Sectoral Shocks, the Beveridge Curve and Monetary Policy*

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Abstract

The slow recovery of the US labor market and the observed shift in the Beveridge curve has prompted speculation that sector-specific shocks are responsible for the current recession. We document a significant correlation between shifts in the US Beveridge curve in postwar data and periods of elevated sectoral shocks relying on a factor analysis of sectoral employment to derive our sectoral shock index. We provide conditions under which sector-specific shocks in a multisector model augmented with labor market search generate outward shifts in the Beveridge curve and raise the natural rate of unemployment. Consistent with empirical evidence, our model also generates cyclical movements in aggregate matching function efficiency and mismatch across sectors. We calibrate a two-sector version of our model and demonstrate that a negative shock to construction employment calibrated to match employment shares can fully account for the outward shift in the Beveridge curve. We augment our standard multisector model with financial frictions to demonstrate that financial shocks or a binding zero lower bound can act like sectoral productivity shocks, generating a shift in the Beveridge curve that may be counteracted by expansionary monetary policy.

Keywords: sectoral shocks, Beveridge curve, labor reallocation.

JEL Classification: E24

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1 Introduction

> You can't change the carpenter into a nurse easily, and you can't change the mortgage broker into a computer expert in a manufacturing plant very easily. Eventually that stuff will work itself out . . . [M]onetary policy can't retrain people. Monetary policy can't fix those problems.
>
> Charles Plosser, President of the Federal Reserve Bank of Philadelphia

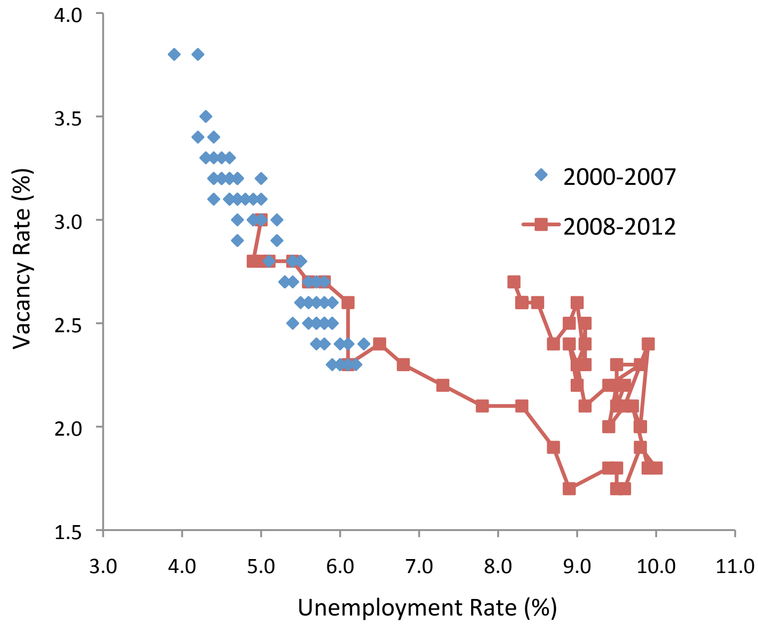
Though the Great Recession ended in the middle of 2009, the US labor market remains weak three years later with an unemployment rate near 8%. Some have speculated that a slow recovery is inevitable as the labor force must reallocate from housing-related sectors to the rest of the economy. Proponents of this view have cited the shift in the US Beveridge curve as evidence for sectoral shocks leading to labor reallocation.[1] The view that Beveridge curve shifts reflect sectoral disruptions and periods of increased labor reallocation was first elucidated by Abraham and Katz (1986) and Blanchard and Diamond (1989). Figure 1 displays unemployment and vacancies since 2000 using vacancy data from the Job Openings and Labor Turnover Survey (JOLTs). The Beveridge curve has shifted during the recovery period with the unemployment rate rising 1.5-2 percentage points at each level of vacancies.[2] Vacancy rates in 2012 are consistent with an unemployment rate of less than 6% on the pre-recession Beveridge curve. The observed shift in the Beveridge curve has prompted disagreement on what implications, if any, this shift may have for monetary policy. Kocherlakota (2010) and Plosser (2011) suggest that, if sectoral shocks require labor reallocation and that process is costly and prolonged, then the natural rate of unemployment has risen, implying that further monetary easing would be inflationary.

We investigate the relationship between sector-specific shocks, shifts in the Beveridge curve, and changes in the natural rate of unemployment. In particular, we address three questions: Has the US labor market experienced sector-specific disruptions? Can sectoral shocks account for the shift in the Beveridge curve? Do sectoral shocks raise the natural rate of unemployment? We build a measure of sector-specific shocks using a factor analysis of sectoral employment and augment a standard multisector model with labor market search to analyze the relationship between sector-specific shocks, the Beveridge curve, and the natural rate of unemployment.

[1] See Kocherlakota (2010), and Plosser (2011)

[2] See Barnichon et al. (2010) for measurement of the shift in the empirical Beveridge curve using JOLTs data. Exact size of the shift depends on the definition of the vacancy rate: job openings rate used in JOLTs is $V/(N+V)$ or alternative is vacancy to labor force ratio V/L (analogous to the unemployment rate).

Figure 1: US Beveridge curve, 2000-2012



Our first contribution is a new index of sector-specific shocks that measures the dispersion of the component of sectoral employment not explained by an aggregate employment factor. Our measure is distinct from the Lilien (1982) measure of employment dispersion and addresses the Abraham and Katz (1986) critique that asymmetric responses of sectoral employment may be attributable to differing sensitivities of sectors to aggregate shocks. We confirm that the recovery from the Great Recession is characterized by a substantial increase in sectoral shocks that matches the timing of the shift in the Beveridge curve. Moreover, we show that shifts in the US Beveridge curve in postwar data are correlated with periods in which sector-specific shocks are elevated as measured by our index.

Our second contribution is to define the Beveridge curve in a multisector model and examine its behavior in the presence of sectoral shocks. The Beveridge curve is defined as the set of unemployment and vacancy combinations traced out by changes in real marginal cost, which captures the effect of a variety of aggregate disturbances. We show that sectoral productivity or demand shocks will, in general, shift the Beveridge curve. Sectoral shocks shift the Beveridge curve through two channels: a composition effect and a mismatch effect. The former channel is operative if a sectoral shock shifts the distribution of vacancies towards a sector with greater hiring costs, thereby increasing unemployment for any given aggregate level of vacancies. The latter channel stems from decreasing returns to the matching function and costly reallocation: a sectoral shock that leaves overall vacancies unchanged raises unemployment because the reduction in vacancies in one sector increases unemployment by more than the corresponding fall in unemployment in the other sector. Our model validates our empirical strategy and verifies the hypothesized relationship between our sector-specific shock index and shifts in the Beveridge curve.

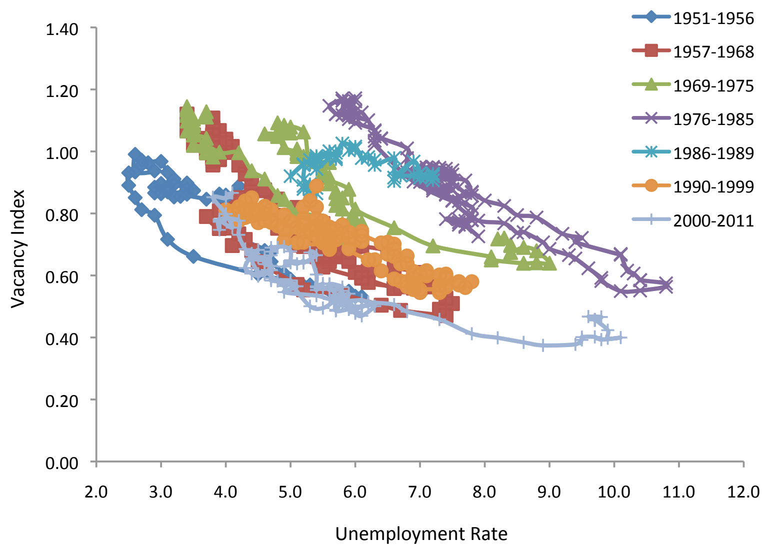
Our third contribution is to clarify the relationship between the Beveridge curve and the natural rate of unemployment. In the baseline model with exogenous sectoral productivity or demand shocks, shifts in the Beveridge curve necessarily imply a movement in the natural rate of unemployment in the same direction as the shift in the Beveridge curve. However, the converse need not hold: for example, a negative aggregate productivity shock raises the natural rate of unemployment without shifting the Beveridge curve. Changes in the natural rate affect monetary policy by changing the inflation-employment tradeoff for the central bank.

We calibrate a two-sector version of our model to data on the construction and non-construction sectors of the US labor market to quantify the effect of sectoral shocks on the Beveridge curve and the natural rate of unemployment. A sector-specific shock to construction of sufficient magnitude to match movements in construction's employment share generates a shift in the Beveridge curve that quantitatively matches the shift observed in the US. Moreover, the shock to construction raises the natural rate of unemployment by 1.4 percentage points - insufficient to fully explain the rise in unemployment observed in the current recession and of similar magnitude to the estimates in Sahin et al. (2010) who examine the contribution of mismatch to overall unemployment.

Our final contribution is an extension of the model to incorporate financial frictions. In this environment, it is no longer the case that a Beveridge curve shift implies a change in the natural rate. We show that financial shocks or systematic changes in monetary policy increase mismatch in the same way as a sector-specific productivity or demand shocks. Events like a binding zero lower bound could act like a sector-specific shock, generating a shift in the Beveridge curve while not implying any change in the natural rate of unemployment. Given our analysis, we conclude that a Beveridge curve shift is not sufficient to draw any conclusions about the behavior of the natural rate of unemployment.

Our paper is organized as follows. Section 2 describes our method for constructing a long-run sector-specific shock index and its correlation with historic shifts in the Beveridge curve. Section 3 lays out our baseline model: a sticky price multisector model augmented with labor market search within sectors and costly reallocation across sectors. Analytical results establishing the relationship between sectoral shocks, labor reallocation, and the Beveridge curve along with implications for

Figure 2: US Beveridge curve, 1950-2011



the natural rate are described in Section 4. Section 5 describes our calibration strategy and shows the effect of sectoral productivity shocks in a two-sector model. Section 6 extends the multisector model to incorporate financial frictions and illustrates how financial frictions and changes in the monetary policy rule can act as sectoral shocks and shift the Beveridge curve. Section 7 concludes.

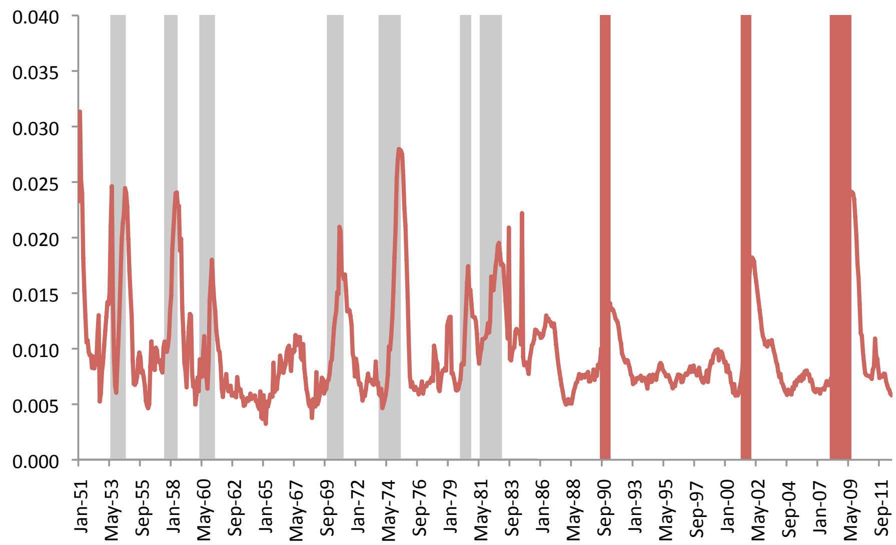
2 Empirical Evidence on Sectoral Shocks and the Beveridge Curve

To examine the relationship between sectoral shocks and the Beveridge curve, we construct the long-run US Beveridge curve and build a summary measure of sector-specific shocks. Since vacancies data from the JOLTs survey is only available after 2000, the Conference Board's Help-Wanted Index is frequently used as a proxy for the vacancy rate prior to 2000. Figure 2 displays the Beveridge curve using the Help-Wanted Index (HWI) normalized by the labor force as a proxy for the vacancy rate.[3] Figure 2 shows that the historic Beveridge curve exhibits periods when the vacancy-unemployment relationship is stable and periods when it appears to shift.

Historic shifts in the US Beveridge curve are documented in Bleakley and Fuhrer (1997) and Valletta and Kuang (2010). Importantly, shifts in the Beveridge curve are not a business cycle phenomenon with some recessions accompanied by shifts but other shifts occuring during expansions - the behavior of vacancies and unemployment in the mid 1980s provides a good example. Like

[3]After 1996, the HWI is the composite index derived in Barnichon (2010) and updated to 2011, which adjusts for the shift away from newspaper advertising of vacancies to online advertising.

Figure 3: Lilien measure of dispersion in employment growth



the Beveridge curve obtained using JOLTs data, the composite HWI Beveridge curve exhibits an upward shift since 2009.

2.1 Existing Measures of Sector-Specific Shocks

Lilien (1982) proposed the dispersion in sectoral employment growth as a measure for sector-specific shocks, arguing that these shocks are an important driver of the business cycle given the strong countercyclical behavior of his measure. Figure 3 plots the Lilien measure using monthly sectoral employment data.[4] The figure demonstrates the strongly countercyclical behavior of the series including most recent recessions that have featured a slower recovery in the labor market in comparison to past recessions. In the current recession, the Lilien measure peaks in the summer of 2009 at the recession trough.

Abraham and Katz (1986) questioned the Lilien measure by arguing that increases in the dispersion of employment growth could be attributed to differences in the elasticity of sectoral employment to aggregate shocks. As an alternative, Abraham and Katz argued that sector-specific shocks should result in periods in which vacancies and unemployment are both rising and showed that the Lilien measure does not comove positively with vacancies.

[4]The Lilien measure is: $\sigma_t = \left(\sum_{i=1}^{K} (g_{it} - g_t)^2\right)^{1/2}$ where g_{it} is the growth rate of employment in sector i and g_t is the growth rate of aggregate employment.

2.2 Constructing Sector-Specific Shock Index

To derive a measure of sector-specific shocks, we conduct a factor analysis of sectoral employment. The factor analysis addresses the Abraham and Katz critique by allowing sectoral employment to respond differently to aggregate shocks.

We estimate the following approximate factor model:

$$n_t = \epsilon_t + \lambda F_t,$$

where n_t is a $N \times 1$ vector of employment by sector, ϵ_t is a $N \times 1$ vector of mean-zero sector-specific shocks, F_t is a $K \times 1$ vector of factors, and λ is a $N \times K$ matrix of factor loadings.

As is standard in the approximate factor model discussed in Stock and Watson (2002), we assume that n_t and F_t are covariance stationary processes, with $Cov\left(F_t,\, \epsilon_t\right) = 0$. As shown by Stock and Watson (2002), the approximate factor model allows for serial correlation in F_t, ϵ_t, and weak cross-sectional correlation in ϵ_t - the variance-covariance matrix of ϵ_t need not be diagonal. The factor analysis implicitly identifies the sector-specific shock by assuming that loadings on the aggregate factor are invariant over time; that is, sectoral employment responds in a similar manner over the business cycle to aggregate fluctuations.

The sectoral residual ϵ_{it} represents the sector-specific shock, and we construct an index to examine the time variation in sector-specific shocks by measuring cross-sectional dispersion, squaring the sectoral employment residuals from our factor analysis:

$$S_t^{dis} = \frac{1}{K}\left(\sum_{i=1}^{K} \epsilon_{it}^2\right)^{1/2}.$$

Given that variances are normalized to unity before estimating, the sector specific shocks need not be weighted by their employment shares. We also construct an alternative measure of employment dispersion as the sum of the absolute values of the residuals from our factor analysis:

$$S_t^{abs} = \frac{1}{K}\sum_{i=1}^{K} \left|\epsilon_{it}\right|.$$

This measure of sector-specific shocks is always positive and weights all sectors equally.

2.3 Data

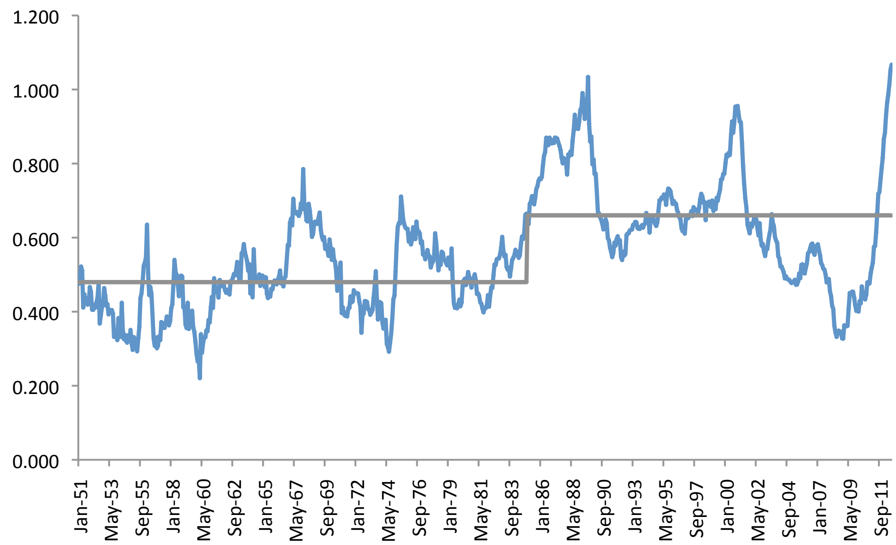
To estimate the sectoral shock index, we use long-run US data on sectoral employment. These data are available for the US from January 1950 to July 2012 on a monthly basis for 14 sectors that represent the first level of disaggregation for US employment data. Due to its relatively small share of employment, we drop the mining and natural resources sector. The sectoral data is taken from the Bureau of Labor Statistics establishment survey. While, in principle, we could use sectoral data on variables like real output, relative prices, or relative wages, employment data offers the longest available history at the highest frequency and is presumably measured with the least error. The principal concern with this data set is the small number of cross-sectional observations relative to the number of observations in the time dimension. While traditional factor analyses draw on highly disaggregated price, output, or employment data, these series are not available before the 1970s. Given our aim of investigating shifts in the Beveridge curve and the relative infrequency of these events, we try to construct the longest possible series for sector-specific shocks.

The log of monthly sectoral employment is detrended to obtain a mean-zero stationary series and the variance of each series is normalized to unity. This normalization ensures that no series has a disproportionate effect on the estimation of the national factor.

We detrend employment in each sector by means of a cubic deterministic trend. The underlying trend in sectoral employment differs substantially among sectors, and employment shares are nonstationary over the postwar period. For example, manufacturing employment falls as a share of total employment over the whole period, but even decreases in absolute terms starting in the 1980s. Sectors, such as construction and information services show a general upward trend in levels characterized by very large and long swings in employment that are longer than simple business cycle variation. Higher-order deterministic trends fit certain sectors much better than a simple linear or quadratic trend. Moreover, most of the sectoral employment series obtained by removing a linear or quadratic trend fail a Dickey-Fuller test at standard confidence levels. For robustness, as will been shown in the next section, we also consider detrending by first-differences, computing quarter-over-quarter or year-over-year growth rates for each sector, normalizing variances, and then estimating the factor model.

Given that our full sample from 1950-2012 has a small number of cross-section observations relative to the time dimension, we also estimate the same model using a larger cross-section of 85 sectoral employment series at the 2-digit NAICS level available monthly since 1990. We find the

Figure 4: Sector-specific shock index



same pattern for the shock index as in our larger sample.

2.4 Sectoral Shock Index and Shifts in the Beveridge Curve

The sector-specific shock index shown in Figure 4 displays several notable features. First, the shock index rises rapidly in late 2009. The rise in the shock index occurs at the beginning of the recovery, not at the beginning of the recession, matching the timing of the shift in the Beveridge curve.[5] Second, the sector-specific shock index is not a business cycle measure. Its correlation with various monthly measures of the business cycle is highlighted in Table 1, with all correlations below 0.15. Third, the sectoral shock index displays a low and negative correlation with the Lilien measure.[6] Finally, the average level of the shock index is higher in the Great Moderation period as shown by the gray line in Figure 4. This behavior is consistent with the behavior of sectoral employment documented in Garin, Pries and Sims (2010).

Just as the current shift in the Beveridge curve coincides with a rise in the sector-specific shock index, historic shifts in the Beveridge curve are also correlated with elevated levels of sector-specific shocks. We illustrate this correlation between shifts in the Beveridge curve and the sector-specific

[5]The rise in the index in the recovery period after the Great Recession is also consistent with the elevated dispersion in labor market conditions highlighted by Barnichon and Figura (2011) and sectoral dispersion measures computed by Rissman (2009).

[6]For the index obtained using growth rates, the correlation with business cycle measures and the Lilien measure is markedly higher than the time trend specifications. This correlation is driven by the behavior of the index in the first half of the sample. The correlation of the sectoral shock index with the Lilien measure drops to 0.18 from 0.56 in the Great Moderation period.

Table 1: Correlation of shock index with business cycle measures

	Business Cycle Measures: Industrial production growth	Employment growth	Unemployment rate	Lilien measure	Beveridge Curve
Detrend with time trend					
Cubic	**-0.020**	**0.123**	**0.000**	**-0.181**	**0.363**
Quartic	-0.049	0.075	0.009	-0.058	0.179
Detrend with growth rates					
Quarter over quarter	-0.175	-0.119	-0.068	0.284	-0.063
Year over year	-0.319	-0.355	0.092	0.569	-0.088

shock index by plotting the shock index against the intercept of a 5-year rolling regression of vacancies on unemployment (five-year trailing window). Absent any shifts in the Beveridge curve, the intercept should be constant. Therefore, variation in the intercept series captures movements in the Beveridge curve. Figure 5 shows a clear correlation between movements in the intercept of the Beveridge curve and the sector-specific shock index. This correlation in monthly data calculated from 1956-2012 is 0.363 and is shown in the last column in Table 1. This result is robust to the use of a 4th order trend, though somewhat weaker. Our evidence provides support for the mechanism described by Abraham and Katz where sector-specific shocks generate a shift the Beveridge curve.

