benchmark for comparison. In a representative agent model, marginal utilities must be equalized across agents implying that $c_t^a = c_t^b = c_t$. Solving each agent’s labor
supply equation in terms of \( n_t \) and aggregating labor using (18) gives an aggregate labor supply relation:

\[
\begin{align*}
    w_t &= \frac{1}{\tilde{\phi}} n_t + \frac{1}{\tilde{\sigma}} c_t \\
    \tilde{\phi} &= l_b \varphi_b + (1 - l_b) \varphi_s \\
    \tilde{\sigma} &= \frac{\tilde{\phi}}{(l_b \frac{s_b}{\sigma_b} + (1 - l_b) \frac{s_s}{\sigma_s})}
\end{align*}
\]

where \( l_b \) is the share of borrower’s hours in total hours worked. Given this aggregate labor supply condition, the output multiplier can be obtained by solving for consumption and the wage in terms of output (since \( mc_t = 0 \)) and substituting into the resource constraint (17):

\[
y_t = \frac{\alpha}{\alpha + s_c \tilde{\sigma} \left( 1 - \alpha + \frac{1}{\tilde{\phi}} \right)} g_t
\]

where \( s_c \) is the share of consumption in GDP and \( \tilde{\phi} \) is the average Frisch elasticity and \( \tilde{\sigma} \) is the representative agent’s intertemporal elasticity of substitution. Government spending increases output via a negative wealth effect, but the government spending multiplier is necessarily less than one. Transfers and deficit-financing have no effect on output.

The representative agent model also admits a representation for the Phillips curve. Eliminating \( mc_t \) using the labor demand equation (15) and eliminating \( n_t \) using the production function (16) provides a Phillips curve representation:

\[
\pi_t = \kappa \left( w_t + \frac{1 - \alpha}{\alpha} y_t \right) + \beta E_t \pi_{t+1}
\]

Using the resource constraint (17) to eliminate \( c_t \) and the production function, wages can be expressed in terms of output and government purchases. Replacing the wage in the Phillips curve provides the relationship between output and inflation:

\[
\pi_t = \frac{\kappa}{\alpha} \left( \frac{1}{\tilde{\phi}} + \frac{1}{s_c \tilde{\sigma}} + 1 - \alpha \right) y_t - \frac{\kappa}{s_c \tilde{\sigma}} g_t + \beta E_t \pi_{t+1}
\]

An increase in government purchases shifts back the Phillips curve by increasing labor supply and lowering wages - government purchases raise the natural rate of output.
5.2 Flexible Prices

In the case of the multiple agent model, the labor supply relations can be solved for consumption $c_i^t$ in terms of the wage $w_t$ and hours worked $n_i^t$ for each agent. Substituting into the resource constraint and eliminating the wage using (15), output can be expressed in terms of purchases and hours worked by each agent:

$$y_t = \frac{\alpha}{\alpha + s_c \sigma (1 - \alpha) g_t}$$

$$- \frac{s_c}{\alpha + s_c \sigma (1 - \alpha)} \left( s_b \sigma_b \frac{n_b}{\varphi_b} + (1 - s_b) \sigma_s \frac{n_s}{\varphi_s} \right)$$

where $\sigma = s_b \sigma_b + (1 - s_b) \sigma_s$ is a weighted average elasticity of intertemporal substitution and other parameters are as defined earlier.

The expression for output can be further simplified by solving for $n_b^t$ from labor market clearing (18), giving output as a function of government purchases and the saver household’s labor supply:

$$y_t = \frac{\alpha}{\alpha + s_c \sigma (1 - \alpha) + s_c \frac{s_b \sigma_b}{\varphi_b} \frac{g_t}{\alpha}}$$

$$+ \frac{\alpha s_c}{\alpha + s_c \sigma (1 - \alpha)} \left( \frac{1 - l_b}{l_b} \frac{s_b \sigma_b}{\varphi_b} - \frac{1 - s_b}{\varphi_s} \right) n_s^t$$

**Proposition 3.** Transfers and the means of financing any government expenditure have no effect on output and employment if:

1. Preferences are linear in hours worked as in Hansen (1985) and Rogerson (1988)

2. Labor supply by households is coordinated: $n_s^t = n_b^t$

3. Preferences satisfy the following condition:

$$\frac{1 - l_b}{l_b} \frac{\varphi_s}{\varphi_b} = \frac{1 - s_b}{s_b} \frac{\sigma_s}{\sigma_b}$$

**Proof.** In the first case, as the Frisch elasticities $\varphi_s = \varphi_b \rightarrow \infty$, the coefficient on the second term in the expression for output goes to zero, and output is only affected by purchases. In the second case, hours worked by the saver hours equal aggregate hours: $n_s^t = n_t^t = \frac{1}{\alpha} y_t$ and output is solely a function of purchases. In the last case, the coefficient on hours of the saver household is zero. ■

The proposition illustrates that, even with wealth effects on labor supply, transfers and deficit-
financing may have little effect on output or employment. The deviations from the representative agent benchmark stem solely from the second term in the output expression. If households are sufficiently homogenous - that is, if household do not differ appreciably in underlying parameters and shares of consumption and hours, the coefficient on the second term is likely to be small. If this coefficient is positive, fiscal policies that strengthen the negative wealth effect on the saver household will boost the output multiplier relative to the representative agent benchmark. In particular transfers away from the saver household should boost multipliers. However, if the coefficient is negative, transfers that increase the negative wealth effect on borrowers will boost multipliers.

5.3 Sticky Prices

In the case of sticky prices, fiscal policy has both an aggregate supply element that reduces marginal costs and an aggregate demand element that raises marginal costs. Monetary policy does not face a stable Phillips curve relation between inflation and output, and the choice of fiscal policy may shift the Phillips curve in favorable or unfavorable ways. As before, the Phillips curve can be expressed in terms of both output and wages:

\[ \pi_t = \kappa \left( w_t + \frac{1 - \alpha}{\alpha} y_t \right) + \beta E_t \pi_{t+1} \]

However, unlike the representative agent model, in the presence of wealth effects, wages cannot generally be expressed in terms of aggregate output.

In the cases considered in the previous proposition, transfers have no effect on aggregate output and the Phillips curve can be represented in terms of inflation and output as in the representative agent model. Since transfers do not shift the Phillips curve, credit spreads do not affect the Phillips curve and the output-inflation tradeoff is unchanged. As before, households labor supply equations can be aggregated into an aggregate labor supply equation:

\[ w_t = \frac{1}{s_c \sigma} (y_t - g_t) + \frac{1}{\sigma} \left( \frac{s_b \sigma_b}{\varphi_b} n_t^b + \frac{(1 - s_b) \sigma_s}{\varphi_s} n_t^s \right) \]

where the first term gives the wealth effect on labor supply and the second term gives the substitution effect. Because government purchases act to directly lower the wage while transfers cause offsetting movements in hours between households, purchases are likely to have a greater downward effect on wages. A reduction in wages will provide the monetary authority with a more favorable output and inflation tradeoff and allow for a less restrictive monetary policy. In this sense, one


can claim that purchases may be better than transfers for boosting output and employment by improving the inflation-output tradeoff for the central bank.

### 5.4 Calibration

As I have shown, in the presence of wealth effects on labor supply, fiscal policy will have both aggregate supply and aggregate demand channels. To assess the degree to which the multiple agent model differs from the representative agent model, I calibrate the model with wealth effects and examine the effect of deficit-financed purchases and tax rebates. While each deficit-financed policy can be expressed as a balanced budget combination of purchases and transfers, the deficit-financed policies considered here are closest to fiscal policy in practice and avoid issues of incentive compatibility.

