# System-Optimal Routing of Traffic Flows with User Constraints in Networks with Congestion

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# 2002 Urban Mobility Study

http://mobility.tamu.edu/ums

"The bad news is that even if transportation officials do all the right things, the likely effect is that congestion will continue to grow. . . "

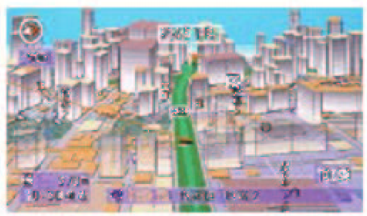
- Total congestion "bill" in 2000 was $67.5 billion

(= 3.6 billion hours delay + 5.7 billion gallons gas)

| | 1982 | 2000 |
|---|---|---|
| time penalty for peak period travelers | 16 hours | 62 hours |

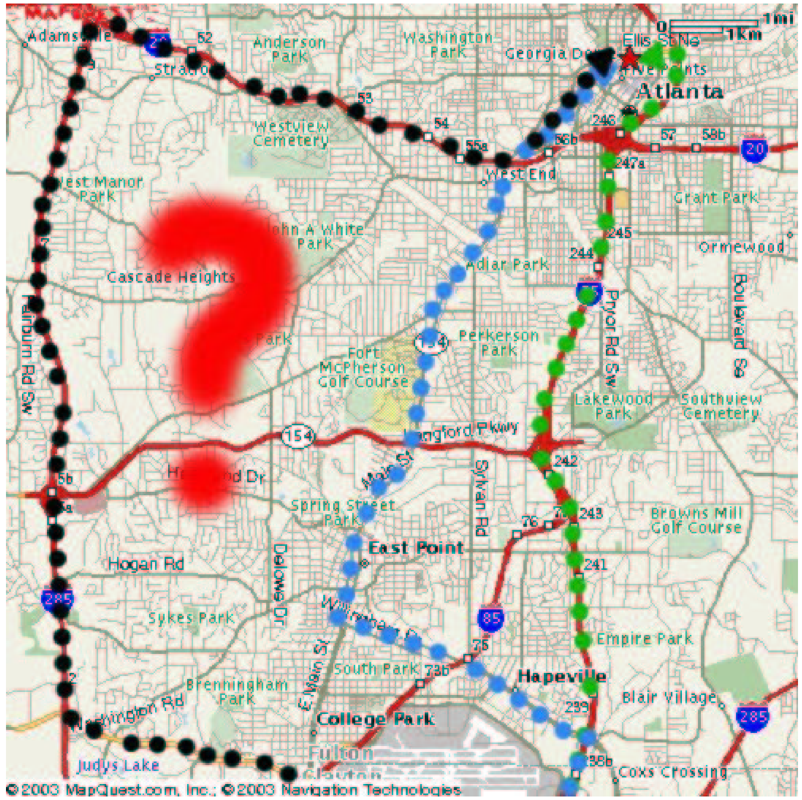
# In-Car Navigation Systems






# Shall we guide users? How?



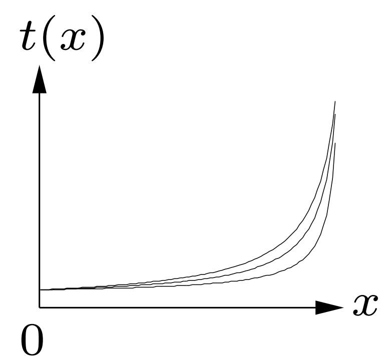

| **selfish users** | **central planner** | **our goal** |
|---|---|---|
| optimize own travel time | optimize system welfare | |
| fair, not efficient | efficient, not fair | **fair, efficient** |

# Outline

- Route Guidance
- Theoretical Results
- Computational Experiments

# The Traffic Model

- Directed graph $G = (V, A)$
- OD pairs $(o_i, d_i)$ of rate $r_i$, $i = 1, \ldots, k$
- Flows on paths $x_P$. Can be non-integral.
- Traversal times: *latency functions* $t_a(\cdot)$ for all $a \in A$
  - flow $x_a$ in $a \to$ traversal time $t_a(x_a)$
  - *continuous*
  - *nondecreasing*
  - $x\,t(x)$ *convex*