To examine the robustness of this correlation, we also estimate the Beveridge curve augmented with our sector-specific shock index:

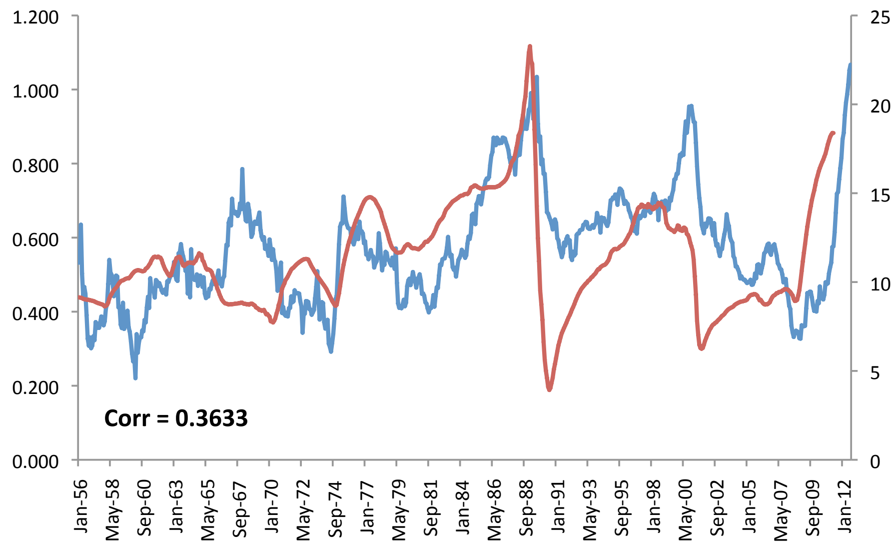
$$v_t = c + \beta(L) u_t + \gamma(L) S_t + \eta_t$$

where v_t is log vacancies, u_t is log unemployment, $\beta(L)$ and $\gamma(L)$ are lag polynomials, c is a constant, and η_t is a mean zero error term.[7] The Beveridge curve is estimated with four lags of unemployment to control for the persistence of both vacancies and unemployment and with Newey-West standard errors (4 lags) to account for serial correlation in η_t. We consider several variants of our sector-specific shock index using both the dispersion measure (Panel A) and the absolute-value measure (Panel B). Employment is detrended with either time trends and growth rate trends. Given the persistence exhibited by the sector-specific shock indices obtained from time detrending, we estimate specifications both with and without an additional lag of the shock index.

Table 2 displays the estimates for the coefficient γ on the sector-specific shock index. This

[7] An earlier version of this paper estimates the Beveridge curve using vacancies and unemployment rates in levels. Given the nonlinear nature of the Beveridge curve, the log specification is preferred. However, the use of log or levels does not greatly affect the estimation.

Figure 5: Correlation of Beveridge curve shifts and sector-specific shocks



coefficient enters significantly for most of the time trend specifications we consider. Our baseline cubic detrending is highlighted in bold in the table with positive and statistically significant coefficients in all cases. The shock index based on growth rate detrending delivers a significant negative coefficient in the case of the year-over-year specification. While our reduced form model makes no prediction about the sign of the coefficient γ, we show in section 4.2 that our model-implied measure of Beveridge curve shifts delivers coefficients that are consistent in sign across all specifications. We defer further discussion until then.

Table 2: Effect of shock index on Beveridge curve intercept

Panel A: Dispersion Index	Coeff	Std Error	T-stat	Panel B: Absolute Value Index	Coeff	Std Error	T-stat
Detrend with time trend				Detrend with time trend			
Cubic**	**0.816**	**0.309**	**2.64**	**Cubic****	**0.328**	**0.098**	**3.35**
Cubic w/1 lag**	**0.804**	**0.312**	**2.58**	**Cubic w/1 lag****	**0.327**	**0.099**	**3.31**
Quartic**	0.635	0.300	2.11	Quartic	0.209	0.109	1.92
Quartic w/1 lag**	0.624	0.302	2.07	Quartic w/1 lag	0.207	0.110	1.89
Detrend with growth rates				Detrend with growth rates			
Quarter over quarter	-0.084	0.184	-0.45	Quarter over quarter	-0.089	0.072	-1.24
Year over year**	-1.360	0.326	-4.18	Year over year**	-0.530	0.111	-4.76

T = 726
** Indicates significance at 5% level

3 Multisector Model with Labor Reallocation

Our model incorporates labor market search into a sticky-price multisector model, similar to Aoki (2001) or Carvalho and Lee (2011). Each sector hires from a labor force specific to that sector, where sectors may, in principle, conform to geographies, industries, occupations or other dimensions of worker heterogeneity. Households reallocate their workers across sectors subject to a utility cost of changing the distribution of the labor force.

3.1 Retailers and Wholesale Firms

The consumption goods are sold by a set of monopolistically competitive retailers who can costlessly differentiate the single final good assembled by wholesale firms. These retailers periodically set prices a la Calvo at a markup to marginal cost, which is the real cost of the final good P_{ft}/P_t. The retailers problem is standard to any New Keynesian model and discussed at length in Benigno and Woodford (2005). We relegate the statement of the retailers price-setting problem to the Appendix and simply specify the nonlinear equilibrium conditions:

$$1 = \chi \Pi_t^{\zeta-1} + (1-\chi)\left(\frac{H_t}{T_t}\right)^{\zeta-1} \tag{1}$$

$$H_t = \frac{\zeta}{\zeta-1}\left(\frac{P_{ft}}{P_t}\right) u_c\left(C_t, N_t\right) Y_t + \chi\beta E_t \Pi_{t+1}^{\zeta} H_{t+1} \tag{2}$$

$$T_t = u_c\left(C_t, N_t\right) Y_t + \chi\beta E_t \Pi_{t+1}^{\zeta-1} T_{t+1} \tag{3}$$

where Π_t is the gross inflation rate, T_t and H_t are state variables summarizing the cost and benefit of resetting the firm's price, Y_t is output, C_t is consumption, N_t is employment and P_{ft}/P_t is the real marginal cost of the final good. The parameter χ is the Calvo parameter governing the degree of price stickiness, $\zeta > 1$ is the elasticity of substitution among the differentiated goods produced by retailers, and β is the household's rate of time preference. In a zero inflation steady state, a log-linearization of these equilibrium conditions delivers the standard New Keynesian Phillips curve.

The final good purchased by retailers is sold by wholesale firms who purchase an intermediate output good produced by firms in each sector. We assume a finite set of sectors that produce an intermediate good that is transformed into the final good using a constant elasticity of substitution aggregator.

$$\Pi_t^f = \max_{Y_{it}} \frac{P_{ft}}{P_t} Y_t - \sum_{i=1}^{K} \frac{P_{it}}{P_t} Y_{it} \tag{4}$$

$$\text{subject to } Y_t = \left(\sum_{i=1}^{K} \phi_{it}^{\frac{1}{\eta}} Y_{it}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \tag{5}$$

where ϕ_{it} represents a relative preference shock (or relative demand shock) and η is the elasticity of substitution among intermediate goods. Optimization by final good firms provides demand functions for each intermediate good and an aggregate price index for the final good:

$$Y_{it} = \phi_{it} Y_t \left(\frac{P_{it}}{P_{ft}} \right)^{-\eta} \; \forall i \epsilon \{1, \ldots, K\} \tag{6}$$

$$\frac{P_{ft}}{P_t} = \left\{ \sum_{i=1}^{K} \phi_{it} \left(\frac{P_{it}}{P_t} \right)^{1-\eta} \right\}^{\frac{1}{1-\eta}} \tag{7}$$

For $\eta = 1$, the CES aggregator is Cobb-Douglas and intermediate goods are neither complements nor substitutes. If $\eta < 1$, intermediate goods are complements, while if $\eta > 1$, intermediate goods are substitutes.

3.2 Intermediate Good Firms and Hiring

Intermediate goods are produced by competitive firms in each sector who hire labor and post vacancies subject to a linear production function and a law of motion for firm employment. Firms in each sector face sectoral productivity shocks with wages, separation rates, and a job-filling rate that may be unique to the sector. The firm's intertemporal problem is given below:

$$\Lambda_{it} = \max_{V_{it}} E_t \sum_{T=0}^{\infty} Q_{t,T} \left(\left(\frac{P_{iT}}{P_t} \right) Y_{iT} - W_{iT} N_{iT} - \kappa V_{iT} \right) \tag{8}$$

$$\text{subject to } N_{it} = (1 - \delta_i) N_{it-1} + q_{it} V_{it} \tag{9}$$

$$Y_{it} = A_{it} N_{it} \tag{10}$$

where q_{it} is the vacancy yield or job-filling rate. The firm's vacancy posting condition is given below:

$$\frac{P_{it}}{P_t} A_{it} = W_{it} + \frac{\kappa}{q_{it}} - E_t Q_{t,t+1} (1 - \delta_i) \frac{\kappa}{q_{it+1}} \tag{11}$$

where $Q_{t,t+1}$ is the stochastic discount factor of the representative household between period t and $t+1$. The vacancy posting condition equalizes the marginal product of labor on the left-hand side of (11) and the marginal cost of labor on the right-hand side, which is wages inclusive of hiring costs. This vacancy posting condition is identical to the vacancy posting condition in a standard Diamond-Mortensen-Pissarides model when $K = 1$.

Hiring is mediated by a sectoral matching function that depends on the level of vacancies and unemployment in each sector. We allow sectoral matching functions to differ in matching function productivity, but require the matching function to display constant returns to scale and share a common matching function elasticity α:

$$q_{it} \equiv \frac{H_{it}}{V_{it}} = \varphi_i \left(\frac{V_{it}}{U_{it}}\right)^{-\alpha} \tag{12}$$

3.3 Households

Households supply labor across K distinct sectors and invest in a full-set of state-contingent securities. While hiring in each sector is subject to search frictions, the household is free to reallocate workers across sectors subject to a utility cost of changing the distribution of labor. This utility cost captures costs associated with worker retraining, relocation, or the loss of industry-specific skills. As a result, both the initial distribution of the labor force and initial distribution of employment are state variables for the household.

With costly reallocation, the household's problem differs from the standard labor market search model since the household has an active margin of adjustment by reallocating the pool of available workers across sectors. Additionally, the household's surplus for an additional worker in each sector determines the Nash-bargained wage:

$$\begin{aligned}
V\left(\mathbf{N_{t-1}},\, \mathbf{L_{t-1}}\right) &= \max\nolimits_{\mathbf{L_t}} u\left(C_t, N_t\right) - \sum_{i=1}^{K-1} R\left(L_{it-1}, L_{it}\right) && (13)\\
&\quad + \beta E_t V\left(\mathbf{N_t}, \mathbf{L_t}\right) && (14)\\
\text{subject to } C_t &= \sum_{i=1}^{K}\left(W_{it}N_{it} + \Pi_{it}\right) + B_t - E_t Q_{t,t+1} B_{t+1} && (15)\\
N_{it} &= \left(1-\delta_i\right) N_{it-1} + p_{it} U_{it} && (16)\\
L_{it} &= N_{it-1} + U_{it} && (17)\\
1 &= \sum_{i=1}^{K} L_{it} && (18)
\end{aligned}$$

where $\mathbf{N_t}$ and $\mathbf{L_t}$ are $K \times 1$ vectors of sectoral employment and sectoral distribution of the labor force respectively. The household maximizes utility net of reallocation costs subject to a standard budget constrant (equation (15)) where Π_{it} represents firm profits distributed to households and B_t are payments from state contingent securities. For each sector, sectoral employment N_{it} evolves by a law of motion (equation (16)) where p_{it} is the job-finding rate in sector i. Sectoral unemployment is the difference between the labor force allocated in that sector L_{it} and last period sectoral employment (equation (17)). The total labor force of the household is normalized to unity.

The household takes the job-finding rate in each sector and profits from intermediate goods firms as exogenous. We make minimal assumptions on the cost function for labor reallocation. The cost function $R(\cdot,\cdot)$ is assumed to be continuous and differentiable in its arguments and minimized when $L_{it-1} = L_{it}$ for any sector i.

To determine the sectoral wage and the optimality condition for the distribution of the labor force, we define the household surplus J_{it} from an additional worker employed in sector i. This value is given by the derivative of the value function with respect to N_{it-1}:[8]

$$\begin{aligned} J_{it} &= W_{it} - U_{nt} + E_t Q_{t,t+1}\left(1 - \delta_i - p_{it+1}\right) J_{it+1} \\ \text{with } U_{nt} &= -\frac{u_n\left(C_t, N_t\right)}{u_c\left(C_t, N_t\right)} \\ \text{and } Q_{t,t+1} &= \beta \frac{u_c\left(C_{t+1}, N_{t+1}\right)}{u_c\left(C_t, N_t\right)} \end{aligned}$$

where $N_t = \sum_{i=1}^{K} N_{it}$.

The household's intertemporal consumption choice delivers a standard Euler equation that determines the pricing of a risk-free bond:

$$1 = E_t Q_{t,t+1}(1 + i_t^d)/\Pi_{t+1}$$

The allocation of the labor force relates differences in the household surplus across sectors to the expected path of the labor force in each sector:

$$p_{it}J_{it} - p_{Kt}J_{Kt} = \frac{1}{u_c\left(C_t, N_t\right)} E_t\left(R_2\left(L_{it-1}, L_{it}\right) - \beta R_1\left(L_{it}, L_{it+1}\right)\right) \text{ for } \forall i \epsilon \{1, \ldots K-1\} \qquad (19)$$

In this optimality condition, the household surplus in any given sector i (weighted by the probability

[8] We use the variable $J_{it} = \frac{\partial V}{\partial N_i}$ instead of V_{it} to avoid confusion with vacancies.

of finding a job in that sector) may differ from its counterpart in sector K (the numeraire sector) by the costs of reallocation (expressed in units of consumption). That is, the household will tolerate differences in expected surpluses depending on the marginal cost of adjusting the labor force.

Two extreme cases for labor reallocation will prove useful in our analysis and are defined here. If the cost of reallocation is zero, the the right-hand side of equation (19) is always zero. Alternatively, if reallocation costs are infinitely large, the labor force is effectively fixed and equation (19) becomes redundant. For future reference, we define the costless reallocation and no reallocation cases here:

Definition 1. *Let $R\left(L_{it-1},\ L_{it}\right) = 0$ for all combinations of L_{it-1} and L_{it}. Then, labor reallocation is costless.*

Importantly, the case of costless reallocation corresponds to the model steady state where the labor force distribution has settled.

Definition 2. *Let $R\left(L_{it-1},\ L_{it}\right) \rightarrow \infty$ for any $L_{it-1} \neq L_{it}$. Then, there is no labor reallocation.*

Also, in later sections, we will refer to the case of no wealth effects on labor supply and define that case here.

Definition 3. *Let $U_{nt} = f\left(N_t\right)$ for some function f. That is, the marginal rate of substitution does not depend on consumption C_t. Then, labor supply does not exhibit wealth effects.*

The standard Diamond-Mortensen-Pissarides model assumes neither wealth effects nor any variable disutility of labor supply. This conforms to the case of $f(N) = z$ for some constant reservation wage z.

3.4 Wages and Labor Market Equilibrium

The job-finding probability is taken as exogenous by the household and is determined in equilbrium by the sectoral matching function and the level of vacancies and unemployed persons in each sector:

$$p_{it} \equiv \frac{H_{it}}{U_{it}} = \varphi_i \left(\frac{V_{it}}{U_{it}}\right)^{1-\alpha} \tag{20}$$

Wages are determined via Nash bargaining in each sector. The firm's surplus is equal to the cost of hiring a new worker, which is the cost of posting a vacancy scaled by the probability of filling the vacancy:

$$J_{it}^f = \frac{\kappa}{q_{it}}$$

where J_{it}^{f} is the surplus of intermediate goods firms in sector i.[9] Nash-bargaining implies that the sectoral wage satisfies the following condition equating the household's surplus and the firm's surplus:

$$\nu J_{it}^{f} = (1-\nu) J_{it}$$

Substituting into the dynamic equation for the household surplus, we can express the wage in terms of the job-filling rate and job-finding rates in each sector:

$$W_{it} = U_{nt} + \frac{\nu}{1-\nu}\kappa\left(\frac{1}{q_{it}} - E_t Q_{t,t+1}\left(1-\delta_i - p_{it+1}\right)\frac{1}{q_{it+1}}\right) \tag{21}$$

While the optimality condition for worker reallocation (equation (19)) may appear cumbersome, the costless reallocation limit is instructive. When reallocation is costless or in the nonstochastic steady state, the right hand side of the reallocation condition is zero and household surpluses are equalized for all sectors. In particular, this condition implies the Jackman-Roper condition that labor market tightness must be equalized across sectors.[10]

Proposition 1. *Let $R\left(L_{it-1}, L_{it}\right) = 0$ for all L_{it-1} and L_{it} or $L_{it-1} = L_{it}$. Then, for any sectors i and j, $\theta_{it} = \theta_{jt}$ where $\theta_{it} = V_{it}/U_{it}$.*

Proof. Observe that for any two sectors, household optimality and Nash-bargaining imply:

$$\begin{aligned} p_{it} J_{it} &= p_{jt} J_{jt} \\ \Rightarrow \kappa\frac{\nu}{1-\nu}\frac{p_{it}}{q_{it}} &= \kappa\frac{\nu}{1-\nu}\frac{p_{jt}}{q_{jt}} \\ \Rightarrow \frac{V_{it}}{U_{it}} &= \frac{V_{jt}}{U_{jt}} \end{aligned}$$

where the first equality follows from the relation of firm surplus and household surplus from Nash-bargaining and the second equality follows from the definition of p_{it} and q_{it}. □

This result requires bargaining power and flow vacancy costs to be equalized across sectors but places no restriction on the parameters of the matching function or separation rates. In contrast

[9] Formally, $J_{it}^{f} = \frac{\partial \Lambda_i}{\partial N_i}$ holding V_{it} constant. Intuitively, it the value to the firm of a hired worker once vacancy costs are sunk, which is the relevant quantity for determining the firm's value from a match.

[10] The condition that labor market tightness be equalized across sectors was posulated in Jackman and Roper (1987) as a benchmark for measuring the degree of structural unemployment.

to the environment considered by Jackman and Roper (1987), our results show that this condition continues to hold in a fully dynamic setting and allowing for greater heterogeneity in hiring costs across sectors. More generally, if bargaining power or vacancy posting costs differ across sectors, a generalized Jackman-Roper condition will obtain where sectoral tightness will be equalized up to a wedge term reflecting differences in bargaining power and vacancy costs. This condition is analogous to the generalized Jackman-Roper condition derived in Sahin et al. (2010).

When reallocation is costly, the probability-weighted household surplus will generally fail to be equalized across sectors and the household will have an incentive to transfer workers to sectors with a higher surplus or a greater job-finding rate. In the no reallocation limit with a fixed labor force distribution, tightness across sectors will generically depart from the Jackman-Roper condition.

3.5 Shocks

Our model features both aggregate and sector-specific Markov shocks. We consider two types of sector-specific shocks: sectoral productivity shocks A_{it} and sectoral preferences (or demand) shocks ϕ_{it}. Fluctuations in government purchases G_t provide an aggregate demand shock, though, as we will show, other types of demand shocks like preference shocks or monetary shocks that impact the household's Euler equation or the monetary policy rule could be considered without affecting the conclusions of our model.

Since our model features a finite number of sectors, it is necessary to account for the aggregate component of variation in A_{it} and ϕ_{it}. In the absence of productivity shocks and assuming a uniform level of productivity (i.e. $A_{it} = A_{ht} = A_t$ for $\forall i, h$), the only sector-specific shock is the product share ϕ_{it} in the CES aggregator. Naturally, a sector specific shock is any change in the distribution of ϕ_{it} subject to the restriction that $\sum_{i=1}^{K} \phi_{it} = 1$. However, given that sectors have nonzero mass, an increase in sectoral productivity will have aggregate effects if not offset by declines in sectoral productivity elsewhere. Moreover, the size of the offsetting shock depends on the degree of substitutability for goods across sectors. For example, if goods are perfect complements and productivity is initially equalized across sectors, a negative shock to one sector shifts in the production possibilities frontier of the economy even if offset by a corresponding positive shock to the other sector. We address this issue by defining aggregate productivity and sectoral shocks as follows:

Definition 4. *Aggregate total factor productivity is* $A_t \equiv \left\{\sum_{i=1}^{K} \phi_{it} A_{it}^{\eta-1}\right\}^{\frac{1}{\eta-1}}$. *Then, a sector specific productivity or preference shock is a linear combination of shocks* $\{A_{it}, \phi_{it}\}_{i=1}^{K}$ *such that*

$1 = \sum_{i=1}^{K} \tilde{\phi}_{it}$ *where* $\tilde{\phi}_{it} = \phi_{it} \left(A_{it}/A_t \right)^{\eta - 1}$.