The baseline calibration assumes standard separable utility function of the form

$$U(C, N) = \frac{C^{1-\sigma^{-1}}}{1 - \sigma^{-1}} - \nu N^{1+\phi^{-1}}$$

with standard values for the Frisch elasticities and intertemporal elasticities of substitution. In the baseline calibration these values are equal across agents with $\varphi_b = \varphi_s = 2$ and $\sigma_b = \sigma_s = 1$. In steady state, output $Y$ is normalized to 1 and the disutility of labor supply for each household $\nu_s$ and $\nu_b$ is set to ensure that each household supplies labor such that $\overline{N}_b = \overline{N}_s = 1$. The markup due to monopolistic competition is set at 25% and the labor share $\alpha$ is set to ensure that the wage bill is equal to 70% of GDP, consistent with U.S. data. The Calvo parameter $\theta$ is set to 0.75 so that firms

---

*In the case of household specific taxes and transfers, household have an incentive to mask their type and represent themselves as borrowers or lenders based on the proposed policy.*

---

**Table 2: Calibration Summary**

<table>
<thead>
<tr>
<th>Parameter Description</th>
<th>Parameter</th>
<th>Value</th>
<th>Parameter Description</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intertemporal elasticity</td>
<td>$\sigma_i$</td>
<td>1</td>
<td>Deposit rate</td>
<td>$i_d$</td>
<td>1.02^{0.25}</td>
</tr>
<tr>
<td>Frisch elasticity</td>
<td>$\varphi_i$</td>
<td>2</td>
<td>Borrowing rate</td>
<td>$i_b$</td>
<td>1.06^{0.25}</td>
</tr>
<tr>
<td>Calvo parameter</td>
<td>$\theta$</td>
<td>0.75</td>
<td>Borrower share</td>
<td>$\eta$</td>
<td>50%</td>
</tr>
<tr>
<td>Markup</td>
<td>$\mu_p$</td>
<td>0.25</td>
<td>Debt elasticity</td>
<td>$\chi_b$</td>
<td>0.1</td>
</tr>
<tr>
<td>Wage bill</td>
<td>$\frac{W N}{Y}$</td>
<td>0.70</td>
<td>Income elasticity</td>
<td>$\chi_n$</td>
<td>0</td>
</tr>
<tr>
<td>Gov’t purchases</td>
<td>$\frac{G}{Y}$</td>
<td>0.20</td>
<td>Taylor rule (inflation)</td>
<td>$\phi_\pi$</td>
<td>1.5</td>
</tr>
<tr>
<td>Debt/GDP</td>
<td>$\frac{B_g}{Y}$</td>
<td>2</td>
<td>Taylor rule (output)</td>
<td>$\phi_y$</td>
<td>0</td>
</tr>
<tr>
<td>Household debt</td>
<td>$\frac{B}{WN_b}$</td>
<td>4</td>
<td>Fiscal rule</td>
<td>$\phi_b$</td>
<td>0.2</td>
</tr>
</tbody>
</table>
change prices every 4 quarters on average. The rates of time preference $\beta$ and $\gamma$ are set to target an annual deposit rate of 2% and an annual borrowing rate of 6%. The disutilities of labor supply, the rates of time preference, and the markup do not enter the log-linearized equilibrium conditions and, therefore, do not affect the dynamics of the model. In steady state, the consumption of the borrower household is less than that of the saver household since the saver household earns both wage income and profits from the firm. Government spending is 20% of GDP in steady state. The steady state public debt is 50% of GDP consistent with recent U.S. levels. In steady state, the household debt for the borrower household is equal to annual household income, consistent with data on household wealth from the Survey on Consumer Finances.

The nonstandard parameters for the model include the credit spread parameters $\chi_b$ and $\chi_n$ that control the endogenous response of spreads to private sector debt and expected borrower income respectively and the share of borrower households $\eta$ in the economy. In the baseline case, I will consider a debt-elastic spread such that $\chi_b = 0.1$ and $\chi_n = 0$ - a calibration that implies a 1% increase in debt raises spreads by roughly 50 basis points. In general, a regression of spreads on measures of indebtedness and income in aggregate data is unlikely to accurately estimate these elasticities given that common shocks may induce a comovement of income and spreads even though $\chi_n = 0$. As shown in Section 6, the financial shock $z_t$ causes income and spreads to comove even with $\chi_n = 0$. As it turns out, these credit spread elasticities have little effect on the experiments here suggesting that spreads may have a fairly small effect on fiscal policy transmission away
from the zero lower bound. The share of borrower household $\eta$ is set to 50% as in Curdia and Woodford (2010); this parameter has no obvious analogue in the data and is selected conservatively to minimize heterogeneity. The calibration values are summarized in Table 2.

5.5 Fiscal Policy Experiments and Sensitivity

The first experiment in Figure 1 considers the effect of a 1% of GDP increase in government purchases (top panel) and a 1% of GDP increase in tax rebates (bottom panel), each with a persistence of $\rho = 0.9$. The figure also shows the response of the representative agent economy with parameters as defined in Section 5.1. The fiscal authority runs a budget deficit and taxes follow a fiscal rule - taxes adjust upwards to return the public debt to its steady state level. The response parameter in the fiscal rule $\phi_b$ is close to the rule used in Gali, Lopez-Salido and Valles (2007), which is based on VAR estimates for U.S. data. Prices are reset each period and, therefore, firm markups are constant.

In this environment, the effect of purchases and rebates is driven by wealth effects on labor supply. Under the baseline calibration where the Frisch elasticity and intertemporal elasticity of substitution are equal, the only source of heterogeneity is the share of borrower consumption $s_b < \frac{1}{2}$ since the borrower household pays interest to the intermediary and does not receive any profits from firms\(^5\). Under this calibration, the coefficient on saver’s hours (in the output expression in Section 5.2) is negative. As a result, the tax rebate multiplier is slightly negative - the fall in hours worked by the borrower household is not fully offset by the rise in hours by the saver household. The transfer acts to dampen incentives to work. Likewise, the government purchases multiplier on output is slightly lower than the representative agent multiplier since the labor supply effects for the borrower are dampened by the increase in the deficit. As the second column shows, the response in hours worked by each household is quite different reflecting the transfer component of fiscal policy. However, these movements wash out in the aggregate - the difference in aggregate hours between the representative agent model and the multiagent model is miniscule. The dynamics of public debt illustrate the degree of transfers from the saver household - periods of increasing debt represent net transfers to borrowers, while periods of stabilizing and falling debt represent transfers from borrowers back to savers. Importantly, these policies do not imply the same debt

\(^5\)Steady state government purchases are financed by a tax on patient households to reduce differences in steady state levels of consumption (through a tax on capital holdings). However, it is assumed that both household pay taxes proportional to their size in the economy to finance government purchases in excess of steady state levels. In steady state, $C_s/C_b \sim 1.3$. 

22
dynamics since changes in the interest rate have an effect on debt accumulation in a calibration with a positive steady state level of debt. With zero debt, both policies would imply the same path of the public debt in a linear approximation. Government purchases have larger output multipliers than tax rebates simply because purchases have a larger wealth effect on labor supply. Output and employment rise as the wage falls due to the increased willingness of both households to work.

Figures 2 and 3 examine how sensitive these results are to the credit spread elasticities $\chi_b$ and $\chi_n$ and to heterogeneity in wealth effects across households by adjusting the relative intertemporal elasticities of substitution. Figure 2 show that different models of the spread have little effect on the deviations of output multipliers from the representative agent benchmark - in particular the tax rebate multiplier is still negative and close to zero. Figure 2 considers three cases: debt elastic spreads ($\chi_b = 0.5$, $\chi_n = 0$), leverage elastic spread ($\chi_b = \chi_n = 0.5$), and income elastic spread ($\chi_b = 0.1$, $\chi_n = 0.5$). In all cases, the purchases and rebate multipliers deviates by less than 5% from the representative agent benchmark. In each case, the behavior of hours and spreads differs, but the aggregate effect on output, hours, wages, and consumption are all close to the representative agent benchmark. The second column shows that saver’s hours respond strongly to the tax rebate shock, but the borrower’s response almost fully offsets this rise in hours resulting in little net effect.