# The system optimum (SO) is the flow that minimizes the total travel time

$$\min \quad C(x) \quad := \quad \sum_{a \in A} \quad t_a(x_a)x_a$$

$$\text{s.t.} \quad \sum_{P \ni a} \quad x_P = x_a \qquad \text{for all } a \in A$$

$$\sum_{P \in \mathcal{P}_i} \quad x_P = r_i \qquad \text{for all } i = 1, \ldots, k$$

$$x_P \geq 0 \qquad \text{for all } P \in \mathcal{P}$$

where $\mathcal{P}_i$ : set of paths from $o_i$ to $d_i$

$\mathcal{P} = \bigcup_i \mathcal{P}_i$

# User Equilibrium

- Users optimize their own travel times: Wardrop 1952

> A flow is a **user equilibrium** (**UE**) iff nobody can switch to a path with smaller travel time

- Travel times of users between the same OD pair are equal

- **UE** always exists and is essentially unique

  Beckmann, McGuire & Winsten 1956

# Our Approach

- **SO** cannot be implemented in practice due to unfairness

- **UE** does not take into account the global welfare

**Use constrained SO instead!**

- **CSO** = min total travel time
  s.t. demand satisfied
  users are assigned to "fair" routes

# Normal Lengths

Normal lengths: a-priori belief of network

- Geographic distances
- Free-flow travel times (times in empty network)
- Travel times under **UE**

Notation:

- normal length of arc: $\ell_a$
- normal length of path: $\ell_P = \sum_{a \in P} \ell_a$

# CSO Definition

- Fix a tolerance factor $\varepsilon \geq 0$
- A path $P \in \mathcal{P}_i$ is valid if $\qquad \ell_P \leq (1+\varepsilon) \times \min_{Q \in \mathcal{P}_i} \ell_Q$
- Definition:

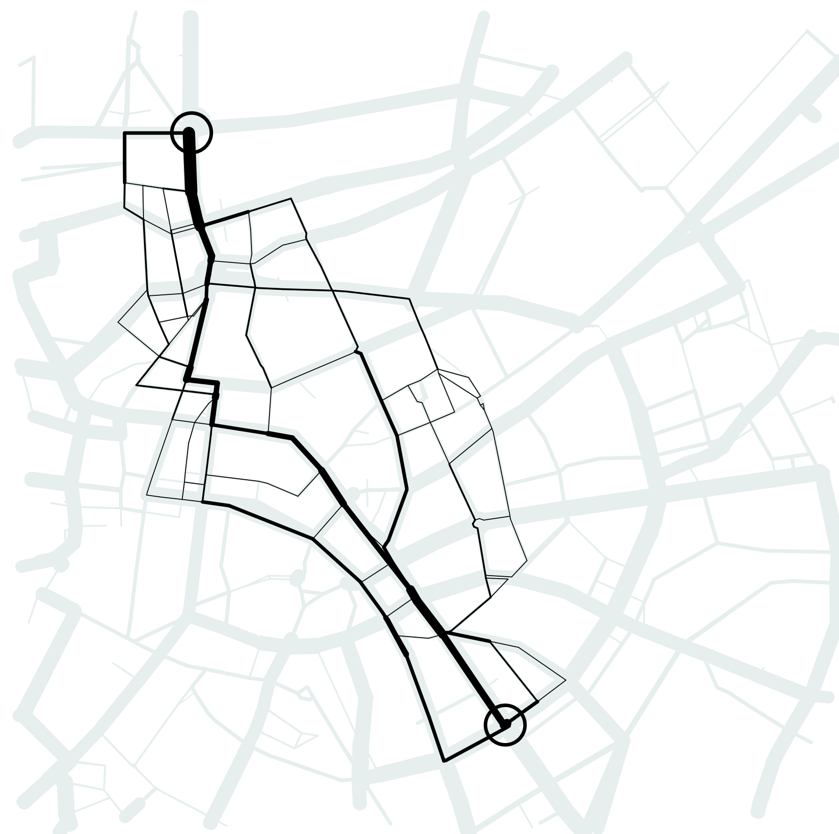
$$\textbf{CSO}_\varepsilon := \arg\min \quad C(x)$$

$$\text{s.t.} \sum_{P \in \mathcal{P}_i : P \text{ valid}} x_P = r_i \qquad \text{for all } i$$

$$x_P \geq 0$$

# CSO Example

SO



CSO

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# Performance evaluation of route guidance systems