This definition of aggregate productivity and sector-specific shocks is motivated by a simple decomposition of the CES aggregator where output can be expressed in terms of aggregate productivity, aggregate employment, and a misallocation term that reflects the output costs of deviations from an optimal distribution of employment. As shown in the proof for Proposition 6, the misallocation term is minimized when the distribution of labor mirrors the distribution of shocks and, hence, output is maximized for a given level of labor and aggregate productivity.

$$
\begin{aligned}
Y &= AN \left\{ \sum_{i=1}^{K} \phi_i^{\frac{1}{\eta}} \left(\frac{A_i N_i}{AN} \right)^{\frac{\eta-1}{\eta}} \right\}^{\frac{\eta}{\eta-1}} \\
&= AN \left\{ \sum_{i=1}^{K} \tilde{\phi}_i^{\frac{1}{\eta}} \left(\frac{N_i}{N} \right)^{\frac{\eta-1}{\eta}} \right\}^{\frac{\eta}{\eta-1}} \\
&\leq AN
\end{aligned}
$$

where the last inequality follows from the fact that both $\tilde{\phi}_i$ and N_i/N must sum to unity. When the distribution of productivity is uniform, a sector-specific preference shock satisfies the typical CES condition that product shares sum to one. Moreover, our definition ensures that an aggregate productivity shock leaves the pairwise ratios of sectoral productivities unchanged.

3.6 Equilibrium

We assume that monetary policy follows a simple Taylor rule. Market-clearing in the asset market implies a standard resource constraint augmented by the real costs of posting vacancies for all sectors:

$$
\begin{aligned}
log\left(\frac{1 + i_t^d}{1 + \overline{\overline{i}}_d} \right) &= \phi_\pi log(\Pi_t) + \phi_y log\left(\frac{Y_t}{\overline{\overline{Y}}} \right) && (22) \\
Y_t &= C_t + \sum_{i=1}^{K} \kappa V_{it} + G_t && (23)
\end{aligned}
$$

We define a competitive equilibrium for the economy with costly reallocation:

Definition 5. *A competitive equilibrium for the economy with costly reallocation is a set of aggregate allocations* $\{Y_t,\, N_t,\, C_t,\, H_t,\, T_t\}$*, sectoral allocations* $\{Y_{it},\, N_{it},\, U_{it},\, V_{it},\, L_{it}\}_{i=1}^{K}$*, a set of sectoral prices for* $\{W_{it},\, P_{it}/P_t\}_{i=1}^{K}$ *and aggregate prices* $\left\{P_{ft}/P_t,\, i_t^d,\, \Pi_t\right\}$*, a set of job-finding and job-*

filling rates $\{p_{it},\, q_{it}\}_{i=1}^{K}$*, and initial values of sectoral employment, unemployment, and the labor force* $\{N_{i,-1},\, U_{i,-1},\, L_{i,-1}\}_{i=1}^{K}$ *that jointly satisfy:*

1. *Retailers' price-setting conditions and inflation dynamics (1) - (3),*
2. *Final good firm production and demand functions (5) - (6) for* K *sectors,*
3. *Intermediate goods production and vacancy posting conditions (10) - (11) for* K *sectors,*
4. *Job-filling and job-finding rates (12) and (20) for* K *sectors,*
5. *Employment flows (16) for* K *sectors,*
6. *Unemployment identities (17) - (18),*
7. *Jackman-Roper conditions (19) for* $K-1$ *sectors,*
8. *Wage equation (21) for* K *sectors,*
9. *Monetary policy rule (22),*
10. *Goods-market clearing (23),*

subject to an exogenous government purchases shock: G_t *and exogenous sectoral shocks:* $\{A_{it},\, \phi_{it}\}_{i=1}^{K}$.

In total, the economy is characterized by $9K+6$ endogenous variables with $9K+6$ equilibrium conditions and $2K+1$ exogenous shocks. The aggregate productivity shock is derived from the sectoral shocks using Definition 4.

4 Sectoral Shocks, the Beveridge Curve and the Natural Rate of Unemployment in Theory

In this section, we characterize the Beveridge curve in a multisector model and provide analytical results relating sectoral shocks, the Beveridge curve, and the natural rate of unemployment.

4.1 Defining the Beveridge Curve

For the US, labor market flows are large and vacancies and unemployment quickly converge to their flow steady state. To derive the Beveridge curve, we treat the sectoral equations determining vacancies, unemployment and employment as steady state conditions. In particular, in the analysis that follows, equations (11), (16) - (18) and (21) are assumed to be at their flow steady state.[11]

In the standard one-sector model (i.e. $K = 1$), the Beveridge curve is a single equation defining the relationship between unemployment and vacancies and given by the steady state of the employment flow equation (16):

$$\delta(1 - U) = \varphi U^{\alpha} V^{1-\alpha}$$

Only changes in the separation rate δ and matching function productivity φ shift the Beveridge curve, while other shocks like aggregate productivity shocks simply move unemployment and vacancies along the pair of points defined by this equation. This relation also explains why the one-sector Beveridge curve is the same irrespective of real or demand-driven business cycles.

In a multi-sector model, an analytical relationship between U and V does not exist, and the aggregate steady state Beveridge curve is an equilibrium object. It is useful to construct the multisector analog of the one-sector steady state employment flow equation. Summing over sectoral employment in equation (16), we obtain a single equation relating sectoral vacancies and sectoral unemployment:

$$\begin{aligned} L - U &= \sum_{i=1}^{K} \frac{\varphi_i}{\delta_i} U_i^{\alpha} V_i^{1-\alpha} \\ \Rightarrow \frac{L - U}{V} \theta^{\alpha} &= \sum_{i=1}^{K} \frac{\varphi_i}{\delta_i} \left(\frac{\theta_i}{\theta}\right)^{-\alpha} \frac{V_i}{V} \end{aligned}$$

where $\theta = V/U$ is aggregate labor market tightness and $\theta_i = V_i/U_i$ is sectoral labor market tightness. The left-hand side is an expression solely in terms of aggregate unemployment and vacancies but the right-hand side will generally depend on both the type of aggregate shocks and the distribution of sectoral shocks. This term is the source of shifts in the Beveridge curve.

In a solution to our model, aggregate vacancies and unemployment are a function of the exogenous shocks: government purchases, aggregate productivity and the full set of sectoral productivities

[11] Impulse responses for the multisector model calibrated to monthly data show that unemployment and vacancies converge to the log-linearized Beveridge curve within 3 months. The rapid convergence of the labor market to the steady state Beveridge curve explain the high correlation of vacancies and unemployment in the calibration exercise in Shimer (2005).

A_{it} and preferences ϕ_{it}:

$$
\begin{aligned}
U &= U\left(G_t,\ A_t,\ A_{it},\ \phi_{it}\right) \\
V &= V\left(G_t,\ A_t,\ A_{it},\ \phi_{it}\right)
\end{aligned}
$$

The full set of equations that determine unemployment and vacancies are listed at the beginning of Appendix A. We use variations in G_t as the variable that traces out the Beveridge curve and drop time subscripts:

Definition 6. *The Beveridge curve is a function $f(\cdot)$ given by $V(G;\ A,\ A_i,\ \phi_i) = f(U(G;\ A,\ A_i,\ \phi_i))$ where G is the parameter varying U and V, holding constant aggregate productivity, sectoral productivity and preferences: A, A_i and ϕ_i.*

4.1.1 Aggregate Shocks and the Beveridge Curve

To separate movements along the Beveridge curve from shifts in the Beveridge curve, it is necessary to choose a single shock as the source of business cycles. Indeed, in the absence of any other aggregate or sectoral shocks, the Beveridge curve in a multisector model never shifts. However, in the presence of several different types of aggregate and sectoral shocks, the Beveridge curve could be equally well-defined as the locus of points in the U-V space traced out by aggregate productivity shocks or shocks to any given sector.

While our definition of the Beveridge curve as the locus of points in the U-V space traced out by government purchases shocks may seem fairly restrictive, a variety of real and nominal shocks trace out the same Beveridge curve. In the absence of wealth effects on labor supply, the equations that determine aggregate vacancies and unemployment and the sectoral distribution of vacancies and unemployment can be decoupled from the remaining equations that determine other endogenous variables.

Proposition 2. *Assume no wealth effects and either costless labor reallocation or no reallocation. For any value of government spending shock G, there exists an A such that $V(G,\ 1,\ A_i,\ \phi_i) = V(1,\ A,\ A_i,\ \phi_i)$ and $U(G,\ 1,\ A_i,\ \phi_i) = U(1,\ A,\ A_i,\ \phi_i)$ holding constant $\{A_i,\ \phi_i\}_{i=1}^{K}$.*

Proof. See Appendix. □

This proposition shows that an aggregate productivity shock traces out the same Beveridge curve as a government purchases shock. Moreover, the same proposition applies to other types of demand shocks like monetary policy shocks not specified in our model. Indeed, any shock, real or nominal, that does not enter the steady state labor market equations that determine vacancies and unemployment, traces out the same Beveridge curve.

In the absence of wealth effects, holding constant sectoral productivity and preferences, aggregate vacancies and unemployment can be parameterized by real marginal cost times aggregate productivity: $\frac{P_{ft}}{P_t}A_t$. Real marginal cost, an endogenous variable, is the only link between the block of equations that determine aggregate vacancies and unemployment and the rest of the model equations. Under no wealth effects on labor supply (as in Shimer (2005) or Hagedorn and Manovskii (2008)), our multisector model effectively generalizes the behavior of the one-sector Beveridge curve under aggregate shocks.

Moreover, given the results on aggregate productivity shocks in Proposition 2, our conclusions about the relationship between sectoral shocks and shifts in the Beveridge curve continue to hold in a model without sticky prices where business cycle fluctuations are driven by real shocks instead of demand shocks.

4.1.2 Neutrality of Sector-Specific Shocks

As our derivation of the Beveridge curve suggests, sectoral shocks can shift the Beveridge curve if these shocks alter the distribution of vacancies or generates mismatch across sectors. However, as showed earlier, when labor reallocation is costless, the Jackman-Roper condition obtains and tightness is equalized across sectors. In this case, we can once again obtain an aggregate Beveridge curve that is identical to the one-sector Beveridge curve:

Proposition 3. *If labor reallocation is costless across sectors and separation rates and matching function efficiencies are the same across sectors (i.e. $\delta_i = \delta$, $\varphi_i = \varphi$), then sector-specific shocks do not shift the Beveridge curve.*

Proof. Under costless labor reallocation, the Jackman-Roper condition holds and labor market tightness across sectors is equalized: $V_{it}/U_{it} = V_{ht}/U_{ht}$ for all i, $h \epsilon \{1, \ldots, K\}$. Summing over the steady state sectoral Beveridge curves (steady state version of (16)):

$$\begin{aligned} \sum_{i=1}^{K} N_i &= \sum_{i=1}^{K} \frac{\varphi}{\delta} \theta^{-\alpha} V_i \\ \Rightarrow 1 - U &= \frac{\varphi}{\delta} \left(\frac{V}{U} \right)^{-\alpha} V \end{aligned}$$

as required. $\square$

As a result, neither aggregate nor sector-specific shocks generate a shift in the Beveridge curve, providing a useful benchmark for our analysis of the effects of sector-specific shocks when reallocation is costly.

The conditions that recover the aggregate Beveridge curve in Proposition 3 highlight the two channels through which sector-specific shocks shift the Beveridge curve: the mismatch channel and the composition channel. If sectors share identical hiring technologies and separation rates, a sector-specific shock can only shift the Beveridge curve by changing the distribution of θ_i/θ - in other words, by generating mismatch. When labor market reallocation is costly, a sector-specific shock increases tightness in one sector while decreasing tightness in the other. Because of the decreasing returns to scale of the matching function, the rise in vacancies for the sector experiencing a positive shock exceeds the fall in vacancies for the sector with a negative shock. In contrast, an aggregate shock depresses tightnesses more or less uniformly, lowering vacancies in all sectors. The composition effect is present even when labor reallocation is costless. If some sectors feature greater hiring frictions, a shock favoring those sectors will shift the distribution of vacancies toward that sector, raising overall vacancies relative to a shock that leaves the distribution unchanged. Together, these two channels account for the effect of sector-specific shocks on the Beveridge curve.

4.2 Model-Implied Measures of Sectoral Shocks and Beveridge Curve Shifts

Our multisector model provides a useful framework for assessing the validity of empirical measures that rely on the labor market to measure sector-specific disturbances. As discussed earlier, Lilien (1982) argued that sector-specific shocks could be measured by dispersion in employment growth across sectors, with Abraham and Katz (1986) countering that increases in employment growth dispersion could be generated by aggregate shocks if sectors feature asymmetric responses to aggregate shocks.

Our model verifies that the Lilien measure is a biased measure of sector-specific shocks validating the Abraham and Katz critique. To a log-linear approximation, sectoral employment can be expressed as a function of sectoral shocks and aggregate output. Below, we express sectoral employment under the polar cases of no reallocation n_{it}^{nr} and costless reallocation n_{it}^{r} respectively, assuming no wealth effects on labor supply:

$$\begin{aligned}
n_{it}^{nr} &= \underbrace{\lambda_i \left(\left(\phi_{it} - (1-\eta)\, a_{it} \right) \right)}_{\epsilon_{it}} + \lambda_i \underbrace{\left(y_t - (1-\eta)\, a_t \right)}_{F_t} \\
n_{it}^{r} &= \left(\phi_{it} - (1-\eta)\, a_{it} \right) + y_t - (1-\eta)\, a_t - \eta \left(s_i \varphi_i + (1-s_i)\, \alpha \right) \theta_t \\
\text{where } \lambda_i &= \frac{1}{1 + \eta \left(s_i \varphi_i + (1-s_i)\, \alpha \right) \frac{\overline{L}_i / \overline{U}_i}{1-\alpha}}
\end{aligned}$$

where φ_i is a macro Frisch elasticity that reflects the dependence of the Nash-bargained sectoral wages on labor market tightness and $1 - s_i$ is the steady state size of the surplus.[12] This parameter is a function of steady-state job-finding rates and vacancy-filling rates along with others parameters of the model such as the sectoral separation rate, etc. These expressions for sectoral employment are not materially changed by allowing for wealth effects or convex disutility of labor supply, which would simply add linear functions of y_t and n_t to each expression.

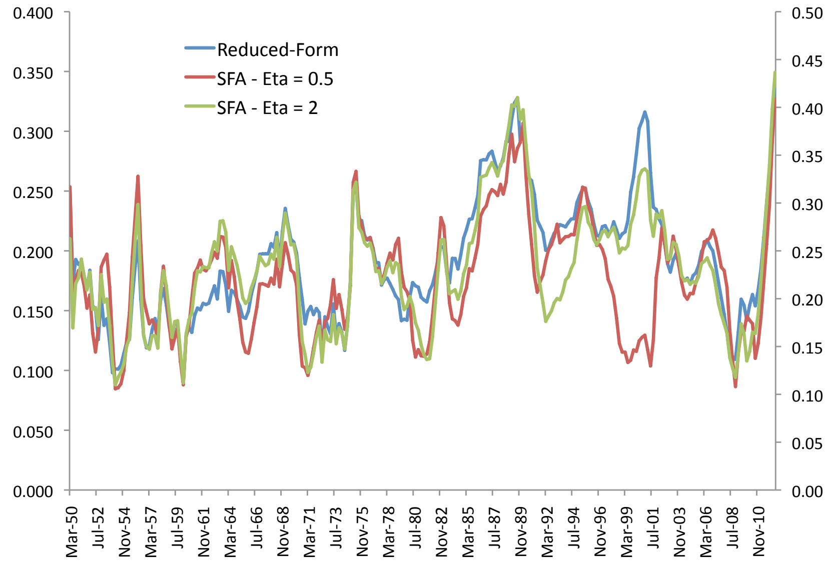
These expressions for sectoral employment show that both sector-specific shocks and aggregate shocks will increase employment dispersion in both the costless reallocation and no reallocation cases. In the case of the latter, the sensitivity of a sector to aggregate and sector-specific shocks increases with the elasticity λ_i which is larger for sectors with a lower Frisch elasticity. For example, if household's bargaining power is zero, wages are set at a constant level and $\varphi_i = 0$ for all sectors. Then sectors with a lower surplus display greater sensitivity to aggregate shocks consistent with the volatility of employment in a one-sector search model as discussed by Hall (2005) and Hagedorn and Manovskii (2008).

Since sector-specific shocks are generally correlated with output, our model shows that the assumptions underlying our factor analysis in Section 2 will generally not be satisfied. In short, simply allowing for differential elasticities to aggregate shocks is insufficient to identify sector-specific shocks. However, following the procedure in Foerster, Sarte and Watson (2011), we can conduct a structural factor analysis by using a calibrated version of the model to correct for the endogeneity problem. For simplicity, assume only sectoral productivity shocks a_{it} and assume that aggregate productivity shocks are simply a linear combination of sectoral productivity shocks. Let $a_t = (a_{1t}, \ldots, a_{Kt})'$ be the vector of sectoral productivity shocks taken as exogenous. Assume a factor decomposition of this exogenous process such that:

$$a_t = \Phi z_t + \epsilon_t$$

[12] Specifically, $s_i = \overline{W}_i / \overline{P}_i \overline{AA}_i$.

Figure 6: Sectoral shock index using structural factor analysis



where ϵ_t is a $K \times 1$ vector of sector-specific productivity shocks and z_t is a scalar defined as the aggregate productivity shock with $Cov\left(z_t,\, \epsilon_t\right) = 0$. Combining the expressions for sectoral employment and output, sectoral employment is a function of the vector of sectoral productivity shocks :

$$Mn_t = Ha_t$$

where M is a nondiagonal matrix with $1/\lambda_i - \gamma_i$ as its diagonal elements and -γ_j as its off-diagonal elements. Similarly H is a nondiagonal matrix with $\eta - 1 + \gamma_i$ as its diagonal elements and γ_j as its off diagonal elements. The coefficient $\gamma_i = \overline{\phi}_i^{1/\eta}\left(\frac{\overline{Y}_i}{\overline{Y}}\right)^{\frac{\eta-1}{\eta}}$ - the steady state share of output for each sector - enters the solution for sectoral employment since $y_t = \sum_{i=1}^{K} \gamma_i \left(a_{it} + n_{it}\right)$. Unless M is diagonal, a factor analysis of n_t will not accurately identify the sectoral shocks ϵ_t. However, for higher degrees of substitutability, the off-diagonal elements of M and H are dominated by the diagonal elements and the endogeneity correction becomes less important. In the limit, when goods are perfect substitutes, the reduced-form analysis in Section 2 is the correct procedure for identifying sector-specific shocks.

Proposition 4. *Assume the case of no labor reallocation and let $\eta \to \infty$. Then $n_t = Ha_t$ and a factor analysis of employment identifies the sector-specific shock ϵ_t.*

Proof. See Appendix, Section C. ◻

To correct for possible endogeneity in our estimates of sector-specific shocks, we calibrate our

Table 3: Correlation of reduced-form and structural sectoral shock index

Detrending and index type	$\eta = 0.5$	$\eta = 2$
Cubic Trend, absolute value index	0.586	0.912
Cubic Trend, dispersion index	0.648	0.908
Quartic Trend, absolute value index	0.812	0.945
Quartic Trend, dispersion index	0.834	0.959
Growth rates, year-over-year, absolute value index	0.921	0.952
Growth rates, year-over-year, dispersion index	0.886	0.959

model to derive the rotation matrix M, apply this rotation to sectoral employment data, and then perform a factor analysis on this rotation of the data. The calibration used to derive the matrix M is discussed in the Appendix. Our structural factor analysis follows the same procedure as in Section 2 with the exception of applying the rotation M to the data and using quarterly data instead of monthly data before removing the first principal component and computing the sector-specific shock index.[13] As shown in Figure 6, the model-implied sectoral shock index displays a strong correlation with our reduced form shock index. As hypothesized, the correlation is stronger when goods are moderate substitutes (the case of $\eta = 2$) because the off-diagonal elements of M are less important. Table 3 provides the correlation for alternative specifications of the sector-specific shock index obtained using 4th order detrending or year-over-year growth rates.

4.2.1 Sectoral Shock Index and Shifts in the Beveridge Curve

In Section 2, we correlated our sector-specific shock index with movements in the Beveridge curve intercept and showed that the index appears significant in explaining variation in vacancies controlling for the the variation explained by unemployment. Our model can also be used to think about the relationship between sector-specific shocks and movements in the Beveridge curve.

Under the assumption of no reallocation across sectors and log-linearizing around a steady state with $\overline{\theta}_i = \overline{\theta}_h$ for all i, $h \epsilon \{1, \dots K\}$, we can derive an expression for the Beveridge curve augmented with sectoral dispersion:

$$v_t = -\frac{1}{1-\alpha}\left(\alpha + \frac{\overline{U}}{\overline{N}}\right) u_t + \frac{1}{1-\alpha}\sum_{i=1}^{K}\left(\frac{\overline{U}_i}{\overline{U}} - \frac{\overline{N}_i}{\overline{N}}\right) n_{it}$$

[13]We use quarterly data instead of monthly data since, in our model, we assume the labor market is in its flow steady state.

Table 4: Regression analysis for reduced-form and model-implied index

Panel A: Dispersion Index (Full Sample)

	Coeff	Std Error	T-stat
Detrend with time trend			
Cubic	0.595	0.470	1.27
Cubic w/1 lag	0.582	0.481	1.21
Quartic	0.528	0.486	1.09
Quartic w/1 lag	0.511	0.492	1.04
Detrend with growth rates			
Quarter over quarter	-0.109	0.315	-0.35
Year over year*	-0.985	0.511	-1.93

T = 242

Panel B: Model-Implied Index (Full Sample)

	Coeff	Std Error	T-stat
Detrend with time trend			
Cubic**	-0.723	0.298	-2.43
Cubic w/1 lag**	-0.727	0.303	-2.40
Quartic	0.169	0.460	0.37
Quartic w/1 lag	0.187	0.469	0.40
Detrend with growth rates			
Quarter over quarter***	0.584	0.201	2.90
Year over year***	1.096	0.268	4.09

Panel C: Dispersion Index (After 1980)

	Coeff	Std Error	T-stat
Detrend with time trend			
Cubic***	2.244	0.564	3.98
Cubic w/1 lag***	2.193	0.581	3.77
Quartic**	1.779	0.752	2.37
Quartic w/1 lag**	1.735	0.748	2.32
Detrend with growth rates			
Quarter over quarter	0.292	0.580	0.50
Year over year	-1.010	0.834	-1.21

Panel D: Model-Implied Index (After 1980)

	Coeff	Std Error	T-stat
Detrend with time trend			
Cubic	1.380	0.883	1.56
Cubic w/1 lag	1.328	0.907	1.47
Quartic* **	**1.933**	**0.250**	**7.78**
Quartic w/1 lag* **	**1.948**	**0.253**	**7.70**
Detrend with growth rates			
Quarter over quarter* **	**1.391**	**0.333**	**4.18**
Year over year* **	**2.360**	**0.331**	**7.14**

T = 126

*** Indicates significance at 1% level

** Indicates significance at 5% level

* Indicates significance at 10% level

where α is the matching function elasticity, and the weights on sectoral employment are difference between the unemployment share and employment share in each sector. When matching function parameters are identical, these weights are all zero, and we obtain a standard log-linearized Beveridge curve relating vacancies and unemployment. Positive shocks to sectors with a higher share of employment than unemployment shift in the Beveridge curve since these sectors have lower search frictions while the opposite happens to sectors with a lower employment share then unemployment share.