Figure 3 examines the effect of variations in the relative intertemporal elasticity of substitution holding the average intertemporal elasticity fixed at $\tilde{\sigma} = 1$ where $\tilde{\sigma}$. In the case of “high borrower...
elasticity," wealth effects for the borrower household are diminished by choosing an intertemporal elasticity of substitution three times higher than that of the saver household. Alternatively, in the case of “high saver elasticity,” the borrower household has an intertemporal elasticity of substitution one-third the size of the saver household and, therefore, the borrower’s labor supply is more sensitive to changes in wealth. When the borrower household exhibits smaller wealth effects, the tax rebate multiplier becomes positive. Savers respond to the negative wealth shock by working harder while borrowers reduce their hours but by less than in the baseline case. As a result, aggregate hours and output rises. The opposite occurs in the case of high saver elasticity. As before, the government purchases multiplier is an order of magnitude higher than the tax rebate multiplier simply because of the stronger wealth effects on aggregate labor supply under purchases.

Figure 4 relaxes the assumption of flexible prices and examines the effect of an increase in purchases and tax rebates when monetary policy follows a Taylor rule. To ensure that a tax rebate is expansionary, the calibration used in Figure 4 assumes the case of a high borrower elasticity of intertemporal substitution - that is, \( \sigma_b / \sigma_s = 3 \). As the experiment demonstrates, fiscal multipliers rise sizably under an operative aggregate demand channel. Moreover, a more elastic credit spread raises multipliers further - when the elasticity of the spread to debt rises from \( \chi_b = 0.1 \) to \( \chi_b = 0.5 \), the output multiplier on purchases rises from 0.69 to 0.82. Likewise, the output multiplier for tax rebates rises from 0.05 to 0.15. The falling credit spread dampens the transmission of monetary policy as the rise in the deposit rate is not fully incorporated into the borrowing rate (since spreads
are falling). However, as noted earlier, bigger multipliers come only at the cost of higher inflation as seen in the last column. This rise in inflation is due to the fact that the Phillips curve has not shifted, and larger multipliers are the product of an accommodative stance of monetary policy.

As before, the purchases multiplier is an order of magnitude larger than the transfers multiplier. However, if monetary policy responds asymmetrically to different fiscal shocks, it is possible to obtain cases where the tax rebate multiplier is as high or higher than the purchases multiplier. Finally, purchases are preferred to tax rebates in that sense that purchases generate a larger rise in output and employment for a given amount of inflation. The negative wealth effect of purchases raises labor supply, reduces marginal costs, and improves the Phillips curve tradeoff.

6 Zero Lower Bound

In this section, I examine how credit spread shocks may cause the zero lower bound to bind and consider the effect of government purchases and transfers on output and consumption. Consistent with evidence from representative agent models, the government purchases multiplier is above unity at the zero lower bound. Additionally, transfers (implemented by tax rebates) may be similarly effective as purchases in stabilizing output and consumption. The choice among policies depends on the endogenous feedback of debt and income on the credit spread.

Representative agent models typically rely on preference shocks or other reduced form shocks
to the natural rate of interest to cause the zero lower bound to bind. However, in a model with multiple agents, disruptions to the financial system that raise the credit spread may also cause the zero lower bound to bind. As shown in Section 4.2, an aggregate IS-equation can be obtained by summing the agent’s Euler equations:

\[ i^d_t = E_t \pi_{t+1} - \frac{1}{s_c \sigma} (y_t - g_t - E_t (y_{t+1} - g_{t+1})) - \frac{s_b \sigma_b}{\sigma} \omega_t \]

Any shock to the credit spread \( \omega_t \) will drive down the interest rate when monetary policy seeks to maintain \( y_t = \pi_t = 0 \), and for sufficiently large shocks, the interest rate will fall to the zero bound. The reduced form credit spread depends endogenously on debt and borrower income and, exogenously, on a financial shock. Any underlying shock that drives up debt and/or decreases borrower income may reduce the deposit rate, but I will consider an exogenous financial shock as the shock that causes the zero lower bound to bind.

The special case of real wage rigidity and a credit spread with zero debt elasticity \( \chi_b = 0 \) illustrates the role of purchases versus transfers in determining output and inflation. Under these conditions, output and inflation are solely determined by the Phillips curve and the intertemporal IS curve:

\[
\begin{align*}
\pi_t &= \kappa \frac{(1 - \alpha)}{\alpha} y_t + \beta E_t \pi_{t+1} \\
y_t &= E_t (y_{t+1}) + E_t (g_{t+1} - g_t) - s_c \sigma \left( i^d_t - E_t \pi_{t+1} \right) - s_c s_b \sigma_b \left( z_t - \frac{X_n}{\alpha} E_t y_{t+1} \right)
\end{align*}
\]

where the credit spread is replaced by the log-linearized version of (9). By setting the debt-elasticity of the spread to zero, the law of motion for debt and the distribution of consumption between saver and borrower households is decoupled from the determination inflation and output. A zero lower bound episode is caused by a temporary increase in \( z_t \) to \( z \) that reverts to zero with probability \( 1 - \rho \) in each period causing the zero lower bound to bind: \( i^d_t = -\tau \). Given the absence of any state variables, this two-equation system is forward-looking and multipliers may be computed explicitly as in Woodford (2010):

\[
\begin{align*}
y_{zlb} &= \nu_g \bar{y} - \zeta \\
\nu_g &= \frac{(1 - \rho) (1 - \beta \rho)}{(1 - \beta \rho) \left( 1 - \rho - s_c s_b \sigma_b \rho \frac{X_n}{\alpha} \right) - s_c \sigma \rho \left( 1 - \alpha \right) \rho} \\
\zeta &= \frac{s_c (1 - \rho) (s_b \sigma_b \bar{z} - \bar{\sigma} \tau)}{(1 - \beta \rho) \left( 1 - \rho - s_c s_b \sigma_b \rho \frac{X_n}{\alpha} \right) - s_c \sigma \rho \left( 1 - \alpha \right) \rho}
\end{align*}
\]
The constant term gives the decrease in output in the absence of any policy intervention and under the assumption that monetary policy ensures that $y_t = \pi_t = 0$ after the financial shock dissipates. If the credit spread does not respond (or responds weakly) to changes in private debt, transfers and tax rebates have no effect on output and inflation and are ineffective as fiscal stimulus. Government purchases are effective in counteracting the effects of a financial shock, but the mechanism is essentially the same as any representative agent model of government purchases at the zero lower bound. The means of financing the increase in purchases are irrelevant. In fact, the multiplier is identical to the multiplier in Woodford (2010) except for the endogenous effect of output on the spread through $\chi_n$. When $\chi_n > 0$, credit spreads amplify shock and the multiplier on government purchases is higher as is the negative effect of the financial shock.

This simple example highlights how transfers operate through the credit spread. To the extent that transfers decrease the credit spread by lowering household indebtedness, transfers will have a positive multiplier. This analysis suggests that deficit-financed government purchases will be preferred to purchases financed by taxes because the transfer component of the policy further reduces credit spreads; indeed, this result holds in the numerical examples considered in the next section. Moreover, since transfers only operate through the spread, the transfers multiplier is unlikely to exceed the government purchases multiplier unless purchases worsen the rise in spreads. In the calibrated examples considered next, purchases reduce private sector indebtedness and the credit spread.

### 6.1 No Policy Intervention

The experiment here roughly attempts to capture the type of disruption experienced in the U.S. after the collapse of Lehman Brothers in 2008. The model and calibration are the same as considered in the previous section, however, for simplicity, the monetary authority is assumed to follow perfect inflation stabilization $\pi_t = 0$ for all periods after the zero lower bound ceases to bind. While monetary policy may, in principle, mitigate the effects of the financial shock by committing to higher future inflation (as discussed extensively in Eggertsson and Woodford (2003)), I assume that time inconsistency diminishes the effectiveness of these commitments. Additionally, steady state public debt is assumed to be zero to ensure that each policy implies the same path for the public debt (to a linear approximation) and the shock generates no endogenous movements in debt or taxes.

Figure 5 shows the effect of a financial shock that raises (annualized) credit spreads 16 percentage
points\textsuperscript{6}. The model is solved using the solution algorithm described in the appendix of Eggertsson and Woodford (2003) and the financial shock is assumed to have a persistence of $\rho = 0.8$. The point at which the economy exits the zero lower bound depends on the endogenous behavior of private sector debt and the value of the elasticity $\chi_b$. For high elasticities, a faster rate of deleveraging will cause the credit spread to fall faster hastening the exit from the zero lower bound. However, as shown in Figure 5, agents actually increase their debt loads since the elasticity of the spread to debt is fairly low ($\chi_b = 0.1$).