- The **Price of Anarchy** measures the impact of lack of Central Coordination — Papadimitriou 2001

- Let inst($\mathcal{T}$) be the set of instances with latencies drawn from $\mathcal{T}$

$$\text{Price of Anarchy is} \quad \max_{\text{inst}(\mathcal{T})} \frac{C(\textbf{UE})}{C(\textbf{optimum})}$$

- Previously, this *price* was evaluated using **SO** as solution concept — Roughgarden, Tardos, Correa, Schulz, St., . . .

- But it is a *pessimistic* measure because **SO** cannot be achieved!

# New Guarantees

Instead, study relation between $C(\textbf{UE})$ and $C(\textbf{CSO}_\varepsilon)$
(and also $C(\textbf{SO})$ )

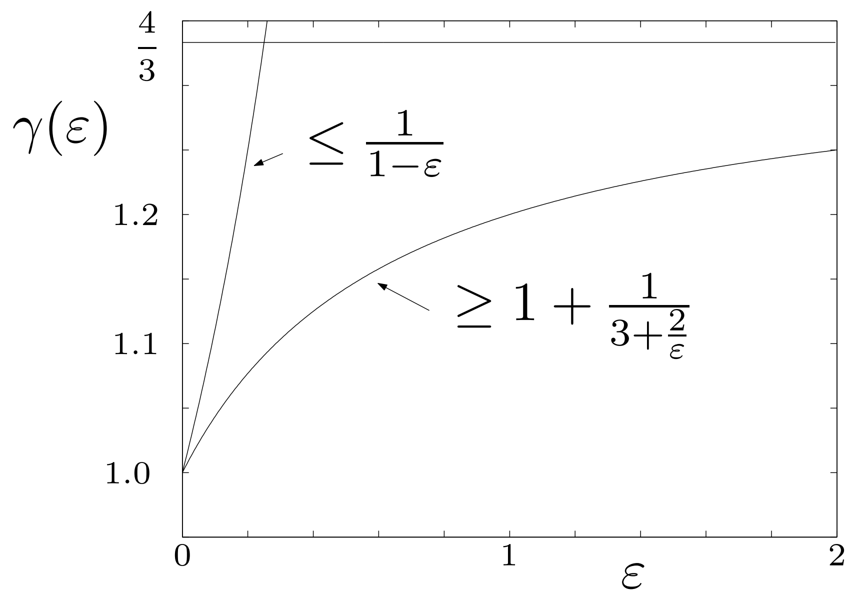
- Guarantee: $\gamma(\varepsilon) := \sup_{\text{inst}(\mathcal{T})} \frac{C(\textbf{UE})}{C(\textbf{CSO}_\varepsilon)}$

- $1 \leq \gamma(\varepsilon) \leq \alpha(\mathcal{T})$ for all $\varepsilon$, where $\alpha(\mathcal{T}) := \sup_{\text{inst}(\mathcal{T})} \frac{C(\textbf{UE})}{C(\textbf{SO})}$

- The function $\gamma(\varepsilon)$ is non-decreasing

# Linear Case and Free-flow Normal Lengths

- $\gamma(\varepsilon) \leq \alpha(\{\text{linear latencies}\}) = \frac{4}{3}$




- In the limit we recover unconstrained case ( $\lim_{\varepsilon \to \infty} \gamma(\varepsilon) = \frac{4}{3}$ )

- **CSO**$_\varepsilon$ not efficient when $\varepsilon$ small ($\gamma(\varepsilon) \approx 1$ if $\varepsilon \approx 0$)

# UE Travel Times as Normal Lengths

- **UE** is feasible in **CSO** problem:

$$C(\mathbf{CSO}_\varepsilon) \leq C(\mathbf{UE}) \quad \text{for all } \varepsilon$$