Using our calibration described in the Appendix, we compute the model-based distribution of unemployment and run a regression of vacancies on unemployment and the model-based measure of shifts in the Beveridge curve. Log vacancies (measured by the HWI) and log unemployment are quarterly from 1951 to 2011. We replicate the regression in section 2 using quarterly instead of monthly data.

Our results are presented in Table 4. The top panels A and B compute the Beveridge curve estimate using the reduced form shock index from section 2 and the model-implied index respectively using the full sample. In quarterly data, the reduced-form regressions are similar to the regressions presented in Table 2 but feature higher standard errors. Panel B shows that sectoral employment detrended with time trends displays coefficients that are negative and often insignificant, inconsistent with the predictions of our model. However, in the case of growth rate detrending, coefficients are positive and significant.

Panels D shows that the negative coefficients on the specifications using detrending via time trends are driven by the early part of the sample. If we consider a sample only after 1980, the coefficients are positive, consistent with our model, and frequently greater than one as predicted by the model. Given that our model is a log-linearization around a steady state and that our calibration relies on unemployment and employment weights computing averages in the last decade, our model-implied measure is likely to be less accurate farther back in time. Given the large movements in employment share across sectors over time, our model-implied measure should fit better in more recent data. It is also worth noting that our model-implied measure delivers positive coefficient across all detrending procedures in Panel D, in contrast to the reduced-form measure considered in section 2.

4.3 Beveridge Curve and the Natural Rate of Unemployment

We define the natural rate of unemployment as the unemployment rate at which inflation is stabilized. This is a policy-relevant variable for a central bank that seeks to lower unemployment to a point at which inflation remains stable.

Definition 7. *The natural rate of unemployment is the unemployment rate when $P_{ft}/P_t = 1$.*

4.3.1 Undistorted Initial State

A useful benchmark for assessing the relationship between sector-specific shocks, Beveridge curve shifts, and the natural rate is the case of an undistorted initial state with no misallocation of output and no differences in labor market tightness across sectors. The household's marginal rate of substitution is assumed to be constant at $z < 1$. If sectors share the same separation rates δ and matching function efficiencies φ, then hiring costs are equalized, relatives prices P_i/P are equalized and determined by the inverse markup. In this setting, the model admits a symmetric solution with $Y = AN$, $\frac{P_i}{P}A_i = \mu^{-1}A$, $N_i = \tilde{\phi}_i N$ where $\tilde{\phi}_i$ is the productivity-adjusted product share defined in Section 3.5. Aggregate employment N and labor market tightness θ are implicitly defined by a common vacancy posting condition and labor market clearing:

$$\mu^{-1}A = z + \frac{\kappa}{\varphi} g(\theta) \tag{24}$$

$$N = \frac{\varphi\theta^{1-\alpha}}{\delta + \varphi\theta^{1-\alpha}} \tag{25}$$

where g is an increasing and concave function of labor market tightness θ. Total employment is simply the job-finding rate over the sum of job-finding rate and the separation rate. Moreover, the distribution of labor market variables: employment, unemployment, vacancies and the labor force all equal the productivity-adjusted product share $\tilde{\phi}_i$.

Proposition 5. *Assume costless labor reallocation and for* $\forall i \epsilon \{1, \ldots, K\}$, $\delta = \delta_i$ *and* $\varphi = \varphi_i$. *Then a sector-specific demand or productivity shock does not change the natural rate of unemployment and does not shift the Beveridge curve.*

Proof. The first result follows from the solution for the undistorted steady state and the joint determination of employment and tightness in the equations (24) and (25). Observe that sector-specific productivity and preferences shares do not enter these equilbrium conditions implying that total employment is determined independently of any sector-specific shock. The second result is an application of Proposition 3. □

With costless reallocation, a sector-specific shock results in an immediate redistribution of the labor force. Because the cost of hiring is equalized across sectors, a sector-specific shock does not shift the production possibilities frontier leaving aggregate tightness and employment unchanged. Thus, both the Beveridge curve and the natural rate after left unchanged by a sector-specific shock. While the Beveridge curve does not shift under sectoral or aggregate shocks (due to Proposition 3), the natural rate of unemployment may change under real aggregate shocks. A negative productivity shock raises the natural rate, but an increase in markups due to a negative aggregate demand disturbance will leave the natural rate of unemployment unchanged. This provides a simple instance in which changes in the natural rate do not imply a shift in the Beveridge curve.

However, the neutrality of sector-specific shocks for both the Beveridge curve and the natural rate of unemployment hinge on the assumption of costless labor reallocation.

Proposition 6. *Assume no reallocation of labor with* $\delta_i = \delta =$ *and* $\varphi_i = \varphi$ *for* $\forall i \epsilon \{1, \ldots, K\}$. *Then a sector-specific demand or productivity shock such that* $L_i \neq \tilde{\phi}_i$ *raises the natural rate of* *unemployment and shifts the Beveridge curve outward (i.e. for any level of unemployment, aggregate vacancies rise).*

Proof. See Appendix. □

In this case, shifts in the Beveridge curve and changes in the natural rate are tightly connected, with an outward shift in the Beveridge curve implying an increase in the natural rate of unemployment. Our proof relies on the properties of convex functions to show how mismatch raises the unemployment rate. Intuitively, a sector-specific shock generates mismatch since labor must be reallocated across sectors to ensure that employment shares equals the product shares. If the labor force cannot be reallocated, tightness rises in the sector where desired employment rises and falls in the other sector. This causes aggregate employment to fall since hiring costs rise faster in the sector that is positively impacted relative to the fall in costs for the sector that is negatively impacted. Similarly, due to the convexity of the matching function, vacancies in the sector with a positive shock rise more than the fall in vacancies in the sector that is negatively hit.

4.3.2 Distorted Initial State

When separation rates or matching function efficiency differ across sectors, the relationship between shifts in the Beveridge curve and changes in the natural rate are not as straightforward. Assuming that labor market reallocation is costless, the steady state of the two-sector version of the model can be summarized in three equation:

$$\mu^{-1}A = \left\{\tilde{\phi} g_A(\theta)^{1-\eta} + \left(1-\tilde{\phi}\right) g(\theta)^{1-\eta}\right\}^{\frac{1}{1-\eta}} \tag{26}$$

$$1 = N\left(1+\theta^{\alpha-1}\left(n_A\frac{\delta_A}{\varphi_A} + (1-n_A)\frac{\delta_B}{\varphi_B}\right)\right) \tag{27}$$

$$\frac{n_A}{1-n_A} = \frac{\tilde{\phi}}{1-\tilde{\phi}}\left(\frac{g_A(\theta)}{g_B(\theta)}\right)^{-\eta} \tag{28}$$

where labor market tightness θ, total employment N, and employment share n_A are the endogenous variables. The function g_i measures hiring costs (inclusive of wages) and is increasing and concave in labor market tightness. Without loss of generality, if sector A has a higher relative matching function efficiency or lower relative separation rate, then $g_A < g_B$ for $\theta > 0$.

Differences in hiring frictions across sectors imply that even in the absence of sectoral shocks, employment shares respond asymmetrically to changes in labor market tightness as can be discerned from equation (28). If sector A has lower hiring costs, it follows that $n_A > \tilde{\phi}$ since relative prices are distorted by the asymmetry in hiring costs. Effectively, sector A has higher productivity than sector B and the competitive allocations of labor are distorted toward that sector. A sector-specific shock favoring sector A lowers hiring costs and shifts out the production possibilities frontier for the economy thereby reducing the natural rate of unemployment. Moreover, this reduction in the natural rate is accompanied by a decrease in the aggregate quantity of vacancies needed to attain a particular level of employment. Since labor market tightness is equalized, shifts in the Beveridge curve due to sectoral shocks in this case stem from a composition channel. Moreover, shifts in the Beveridge curve and changes in the natural rate of unemployment move in the same direction; the Beveridge curve may shift inward or outward depending on the whether or not the sector-specific shock favors the sector with lower hiring costs. The following proposition summarizes this result:

Proposition 7. *Consider the two-sector version of the model with costless labor reallocation and zero bargaining power for households $\nu = 0$. Without loss of generality, assume that $\varphi_A > \varphi_B$ and $\delta_A = \delta_B$ or vice versa (i.e. sector A has lower hiring costs than sector B). Then, a positive sector sector-specific shock to sector A lowers the natural rate of unemployment (i.e. if $\tilde{\phi}_A < \tilde{\phi}'_A \Rightarrow N < N'$) and shifts the Beveridge curve inward.*

Proof. See Appendix. □

The assumption of zero bargaining power simply guarantees that the ratio $\frac{g_A}{g_B}$ as a function of θ is monotonic. For moderate values of θ, the ratio of hiring costs will be locally monotonic with nonzero bargaining power as confirmed in numerical experiments.

With the combination of costly reallocation and asymmetric hiring costs, the connection between the direction of Beveridge curve and the natural rate of unemployment appears to hold in our numerical examples. However, we cannot analytically rule out cases in which a sector-specific shock lowers the natural rate but shifts out the Beveridge curve or vice versa. The analysis here however suggests that this would be the exception rather than the rule.

4.4 Alternative Labor Market Measures and Sectoral Shocks

Our model also provides a framework for assessing how well alternative labor market measures capture sector-specific shocks and shifts in the Beveridge curve.

4.4.1 Aggregate Matching Function Efficiency and Mismatch

Recent papers by Sedlacek (2011) and Barnichon and Figura (2011) perform a decomposition analysis of the matching function analagous to measuring the Solow residual in a growth accounting exercise. Constructing measures of unemployment, vacancies, and hires, these authors measure aggregate matching function efficiency as the residual relating these variables

$$\varphi = \frac{H}{U^{\alpha}V^{1-\alpha}}$$

and show that aggregate matching function efficiency is procyclical. In our multisector model, aggregate matching function efficiency can be expressed in terms of mismatch and the distribution of vacancies:

$$\begin{aligned} H &= \sum_{i=1}^{K} \varphi_i U_i^{\alpha} V_i^{1-\alpha} \\ \Rightarrow \frac{H}{\bar{\varphi} U^{\alpha} V^{1-\alpha}} &= \sum_{i=1}^{K} \frac{\varphi_i}{\bar{\varphi}} \left(\frac{\theta_{it}}{\theta}\right)^{-\alpha} \frac{V_i}{V} \end{aligned}$$

where $\bar{\varphi}$ is the average level of matching function efficiency. Changes in mismatch and the distribution of vacancies will lead to variations in measured aggregate matching function efficiency. To a log-linear approximation, mismatch is a function of sectoral employment in our model:

$$\theta_{it} = \frac{1 + \frac{\overline{N}_i}{\overline{U}_i}}{1-\alpha} n_{it}^{nr}$$

Since sectoral employment is a function of both aggregate and sector-specific shocks, dispersion in mismatch will also be subject to the Abraham and Katz critique. Therefore, fluctuations in matching function efficiency are not, as such, an indicator of either sector-specific shocks or shifts in the Beveridge curve. For a suitably long time series, if the relationship between matching function efficiency and aggregate shocks is stable, then sector-specific shocks could be identified as periods where movements in matching function efficiency are not explained by the business cycle. Unlike a one-sector model with constant matching function efficiency, our multisector model with costly reallocation is consistent with the empirical observation of movements over the cycle in aggregate matching efficiency.

Similarly, work by Sahin et al. (2012) and Lazear and Spletzer (2012) construct mismatch indices by industry, region and occupation to examine whether mismatch has increased in the current recession. Like measurements of matching function efficiency, our model shows that variation in these measures over the cycle is not sufficient to identify sector-specific shocks or Beveridge curve shifts. Instead, these measures are evidence of the feature in our model that generates mismatch: costly labor reallocation. These empirical mismatch indices rely on direct measures of labor market tightness with vacancies data from either the JOLTs or from online vacancy postings collected by the Conference Board. Measures of sectoral or regional unemployment are constructed from the Current Population Survey (CPS). Data availablity limits the time series dimension of these measures, with the mismatch indices begining in either 2001 or 2006. Since, mismatch can be driven by either aggregate or sectoral shocks, the cyclical increase in mismatch shown in Sahin et al. is consistent with either aggregate or sectoral shocks.

4.4.2 Labor Productivity

Garin, Pries and Sims (2010) document systematic changes in the behavior of labor productivity in post Great Moderation recessions. Our model supports the view that measured labor productivity behaves differently under sectoral shocks than aggregate shocks. To a log-linear appoximation, measured labor productivity is a function of sectoral employment:

$$y_t - n_t = a_t + \sum_{i=1}^{K} \left(\gamma_i - \overline{N}_i/\overline{N}\right) n_{it}$$

where γ_i is the share of sector i's output in total output and $\overline{N}_i/\overline{N}$ is sector i's employment share. In an undistorted state where these shares are equalized, measured labor productivity equals true productivity, but if these shares are not equalized, measured labor productivity will be a biased indicator of labor productivity and sectoral shocks can both raise or lower labor productivity depending on whether the sector experiencing a positive shock has a larger output share than its employment share. To the extent that sector-specific shocks contribute more to business cycles in the Great Moderation, labor productivity's correlation with the business cycle will be weakened.

4.4.3 Okun's Law

Our multisector model provides a straightforward relationship between output and the unemployment rate. The typically stable relationship between output growth and the changes in the unemployment rate is labeled as Okun's Law and, like the Beveridge curve, is a reduced form relationship that occasionally breaks down. Combining the CES aggregator with our definition of sector-specfic shocks and total employment, a structural relationship between output and unemployment can be obtained:

$$
\begin{aligned}
Y_t &= A_t N_t \left\{ \sum_{i=1}^{K} \tilde{\phi}_i^{\frac{1}{\eta}} \left(\frac{N_{it}}{N_t} \right)^{\frac{\eta-1}{\eta}} \right\}^{\frac{\eta}{\eta-1}} \\
&= A_t \left(1 - U_t\right) \left\{ \sum_{i=1}^{K} \tilde{\phi}_i^{\frac{1}{\eta}} \left(\frac{N_{it}}{N_t} \right)^{\frac{\eta-1}{\eta}} \right\}^{\frac{\eta}{\eta-1}}
\end{aligned}
$$

where the last term reflects the effect of labor misallocation on output.

The misallocation term is maximized at one - any misallocation must reduce output holding constant the level of unemployment. In this case, sector-specific shocks can disrupt the Okun's law relationship between output growth and changes in the unemployment rate. If the economy is typically characterized by some steady state level of misallocation, then sectoral shocks can shift Okun's law relationship in either direction. For example, a sectoral shock that improves the allocation of labor raises output for any level of unemployment - as shown in Proposition 7, this case would conform to an inward shift in the Beveridge curve. However, without a direct measure of aggegate productivity, it is not clear how to separate the misallocation channel from changes in aggregate productivity.

4.5 Reservation Wage Shocks and Implications for Structural Change

We can readily extend our model to consider the effect of exogenous shocks to the reservation wage with no wealth effects. Now, a solution for vacancies and unemployment is a function of the reservation wage z in addition to the other exogenous shocks described earlier.

Proposition 8. *Assume no reallocation and no wealth effects. Assume that* $A_i = A_j = A$, $\delta_i = \delta_j$ *and* $\varphi_i = \varphi_j$ *for* $\forall i,\ j \epsilon \{1 \dots K\}$. *For any value of the government spending shock* G, *there exists a* z *such that* $V\left(G,\ z_0,\ A,\ \phi_i\right) = V\left(1,\ z,\ A,\ \phi_i\right)$ *and* $U\left(G,\ z_0,\ A,\ \phi_i\right) = U\left(1,\ z,\ A,\ \phi_i\right)$.

Proof. See Appendix. $\square$

A uniform increase in the reservation wage reduces the surplus in each sector in the same way as a productivity or demand shock leaving aggregate vacancies and unemployment on the same Beveridge curve. This proposition shows that, to the extent that unemployment benefits act as an increase in the household's reservation wage, extensions in the duration of unemployment insurance cannot generate a shift in the Beveridge curve.

With some assumptions on functional forms, our multisector model can be augmented to address the effect of structural change in the long-run on labor market variables and employment shares. Structural change refers to the long-run trends in employment and output shares across sectors. Over the postwar period, employment in manufacturing has steadily dropped from nearly 1/3 of total employment to less than 10%. Over the same period, sectors like education, health care and professional services have all steadily grown. Alternatively, sectors like construction have displayed highly persistent fluctuations without any clear time trend. A recent literature highlighted by Acemoglu and Guerrieri (2008) and Ngai and Pissarides (2007) consider the implications of structural change for aggregate growth rates in models without labor market search. Our model extends these models to allow for consideration of structural change on unemployment and vacancies.

Under the assumption of balanced growth preferences (i.e King-Plosser-Rebelo) and vacancy posting costs that are proportional to the household's marginal rate of substitution, our model admits a balanced growth path with constant unemployment and vacancy rates and constant growth rates for employment. Wages and output grow at the same rate as aggregate productivity, though, aggregate productivity growth is only asymptotically constant if sectors diverge in their growth rates of productivity. The assumption that vacancy posting costs are proportional to the household's MRS is a natural one if hiring is an activity that requires labor. Similar assumptions in Blanchard and Gali (2010) and Michaillat (2012) on vacancy costs are justified by assuming that the cost of hiring is proportional to the wage paid to workers.

Proposition 9. *Consider the K sector flexible-price version of the model with costless labor reallocation and identical separation rates and matching function parameters. Additionally, assume that vacancy posting costs are proportional to the households marginal rate of substitution: $\kappa_t = -\chi_c U_n (C_t, N_t) / U_c (C_t, N_t)$, preferences are King-Plosser-Rebelo: $U(C, N) = log(C) - v(N)$, and the number of households grows at a constant rate g_l with each household supplying a unit measure of labor inelastically. Then, in the labor market steady state:*

1. *Employment shares equal product shares: $N_{it}/N_t = \tilde{\phi}_i$*

2. *Unemployment rates U_t/L_t and vacancy rates V_t/L_t are constant*

3. *Employment growth $\Delta N/N$ equals labor force growth g_l*

4. *Aggregate output* $\Delta Y/Y$ *and consumption growth* $\Delta C/C$ *is equal to productivity plus labor force growth:* $g_y = g_c = g_A + g_l$

5. *Wage growth equals productivity growth:* $g_w = g_A$

If initial productivity is equalized across sectors and grows at the same rate or if $\eta = 1$*, then* g_A *is constant and equal to input-share average of productivity growth across sectors. If sectors grow at different rates, productivity growth is asymtotically constant with* $g_A = \gamma_j$ *where* $j \epsilon \{1 \ldots K\}$ *is the sector with the highest growth rate if* $\eta > 1$ *or* j *is the sector with the lowest growth rate if* $\eta < 1$*.*

Proof. See Appendix. □

Under KPR preferences and symmetry across sectors in hiring costs, the household reallocates labor to mirror the movements in productivity-adjusted product shares. Since the cost of labor is equalized across sectors, relative prices are equalized and an aggregate vacancy posting condition obtains. The assumption that vacancy posting costs are proportional to the household's MRS ensures that market tightness and employment have no trend. If real vacancy posting costs did not change over time, productivity growth would result in a downward trend for unemployment. In contrast, US unemployment exhibits, if anything, a slight upward trend. In general, if sectors exhibit persistent differences in matching function efficiency or separation rates, unemployment, vacancies and employment would not exhibit constant growth rates. However, the proposition presented here establishes a useful benchmark for thinking about long-run trends in unemployment and vacancies.

5 Quantitative Predictions of the Model

To examine whether a sector-specific shock can account for the observed shift in the Beveridge curve and the rise in the unemployment rate in the Great Recession, we calibrate a two-sector version of our model. In this recession, the construction sector is the largest contributor to the sector-specific shock index and is frequently identified as the sector where the employment dislocation has been most severe and persistent. We calibrate the two-sector model to match various moments on employment, unemployment and vacancies across construction and non-construction sectors. Since construction displays a far higher job-filling rate than the rest of the economy, our calibration requires that construction either feature markedly lower hiring costs or reduced labor market tightness relative to the non-construction sector. We consider each explanation in turn.

5.1 Calibration Strategy

The economy is partitioned into construction and non-construction sectors with initial labor market tightness equalized across sectors as would be the case in the model steady state. Several standard parameters in search models are chosen exogenously: the discount rate $\beta = 0.96^{(1/12)}$ to target an annual interest rate of 4%, and the matching function elasticity $\alpha = 0.5$ is assumed to be the same across sectors consistent with evidence from Petrongolo and Pissarides (2001). We also assume that sectoral productivity is equalized and normalized to unity along with the price markup.[14]

Parameters unique to our model determine hiring costs in each sector: the sectoral separation rates δ_c and δ_{nc}, sectoral matching function efficiencies φ_c and φ_{nc}, the cost of posting vacancies κ, the reservation wage z, and the household's bargaining power ν. Moreover, we must also choose parameters in the CES aggregate - namely the input share of construction ϕ in the CES aggregator and the elasticity of substitution η that determines the degree of complementarity or substitutability across goods. We fix $\eta = 0.5$ so that construction and non-construction goods are moderate complements. However, we consider other values of η in our robustness checks.