A 16 percentage point financial shock raises credit spreads by 20 percentage points and leads to a very large fall in output and consumption in excess of 20%. The negative wealth effect drives down wages 40% and the fall in demand and wages combines to cause a very steep deflation. The zero lower bound episode lasts for 10 quarters or two and half years, and rates gradually normalize as debt and spreads fall. After the ZLB ceases to bind, inflation remains at zero and output is marginally positive due to wealth effects that keep wages lower than their steady state level.

Given the role of wealth effects in determining the behavior of wages and inflation, I also consider a similar shock in a model without wealth effects on labor supply. In the presence of a perfectly rigid wage, changes in household wealth have no affect on output or employment. Figure 6 provides the impulse responses to a larger 20 percentage point financial shock\textsuperscript{7}.  

\textsuperscript{6}The actual rise in the credit spread is larger because of the endogenous component due to the increase in private sector debt.

\textsuperscript{7}Relative to the calibration with flexible wages, the persistence of the financial shock is increased to $\rho = 0.9$ and...
Under rigid wages, the fall in output and inflation are significantly dampened with output falling 8% and (annualized) inflation falling 1.5% on impact - values that are comparable to the U.S. output and inflation response in the fourth quarter of 2008. The zero lower bound ceases to bind in 14 quarters and, since the Phillips curve is independent of spreads, output, consumption and inflation jump to their steady state levels. Households deleverage throughout the crisis period and only begin to releverage after four years; interest rates remain below their steady state level for the entire period of 24 quarters or six years. The output and inflation response in the rigid wage model suggest that some degree of wage rigidity might be desirable for fitting the model to the current recession.

6.2 Policy Intervention

I consider deficit financed fiscal policies where the intervention ends as soon as the zero lower bound ceases to bind. Fiscal policy may either raise government purchases or reduce taxes by some level so long as the zero lower bound binds. The choice of a flexible or rigid wage has significant implications for the efficacy of policy. Figure 7 shows the effect of a 1% of GDP increase in the saver's intertemporal elasticity of substitution is set at $\sigma_s = 2$ so that saver households are more responsive to changes in the real interest rate than borrower households.
government purchases and a 1% decrease in taxes for all periods that the zero lower bound binds. Small policy interventions have very large effects relative to no intervention. For both government purchases and tax rebates, the economy exits the zero lower bound within a year instead of 2.5 years. Under the tax rebate, the fall in output is 2.5% versus a 22% fall absent intervention. For government purchases, the fall in output only last a quarter with output falling by only 0.5%. Instead of continuing to increase leverage, households deleverage between 3% and 5% and the intervention reduces the rise in spreads by 1/3. Deficit-financed purchases are preferred to tax rebates both in terms of output and consumption. Purchases act more directly to raise output and inflation, reducing the real interest rate faced by saver households and “crowding-in” consumption.

Figure 8 shows the much more limited effect of a 1% increase in purchases and tax rebates in the rigid wage model. Both policies are successful in boosting both output and consumption relative to a policy of inaction, but these policies carry much smaller multipliers than the case of flexible wages. Government purchases limit the fall in output to 6.1% and rebates limit the fall in output to 7.6% relative to the 8% fall absent any intervention. Purchases, once again, act more directly to boost inflation and raise the consumption profile of the saver household while the borrowers consumption path is a function primarily of the credit spread. Output and inflation actually rise above their steady state values before jumping to zero once the zero lower bound stops binding. Relative to no intervention, the economy exits the zero lower bound one period earlier.
6.3 Role of Credit Spreads

The choice between purchases and transfers/tax rebates depends, in part, on which policy is more effective in reducing credit spreads. Since the solution for the model at the zero lower bound is nonlinear, the model response to various fiscal shocks will not be invariant to the size of the shock. In other words, a 2% of GDP deficit-financed increase in purchases is not simply a scaled version of the 1% of GDP experiment; therefore, a fiscal multiplier is not readily defined. Moreover, given that the model features endogenous state variables, the response of various variables of interest will depend on the persistence and shape of the fiscal response, even if the total size of the fiscal intervention is held constant. However, to provide insight into the role of credit spreads, I consider a simple metric for computing output multipliers:

\[ M_t = \frac{\sum_{t=0}^{24} (y_{t}^{pol} - y_{t}^{nopol})}{\sum_{t=0}^{24} x_t} \]

where \( y_{t}^{pol} \) is the response of output, in log-deviations, under a particular fiscal intervention, \( y_{t}^{nopol} \) is the response of output under no intervention, and \( x_t \) is the total expenditure on the policy. The multiplier is a equally weighted sum over 24 quarters (6 years) of the deviations of output from the path it would have taken absent any intervention, conditional on the same underlying shock.
Two natural variables of interest are output (which is also employment in this model) and aggregate consumption. To isolate the effect of variations in the debt and income elasticity of the credit spread, I consider fiscal interventions of the following form: a 1% of GDP increase in government purchases financed by current taxes or a 1% transfer from saver to borrower households in all periods. The underlying shock is a 5 percentage point increase in the financial shock that decays deterministically at rate $\rho = 0.9$, and fiscal interventions end once the zero lower bound stops binding. Only the model with perfect wage rigidity is considered.

Table 3 provides output and consumption multipliers and the time to exit for different values of the debt and income elasticity parameters in the credit spread. As Table 3 shows, the output multiplier on government purchases exceeds the output multiplier on transfers, though transfers may be more effective in boosting aggregate consumption than government purchases. The effectiveness of transfers relative to purchases rises with the debt-elasticity of the credit spread $\chi_b$, and the time to exit the zero lower bound falls with $\chi_b$. Absent any fiscal intervention, a higher debt elasticity leads to faster deleveraging by borrowers and a quicker exit from the zero lower bound as the endogenous component offsets the financial shock component in the credit spread. Transfers facilitate this process of deleveraging allowing for quicker exits from the zero lower bound. As the debt elasticity rises, transfers are more effective in reducing credit spreads and mitigating the effects of the financial shock. Unlike transfers, an increase in government purchases leaves the path of the credit spread largely unchanged along with the timing of exit. Consistent with the analytical results shown earlier for a debt inelastic spread, transfers are ineffective when $\chi_b = 0$; while transfers redistribute consumption from savers to borrowers, aggregate output and inflation will be unaffected.

An increase in the income-elasticity of the credit spread $\chi_n$ (shown on the right-hand side of Table 3) tends to magnify the effect of either type of fiscal intervention with output multipliers approaching the values estimated by the Congressional Budget Office. The time to exit the zero lower bound increases because the fall in output from the financial shock feeds back into the credit spread, amplifying its effect. The amplification of both the underlying shock and the effect of fiscal policy is consistent with the analytical results shown earlier. Despite the importance of the debt

---

8 Evidence from Edelberg (2003) suggest very low elasticities of risk premia on consumer loans with respect to debt and borrowing. Using data from the Survey on Consumer Finances, even for large differences in income and personal debt, spreads vary by less than two percentage points. A naive extrapolation suggests elasticity of spreads with respect to borrowing and income of less than 0.01 - an order of magnitude lower than shown in the numerical experiments in this section. However, given the extensive use of non-price tools such as credit limits, downpayments, and credit history in rationing credit, these elasticities should be viewed as a lower bound rather than an upper bound on credit spread elasticities.
Table 3: Output and Consumption Multipliers