Separation rates are set using the 2001-2006 averages of employment-weighted sectoral separation rates in the Job Openings and Labor Turnover survey; construction exhibits a significantly higher separation rate than other sectors. Bargaining power is set at $\nu = 0$ to deliver real wage rigidity as in Hall (2005) to ensure large employment effects from small changes in markups or aggregate productivity. As Hagedorn and Manovskii (2008) emphasize, the key variable determining the variability of employment is the size of the surplus rather than the bargaining power. Moreover, since bargaining power is the same across sectors, the level of bargaining power does not affect the mismatch channel by which sector-specific shocks shift the Beveridge curve.

The remaining five parameters - matching function efficiencies, reservation wage, vacancy posting cost, and product share - are jointly chosen to match the following targets: unemployment rate $U/L = 5\%$, vacancy rate $V/L = 2.5\%$, construction's share in total employment $N_c/N = 5.7\%$, construction's share in total vacancies $V_c/V = 3.7\%$, and a product share-weighted average accounting surplus of 10% as in Monacelli, Perotti and Trigari (2010) and close to the surplus delivered in the calibration of Hagedorn and Manovskii (2008).[15] The construction share of employment is chosen

[14]A positive markup has no effect on our calibration other than changing the average price of each good. Alternatively, if the fiscal authority provides a production subsidy to retailers, the markup will be fully offset in steady state with the price index equal to unity.

[15]The surplus is defined as $\frac{P_i}{P} A_i - z$, the difference between the marginal product of labor and the household's marginal rate of substitution.

Table 5: Summary of Calibration Parameters

Aggregate Parameters	Targets	Value	Aggregate Parameters	Targets	Value
Discount rate, β		0.96ˆ(1/12)	Reservation wage, z	U/L	0.9
Bargaining power, ν		0	Construction share, ϕ	N_c/N	0.056
Matching function elasticity, α		0.5	Vacancy posting cost, κ	$P_iA_i - z$	3.49
Elasticity of substitution, η		0.5			
Sectoral Parameters	Construction	Non-Construction			
Monthly separation rates, δ_i	6.0%	3.6%			
Matching function efficiency, φ_i	2.48	0.95			
Model-Implied Moments	Construction	Non-Construction			
Labor force shares, L_i	5.6%	94.4%			
Unemployment rates, U_i/L_i	3.3%	5.1%			
Job-filling rates, q_i	3.51	1.34			

to match the peak of construction employment in 2007 and the vacancy share is the average level of vacancies from 2001-2006. Parameter values and targets are summarized in the Table 5.

Under the assumption that labor market tightness is equalized across sectors, the model generates a lower unemployment rate for construction relative to non-construction sectors. Because hiring costs are considerably lower in the construction sector under this calibration, the household allocates fewer worker to the construction sector to search in order to equalize labor market tightness. However, using sectoral unemployment shares calculated in the CPS, the level of unemployment in the construction sector appears counterfactually low. Nevertheless, the correspondence between the CPS measure of sectoral unemployment and the economic concept of sectoral unemployment in the model is unclear. The CPS measures sectoral unemployment by assigning workers to sectors based on the industry of previous employment with those workers outside the labor force or entering the labor force unassigned to any sector. In the model, a worker is unemployed in sector i if that worker is searching for jobs in sector i. The CPS measure may not accurately capture the sector in which a worker is searching, particularly among those workers transiting between participation and non-participation. In any case, in the next section, we show that an alternative calibration matching the unemployment and vacancy shares of workers does not substantially alter our results.

5.2 Experiment

We depict the shift in the steady state Beveridge curve generated by a permanent shock to the construction share ϕ that reduces the share to $\phi' = 0.04$. This reduction in construction share is chosen to match the observed drop in construction employment shares from a pre-recession peak of 5.7% to its 2012 level of 4.1%. The pre-shock Beveridge curve traces out the locus of aggregate vacancies and unemployment rates for different levels of real marginal cost, while the post-shock Beveridge curve traces the same locus with $\phi = \phi'$ leaving the distribution of the labor force either unchanged (in the case of no reallocation) or shifting the distribution to ensure equalized labor market tightness across sectors (in the case of perfect reallocation).

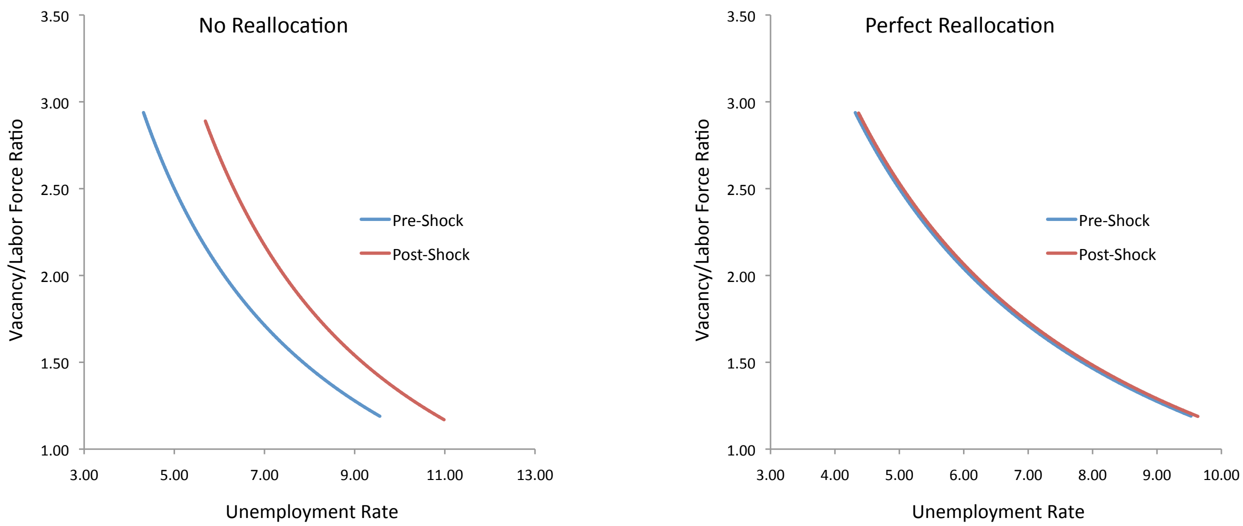
Figure 7 illustrates our main quantitative results. We show that, in the absence of reallocation (left-hand panel of Figure 7), a sector-specific shock to the construction sector generates a shift in the Beveridge curve of about 1.3% (horizontal shift - the rise in the unemployment rate at each level of vacancies). This matches the observed shift in the US data on unemployment rates and the vacancy to labor force ratio. A comparison of simple trend lines of V/L on U/L before and after 2009 (using data from December 2001-November 2011) reveals a shift in the horizontal intercept of 1.4%. While analyses using the job-openings rate (a slightly different measure of vacancies then the vacancy to labor force ratio) reveal a somewhat larger shift of 2%, the shift generated in our baseline calibration with no labor reallocation explains a substantial fraction of the observed shift in either case.

In contrast, when reallocation is costless, the Beveridge curve is essentially unchanged after the sector-specific shock. We take each case as bounds on the shift in the Beveridge curve and, as we will argue, the case of no reallocation is both a good approximation for the short-run behavior of the Beveridge curve and will continue to hold over the medium run given evidence on the costs of labor reallocation for displaced workers. So long as the labor force does not overshoot its long-run distribution, vacancies and unemployment along the transition path will lie in the region between these curves.[16]

In our model, employment shares vary with both changes in the markup and sector-specific shocks, though the movement in employment shares for aggregate shocks is quite small. For a markup shock, employment shares in construction drop because the surplus in construction is lower than that of the non-construction sectors. Lower hiring costs ensure a smaller surplus and, therefore, a greater decrease in the relative surplus for the construction sector. While construction shares displayed somewhat larger cyclical movements in employment shares before 1984, construction shares did not fall in the last recession and recovered quite slowly after the 1990s recession.

[16]Numerical simulations using a quadratic cost of reallocation in a two-sector model show that the labor force moves monotonically after a permanent shock towards the labor force distribution that equates tightnesses.

Figure 7: Model-implied Beveridge curves



Our calibration is consistent with small cyclical effects of aggregate shocks on employment shares consistent with evidence in the past three recessions where shares show little systematic movement in recessions. In this experiment, construction's employment share falls to 4.2% when overall unemployment is at 9% and predicts that the share would only rise to 4.3% at a 5% unemployment rate (with no labor reallocation). Once reallocation takes place, this sector-specific shock lowers construction's employment share further to 4.1%.

5.3 Distorted Initial State and Substitutability

As mentioned, the restriction that initial labor market tightness is equalized across sectors results in a counterfactual sectoral unemployment rate and labor force distribution using measures of these moments from the CPS. If we relax the assumption of equalized labor market tightness, an alternative calibration matches the distribution of employment, unemployment and vacancies. As before, five parameters - matching function efficiencies, the reservation wage, the vacancy posting cost and the product share - are jointly chosen to match the same targets as in Section 5.1. For consistency, we modify the targeted employment share of construction at 5.3%, it's 2000-2006 average. As Table 6 shows, aside from the matching function efficiencies, the remaining parameters are largely unchanged.

Figure 8 shows the shift in the Beveridge curve for a preference shock that reduces the construction share to $\phi' = 0.04$. This shock generates a shift in the Beveridge curve slightly smaller than the previous calibration with an average 1% shift in the unemployment rate at each vacancy rate.

Table 6: Summary of Calibration Parameters - $\theta_c \neq \theta_{nc}$

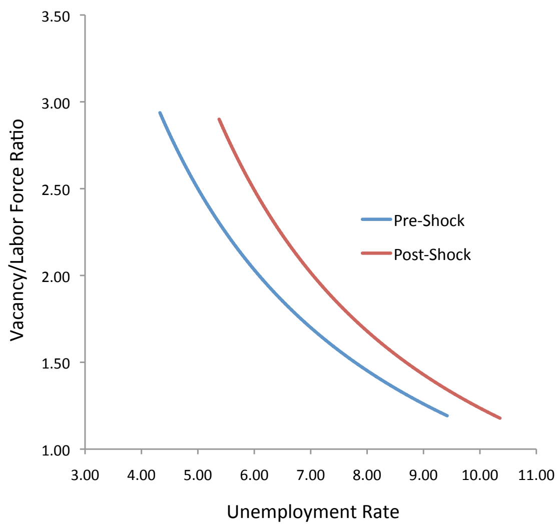
Aggregate Parameters	Targets	Value
Reservation wage, z	U/L	0.9
Construction share, ϕ	N_c/N	0.052
Vacancy posting cost, κ	$P_iA_i - z$	3.49
Sectoral Parameters	**Construction**	**Non-Construction**
Monthly separation rates, δ_i	6.0%	3.6%
Matching function efficiency, φ_i	1.38	0.99
Model-Implied Moments	**Construction**	**Non-Construction**
Labor force shares, L_i	5.6%	94.4%
Unemployment rates, U_i/L_i	9.5%	4.7%
Job-filling rates, q_i	3.36	1.34

At higher levels of unemployment, the shift is mitigated since the sector-specific shock favors the non-construction sector which has a lower cost of hiring for a given level of market tightness. Even though job-filling rates are similar under both calibrations, the reasons for the higher job-filling rate for construction in each calibration are quite different. In our baseline calibration, job-filling rates in the construction sector are higher solely due to higher matching function productivity (even after accounting for the higher separation rate). However, in the distorted steady state calibration, job-filling rates are higher because of lower labor market tightness in the construction sector - effectively the labor force is misallocated with too many workers in construction. Absent labor reallocation, the sector-specific shock still shifts the Beveridge curve outward because a negative sector-specific shock worsens the mismatch between construction and non-construction sectors.

In addition to generating a similar shift in the Beveridge curve, employment shares exhibit somewhat greater volatility under aggregate shocks, though the overall volatility remains low. Since the initial level of mismatch is elevated in this case, the surplus is lower in construction than in non-construction sectors. As a result, aggregate shocks have a greater effect on employment and generate larger increases in mismatch and movement in employment shares.

The behavior of employment shares under aggregate shocks is also affected by the degree of complementarity among goods. When goods are complements, aggregate shocks generate relatively small movements in employment shares. This is due to the limited effect of prices on relative

Figure 8: Effect of SS-Shock in Distorted Initial State



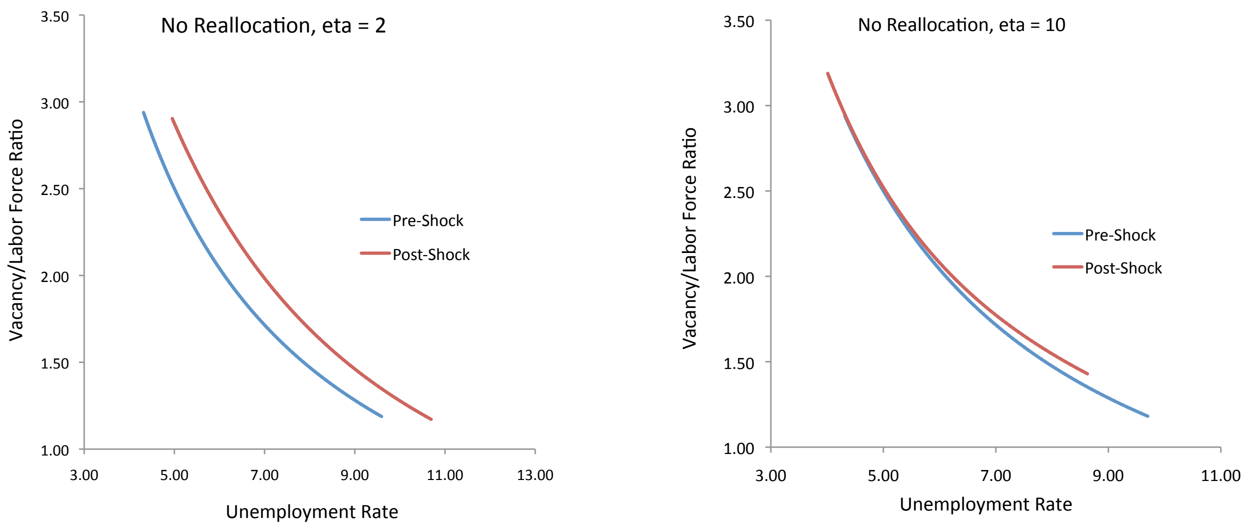
employment shares and can be seen by combining input demand conditions:

$$\frac{N_A}{N_B} = \frac{\phi}{1-\phi}\left(\frac{P_A}{P_B}\right)^{-\eta}$$

In the limit, when $\eta \rightarrow 0$, goods are perfect complements and employment shares are constant irrespective of any aggregate shocks. For higher levels of substitutability, employment shares exhibit greater variation with aggregate shocks, but the magnitude of the shift in the Beveridge curve induced by a sector-specific shock decreases. Figure 9 displays the shift in the Beveridge curve when $\eta = 2$ and $\eta = 10$ - moderate and high degrees of substitutability. For the alternative values of η, we recalibrate the five parameters discussed earlier to maintain the same aggregate and distributional targets. With a higher degree of substitutability, sectors exhibit greater variation in employment shares over the business cycle but show a smaller shift in the Beveridge curve conditional on a sector-specific shock that delivers the same movement in employment shares from 5.3% to about 4% after labor reallocation. However, in the absence of labor reallocation, sector-specific shocks do not match the observed fall in construction employment shares. With $\eta = 2$, construction's employment share is 4.5% at an unemployment rate of 8% - too high relative to the data. Similarly, for $\eta = 10$, construction's share is 5.3%.

Aside from counterfactually high employment shares in the short-run, high degrees of substitutability imply business cycle variation in employment shares inconsistent with evidence in the Great Moderation period. Aside from trends, employment shares across sectors are typically stable

Figure 9: Effect of Substitutability



over the cycle with durable goods and service sectors displaying the strongest business cycle movements (durables are countercyclical while services are countercyclical). While construction's share of employment fell in the early 1990s recession, the construction share remained stable in the 2001 recession before rising and falling with the housing bubble. This suggests that the assumption of mild complementarity or substitutability is not unreasonable in the current recession. Moreover, evidence cited in the growth literature and in studies of durable versus nondurable goods do not support very high levels of substitutability in the CES aggregator.[17] In short, our conclusions that a sector-specific shock to construction account for over 2/3 of the shift in the Beveridge curve hold under alternative assumptions of labor market tightness and for reasonable values of the degree of substitutability.

5.4 Natural Rate of Unemployment

The experiments considered here also allows for an examination of the quantitative relationship between shifts in the Beveridge curve and changes in the natural rate of unemployment. Table 7 shows the natural rate of unemployment before and after a sector specific shock for various specifications of our model. The baseline calibration, which fully accounts for the shift in the Beveridge curve, finds a rise of 1.4 percentage points in the natural rate of unemployment to 6.4%. Once labor reallocation takes place, the sectoral shock to construction has a trivial effect on the unemployment rate, raising the rate to 5.06%. The initial rise in the natural rate of unemployment is similar in

[17]See Acemoglu and Guerrieri (2008), Carvalho and Lee (2011), and Monacelli (2009).

Table 7: Natural Rate and BC Shift

Undistorted state	No reallocation - BC shift	No reallocation - nat. rate	Full reallocation - nat. rate
$\eta = 0.5$	1.3	6.40	5.06
$\eta = 2$	0.9	6.00	5.06
$\eta = 10$	0.4	5.12	5.06
Distorted state			
$\eta = 0.5$	0.9	6.04	4.88
$\eta = 2$	0.5	5.50	4.88
$\eta = 10$	-0.2	4.92	4.87

magnitude to the estimate in Sahin et al. (2012) of the contribution of mismatch unemployment in the Great Recession. The absence of labor reallocation is responsible for most of the rise in the unemployment rate, while the composition effect accounts for the increase in the unemployment rate once reallocation takes place. This slight long-run rise in the unemployment rate is due to the fact that a sectoral shock shifts employment away from the sector with lower hiring costs. For higher degrees of substitutability, sectoral shocks that deliver the same employment share once reallocation takes place imply similar long-run unemployment rates but also a lower rise in the natural rate even in the absence of labor reallocation. In each case, a higher degree of substitutability implies less movement in employment shares as agents tolerate greater deviations of employment shares from product shares leading to a smaller rise in the natural rate of unemployment. Greater substitutability also generates a smaller shift in the Beveridge curve.

As the second panel of Table 7 illustrates, in the presence of some initial degree of mismatch, the quantitative relationship between the natural rate and the shift in the Beveridge curve is somewhat weaker. In the baseline case of $\eta = 0.5$, both the shift in the Beveridge curve and the rise in the natural rate are somewhat lower than the undistorted case with a somewhat larger increase in the natural rate than implied by the shift in the Beveridge curve. Moreover, once reallocation takes place, the natural rate actually falls to 4.88% relative to the initial unemployment rate. This reduction in the long-run unemployment rate differs from the undistorted case because hiring costs are now greater in the construction sector relative to the non-construction sector. Therefore, the sectoral shock favors the sector with lower costs. For higher levels of substitutability, movements in the natural rate are attenuated, consistent with the smaller shifts in the Beveridge curve.

Our experiment reveals an approximate one-to-one relationship between shifts in the Beveridge curve and changes in the natural rate of unemployment. Moreover, when labor reallocation is

Table 8: Reallocation Rates by Education Level

	Industry			Occupation		
Education Level	LF	Emp	Unemp	LF	Emp	Unemp
HS Dropout	2.83	2.85	2.63	3.09	3.13	2.67
HS Graduate	2.24	2.21	2.71	2.58	2.56	3.03
Some College	2.15	2.14	2.40	2.52	2.52	2.56
College Graduate	1.86	1.84	2.25	2.07	2.07	2.26
Advanced Degrees	1.28	1.27	1.80	1.30	1.29	1.78

complete, the natural rate of unemployment returns to approximately the same level despite a permanent sector-specific shock and differences across sectors in hiring costs and matching function technology. However, the one-to-one link between Beveridge curve shifts and the natural rate of unemployment does not hold under extensions of the model considered in Section 6.

5.5 Labor Reallocation

As our quantitative results have emphasized, the ability of sector-specific shocks to explain the shift in the Beveridge curve and generate any economically significant fluctuations in the natural rate of unemployment depends crucially on the speed of labor reallocation across sectors. The available evidence supports slow labor reallocation in the short-run (1-2 years) but evidence on the pace of labor reallocation over the medium-run (2-8 years) is more mixed. We review the available evidence on labor reallocation in both the short-run and medium-run.

Costless labor reallocation is likely to be a poor approximation for the short-run behavior of the labor market. Given the quantitatively small role played by composition effects, costless reallocation would imply no mismatch across sectors and nearly constant aggregate matching function efficiency over the business cycle. However, the empirical measures constructed in Sahin et al. (2010), Barnichon and Figura (2011), and Sedlacek (2011) show that these variables fluctuate significantly over the business cycle. Moreover, observed vacancy to unemployment ratios using JOLTs and CPS data are not equalized across sectors, which is also inconsistent with the view that labor market reallocation is costless.

However, to explain a persistent shift in the Beveridge curve, labor reallocation must also be costly over the medium run. Transition rates for workers across sectors suggest large rates of reallocation, while evidence for displaced workers suggest substantial and persistent barriers to reallocation. The most natural measure of reallocation rates across sectors is monthly transition rates for employed and unemployed workers in the CPS. Since the CPS features a rotating panel design, households are tracked for four consecutive months and interviewed again a year later for another four consecutive months. Using matched CPS data from 2003-2006, we measure monthly reallocation rates for both employed and unemployed workers across major industries and major occupations. These monthly transition rates averaged 2.1% and 2.4% for the industry and occuption reallocation rates respectively. As Table 8 shows, reallocation rates are decreasing with educational attainment and are generally higher for workers who are currently unemployed. Interestingly, for workers with less than a high school degree, reallocation rates drop for unemployed workers relative to employed workers. This fact may be salient for construction workers since construction exhibits the lowest skill attainment of any major industry. The left-hand column of Table 9 gives the fraction of workers in each industry who are college graduates or higher.