<table>
<thead>
<tr>
<th>( \chi_n = 0.0 )</th>
<th>( \chi_b = 0.0 )</th>
<th>( \chi_b = 0.1 )</th>
<th>( \chi_b = 0.2 )</th>
<th>( \chi_b = 0.3 )</th>
<th>( \chi_n = 0.2 )</th>
<th>( \chi_b = 0.0 )</th>
<th>( \chi_b = 0.1 )</th>
<th>( \chi_b = 0.2 )</th>
<th>( \chi_b = 0.3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Purchases</td>
<td>= 1% of GDP</td>
<td>Exit</td>
<td>16 qtr</td>
<td>15 qtr</td>
<td>14 qtr</td>
<td>13 qtr</td>
<td>Exit</td>
<td>17 qtr</td>
<td>16 qtr</td>
</tr>
<tr>
<td>Output</td>
<td>1.35</td>
<td>1.22</td>
<td>1.19</td>
<td>1.15</td>
<td>2.53</td>
<td>2.37</td>
<td>2.12</td>
<td>1.92</td>
<td>1.53</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.35</td>
<td>0.22</td>
<td>0.19</td>
<td>0.15</td>
<td>1.53</td>
<td>1.37</td>
<td>1.12</td>
<td>0.92</td>
<td>0.00</td>
</tr>
<tr>
<td>Transfers</td>
<td>= 1% of GDP</td>
<td>Exit</td>
<td>16 qtr</td>
<td>14 qtr</td>
<td>13 qtr</td>
<td>12 qtr</td>
<td>Exit</td>
<td>17 qtr</td>
<td>15 qtr</td>
</tr>
<tr>
<td>Output</td>
<td>0.00</td>
<td>0.26</td>
<td>0.29</td>
<td>0.30</td>
<td>0.00</td>
<td>0.48</td>
<td>0.50</td>
<td>0.52</td>
<td>0.26</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.00</td>
<td>0.26</td>
<td>0.29</td>
<td>0.30</td>
<td>0.00</td>
<td>0.48</td>
<td>0.50</td>
<td>0.52</td>
<td>0.00</td>
</tr>
</tbody>
</table>

and income elasticity parameters for fiscal policy, a simple regression of credit spread measures on output and private sector borrowing is unlikely to accurately estimate these parameters since the error term is likely to be highly correlated with output and borrowing.

6.4 Savers and Borrowers Policy Preferences

Given that the model features distinct agents, optimal fiscal and monetary policy will, in general, depend on the weighting of each agent in the social welfare function. Moreover, even in the absence of financial shocks, the fiscal authority may wish to redistribute income among agents by changes in the level of the public debt\(^9\). However, even at the zero lower bound - where fiscal policy is most relevant - savers and borrowers may disagree on which fiscal interventions they prefer and may, in some cases, prefer no intervention at all. Furthermore, when fiscal policies are financed by deficits, preferences will differ on the rate with which the deficit is returned to its steady state. As shown earlier, when the credit spread is fairly debt inelastic, tax rebates and transfers simply have the effect of redistributing income and consumption with little effect on aggregates and savers may prefer inaction to any fiscal intervention. Importantly, the financial shock itself is redistributive - saver households benefit from the increase in credit spreads because, under the assumption that intermediary profits flow to the savers and absent no default (no real costs of intermediation), intermediary profits increase with a rise in spreads.

A simple metric, analogous to the output multiplier, for gauging household’s preferences over various policy options is the difference in household consumption under a fiscal intervention relative

\(^9\)For instance, a higher government debt implies greater inequality in steady state given higher interest payments to savers. Aiyagari and McGrattan (1998) consider the optimal level of public debt in a model with idiosyncratic risk and incomplete markets.
Table 4: Relative Consumption (% Deviation)

<table>
<thead>
<tr>
<th>Fiscal Parameter:</th>
<th>$\varphi_b = 0.1$</th>
<th>$\varphi_b = 0.2$</th>
<th>$\varphi_b = 0.3$</th>
<th>$\varphi_b = 0.4$</th>
<th>$\varphi_b = 0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Purchases</td>
<td>= 1% of GDP</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Saver Household</td>
<td>-0.03</td>
<td>0.06</td>
<td>0.09</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>Borrower Household</td>
<td>1.01</td>
<td>0.81</td>
<td>0.78</td>
<td>0.71</td>
<td>0.65</td>
</tr>
<tr>
<td>Tax Rebates</td>
<td>= 1% of GDP</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Saver Household</td>
<td>-0.12</td>
<td>-0.04</td>
<td>-0.03</td>
<td>-0.02</td>
<td>-0.02</td>
</tr>
<tr>
<td>Borrower Deviation</td>
<td>0.72</td>
<td>0.52</td>
<td>0.41</td>
<td>0.33</td>
<td>0.28</td>
</tr>
</tbody>
</table>

to no intervention:

$$\frac{1}{24} \sum_{t=0}^{24} \left( c_t^{i,pol} - c_t^{i,nopol} \right) \ i\epsilon \{s, b\}$$

Table 4 displays each households’ consumption relative to the no intervention baseline under two alternative fiscal policies:

1. Debt-financed increase in government purchases by 1% of GDP for all periods the zero lower bound binds

2. Debt-financed tax rebates to all households of 1% of GDP for all periods the zero lower bound binds

Under the assumption of zero initial public debt, these policies imply equivalent fiscal cost for the government with the debt increasing during the ZLB episode and subsequently converging back to zero. The rate at which taxes are raised to pay back the public debt is controlled by the parameter $\varphi_b$ with higher values leading to a faster increase in taxes. As Table 4 indicates, saver households prefer policies that minimize the increase in the public debt, and therefore minimize the degree of transfers. Borrower households instead prefer a greater degree of deficit-financing which increases disposable income in the near term when the cost of credit is high. In the baseline calibration considered ($\chi_b = 0.1, \chi_n = 0$), any fiscal intervention offers a higher consumption path for borrowers relative to no intervention. In contrast, savers often enjoy a higher consumption path absent any fiscal intervention. Under this calibration, savers never prefer tax rebates (even though savers receive this rebate and are paid market rate on public debt), and only prefer purchases if the transfer component is limited. Deficit-financed purchases are the policy that most frequently increases the consumption path of both households relative to the baseline of no intervention. The preference for government purchases in this example is somewhat driven by the low debt elasticity
of the credit spread, which renders transfers less effective for stabilization. Though this analysis abstracts from welfare costs of labor supply and inflation, it illustrates the potential for disagreement over fiscal policy and deficits.

7 Conclusion

Existing representative agent models, by Ricardian equivalence, rule out any role for transfers, tax rebates, or deficit-financing as tools for stabilizing business cycles. This paper analyzes a borrower-lender model with credit spreads to examine transfers as an instrument of fiscal policy and compare transfers to government purchases. I showed that deficit-financed policies such as an increase in purchases or temporary lump sum tax rebates can be expressed as a combination of two fiscal instruments: purchases and transfers, and I distinguished between two important channels for these instruments: wealth effects and a Keynesian demand channel. I find, in general, that government purchases are a more effective means of boosting output and employment than transfers/tax rebates, primarily because of its large wealth effects on labor supply.

Given the role of the credit spread elasticities in determining the multiplier on transfers relative to government spending at the zero lower bound, the details of financial intermediation are likely to be important for determining relative multipliers. While there is no single consensus model of financial intermediation, particularly among households, business cycle models of intermediation have generally focused on the importance of either collateral constraints for the borrower or the net worth of the intermediary\(^\text{10}\). In a model where borrowing is constrained by the value of collateral, fiscal multipliers depend on the effect of policy on the shadow price of the collateral constraint. Both transfers and government purchases, by boosting disposable income, could ease these constraints temporarily or may boost the value of collateral, with the multiplier depending on how much each policy boosts borrower's disposable income. Moreover, given the importance of housing as household collateral, relative multipliers are also likely to depend heavily on the effect of these policies on housing values. Even for relatively large fiscal outlays, these effects are likely to be small absent a policy directed towards housing.

Alternatively, in a model where the cost of financial intermediation depends on the net worth of intermediaries, fiscal multipliers are likely to operate via the default channel. If the net worth of financial intermediaries is low, the cost of credit will only fall if intermediaries are able to restore

\(^{10}\text{For the former, see Kiyotaki and Moore (1997) or Iacoviello (2005). For the latter, see Bernanke, Gertler and Gilchrist (1999).}\)
their net worth. Absent direct recapitalizations or changes in dividend policy, fiscal policy is only likely to impact intermediary net worth by reducing default rates. By raising disposable income either by raising output or through direct transfers, fiscal policy may reduce default rates and increase the rate at which intermediaries recapitalize. Alternatively, if fiscal policy leads to more defaults via households’ decisions to strategically default, fiscal policy could increase the cost of credit resulting in negative multipliers. In the end, more direct fiscal instruments that address the cause of the rise in credit costs - whether declining house prices or insufficient intermediary net worth - are likely to be more effective than indirect policies such as transfers and government purchases.

Given multiple instruments of policy, a natural extension will be analysis of optimal fiscal policy at the zero lower bound and further analysis of disagreement among agents over policy. Endogenizing the credit spread will be another important extension given the dependence of policy on the behavior of the credit spread. These extensions are ongoing research.
References


A  Extensions

In this section, I briefly consider fiscal policy in a model where a subset of the population operates as rule of thumb agents who simply consume current income each period and a model where agents face a borrowing constraint. This section relates to a literature on fiscal policy and rule of thumb agents developed by Mankiw (2000) and Gali, Lopez-Salido and Valles (2007), and a literature examining the effects of monetary policy when agents face borrowing constraints such as Iacoviello (2005) and Monacelli (2009). As this section illustrates, the credit spread model considered in this paper can easily be related to rule of thumb or borrowing constraint models and, therefore, the policy implications are likely to carry over to a broader class of DSGE models.

A.1 Rule of Thumb Agents

Rule of thumb agents face a static optimization problem and choose hours period-by-period facing a simple budget constraint with consumption equal to current disposable income:

\[
U_c (C_y^t, N_y^t) W_t = -U_h (C_y^t, N_y^t)
\]

\[
C_y^t = W_t N_y^t - T_t
\]

Log-linearizing these equilibrium conditions and combining with the equilibrium conditions for the firms and saver households discussed earlier delivers a closed form solution for output in terms of government purchases and taxes:

\[
y_t = \frac{\alpha}{(\alpha + s_c \sigma(1 - \alpha) + s_c \frac{1-s_y}{1-l_y} \frac{\sigma_s}{\varphi_s} - s_c(1 - \alpha)\phi v)} g_t - \left(\frac{\alpha \phi \bar{Y}}{\sigma_y \bar{c}_y} \right)^{tax_t}
\]

\[
\phi = s_y \sigma_y \varphi_y - (1-s_y) \frac{s_y}{\varphi_s} \frac{l_y}{1-l_y}
\]

\[
v = \left(1 - \frac{\bar{w}_m}{\sigma_y \bar{c}_y}\right) / \left(\frac{1}{\varphi_b} + \frac{\bar{w}_m}{\sigma_y \bar{c}_y}\right)
\]

The multiplier on government spending has several terms similar to the multiplier derived in Section 5, with the parameters \(\phi\) and \(v\) as the new terms. For Frisch elasticities less than unity, \(v < 1\), and for households with sufficient symmetry, \(\phi \approx 0\). Therefore, a tax reduction for the borrower household has negligible effect on output for plausible calibrations, and the government spending multiplier remains below unity, consistent with the numerical experiments shown in Figure 1. Intuitively, a transfer from one household to the other has offsetting effects on the labor supply of each household,
leaving total labor supply relatively unchanged and, therefore, output unchanged. To the extent that \( \phi > 0 \), tax rebates will be expansionary and the government spending multiplier will be larger than in the representative agent benchmark.

Under sticky prices, analytical solutions with rule of thumb agents can be obtained under the assumption of GHH preferences or wage rigidity that eliminates a labor supply effect. For simplicity and comparability to the rest of the paper, I consider the case of rigid wages. The Phillips curve from Section 4.2 obtains along with an intertemporal IS curve of the form:

\[
\begin{align*}
y_t - g_t &= E_t (y_{t+1} - g_{t+1}) - s_c \sigma_s (1 - s_y) \left( i_t^d - E_t \pi_{t+1} \right) - s_c s_y \left( E_t c_t^{y} - c_t \right) \\
c_t^{y} &= \frac{\omega_n \alpha}{\alpha} y_t - \frac{\sigma_s}{\sigma_y} \tau_a x_t \\
\pi_t &= \frac{k}{\alpha} (1 - \alpha) y_t + \beta E_t \pi_{t+1}
\end{align*}
\]

The last term in the IS equation can be treated as equivalent to the credit spread \( \omega_t \), and responds to both changes in income and transfers. A temporary increase in transfers that is gradually withdrawn, as in the case of a debt-financed tax rebate, is equivalent to a fall in the credit spread that eventually becomes positive as the transfer turns negative when taxes are raised to return the public debt to its steady state. Relative to the credit spread model, transfers appear directly in the intertemporal IS equation instead of operating indirectly through private sector debt. As before, when monetary policy is unconstrained, the Phillips curve is unchanged and monetary policy is free to target any combination of inflation and output subject to the Phillips curve tradeoff.

When monetary policy is constrained by the zero lower bound, both purchases and transfers may be used for stabilization and an explicit tax rebate multiplier can be derived when there is a constant probability that the shock causing the zero lower bound to bind disappears. While a financial shock no longer appears because of the absence of intermediation, any of the shocks that cause the zero lower bound to bind in representative agent models - like a discount rate shock - would suffice here\(^{11}\). The solution for output at the zero lower bound is similar to the solution

\[^{11}\text{We can easily reintroduce the financial shock and credit spread by simply adding a measure of rule-of-thumb consumers to the existing saver/borrower model. Goods market clearing then implies that } Y_t = \eta_s C_t^s + \eta_b C_t^b + (1 - \eta_s - \eta_b) C_t^{y}. \text{ As before, under the assumption of zero debt elasticity of the credit spread, the log-linearized economy at the zero lower bound is summarized by an aggregate intertemporal IS curve and the standard Phillips curve. Moreover, in a lifecycle model with distinct borrowing and credit spreads, the stochastic steady state would be characterized by saver households, borrower households, and households living in autarky.}\]
derived in Section 6 with the addition of a multiplier on the tax rebate:

\[
y_{zlb} = \nu^*_g (g_{zlb} - tax_{zlb}) - \zeta \\

\nu^*_g = \frac{(1 - \rho)(1 - \beta\rho)}{(1 - \rho)(1 - \beta\rho) \left(1 - inc_g \frac{1}{\alpha}\right) - s_c(1 - s_b) \sigma_c s_b \frac{n}{\alpha} (1 - \alpha) \rho}
\]

where \( inc_g \) is the rule-of-thumb agents share of wage income in national income. Comparison to the multiplier derived in section 6 reveals that the multiplier \( \nu_g \) may be higher or lower; the effect of higher inflation reducing real interest rates (the last term in the denominator) is attenuated relative to the saver/borrower model while the presence of rule-of-thumb agents raises the direct effect of government spending on the consumption of rule-of-thumb agents (the \( inc_g \) term) and the multiplier. Unlike an old-style Keynesian model, the government spending multiplier and tax rebate multiplier are the same, and the balanced budget multiplier is zero.

The reason the multiplier is the same for both government spending and tax rebates is that both affect the savers consumption in the same way. A rise in government spending or equivalent fall in tax rebates raises aggregate demand by the same amount, and equilibrium in the goods market requires either a rise in output or a fall in the savers consumption induced by a rise in the real interest rate. With the nominal rate held constant and no direct effect of either policy on the Phillips curve, the savers consumption response is the same and, therefore, the output multiplier is the same for each policy. When the government’s budget is balanced, the aggregate demand effects cancel out and the savers consumption decision is unchanged.

Finally, it’s worth relating this equilibrium analysis of the zero lower bound with rule-of-thumb agents to the extensive literature on the determinants of consumption and the aggregate consumption function where the real interest rate is taken as fixed and exogenous\(^\text{12}\). The multipliers attached to any particular fiscal policy are heavily dependent on the behavior of the real interest rate, and therefore conclusions regarding fiscal multipliers are inherently general equilibrium questions. In the same way that the credit spread - absent wealth effects - does not alter the Phillips curve, a more complex (and realistic) theory of consumption is unlikely to alter the effects of fiscal policy away from the zero lower bound. Unless fiscal stabilization has large effects on the production side of the economy - that is, incentives to supply labor and capital - monetary policy can achieve the same aggregate demand objectives of fiscal policy away from the zero lower bound. The nature of the aggregate consumption function will only become relevant at the zero lower bound where fiscal

\(^{12}\text{See for example Carroll (2001) and Kaplan and Violante (2011).}\)
policies that have larger effect on desired consumption will be preferred to policies with a smaller effect.