Given our interest in labor reallocation out of construction, we examine transitions for only workers in the construction sector over the same period. Table 9 also shows the distribution of transitions from construction to other industries both unconditionally and conditional on the initial skill level. As Table 9 reveals, low-skilled construction workers reallocate toward other low skill industries like retail trade and leisure and hospitality. Service-sector industries - like education and health services, financial activities, and government - which account for a significant share of aggregate employment, are relatively underrepresented. While a significant fraction of transitions take place into professional and business services, these transitions may reflect movements into low skilled jobs like janitorial services and office support rather high-skilled occupations like lawyers, scientists, and managers which both belong to this sector.

The aggregate industry and occupation transition rates reported here are similar in magnitude to the rates documented in Moscarini and Thompson (2007) who examine transition rates at a higher level of disaggregation across occupations instead of industries. However, Kambourov and Manovskii (2004) argue that classification errors are significant in year-to-year transitions in the CPS, leading to spurious transitions. Indeed, measurement error always biases transition rates upwards since a transition is recorded for any consecutive change in recorded industry. Moreover, these transition rates are silent on whether newly transitioned workers are a good substitute for existing workers with industry experience. Therefore, while the raw transition rates suggest large flows across sectors, these transitions are subject to significant measurement error and may not capture whether workers who reallocate are screened by firms. Measurement error and the absence of any measures of match quality may also bias the mismatch measures of Sahin et al. (2012), who

Table 9: Skill Distribution and Reallocation for Construction

Industry	% College +	Construction Transition	College +	Some College or less
Agriculture	14%	3%	2%	3%
Mining	17%	1%	0%	1%
Construction	11%	N/A	N/A	N/A
Manufacturing	23%	18%	14%	19%
Wholesale and Retail Trade	19%	17%	12%	18%
Transportation and Utilities	17%	9%	6%	10%
Information Services	41%	2%	4%	2%
Financial Activities	40%	5%	11%	5%
Professional & Business Services	42%	18%	23%	17%
Education and Health Services	46%	6%	14%	6%
Leisure and Hospitality	14%	9%	4%	10%
Other Services	20%	8%	5%	8%
Public Administration	39%	3%	6%	2%

record a sharp fall in mismatch in the recovery period despite the shift in the Beveridge curve.

High rates of reallocation in the medium term are also inconsistent with evidence from the literature on displaced workers, which documents persistent effects of job loss on wages and labor force outcomes. Davis and von Wachter (2011) show that, in periods of high unemployment, wage loss is up to three years of pre-displacement earnings. This study and related work relies on higher quality longitudinal data from administrative records that accurately track worker outcomes for extended periods. To the extent that wages accurately reflect a worker's marginal product, the steep decline in wages suggests that, conditional on finding employment, displaced workers are not as well suited for their new jobs. The most recent Displaced Workers Survey - a occasional supplement to the CPS - shows that 62% of long-tenured displaced workers (i.e. workers employed for over 3 years) from 2007-2009 came from construction, manufacturing, wholesale and retail trade, or professional and business services.[18] These are precisely the same sectors into which construction workers reallocate suggesting that weak labor market conditions in these sectors make them unlikely to absorb transitions from construction. Moreover, in the latest wave of the Displaced Workers Survey, displaced construction workers exhibit among the lowest rate of reemployment in another industry at 23.9% - second lowest next to education and health services at 19.4%. In short, evidence on displaced workers suggests significant costs to reallocation over the medium term.

[18] Construction workers alone account for 13% of long-tenured displaced workers with a total of 6.8 million workers displaced over the 2007-2009 period. These findings are also supported by Charles, Hurst and Notowidigdo (2012).

5.6 Skilled and Unskilled Labor

While evidence on the degree of labor reallocation across sectors is mixed, one dimension along which workers cannot readily reallocate is skill level. In this section, we extend our baseline model to include skilled and unskilled workers and show that sector-specific shocks can still shift the Beveridge curve even when industry reallocation is costless. Firms in all sectors now hire both skilled and unskilled workers using a fixed proportions technology to produce sectoral output. Workers at a given skill level can freely reallocate across sectors, but workers cannot reallocate across skill levels.

The intermediate good firm's problem from Section 3.2 is modified as follows:

$$
\begin{aligned}
\Lambda_{it} &= \max E_t \sum_{T=0}^{\infty} Q_{t,T} \left(\left(\frac{P_{iT}}{P_t} \right) Y_{iT} - W_{iT} N_{iT}^u - W_{iT} N_{iT}^s - \kappa V_{iT}^u - \kappa V_{iT}^s \right) && (29) \\
\text{subject to } N_{it}^u &= (1-\delta_i) N_{it-1}^u + q_t^u V_{it}^u && (30) \\
N_{it}^s &= (1-\delta_i) N_{it}^s + q_t^s V_{it}^s && (31) \\
Y_{it} &= A_{it} \min\{N_{it}^s,\ \nu_i N_{it}^u\} && (32)
\end{aligned}
$$

Relative to the baseline model, firms in each sector i hire both skilled workers N_{it}^s and unskilled workers N_{it}^u subject to a fixed proportions technology where a unit of effective labor requires a constant sector-specific combination of skilled and unskilled labor ν_i. Firms post vacancies V_{it}^s and V_{it}^u for both types of workers with skill-specific job-filling rates q_t^s and q_t^u. Given costless reallocation within skill cohorts, the job-filling rates are the same across sector for a given skill level. Wages may differ across skill levels but vacancy posting costs are assumed to be the same.

Optimizing behavior by firms implies a single vacancy posting condition for hiring a fixed proportion of workers across skill levels:

$$
\begin{aligned}
\frac{P_{it}}{P_t} A_{it} &= W_{it}^s + \frac{1}{\nu_i} W_{it}^u + \frac{\kappa}{q_t^s} + \frac{\kappa}{q_t^u \nu_i} - E_t Q_{t,t+1} \frac{\kappa}{q_t^s} (1-\delta_i) - E_t Q_{t,t+1} \frac{\kappa}{q_t^u \nu_i} (1-\delta_i) \\
N_{it}^s &= \nu_i N_{it}^u
\end{aligned}
$$

This vacancy posting condition generalizes the standard vacancy posting condition. For sectors with a higher ratio of skilled to unskilled labor, wages and search costs for skilled workers account for a larger share of the marginal product of labor. Changes in the marginal product for a sector characterized by a relatively high skill workforce have a greater effect on skilled worker employment than unskilled worker employment.

Table 10: Summary of Calibration Parameters for Skilled-Unskilled Model

Aggregate Parameters	Targets	Value	Aggregate Parameters	Targets	Value
Discount rate, β		0.96ˆ(1/12)	Reservation wage, z	U/L	0.17
Bargaining power, ν		0	Reservation wage, z_s	W_s/W_{ls}	0.31
Matching function elasticity, α		0.5	Low-skilled share, ϕ	N_{ls}/N	0.56
Elasticity of substitution, η		0.5	Vacancy posting cost, κ	$P_iA_i - z$	0.80

Sectoral Parameters	Low-Skilled	High-Skilled
Monthly separation rates, δ_i	5.1%	3.1%
Matching function efficiency, φ_s, φ_u	0.63	1.85
Labor share, L_s, L_u	70%	30%
Skill ratio, ν_s, ν_{ls}	0.19	0.64

The household problem is left largely unchanged with households free to assign skilled and unskilled workers to search across sectors but unable to transform unskilled workers into skilled workers or vice versa. At each skill level, workers search in sectors to equate their probability-weighted surplus from finding a job - the same condition as in Section 3.3. This optimality condition implies the Jackman-Roper condition with labor market tightness equated across sectors for a given skill level.

We calibrate a two-sector version of this model to demonstrate that sector-specific shocks to the low-skilled sector can generate a quantitatively significant shift in the Beveridge curve. Following the discussion in Section 5.4, we partition the economy into two sectors and two skill levels, segmenting workers as either college graduates or workers with less than a four-year college degree. As noted in Table 8, sectors differ markedly in the skill composition of their workforce. We define the low-skilled sector as construction, mining, leisure and hospitality, trade and transportation, and other services, assigning all remaining sectors to a composite high-skilled sector. The employment weighted ratio of college graduates to non-college graduates is 0.193 for the low-skilled sectors while this ratio is 0.64 for the other sector and determines the value for the parameter ν_i.

For the remaining parameters, our calibration strategy largely follows our strategy described in Section 5.1. Bargaining power ν, matching function elasticity α, the elasticity of substition η across goods, and the discount rate β are the same as in Section 5.1. Sectoral separation rates are chosen to match the employment weighted separation rates (2000-2006 averages) reported from JOLTs. The remaining parameters to be chosen are the matching function efficiencies for skilled workers φ_s and unskilled workers φ_u, the reservation wages for skilled workers z_s and unskilled workers z, the cost of posting vacancies κ, and the preference for the low-skilled sector's good ϕ.

These parameters are chosen to jointly match the following targets: unemployment rate $U/L = 5\%$, vacancy rate $V/L = 2.5\%$, employment share of low-skilled sector $N_{ls}/N = 38.9\%$, vacancy share of low-skilled sector $V_{ls}/V = 37.1\%$, skill premium $z_s/z = 1.82$, and share-weighted average accounting surplus of 10%. The calibration target for employment shares is 2003-2006 average from the BLS establishment survey, while the calibration target from vacancy shares is the average share of vacancies for low-skilled sectors from the JOLTs data over the same period. The skill premium is chosen from estimates in Goldin and Katz (2007), while the share-weighted average accounting surplus is the same as the baseline calibration.[19] The labor share for the skilled sector $L_s = 30\%$ matches the 2003-2006 average share of college graduates in the CPS. The model generates an unemployment share of 51% for the low-skilled sector (versus 50% in the CPS) and unemployment rates by skill level of 4.5% and 5.2% for high skilled and low-skilled workers respectively. Our calibration is summarized in Table 10.

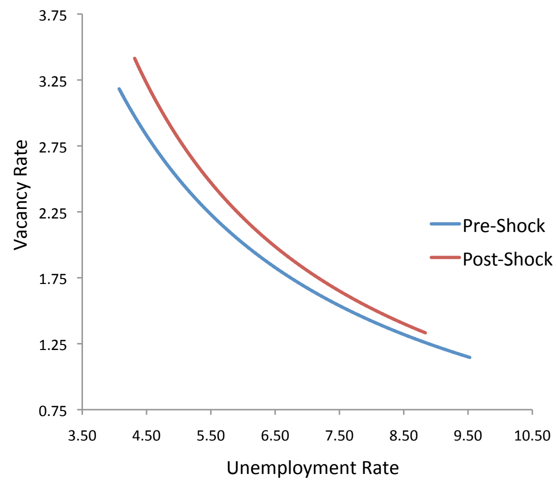
The experiment we conduct is a preference shock that reduces the share of low-skilled employment from 38.9% to 38% corresponding to the reduction observed in the current recession. This fall in employment share is driven largely by construction and partially offset by increases in the other constituent sectors classified as low-skilled. A shock that reduces the input share to $\phi' = 0.547$ reduces the employment share to 38%, raises the unemployment rate to 5.12% and raises vacancies from 2.5% to 2.72% accounting for a sizable outward shift in the Beveridge curve. As seen in Figure 10, this shock increases the unemployment rate by 0.5 percentage points holding vacancies constant, explaining a bit over 1/3 of the observed shift in the Beveridge curve. For higher levels of unemployment, the shift is smaller analogous to the shape of the Beveridge curve observed in the calibration with a distorted initial state. Moreover, in contrast to the construction/non-construction calibration, the sector-specific shock in this calibration delivers an increase in the natural rate of unemployment that is just a quarter of the shift in the Beveridge curve confirming that the size of Beveridge curve shifts and changes in the natural rate are not necessarily one for one.

6 Financial Frictions as Sectoral Shocks

In this section, we extend our baseline model to illustrate how sector-specific shocks could be represented as financial shocks. If financial shocks are responsible for the shift in the Beveridge

[19] See Table A8.1, data for 2005.

Figure 10: Beveridge Curve for Skilled/Unskilled Model



curve, then Beveridge curve shifts no longer necessarily imply any changes in the natural rate of unemployment. In particular, it is now possible for monetary easing to counteract any shift in the Beveridge curve since changes in the conduct of monetary policy in and of itself could generate a shift in the Beveridge curve. We show that a binding zero lower bound on the policy rate - effectively a departure from the unconstrained monetary policy rule - operates as a financial shock that disproportionately impacts the financially constrained sector.

6.1 Financial Frictions on the Firm Side

To model the effect of financial shocks on the production side, we now assume that some sectors face a working capital constraint of the form considered in Christiano, Eichenbaum and Evans (2005). Financially constrained firms have to borrow to pay wages and the cost of posting vacancies.[20] For these firms, their optimization problem is slightly modified from the baseline model by introducing a borrowing rate i_t^b:

$$\Lambda_{it} = \max E_t \sum_{T=0}^{\infty} Q_{t,T}^b \left(\left(\frac{P_{iT}}{P_t} \right) Y_{iT} - \left(1 + i_T^b \right) \left(W_{iT} N_{iT} - \kappa V_{iT} \right) \right) \tag{33}$$

$$\text{subject to } N_{it} = \left(1 - \delta_i\right) N_{it-1} + q_{it} V_{it} \tag{34}$$

$$Y_{it} = A_t N_{it} \tag{35}$$

[20]To introduce financial frictions, we now assume that firms are operated by a distinct set of agents with stochastic discount factor Q_t^b. Given our focus on labor market steady states, the entrepreneur's stochastic discount factor does not enter into the steady state vacancy posting condition.

Financially constrained firms' vacancy posting condition now includes the borrowing rate and changes in expected future borrowing rates:

$$\frac{P_{it}}{P_t}\frac{A_t}{1+i_t^b} = W_{it} + \frac{\kappa}{q_{it}} - E_t Q_{t,t+1}^b \left(1-\delta_i\right) \frac{\kappa}{q_{it+1}} \frac{1+i_{t+1}^b}{1+i_t^b} \tag{36}$$

In steady state, the second term with expected future borrowing rates drops out and changes in the borrowing rate are isomorphic to a negative sector-specific productivity shock as in equation (11). We show in the appendix that a collateral constraint as opposed to a working capital constraint would imply the exact same vacancy posting condition. In Curdia and Woodford (2010), and Mehrotra (2012), the borrowing rate is endogenous to monetary policy as the sum of the nominal deposit rate - the instrument of monetary policy - and an exogenous credit spread less changes in expected inflation:

$$1+i_t^b = \left(1+\omega_t\right)\left(1+i_t^d\right)/E_t\Pi_{t+1}$$

While the credit spread is exogenous, the borrowing rate is not and credit spread shocks may be offset by a reduction in the policy rate or increases in inflation expectations. The presence of a working capital constraint (or other type of financial friction) creates a channel for increasing labor market mismatch between financially constrained and unconstrained sectors, while the effect of the deposit rate on the borrowing rate renders movements in mismatch partially endogenous.

6.2 Financial Frictions on the Household Side

Analogous to the production side, financial frictions on the household side can generate the same change in relative prices as a sector-specific preference shock does in our baseline model. The most realistic financial friction on the household side involves costs of borrowing for purchasing durable goods as modeled in Campbell and Hercowitz (2005) or Monacelli (2009). Since durable goods are lumpy purchases, households typically borrow to make these purchases.

However, the correspondence between sector-specific preference shocks and financial frictions on the household side can be established in a simpler cash-in-advance type setting. We modify our existing model with two types of households and incomplete markets. Assume that a subset of patient households enjoys a fixed share of national income and carries positive wealth from period to period (in the form of government debt). These households provide loanable funds in our setup. The impatient households in our economy supply labor (subject to the search frictions and reallocation frictions detailed earlier) and carry zero wealth from period to period since they are subject to a nonnegative wealth constraint that will bind in the steady state. The impatient household consume two types of goods: C_t and D_t, but the impatient household must borrow at the beginning of the period to purchase D_t, and repay this loan at the end of the period out of income earned from working.

In this setting, the impatient household faces a static optimization problem (in addition to the labor allocation decision detailed in Section 3.3):

$$\begin{aligned} \max \quad & u\left(C_t,\ D_t\right) \\ \text{subject to } \frac{P_{ct}}{P_t}C_t + \left(1+i_t^b\right)B_t &= \sum_{i=1}^{K}\left(W_{it}N_{it}+\Pi_{it}\right) \\ \frac{P_{dt}}{P_t}D_t &= B_t \end{aligned}$$

where i_t^b is the net borrowing rate and the last constraint requires that borrowing inclusive of interest be repaid in full by the end of the period. Instead of a single set of retailers selling a continuum of differentiated goods, we now assume retailers for both types of goods as in Monacelli (2009). These retailers are identical implying the same markup in each sector.

The optimality conditions for the impatient household determine the relative demand for each good. Under the assumption that $u\left(C_t,\ D_t\right)$ is separable:

$$\begin{aligned} \lambda_t u_c\left(C_t\right) &= P_{ct}/P_t \\ \lambda_t u_d\left(D_t\right) &= \left(1+i_t^b\right)P_{dt}/P_t \end{aligned}$$

Relative consumption demand is now a function of both prices and the borrowing rate:

$$\frac{u_c\left(C_t\right)}{u_d\left(D_t\right)} = \frac{P_{ct}}{\left(1+i_t^b\right)P_{dt}}$$

Under log utility and a Cobb-Douglas aggregator for C_t and D_t, we have a relative demand condition that is analogous to the relative employment condition in our baseline model. When patient households relative demand for consumption goods is small or is very similar to that of the impatient household, it follows that a shock to the borrowing rate changes relative employment shares in the same manner as a sector specific shock to ϕ:

$$\frac{N_{ct}}{N_{dt}} \approx \frac{C_t}{D_t} = \frac{\phi}{1-\phi}\left(1+i_t^b\right)\left(\frac{P_{ct}}{P_{dt}}\right)^{-1}$$

While a shock to the borrowing rate is not isomorphic to a preference shock ϕ, changes in borrowing rates shift employment shares and, in the presence of costly labor reallocation, will increase mismatch across sectors.

6.3 Phillips Curve and Mismatch

Since financial frictions on the firm side fits most naturally into our existing model, we illustrate how a change in the monetary policy rule increases mismatch thereby shifting the Beveridge curve. We log-linearize a two-sector version of model where firms in the financially constrained sector are subject to subject to a working capital constraint and there is no reallocation of labor across sectors. When reservation wages are constant, the firms' log-linearized vacancy-posting conditions are given as follows:

$$\begin{aligned} p_{ct} &= i_t^b + s_c \alpha \theta_{ct} \\ p_{ut} &= s_u \alpha \theta_{ut} \end{aligned}$$

where c indexes the financially constrained sector, u indicates the unconstrained sector and $1 - s_i$ is the surplus in sector i. When $s_c = s_u$, the borrowing rate constitutes a wedge between relative prices; an increase in the borrowing rate drives up the prices disproportionately in the financially constrained sector.

An aggregate Phillips curve is obtained by combining the price index and the log-linearized equilibrium conditions of the retailers, with the latter delivering the standard New Keynesian Phillips curve along with equations defining aggregate output and relative employment:

$$\begin{aligned} \pi_t &= \kappa \left(\nu \left(i_t^d + \omega_t + \frac{\alpha}{1-\alpha} s_c \left(1+\epsilon_c\right) n_{ct} \right) + \left(1-\nu\right) \frac{\alpha}{1-\alpha} s_u \left(1+\epsilon_u\right) n_{ut} \right) && (37) \\ &\quad + \beta E_t \pi_{t+1} && (38) \\ y_t &= \gamma n_{ct} + \left(1-\gamma\right) n_{ut} && (39) \\ n_{ct} - n_{ut} &= -\eta \left(i_t^d + \omega_t + s_c \left(1+\epsilon_c\right) \frac{\alpha}{1-\alpha} n_{ct} - s_u \left(1+\epsilon_u\right) \frac{\alpha}{1-\alpha} n_{ut} \right) && (40) \end{aligned}$$

where γ is the steady state share of output for the constrained sector, ν is the steady state share of the price index for the constrained sector, and ϵ_i is the ratio or employment to unemployment in each sector. The three equations summarize the supply block of the two-sector model with financial frictions where labor markets are in their flow steady state. If the intial steady state is distorted (i.e. $P_{ct} \neq P_{ut}$), output and price level shares need not be equalized. Moreover, these shares will generally differ from employment shares and vacancies shares. In the special case where $\gamma = \nu = \phi$, these three equations simplify to two equations:

$$\pi_t = \kappa\left(\nu\left(i_t^d + \omega_t\right) + \frac{\alpha}{1-\alpha} s\left(1+\epsilon\right) y_t\right) + \beta E_t \pi_{t+1} \tag{41}$$

$$n_{ct} - n_{ut} = -\frac{\eta}{1 + \eta s (1+\epsilon) \frac{\alpha}{1-\alpha}} \left(i_t^d + \omega_t\right) \tag{42}$$

and the inflation/output tradeoff is decoupled from the determination of employment shares.

The model is closed by adding the household's aggregate IS condition and specifying a monetary policy rule. We assume that the exogenous credit shock also affects some subset of borrower households as described in the model of Mehrotra (2012). In that setting, an increase the credit spread delivers a business cycle: a decrease in output, inflation, consumption, and employment. Monetary policy is assumed to follow a standard Taylor rule:

$$y_t = E_t y_{t+1} - \sigma\left(i_t^d + E_t \pi_{t+1}\right) - \sigma_b \omega_t \tag{43}$$

$$i_t^d = \phi_\pi \pi_t + \phi_y y_t \tag{44}$$

where σ is the average intertemporal elasticity of substitution and σ_b is the elasticity of substitution for the borrower household. A solution to this five equation system (37) - (39) and (42) - (43) is a process for $\left\{n_{ct},\ n_{ut},\ y_t,\ \pi_t,\ i_t^d\right\}$ as a function of the exogenous shock ω_t.