A.2 Borrowing Constrained Agents

A broad range of models consider a class of agents that are constrained either by an exogenous or endogenous borrowing constraint but assume a single rate for lending and borrowing funds. These model often assume that the borrowing constraint binds at all times and solve for the dynamics of the model by log-linearizing around a binding constraint. Relative to the rule of thumb model in Section 7.1 and assuming an exogenous borrowing constraint, the equilibrium conditions become:

\[
U_c \left( C^b_t, N^b_t \right) W_t = -U_h \left( C^b_t, N^b_t \right)
\]

\[
C^b_t + \frac{1 + i^d_t - 1}{\Pi_t} B_{t-1} = W_t N^b_t + B_t
\]

\[
B_t \geq B
\]

To a log-linear approximation, the borrower’s budget constraint differs from the rule-of-thumb budget constraint only by including the lagged interest rate. If steady state interest payments are small, this term can be safely disregarded and the fiscal multipliers obtained in Section 7.1 remain a good approximation in the case of exogenous constraints. Without further assumptions on the model, a general characterization of fiscal multipliers with an endogenous borrowing constraint is difficult.

Under sticky prices and a demand driven labor market, a similar Phillips curve and intertemporal IS curve determine output and inflation. When borrowers are constrained by an endogenous or exogenous constraint, their optimal choice of borrowing is governed by an Euler equation with nonzero Lagrange multiplier on the binding constraint:

\[
\lambda^b_t = \gamma E_t \lambda^b_{t+1} \frac{1 + i^d_t}{\Pi_{t+1}} + \Theta_t
\]

where the Lagrange multiplier represents the shadow price of the constraint. Since the constraint is assumed to be always binding for sufficiently small shocks, the borrower’s Euler equation can be log-linearized and summed with the saver’s Euler equation to obtain an intertemporal IS curve of the form:

\[
y_t - g_t = E_t (y_{t+1} - g_{t+1}) - s_c \sigma (i^d_t - E_t \pi_{t+1}) - s_c s_b \sigma_b \theta_t
\]
As before, the last term can be regarded as the credit spread, and changes in fiscal policy will shift the credit spread depending on the nature of the borrowing constraint. Importantly, the multiplier is likely to be changed by policy given that any change in income, wages, or taxes will affect the shadow price of the borrowing constraint. Though the mapping of a borrowing constraint model into the credit spread model will depend on further assumptions, the insights on fiscal policy from the credit spreads model should carry over to alternative models of borrowing and lending.

A.3 Housing and Credit Spreads

I maintain the assumption of patient and impatient households, but I now assume a single market interest rate for savers and households. Instead of a credit spread, impatient household are constrained to borrow only a possibly time-varying fraction of the value of their residence. The impatient household’s chooses:

\[
\max \{C^b_t, N^b_t, B_t, H^b_t\} \quad \mathbb{E} \sum_{t=0}^{\infty} \beta^t U \left( C^b_t, N^b_t, H^b_t \right)
\]

subject to

\[
C^b_t = W_t N^b_t - \frac{1 + i^d_t}{\Pi_t} B_{t-1} + B_t - T_t + Q_t \left( H^b_{t-1} - H^b_t \right)
\]

\[
B_t \leq \chi_t Q_t H^b_t
\]

Relative to the equilibrium conditions in Section 3.1, the Euler equation changes and a housing Euler equation is introduced:

\[
\lambda^b_t = \gamma E_t \lambda^b_{t+1} \left( \frac{1 + i^d_t}{\Pi_t} + \Theta_t \right)
\]

\[
\lambda^b_t Q_t = \gamma E_t \lambda^b_{t+1} \left( r^h_{h,t+1} + Q_{t+1} \right) + \Theta_t \chi_t Q_t
\]

Furthermore, if impatient households are the only agents that demand housing services and the supply of housing is fixed, the housing Euler equation will determine the market-clearing price of housing. We can log-linearize the model around a steady state assuming that the collateral constraint is always binding. Under these assumptions, an aggregate IS equation of the same form
as the rule-of-thumb case emerges. To a log-linear approximation:

\[ y_t - g_t = E_t (y_{t+1} - g_{t+1}) - s_c \sigma_s (1 - s_b) \left( \frac{\beta}{\gamma_b} \pi_{t+1} - E_t \pi_{t+1} \right) - s_c s_b \left( E_t c^b_{t+1} - c_t^b \right) \]

\[ c_t^b = \frac{\bar{\pi}}{\bar{c}_b} \left( \gamma_y y_t + \gamma_{\text{tax}} \pi_t + \gamma_b \left( \chi_t + q_t \right) \right) \]

\[ \pi_t = \frac{\kappa}{\alpha} (1 - \alpha) y_t + \beta E_t \pi_{t+1} \]

The preceding equations along with a monetary policy rule do not fully specify the equilibrium of the economy; the borrower household’s Euler equation and housing Euler equation are needed to determine the dynamics of housing prices and the Lagrange multiplier on the borrowing constraint.

The growth rate of borrower’s consumption takes the place of the credit spread in the aggregate IS equation just as in the case of rule-of-thumb households:

\[ E_t \left( c^b_{t+1} - c_t^b \right) = \gamma_y E_t (y_{t+1} - y_t) - \gamma_{\text{tax}} E_t (\text{tax}_{t+1} - \text{tax}_t) \]

\[ + \gamma_b \left( E_t \chi_{t+1} - (2 + \bar{\pi}) \chi_t + (1 + \bar{\pi}) \chi_{t-1} \right) \]

\[ + \gamma_b \left( E_t q_{t+1} - (2 + \bar{\pi}) q_t + (1 + \bar{\pi}) q_{t-1} \right) \]

where \( \gamma_y, \gamma_{\text{tax}}, \) and \( \gamma_b \) are the appropriate constants. An exogenous tightening of the collateral constraint can be represented as a fall in \( \chi_t \) and, ignoring the equilibrium dynamics of housing prices, will act like an increase in the credit spread so long as the stochastic process for \( \chi_t \) is dominated by the middle term for some period of time. In particular, an AR(3) process of \( \chi_t \) could generate an AR(1) process for the “interest-rate” shock represented by borrower consumption growth in the aggregate IS equation.

The inclusion of housing dynamics further complicates matters since simply a fall in housing prices does not guarantee a rise in borrower consumption growth beyond the initial period. Nevertheless, it appears plausible that a collateral shock could cause act in the same manner as a credit spread shock in the aggregate IS equation even with endogenous house prices. Stronger conclusions require greater structure placed on the saver household’s demand for housing and residential investment which will both determine the market clearing housing price.
B Equivalence with Overlapping Generations Model

In this section, I show that the steady state of the model with infinitely-lived agents with differing degrees of time preference is isomorphic to the steady state of a model with finitely lived agents who share the same rate of time preference but differ in effective labor over the life cycle.

Household live $T$ periods with variation in the disutility of labor supply over the life cycle, and each generation that dies in a period is replaced by a generation of equal measure in the next period so that the total population is constant. Household choose consumption, hours worked, and whether to borrow or save in each period. Formally, for each generation $i \in \{0, 1, \ldots, T\}$, household’s optimization problem is:

$$\max \ E_0 \sum_{t=0}^{T-i} \beta^t \{u(C_t(i)) - \theta_t\}$$

$$C_t(i) = W_tN_t(i) + B_t(i) - \frac{1 + \theta_t}{\Pi_t} B_{t-1}(i) - D_t(i) + \frac{1 + \phi_t}{\Pi_t} D_{t-1}(i) + \Pi_f - T_t$$

$$D_t(i) \geq 0$$

$$B_t(i) \geq 0$$

$$B_{T-i}(i) = 0$$

where $\theta_t$ is an exogenous process for effective labor supply that captures the hump-shaped profile of earnings over the lifecycle. The household is prohibited from borrowing in the final period of life. The first-order conditions characterizing the household’s optimal consumption and savings decisions are given below:

$$u_c(C_t(i), N_t(i)) = \lambda_t(i)$$  \hspace{1cm} (21)

$$-u_n(C_t(i), N_t(i)) = \lambda_t(i)W_t\theta_t$$  \hspace{1cm} (22)

$$\lambda_t(i) = \beta E_t \lambda_{t+1}(i) \left(1 + \frac{i^{d_t}}{\Pi_{t+1}} \right) - \phi^{b_t}_t(i)$$  \hspace{1cm} (23)

$$\lambda_t(i) = \beta E_t \lambda_{t+1}(i) \frac{1 + i^{d_t}}{\Pi_{t+1}} + \phi^{d_t}_t(i)$$  \hspace{1cm} (24)

$$\lambda_{T-i}(i) = -\phi^{b_t}_{T-i}(i)$$  \hspace{1cm} (25)

$$\lambda_{T-i}(i) = \phi^{d_t}_{T-i}(i)$$  \hspace{1cm} (26)

$$\phi^{b_t}_t(i)B_t(i) = 0$$  \hspace{1cm} (27)

$$\phi^{d_t}_t(i)D_t(i) = 0$$  \hspace{1cm} (28)
Household optimality requires that households do not borrow or save in the final period. Subtracting the Euler equation for borrowing from the Euler equation for deposits shows that households never simultaneously borrow and save, but may find it optimal to live in autarky:

\[ 0 = \beta E_t \frac{\lambda_{t+1}(i)}{\Pi_{t+1}} \omega_t - \left( \phi^d_t + \phi^b_t \right) \]

I consider a steady allocation of consumption, borrowing and labor supply across generations where wages, interest rates, and the price level are constant, and assume that the utility functions and distribution of \( \theta_i \) over the generations are sufficient to guarantee that a steady state exists.

The firm’s problem, the intermediaries problem, fiscal policy, and monetary policy are unchanged from the discussion in Section 3. Market clearing requires:

\[ Y_t = \sum_{i=0}^{T} C_t(i) + G_t \]  
\[ N_t = \sum_{i=0}^{T} N_t(i) \]

A steady state of the overlapping generations model with credit frictions is a set of aggregate quantities \( \{ Y, N, C, F, K, \Pi_f \} \), a distribution of consumption, labor supply, deposits and borrowings over generations \( \{ C_i, N_i, D_i, B_i, \lambda_i, \phi^d_i, \phi^b_i \}_{i=0}^T \), a set of prices \( \{ \bar{W}, \bar{\Pi}, \bar{\omega}, \bar{MC} \} \), a fiscal policy \( \{ B_g, T, G, reb \} \) that jointly satisfy the steady state versions of:

1. Household optimality conditions (15) - (22)
2. Household budget constraints
3. Firm optimality condition in Footnote 3 and (10) - (11)
4. Government budget constraint, fiscal rule, and solvency condition (7) - (9)
5. Monetary policy rule (6)
6. Market-clearing conditions (23) - (24)

Given a definition for the steady state of the overlapping generations model, for suitable choices of the distribution of \( \theta_i \) and other model parameters, the steady state of the infinite horizon model is equivalent to the steady state of the overlapping generations model.
Proposition 4. Consider a steady state of the overlapping generations model. There exists a set of discount rates and functions for household utility that give the provide steady state in the infinite horizon model.

Proof. Since the firm’s problem, intermediaries’ problem, fiscal and monetary policy are unchanged in the overlapping generation model, a steady state in the OLG model satisfies parts 3-5 of the steady state version of the definition of an equilibrium in the infinite horizon model. It remains to show that household optimality conditions and market clearing conditions may be satisfied.

Let \( \Omega \) is the set of borrowers in \( i e \{0, 1, \ldots T\} \). Savers and borrowers consumption and labor supply can be defined in the OLG model and will satisfy the corresponding market clearing conditions (12) - (13) in the infinite horizon model:

\[
\begin{align*}
C_s &= \frac{1}{1 - \pi_b} \sum_{i \in \Omega^c} C_i \\
C_b &= \frac{1}{\pi_b} \sum_{i \in \Omega} C_i \\
N_s &= \frac{1}{1 - \pi_b} \sum_{i \in \Omega^c} N_i \\
N_b &= \frac{1}{\pi_b} \sum_{i \in \Omega} N_i
\end{align*}
\]

For suitable definitions of the utility functions for each household, household’s labor supply conditions hold in steady state:

\[
\begin{align*}
U^s_c (C_s, N_s) W &= -U^s_h (C_s, N_s) \\
U^b_c (C_b, N_b) W &= -U^b_h (C_b, N_b)
\end{align*}
\]

Under the assumption that firm profits are only paid to savers and the assumption that \( \theta_i \) implies only one switch from borrowing to saving midway through the lifecycle, summing the budget constraints of borrower household:

\[
\begin{align*}
\sum_{i \in \Omega^c} C_i &= \overline{W} \sum_{i \in \Omega^c} N_i + \sum_{i \in \Omega^c} B_i \left( 1 - \frac{1 + \theta_b}{\Pi} \right) - T \sum_{i \in \Omega^c} 1 [i \in \Omega^c] \\
\Rightarrow B &= \frac{1}{\pi_b} \sum_{i \in \Omega^c} B_i
\end{align*}
\]

Finally, the interest rate and borrowing rate from the OLG model determine the discount rates in
the infinite horizon model:

\[
\beta = \frac{1}{1 + \bar{i}_d} \\
\gamma = \frac{1}{(1 + \bar{i}_d)(1 + \bar{\omega})}
\]

C Equilibrium Conditions

- Household equilibrium conditions and relevant transversality conditions for \( i \in \{s, b\} \):

\[
\lambda_i^s = u_c(C_i^s, N_i^s) \\
\lambda_i^b W_t = -u_n(C_i^b, N_i^b) \\
\lambda_t^s = \beta E_t \lambda_{t+1}^s \frac{1 + i_t^d}{\Pi_{t+1}} \\
\lambda_t^b = \gamma E_t \lambda_{t+1}^b \frac{(1 + i_t^f)(1 + \omega_t)}{\Pi_{t+1}}
\]

- Law of motion for public sector and private sector debt:

\[
B_t = C_t^b - W_t N_t^b + \frac{1 + i_{t-1}^b}{\Pi_t} B_{t-1} + T_t \\
B_t^g = G_t + \frac{1 + i_{t-1}^d}{\Pi_t} B_{t-1}^g - T_t
\]

- Firm production, cost minimization, price setting and price level determination:

\[
Y_t = N_t^\alpha \\
W_t = \alpha \frac{Y_t}{N_t} MC_t \\
1 = \theta \Pi_t^{\nu-1} + (1 - \theta) \left( \frac{K_t}{F_t} \right)^{\nu-1} \\
F_t = \frac{\nu}{\nu - 1} \lambda_t^s MC_t Y_t + \theta \beta E_t \Pi_{t+1}^{\nu-1} F_{t+1} \\
K_t = \lambda_t^s Y_t + \theta \beta E_t \Pi_{t+1}^{\nu-1} K_{t+1}
\]
• Monetary and fiscal policy rules and solvency condition:

\[
\left( \frac{i^d_t}{\bar{r}_d} \right) = (\Pi_t)^{\phi_n} \left( \frac{Y_t}{Y^n_t} \right)^{\phi_y}
\]

\[
T_t = \phi_b \left( B^g_{t-1} - B_g \right) - reb_t
\]

\[
0 = \lim_{T \to \infty} E_t \frac{P_t}{P_T} \frac{B^g_T}{\prod_t (1 + i^d_{t-1})}
\]

• Credit spread determination:

\[
1 + \omega_t = E_t \Gamma \left( B_t, W_{t+1}, N^b_{t+1}, Z_t \right)
\]

• Market-clearing conditions:

\[
Y_t = \eta C^b_t + (1 - \eta) C^s_t + G_t
\]

\[
N_t = \eta N^b_t + (1 - \eta) N^s_t
\]

• Exogenous processes:

\[
log \left( \frac{G_t}{\bar{G}} \right) = \rho_g \left( \frac{G_{t-1}}{\bar{G}} \right) + \epsilon_t^g
\]

\[
reb_t - \bar{reb} = \rho_{reb} \left( reb_{t-1} - \bar{reb} \right) + \epsilon_t^{reb}
\]