To see how a change in the monetary policy rule shifts the Beveridge curve, it is useful to fix the level of employment n_t and observe that equation (39) determines the distribution of employment conditional on the response of monetary policy. To a log-linear approximation, steady state employment is $n_t = \tau n_{ct} + (1-\tau) n_{ut}$ where employment shares τ need not match output or price level shares in a distorted steady state. Employment in each sector is given by the expressions:

$$n_{ct} = \frac{1}{\tau}\left\{n_t - (1-\tau) n_{ut}\right\} \tag{45}$$

$$n_{ut} = \frac{\frac{1+\eta\lambda_c}{\tau} n_t + \eta\left(i_t^d + \omega_t\right)}{\left(1+\eta\lambda_c\right)\frac{1-\tau}{\tau} + \left(1+\eta\lambda_u\right)} \tag{46}$$

where λ_i is composite of the other parameters like the sectoral surplus s_c. A weaker policy response (decrease in i_t^d) to the increase in spreads ω_t will increase the share of employment at unconstrained firms so long as similar size shocks ω_t are needed to deliver the same level of employment under each policy.[21] This change in the distribution of employment shifts the Beveridge curve since total vacancies are also a function of the distribution of employment. As shown below, vacancies are equal to:

$$\begin{aligned} v_t &= \frac{\overline{V}_c}{\overline{V}} \frac{1+\alpha\epsilon_c}{1-\alpha} n_{ct} + \frac{\overline{V}_u}{\overline{V}} \frac{1+\alpha\epsilon_u}{1-\alpha} n_{ut} \\ &= \frac{\overline{V}_c}{\overline{V}} \frac{1+\alpha\epsilon_c}{1-\alpha} \frac{1}{\tau} n_t + \left(\frac{\overline{V}_u}{\overline{V}} \frac{1+\alpha\epsilon_u}{1-\alpha} - \frac{1-\tau}{\tau} \frac{\overline{V}_c}{\overline{V}} \frac{1+\alpha\epsilon_c}{1-\alpha} \right) n_{ut} \end{aligned}$$

where the second equality is obtained by expressing employment in the constrained sector in terms of total employment and employment in the unconstrained sector. So long as unconstrained firms face a tighter labor market or account for a disproportionate share of vacancies (relative to their employment share), the coefficient on n_{ut} will be positive and the increase at vacancies at these firms will more than offset the fall in vacancies at the constrained firms shifting the Beveridge curve outward.

In addition to offering an explanation for the shift in the Beveridge curve, the interaction of the zero lower bound and financial frictions at the firm level also offers a potential explanation for the relative stability of inflation in the US despite persistently high unemployment. A credit shock, by affecting firms' costs of production, raises marginal costs for constrained firms. This rise in costs for constrained firms partially offsets the fall in marginal costs from decreasing employment. The financial frictions channels dampens downward pressure on prices, limiting the degree of deflation and, depending on the relative strength of these channels, possibly generating higher inflation. Standard ZLB models in the spirit of Eggertsson and Woodford (2003) have difficulty generating long-lasting zero lower bound episodes without predicting counterfactually high levels of deflation (see Mehrotra (2012)). While extreme downward rigidity in wages could also explain the absence of outright deflation, the presence of a supply-side channel for financial frictions offers another realistic channel to account for stable inflation at the zero lower bound.

7 Conclusion

Discussions about the slow recovery in the US following the Great Recession have raised the possibility of sectoral shocks. Proponents of this view have cited the disproportionate impact of the

[21] In particular, instead of a Taylor rule, assume that monetary policy keeps the borrowing rate constant: $i_t^d = -\omega_t$. Then at the zero lower bound, monetary policy cannot offset the rise in the credit spread and the share of employment in the unconstrained sector rises.

recession on housing-related industries and the shift in the Beveridge curve as evidence of sector-specific shocks. We investigate the role of sector-specific shocks and their impact on the Beveridge curve empirically and theoretically.

On the empirical side, a factor analysis of sectoral employment in the postwar data is used to isolate sector-specific shocks while addressing the Abraham and Katz critique. We derive a sector-specific shock index and show that this index is elevated in the current period and distinct from the business cycle or the Lilien measure of sectoral shocks. Moreover, we show that this measure of sector-specific shocks is elevated in those periods when the Beveridge curve shifts.

On the theoretical side, we build a multisector model with labor market search to investigate how sector-specific shocks affect equilibrium variables like the aggregate Beveridge curve and the level of employment. Our model shows that sector-specific shocks generally shift the Beveridge curve through a composition channel due to differences in hiring costs and hiring technology across sectors and a mismatch channel due to segmentation in labor markets. We show analytically that, through the composition effect, sectoral shocks can raise or lower the natural rate of unemployment, while the mismatch effect always raises the natural rate of unemployment. Moreover, in our baseline model, sectoral shocks that shift the Beveridge curve must also change the natural rate of unemployment.

We calibrate a two-sector version of our model and show that a negative preference shock to the construction sector that matches the distribution of employment shares at the recession trough generates a shift in the Beveridge curve that matches the magnitude of the shift observed in the data. This shock raises the natural rate of unemployment by a quantitatively similar level as the shift in the Beveridge curve - the natural rate rises 1.4 percentage points and results are robust if goods are moderate substitutes instead of complements.

Finally, we show that financial shocks act like sector-specific shocks and can also generate a shift in the Beveridge curve if a subset of firms is financially constrained. In this richer setting, a change in the conduct of monetary policy can generate a shift in the Beveridge curve by magnifying the effect of financial constraints. For example, if monetary policy switches from a Taylor rule to a fixed nominal rate due to a binding zero lower bound, financial constraints will lead to a higher level of mismatch across sectors. These changes in mismatch due to a binding zero lower bound can still be addressed through unconventional monetary policy such as price level targets or credit easing.

As noted in our quantitative results, the assumption of costly or no labor reallocation is crucial in generating the observed persistance of the shift in the Beveridge curve. Existing evidence suggests somewhat contradictory findings on the pace of labor reallocation. Observed transition rates in the CPS and the size of cross-sector flows suggest relatively frequent transitions across sectors. However, evidence from the Displaced Worker Survey and an extensive literature studying labor market outcomes after job loss point to fairly high costs to reallocation. Future research will seek to reconcile these findings to determine the business cycle cost of labor reallocation and dimensions of heterogeneity along which workers do not readily transition.

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A Retailer's Problem

The retailers problem is similar to the standard specification in Woodford (2003). Monopolistically competitive retailers set prices to maximize profits:

$$\begin{aligned}
\max \Lambda_{it}^{ret} &= E_t \sum_{T=t}^{\infty} Q_{t,T} \frac{P_t}{P_T} \chi^{T-t} \left(p_t(i) - P_{fT}\right) y_t(i) \\
\text{subject to } y_t(i) &= Y_t \left(\frac{p_t(i)}{P_t}\right)^{-\zeta} \\
Q_{t,T} &= \beta^{T-t} \frac{u_c\left(C_{t+T},\ N_{t+T}\right)}{u_c\left(C_t,\ N_t\right)} \frac{P_t}{P_T}
\end{aligned}$$

where $p_t\left(i\right)$ is the nominal price chosen by retailers who face a downward sloping demand schedule and discount future profits by the nominal stochastic discount factor $Q_{t,T}$. The optimality condition for price-setting is given below:

$$\begin{aligned}
0 &= E_t \sum_{T=t}^{\infty} Q_{t,T} \chi^{T-t} \left(p_t\left(i\right) P_T^{\zeta} Y_T - \frac{\zeta}{\zeta - 1} P_{fT} P_T^{\zeta} Y_T \right) \\
\Rightarrow \frac{p_t\left(i\right)}{P_t} &= \frac{E_t \sum_{T=t}^{\infty} Q_{t,T} \chi^{T-t} \frac{\zeta}{\zeta-1} \frac{P_{fT}}{P_T} \left(\frac{P_T}{P_t}\right)^{\zeta} Y_T}{E_t \sum_{T=t}^{\infty} Q_{t,T} \chi^{T-t} \left(\frac{P_T}{P_t}\right)^{\zeta-1} Y_T}
\end{aligned}$$

The state variables H_t and T_t are the recursive expressions for the numerator and denominator of the optimal reset price providing equations (2) and (3). The inflation rate is derived from the Calvo assumption with a fraction $1 - \chi$ of firms resetting their prices to $p_t\left(i\right)/P_t$:

$$P_t = \left\{ \chi P_{t-1}^{1-\zeta} + \left(1 - \chi\right) \left(p_t^*\right)^{1-\zeta} \right\}^{\frac{1}{1-\zeta}}$$

Dividing by the price index and defining $\Pi_t = P_t/P_{t-1}$ provides equation (1).

B Additional Proofs

For several proofs, we will refer repeatedly to the equilibrium conditions that determine the steady state Beveridge curve and the natural rate of unemployment in the K-sector model. A solution of the multisector model with "fast-moving" labor markets is a value for aggregate output Y_t, real marginal cost P_{ft}/P_t, consumption C_t, state-variables in the retailers pricing problem H_t, T_t and sectoral prices and quantities $\{Y_{it},\ N_{it},\ U_{it},\ V_{it},\ P_{it}/P_t,\ W_{it},\ p_{it},\ q_{it}\}_{i=1}^{K}$ that satisfy the following static equilibrium conditions

$$
\begin{aligned}
Y_t &= \left\{ \sum_{i=1}^{K} \phi_{it}^{\frac{1}{\eta}} Y_{it}^{\frac{\eta-1}{\eta}} \right\}^{\frac{\eta}{\eta-1}} && (47)\\
\frac{P_{ft}}{P_t} &= \left\{ \sum_{i=1}^{K} \phi_i \left(\frac{P_{it}}{P_t} \right)^{1-\eta} \right\}^{\frac{1}{1-\eta}} && (48)\\
Y_{it} &= \phi_{it} Y_t \left(\frac{P_{it}}{P_{ft}} \right)^{-\eta} && (49)\\
Y_{it} &= A_t A_{it} N_{it} && (50)\\
\frac{P_{it}}{P_t} A_t A_{it} &= W_{it} + \frac{\kappa}{q_{it}} \left(1 - \beta \left(1 - \delta_i\right)\right) && (51)\\
W_{it} &= f\left(N_t\right) + \frac{\nu}{1-\nu} \frac{\kappa}{q_{it}} \left(1 - \beta \left(1 - \delta_i - p_{it}\right)\right) && (52)\\
\delta_i N_{it} &= \varphi_i U_{it}^{\alpha} V_{it}^{1-\alpha} && (53)\\
q_{it} &= \varphi_i \left(\frac{V_{it}}{U_{it}} \right)^{-\alpha} && (54)\\
p_{it} &= \varphi_i \left(\frac{V_{it}}{U_{it}} \right)^{1-\alpha} && (55)
\end{aligned}
$$

and the following dynamic conditions:

$$
\begin{aligned}
1 &= \vartheta \Pi_t^{\theta-1} + (1-\vartheta) \left(\frac{T_t}{H_t} \right)^{\theta-1} && (56)\\
H_t &= \frac{\theta}{\theta-1} \left(\frac{P_{ft}}{P_t} \right) u_c\left(C_t, N_t\right) Y_t + \vartheta \beta E_t \Pi_{t+1}^{\theta} H_{t+1} && (57)\\
T_t &= u_c\left(C_t, N_t\right) Y_t + \vartheta \beta E_t \Pi_{t+1}^{\theta-1} T_{t+1} && (58)\\
1 &= \beta E_t \frac{u_c\left(C_{t+1}, N_{t+1}\right)}{u_c\left(C_t, N_t\right)} (1 + i_t^d)/\Pi_{t+1} && (59)\\
Y_t &= C_t + \sum_{i=1}^{K} \kappa V_{it} + G_t && (60)
\end{aligned}
$$

in terms of the exogenous variables: aggregate productivity A_t, government spending G_t, and sector-specific productivity and demand $\{A_{it},\, \phi_{it}\}_{i=1}^{K}$ subject to the restriction that $1 = \sum_{i=1}^{K} \tilde{\phi}_{it}$ where $\tilde{\phi}_i = \phi_{it} \left(A_{it}\right)^{\eta-1}$. We consider either the case of no reallocation or the case of costless reallocation. With no reallocation

$$
L_{it} = N_{it} + U_{it} \tag{61}
$$

and with costless reallocation

$$
\begin{aligned}
1 &= N_t + U_t & (62) \\
V_{it}/U_{it} &= V_{Kt}/U_{Kt}\,\forall i\epsilon\{1,\ldots,K-1\} & (63)
\end{aligned}
$$

B.1 Proof of Proposition 2: Equivalence under Productivity Shocks

Proof. To that aggregate productivity shocks A_t trace out the same Beveridge curve as government spending shocks G_t, we must show that for any value of the government spending shock G_t, there exists an aggregate productivity shock A_t that implies the same level of aggregate vacancies and unemployment holding constant $\{A_{it},\, \phi_{it}\}_{i=1}^{K}$.

Observe that equations (46) and (48) - (50) can be combined to derive the following modified sectoral demand conditions and vacancy posting conditions:

$$
\begin{aligned}
A_t A_{it} N_{it} &= \phi_{it} A_t \left\{ \sum_{i=1}^{K} \phi_{it}^{\frac{1}{\eta}} \left(A_{it} N_{it}\right)^{\frac{\eta-1}{\eta}} \right\}^{\frac{\eta}{\eta-1}} \left(\frac{P_{it}}{P_{ft}}\right)^{-\eta} & (64) \\
\frac{P_{it}}{P_{ft}} \frac{P_{ft}}{P_t} A_t A_{it} &= W_{it} + \frac{\kappa}{q_{it}} \left(1 - \beta\left(1 - \delta_i\right)\right) & (65)
\end{aligned}
$$

Aggregate productivity cancels out in equation (63). Collectively, equations (51) - (54) for K sectors, the K equations (60) (or (61) and (62)), and the K equations in (63) and (64) define the quantities $\{N_{it},\, U_{it},\, V_{it},\, P_{it}/P_{ft},\, W_{it},\, p_{it},\, q_{it}\}_{i=1}^{K}$ as a function of $\{P_{ft}/P_t,\, A_t\}$. Thus, aggregate vacancies and unemployment are the same conditional on the same combinations of P_{ft}/P_t, an endogenous variable, and A_t, an exogenous variable. The absence of wealth effects on labor supply is important, otherwise household consumption C_t would tie these equations back to the rest of the equilibrium conditions.

Since vacancies and unemployment are functions solely of $\frac{P_{ft}}{P_t} A_t$, any combinations of G_t and A_t that implies the same value for marginal cost times aggregate productivity implies the same values for vacancies and unemployment. Define the function $\frac{P_f}{P}(G,\, A)$ as the endogenous value of real marginal cost for different combinations of the aggregate shocks holding sectoral shocks constant. Choose, $A_t = \overline{A}$ such that $\frac{P_f}{P}\left(\overline{G},\, 1\right) = \frac{P_f}{P}\left(G_0,\, \overline{A}\right)$.

Then, it follows that:

$$\begin{aligned} V\left(\overline{G},\, 1,\, A_i,\, \phi_i\right) &= V\left(G_0,\, \overline{A},\, A_i,\, \phi_i\right) \\ U\left(\overline{G},\, 1,\, A_i,\, \phi_i\right) &= U\left(G_0,\, \overline{A},\, A_i,\, \phi_i\right) \end{aligned}$$

$\square$

B.2 Proposition 6: Undistorted Initial State and No Labor Reallocation

Proof. In this proof, we show that sector-specific shocks raise the natural rate of unemployment and shift outward the Beveridge curve in the absence of labor market reallocation. We begin by listing the equilibrium conditions that determine aggregate employment. Under the assumption of no heterogeneity in matching function efficiencies or separation rates, the system of equations determining employment are given by the following conditions where time subscripts are dropped for simplicity:

$$\begin{aligned} Y &= A\left\{\sum_{i=1}^{K} \tilde{\phi}_i^{\frac{1}{\eta}} N_i^{\frac{\eta-1}{\eta}}\right\}^{\frac{\eta}{\eta-1}} \\ AN_i &= \tilde{\phi}_i Y A^{\eta} g\left(\theta_i\right)^{-\eta} \\ \delta N_i &= \varphi \theta_i^{1-\alpha}\left(L_i - N_i\right) \end{aligned}$$

where g is an increasing and concave function of sectoral labor market tightness. These $2K+1$ equations determine equilbrium output Y, sectoral employment N_i, and sectoral labor market tightness θ_i in terms of the labor force distribution L_i taken as given and constant, sectoral shocks $\tilde{\phi}_i$, and an aggregate productivity shock A that traces out the Beveridge curve.

To prove that the natural rate of unemployment must increase, we normalize $A = 1$ and eliminate θ_i:

$$\begin{aligned} Y &= \left\{\sum_{i=1}^{K} \tilde{\phi}_i^{\frac{1}{\eta}} N_i^{\frac{\eta-1}{\eta}}\right\}^{\frac{\eta}{\eta-1}} \\ N_i &= \tilde{\phi}_i Y g\left(\left(\frac{\delta}{\varphi}\frac{N_i}{L_i - N_i}\right)^{1/(1-\alpha)}\right)^{-\eta} \end{aligned}$$

Eliminating Y, rearranging and summing across sectors, we have:

$$\left\{\sum_{i=1}^{K}\tilde{\phi}_i^{\frac{1}{\eta}}n_i^{\frac{\eta-1}{\eta}}\right\}^{\frac{\eta}{\eta-1}}=\sum_{i=1}^{K}n_i g\left(x_i^{-\frac{1}{1-\alpha}}\right)^{\eta}=\sum_{i=1}^{K}n_i h\left(x_i\right) \tag{66}$$

where $x_i=\frac{L_i/N-n_i}{n_i}$, an expression of aggregate employment N, sectoral employment shares n_i, and the distribution of the labor force L_i. The function h is defined in terms of the function g : $h=g\left(x^{-\frac{1}{1-\alpha}}\right)^{\eta}$ where g is given by:

$$g\left(\theta\right)=z+\frac{1}{1-\nu}\frac{\kappa}{\varphi}\theta^{\alpha}\left(1-\beta\left(1-\delta\right)\right)+\frac{\nu}{1-\nu}\kappa\beta\theta$$

It is readily shown that h is a decreasing and strictly convex function for standard assumptions on the matching function parameters which ensure the coefficents on the polynomial terms of θ in the function g are positive.

Let N_0 be the level of employment when $L_i=\tilde{\phi}_i$ and let N_1 be the level of employment when $L_i\neq\tilde{\phi}_i$. When $L_i=\tilde{\phi}_i$, labor market tightness θ_i is equalized across sectors and the left-hand side of equation (66) is equal to unity. Therefore, N_0 is implicitly defined by the function h:

$$1=h\left(\frac{1}{N_0}-1\right)$$

Using our definitions of x_i and the fact that h is a convex function, we have:

$$\begin{aligned}\left\{\sum_{i=1}^{K}\tilde{\phi}_i^{\frac{1}{\eta}}n_i^{\frac{\eta-1}{\eta}}\right\}^{\frac{\eta}{\eta-1}} &= \sum_{i=1}^{K}n_i h\left(x_i\right)\\ &> h\left(\sum_{i=1}^{K}n_i x_i\right)\\ &= h\left(\frac{1}{N_1}-1\right)\end{aligned}$$

where the first strict inequality follows from the strict convexity of h and the fact for some sectors i and j, it must be the case that $x_i\neq x_j$. The second equality follows from the definition of x_i.

The left-hand side of equation (66) is bounded above by 1. This can be shown by considering the cases of $\eta<1$ and $\eta>1$ separately, and applying the properties of convex or concave functions.

If $\eta < 1$, then:

$$\begin{aligned} \sum_{i=1}^{K} n_i \left(\frac{\tilde{\phi}_i}{n_i} \right)^{1/\eta} &\geq 1 \\ \Rightarrow \left(\sum_{i=1}^{K} n_i \left(\frac{\tilde{\phi}_i}{n_i} \right)^{1/\eta} \right)^{\frac{\eta}{\eta-1}} &\leq 1 \end{aligned}$$

and vice versa in the case of $\eta > 1$.

Thus, we conclude that:

$$\begin{aligned} h\left(\frac{1}{N_0} - 1\right) &> h\left(\frac{1}{N_1} - 1\right) \\ \Rightarrow \frac{1}{N_0} - 1 &< \frac{1}{N_1} - 1 \\ \Rightarrow N_0 &> N_1 \end{aligned}$$

and the natural rate of unemployment must rise in the case that $L_i \neq \tilde{\phi}_i$ as required.

It can be readily verified that when $L_i = \tilde{\phi}_i$, then $N_i = \tilde{\phi}_i N$ with aggregate tightness and employment implicitly defined by the following equations:

$$\begin{aligned} A &= g(\theta) \\ N &= \frac{\varphi\theta^{1-\alpha}}{\varphi\theta^{1-\alpha} + \delta} \end{aligned}$$

Since sectoral shocks do not appears in these equations, aggregate shocks keep tightness equalized across sectors even if reallocation is costly. To show that vacancies rise under a sector-specific shock, we derive an expression for aggregate vacancies in terms of aggregate employment and sectoral tightness:

$$V = \frac{\delta}{\varphi} N \sum_{i=1}^{K} (\theta_i)^{\alpha} \frac{N_i}{N}$$

In the case of aggregate shocks, tightness is equalized across sectors and given by the expression:

$$\theta = \left(\frac{\delta}{\varphi} \frac{N}{1-N} \right)^{\frac{1}{1-\alpha}}$$

Let $N' = N\left(1, \tilde{\phi}'_i\right)$ and $\overline{N} = N\left(A', \tilde{\phi}_i\right)$. Define the share of employment in a given sector under the sectoral shock as $n_i = N'_i/N'$ and ratio of labor to employment as $l_i = L_i/N' = \tilde{\phi}_i/N'$. Then, sectoral tightness is given by:

$$\begin{aligned}\theta_i &= \left(\frac{\delta}{\varphi}\frac{n_i}{l_i - n_i}\right)^{\frac{1}{1-\alpha}} \\ &= \left(\frac{\varphi}{\delta}\frac{l_i - n_i}{n_i}\right)^{-\frac{1}{1-\alpha}}\end{aligned}$$

Define $V' = V\left(1,\, \tilde{\phi}_i'\right)$ and $\overline{V} = V\left(A',\, \tilde{\phi}_i\right)$. Then:

$$\begin{aligned}V' &= \frac{\delta}{\varphi}N'\sum_{i=1}^{K}\left(\frac{\varphi}{\delta}\frac{l_i - n_i}{n_i}\right)^{-\frac{\alpha}{1-\alpha}} n_i \\ &> \frac{\delta}{\varphi}N'\left(\sum_{i=1}^{K}\frac{\varphi}{\delta}\frac{l_i - n_i}{n_i}n_i\right)^{-\frac{\alpha}{1-\alpha}} \\ &= \frac{\delta}{\varphi}N'\left(\frac{\delta}{\varphi}\frac{N'}{1-N'}\right)^{\frac{\alpha}{1-\alpha}} \\ &= \overline{V}\end{aligned}$$

where the first inequality follows from the strict convexity of the inverse labor market tightness and the last equality follows from the fact that $N' = \overline{N}$, which follows from the assumption that unemployment is equalized. □

B.3 Proposition 7: Distorted Initial State and Perfect Reallocation

Proof. Under costless reallocation, the equations determining the steady state Beveridge curve in a two-sector version of the model can be summarized by the following equations:

$$\mu^{-1} = \left\{\tilde{\phi}g_A(\theta)^{1-\eta} + \left(1-\tilde{\phi}\right)g_B(\theta)^{1-\eta}\right\}^{1/1-\eta} \tag{67}$$

$$\frac{n_A}{1-n_A} = \frac{\tilde{\phi}}{1-\tilde{\phi}}\left(\frac{g_A(\theta)}{g_B(\theta)}\right)^{-\eta} \tag{68}$$

$$1 = N\left(1 + \theta^{\alpha-1}\left(n_A\frac{\delta_A}{\varphi_A} + (1-n_A)\frac{\delta_B}{\varphi_B}\right)\right) \tag{69}$$

where $g_i(\theta) = z + \frac{1}{1-\nu}\frac{\kappa}{\varphi_i}\theta^\alpha(1-\beta(1-\delta_i))$. Hiring costs are increasing and strictly concave in θ. Moreover, the condition $\varphi_A > \varphi_B$ (or $\delta_A < \delta_B$) is a sufficient condition for $g_A \leq g_B$ for $\theta \geq 0$ with $g_A = g_B$ at $\theta = 0$. The ratio $g_A/g_B = 1$ at $\theta = 0$ and $lim_{\theta\to\infty}g_A/g_B = \varphi_B/\varphi_A < 1$.

We must first show that, if $\tilde{\phi}' > \tilde{\phi}$, then $N' > N$. For equation (67), if $\tilde{\phi} \to \tilde{\phi}'$, then holding constant θ, the RHS of equation (67) falls. Thus $\theta' > \theta$. If we show that $n_A' > n_A$, then it must be the case that $N' > N$. Since g_A/g_B is monotonic and decreasing, if $\theta \to \theta'$, then $n'_A > n_A$ using equation (68) since the ratio g_A/g_B is less then 1 and falling and $\tilde{\phi}' > \tilde{\phi}$. Thus, we conclude that $N' > N$ and the natural rate of unemployment falls.

To show that the Beveridge curve shifts, we consider the implied level of vacancies for a markup shock and a sector -specific shock that deliver the same level of employment: $N\left(\tilde{\phi}',\, 1\right) = N\left(\tilde{\phi},\, \mu'\right)$. Observe from equation (69), if both shocks deliver the same level of employment, then:

$$\theta^{\alpha-1}\left(n_A\frac{\delta_A}{\varphi_A} + (1-n_A)\frac{\delta_B}{\varphi_B}\right) = \overline{\theta}^{\alpha-1}\left(\overline{n}_A\frac{\delta_A}{\varphi_A} + (1-\overline{n}_A)\frac{\delta_B}{\varphi_B}\right)$$

where the bar superscript signifies the sector-specific shock. It cannot be the case that $\theta = \overline{\theta}$, since equation (68) would not be satisfied. If $\theta < \overline{\theta}$, then $n_A > \overline{n}_A$. Taking ratios of equation (68), it must be the case that:

$$\begin{aligned} \left(\frac{g_A/g_B}{\overline{g}_A/\overline{g}_B}\right)^{-\eta} &< 1 \\ \Rightarrow \left(\frac{\overline{g}_A/\overline{g}_B}{g_A/g_B}\right)^{\eta} &> 1 \end{aligned}$$

which is a contradiction since the ratio g_A/g_B is decreasing in tightness. Therefore, it must be the case that $\theta > \overline{\theta}$ and $n_A < \overline{n}_A$. Under costless reallocation, vacancies simply $V = \theta(1-N)$. Since $N = \overline{N}$, but $\theta > \overline{\theta}$, $V\left(\tilde{\phi}',\, 1\right) < V\left(\tilde{\phi},\, \mu'\right)$ and the Beveridge curve shifts inward. □

B.4 Proof of Proposition 8: Equivalence under Reservation Wage Shocks

Proof. Holding constant $\{A_i,\, \phi_i\}_{i=1}^{K}$, we define $V(G,\, z)$ and $U(G,\, z)$ as aggregate vacancies and unemployment for given values of the government spending shock G and the common reservation wage z. We wish to show that $\forall \overline{G} > 0$, there exists a $\overline{z}$ such that $V\left(\overline{G}, z_0\right) = V(1, \overline{z})$ and $U\left(\overline{G}, z_0\right) = U(1, \overline{z})$.

The government spending shock only affect vacancies and unemployment via the real marginal cost. Let $\bar{\mu}^{-1} = \frac{P_f}{P}\left(\overline{G}\right)$. Relative prices are equalized in steady state since sectoral productivities and hiring costs are equalized. Therefore, the surplus in each sector is the same:

$$\begin{aligned} \frac{P_i}{P}A_i &= z + g(\theta_i) \\ \mu^{-1}A - z &= g(\theta) \end{aligned}$$

For each sector $\theta = g\left(\mu^{-1}A - z\right)^{-1}$ where g is an increasing and concave function. If $\mu = \overline{\mu}$, then $\overline{z} = A - \left(\overline{\mu}^{-1}A - z\right)$ ensures the same labor market tightness in each sector when $\mu^{-1} = 1$, which is the case of no government spending shocks, and tightness is invariant to combinations of μ and z. As a result, aggregate vacancies and unemployment are equalized as required.

If labor reallocation is costless, then Proposition 3 applies. However, in the absence of labor reallocation, sectoral shocks will shift the same Beveridge curve as shown in Proposition 6. Therefore, the fact that two aggregate shocks, government spending shocks and reservation wage shocks, trace out the same Beveridge curve does not follow because no shocks shift the Beveridge curve. □

B.5 Proposition 9: Balanced Growth

Proof. We compute the balanced growth path of the multisector model with costless reallocation. We proceed by stating the equilibrium conditions and solving the model. Under the assumptions in Proposition 9, the model equilibrium conditions given by (46) - (54) and (60) or (61) - (62) can be simplified as follows:

$$
\begin{aligned}
Y_t &= A_t N_t \left\{ \sum_{i=1}^{K} \tilde{\phi}_{it}^{\frac{1}{\eta}} \left(\frac{N_{it}}{N_t} \right)^{\frac{\eta-1}{\eta}} \right\} \\
1 &= \left\{ \sum_{i=1}^{K} \tilde{\phi}_{it} \left(\frac{\tilde{P}_{it}}{P_t} \right)^{1-\eta} \right\}^{\frac{1}{1-\eta}} \\
A_t N_{it} &= \tilde{\phi}_{it} Y_t \left(\frac{\tilde{P}_{it}}{P_t} \right)^{1-\eta} \\
\frac{\tilde{P}_{it}}{P_t} A_t &= W_t + \frac{\kappa_t}{\varphi} \theta_t^{\alpha} \left(1 - \beta\left(1-\delta\right)\right) \\
W_t &= v'\left(N_t\right) C_t + \frac{\nu}{1-\nu} \frac{\kappa_t}{\varphi} \theta_t^{\alpha} \left(1 - \beta\left(1 - \delta - \varphi\theta_t^{1-\alpha}\right)\right) \\
Y_t &= C_t + \kappa_t V_t \\
\theta_t &= V_t / U_t \\
1 &= N_t + U_t
\end{aligned}
$$

where $\tilde{P}_{it}/P_t = P_{it}A_{it}/A_t$ is the productivity-adjusted relative price of sector i's output and $A_t = \left\{\sum_{i=1}^{K} \phi_{it} A_{it}^{\eta-1}\right\}^{\frac{1}{\eta-1}}$

Since hiring costs are equalized, it must be the case that $\tilde{P}_{it}/P_t = 1$ and $N_{it}/N_t = \tilde{\phi}_{it}$. Combining the vacancy-posting condition, Nash-bargained wages and the assumption for vacancy costs, we obtain the following:

$$
\begin{aligned}
A_t &= v'(N_t)\, C_t \left(1 + \frac{\chi}{\varphi} h(\theta_t)\right) \\
\Rightarrow 1 &= v'(N_t) \frac{N_t}{1 + \chi V_t v'(N_t)} \left(1 + \frac{\chi}{\varphi} h(\theta_t)\right)
\end{aligned}
$$

This vacancy posting condition combined with labor market clearing and the definition of market tightness jointly determine labor market variables N, U, V, θ where the time subscript is dropped since none of these variables is a function of exogenous variables that change over time: namely A_t and L_t.

Since growth in the labor force is modeled as a net addition of new households, the labor market variables have a per capita interpretation and each variable grows at the rate $g_l = \Delta L/L$. Thus, the unemployment rate, vacancy rate, and employment rate are constant. It is straightforward to compute the growth rates of per household output, consumption and wages given the resulting expressions:

$$
\begin{aligned}
Y_t &= A_t N \\
Y_t &= C_t + \kappa_t V \\
W_t &= v'(N)\, C_t + \frac{\nu}{1-\nu} \frac{\kappa_t}{\varphi} \theta_t^{\alpha} \left(1 - \beta\left(1 - \delta - \varphi \theta^{1-\alpha}\right)\right)
\end{aligned}
$$

with $g_y = g_c = g_w = g_A$.

However, these growth rates are constant only in the special case when sectoral productivities are equalized and grow at the same rates. Since the expression for aggregate productivity is a sum, different growth rates across sectors will generally change the growth rate of aggregate productivity. Moreover, changes in preference shares over time will also alter productivity growth rates. If all structural change is driven by changes in product shares, all per capita growth rates are zero and all aggregates grow only with the labor force. Employment shares will mirror their productivity-adjusted product shares along the growth path.

More generally, if sectoral TFP growth rates differ, then output, consumption and wage growth will be asymptotically constant. If $\eta > 1$, then $lim_{t \to \infty} \Delta A/A = \gamma_{max}$ where γ_{max} is the TFP growth rate of the fastest growing sector. Alternatively, if $\eta < 1$, then the opposite holds and TFP growth converges to the growth rate of the slowest growing sectors. These results are analogous to the asymptotic growth rates computed in Acemoglu and Guerrieri (2008). If $\eta = 1$, the TFP aggregator is Cobb-Douglas and the aggregate TFP growth rate is a weighted average of each sector's TFP growth rate. □

B.6 Collateral Constraint

Our result demonstrating an equivalence between sector-specific shocks and shocks to the borrowing rate in a model with a working capital constraint can be generalized to other types of financial shocks. A common shock considered in the literature is a Kiyotaki and Moore type shock to the value of collateral. We modify the problem of the intermediate goods producer to include a time-varying collateral constraint that limits the ability of the firm to borrow to finance the wage bill and the cost of posting vacancies:

$$\begin{aligned} \Lambda_{it} &= \max E_t \sum_{T=0}^{\infty} Q_{t,T} \left(\left(\frac{P_{iT}}{P_t} \right) Y_{iT} - \left(1 + i_T^b \right) \left(W_{iT} N_{iT} - \kappa V_{iT} \right) \right) \\ \text{subject to } N_{it} &= \left(1 - \delta_i\right) N_{it-1} + q_{it} V_{it} \\ Y_{it} &= A_t N_{it} \\ \lambda_t \overline{K} &\geq W_{it} N_{it} + \kappa V_{it} \end{aligned}$$

Fluctuation in λ_t can represent a tightening of lending standards by financial institutions or a fall in the value of collateral like real estate or other forms of capital. For simplicity, we continue to assume that labor is the only variable factor of production and that constrained firms have some fixed endowment of capital. The vacancy posting condition in this setting is identical to the vacancy posting condition (29):

$$\frac{P_{it}}{P_t} \frac{A_t}{1 + \varphi_t} = W_{it} + \frac{\kappa}{q_{it}} - E_t Q_{t,t+1} \left(1 - \delta_i\right) \frac{\kappa}{q_{it+1}} \frac{1 + \varphi_{t+1}}{1 + \varphi_t}$$

where φ_t is the Lagrange multiplier on the collateral constraint and replaces the interest rate on borrowed funds. In steady state, the Lagrange multiplier on the constraint enters as a sector-specific productivity shock for any sector that faces a working capital constraint. A decrease in the value of λ_t tightens the constraint and raises the Lagrange multiplier. Therefore, our choice of modeling the financial shock as an interest rate shock instead of a shock to collateral values has no qualitative effects on the behavior of firms.

C Calibration and Model-Based Measures

C.1 Structural Factor Analysis

To a log-linear approximation, sectoral employment can be expressed by solving the equations that determine the steady state Beveridge curve in our model (shown at the beginning of the appendix):

$$\begin{aligned} Mn_t &= Ha_t \\ &= H\left(\Phi z_t + \epsilon_t\right) \end{aligned}$$

where $n_t = (n_{1t}, \ldots, n_{Kt})^{'}$ is the vector of log-linearized sectoral employment expressed in terms of the exogenous variables, the vector $a_t = (a_{1t}, \ldots, a_{Kt})^{'}$ of sectoral productivity shocks. As argued, the exogenous sectoral productivity process can be decomposed into its first principal component and a vector of sectoral shocks $\epsilon_t = (\epsilon_{1t}, \ldots, \epsilon_{Kt})^{'}$ with $Cov\left(z_t, \epsilon_{it}\right) = 0$ for $\forall i \epsilon \left\{1, \ldots, K\right\}$.

The matrix M is determined by the model parameters and the steady state values of labor market variables. To compute this matrix, it is necessary to choose parameters and solve for the model steady state. We calibrate an 11-sector version of our model where the sectors conform to the NAICS supersectors for which there is readily available data on employment, unemployment and vacancies. Our reduced form sector-specific shock index was computed using 13 NAICS sectors, but we use only 11 sectors since retail trade, wholesale trade, transportation and utilities are combined into a single sector in the data on unemployment and vacancies from the CPS and JOLTs respectively.

To calibrate the 11 sector version of the model, some parameters are chosen directly while some parameters are chosen to match targets. As in the calibrations shown earlier, the household's discount rate β, matching function elasticity α, and bargaining power ν are all set to the values described in Section 5.1. Separation rates for the 11 sectors are set to match the 2000-2006 average in the JOLTs data. We chose matching function efficiencies φ_i, CES product shares ϕ_i, reservation wage z, and the vacancy posting cost κ to match the following targets: the distribution of vacancies V_i/V, the distribution of employment N_i/N, an unemployment rate $U/L = 5\%$, a vacancy rate $V/L = 2.5\%$, and a share-weighted accounting surplus of 10%. Vacancy shares and employment shares are set using 2000-2006 averages from the JOLTs and payroll survey respectively. Initial labor market tightness is equalized across sectors so that unemployment shares match vacancy shares. The table below summarizes the calibration targets, parameters, and components of the

Table 11: Summary of Calibration Parameters

Aggregate Parameters	Targets	Value
Discount rate, β		0.96ˆ(1/12)
Bargaining power, ν		0
Matching function elasticity, α		0.5
Elasticity of substitution, η		0.5
Vacancy posting cost, κ	$P_i - z$	3.26
Reservation wage, z	U/L	0.9055

Sectors	Employment Share	Vacancy Share	MFE: φ_i	Separations: δ_i	Product Share: ϕ_i	Output Share: γ_i	Diag(M)
Construction	5.3%	3.6%	2.47	6.2	5.2%	5.2%	1.92
Durables Goods	7.2%	4.6%	1.15	2.8	7.1%	7.0%	1.98
Education and Health Services	12.6%	17.7%	0.53	2.8	12.8%	13.1%	1.94
Financial Activities	6.1%	6.9%	0.66	2.8	6.1%	6.2%	1.95
Government	16.3%	10.7%	0.62	1.5	16.1%	15.9%	2.05
Information Services	2.5%	2.7%	0.77	3.1	2.5%	2.5%	1.94
Leisure and Hospitality	9.4%	12.4%	1.42	7.0	9.5%	9.6%	1.89
Nondurable Goods	4.3%	2.6%	1.37	3.1	4.2%	4.2%	1.97
Other Services	4.1%	4.3%	1.00	3.9	4.1%	4.1%	1.93
Professional and Business Services	12.6%	17.9%	1.07	5.7	12.8%	13.0%	1.89
Trade, Transportation and Utilities	19.6%	16.5%	1.35	4.2	19.5%	19.3%	1.93

matrix M that is used to rotate the sectoral employment data. We consider two possible values for the elasticity of substitution η, with $\eta = 0.5$ and $\eta = 2$. Table 1 summarizes the calibration for the case of complementary goods:

When goods are substitutes the product shares, output shares, and diagonal elements of M are changed. For brevity, the employment shares, vacancy shares, separation rates, and matching function efficiencies are omitted from this table as they are the same as in Table 1. These new steady state values are summarized in Table 2.

C.2 Relation of Sector-Specific Shock Index and the Beveridge Curve

Consider a steady state where $\overline{\theta}_i = \overline{\theta}_h$ for all sectors $i,\ h\epsilon\{1, \ldots,\ K\}$. In the absence of labor market mismatch, it follows that unemployment shares and vacancy shares are equalized. To a log linear approximation, aggregate vacancies, unemployment and employment are given by the following equations:

$$
\begin{aligned}
v_t &= \sum_{i=1}^{K} \frac{\overline{V}_i}{\overline{V}} v_{it} \\
u_t &= \sum_{i=1}^{K} \frac{\overline{U}_i}{\overline{U}} u_{it} \\
n_t &= \sum_{i=1}^{K} \frac{\overline{N}_i}{\overline{N}} n_{it}
\end{aligned}
$$

Table 12: New Parameters: $\eta = 2$

Aggregate Parameters	Targets	Value	
Elasticity of substitution, η		2	
Vacancy posting cost, κ	$P_i - z$	3.26	
Reservation wage, z	U/L	0.9049	

Sectors	Product Share: ϕ_i	Output Share: γ_i	Diag(M)
Construction	5.0%	5.2%	4.67
Durables Goods	6.8%	7.0%	4.90
Education and Health Services	13.6%	13.1%	4.76
Financial Activities	6.3%	6.2%	4.80
Government	15.5%	15.9%	5.22
Information Services	2.6%	2.5%	4.77
Leisure and Hospitality	9.8%	9.6%	4.56
Nondurable Goods	4.0%	4.2%	4.87
Other Services	4.1%	4.1%	4.71
Professional and Business Services	13.5%	13.0%	4.58
Trade, Transportation and Utilities	19.0%	19.3%	4.73

A log-linear approximation to the sectoral Beveridge curve provides the following expression:

$$n_{it} = \alpha u_{it} + (1-\alpha)\, v_{it}$$

Using the expressions for aggregate vacancies and unemployment and the fact that $\overline{U}_i/\overline{U} = \overline{V}_i/\overline{V}$, we have:

$$\sum_{i=1}^{K} \frac{\overline{U}_i}{\overline{U}} n_{it} = \alpha u_t + (1-\alpha)\, v_t$$

Adding and subtracting aggregate employment and rearranging, we obtain the following relation:

$$v_t = \frac{1}{1-\alpha} \left\{ -\left(\alpha + \frac{\overline{U}}{\overline{N}}\right) u_t + \sum_{i=1}^{K} \left(\frac{\overline{U}_i}{\overline{U}} - \frac{\overline{N}_i}{\overline{N}} \right) n_{it} \right\}$$

It is worth noting that in our numerical calibration, the aggregate component of n_{it} approximately cancels out, and we are left with an expression in terms of the sectoral shocks:

$$v_t = \frac{1}{1-\alpha} \left\{ -\left(\alpha + \frac{\overline{U}}{\overline{N}}\right) u_t + \sum_{i=1}^{K} \left(\frac{\overline{U}_i}{\overline{U}} - \frac{\overline{N}_i}{\overline{N}} \right) \epsilon_{it} \right\}$$

C.3 Proof of Proposition 4

Proof. We show that under perfect substitutability, sectoral employment has a factor representation in terms of the exogenous sectoral productivity process. Under perfect reallocation, the relative price of goods across sectors must be equalized. From equation (47), $P_i/P = \mu^{-1}$ for all sectors $i\epsilon\{1,\ldots,\ K\}$. For simplicity, assume no aggregate demand shocks and set $\mu^{-1} = 1$. The firm's vacancy posting condition is given my equation (50):

$$A_i = W_i + \frac{\kappa}{q_i}\left(1 - \beta\left(1 - \delta_i\right)\right)$$

Log-linearizing equations (50) - (55) and combining, we have:

$$a_{it} = \left(1 - s_i\right)\tilde{\alpha}_i \frac{\overline{L}_i/\overline{U}_i}{1 - \alpha} n_{it}$$

where s_i is the steady state surplus and $\tilde{\alpha}_i$ is a composite parameter that depends on the matching function elasticity α and other matching function parameters when bargaining power is nonzero. The diagonal matrix H is obtained by simply inverted the expression to solve for sectoral employment. □