- Therefore, $\displaystyle \sup_{\text{inst}(\mathcal{T})} \frac{C(\mathbf{CSO}_\varepsilon)}{C(\mathbf{SO})} \leq \alpha(\mathcal{T}) \quad \text{for all } \varepsilon$

- **CSO**$_\varepsilon$ efficient for all $\varepsilon$:

$$\gamma(\varepsilon) = \alpha(\mathcal{T})$$

- All the bounds are tight

# Unfairness

- Normal unfairness of path $P$ for OD pair $i = \frac{\ell_P}{\min_{Q \in \mathcal{P}_i} \ell_Q}$

  $\rightarrow$ $1 \leq$ normal unfairness $\leq 1 + \varepsilon$

- Loaded unfairness of path $P$ for OD pair $i = \frac{t_P(x)}{\min_{Q \in \mathcal{P}_i} t_Q(x)}$

  $\rightarrow$ $1 \leq$ loaded unfairness. Equality if **UE**

# The loaded unfairness of CSO is bounded

- Let $f^*$ be a **CSO** (or a **SO**). The loaded unfairness of any user

$$\frac{t_P(f^*)}{\min_{Q \in \mathcal{P}_i} t_Q(f^*)} \;\leq\; \Gamma(\mathcal{T}) \;:=\; 1 + \sup_{t \in \mathcal{T}, x \geq 0} \{x t'(x)/t(x)\}$$

- For example, $\Gamma(\{\text{polynomials of degree } k\}) = k + 1$

- Proof idea:
  - Define new latency functions $t^*(x) := t(x) + x\, t'(x)$
  - For all $i$: travel times $t^*_P(f^*)$ are constant for all valid $P \in \mathcal{P}_i$

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# Computational Experiments

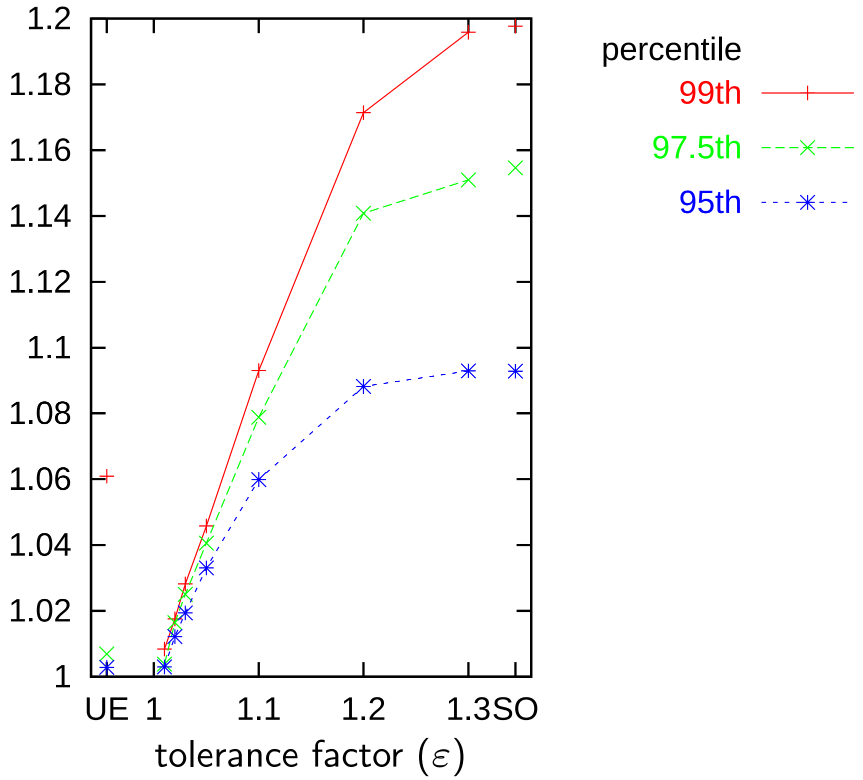
We used real-world instances obtained from *DaimlerChrysler* (Berlin) and from the *Transportation Network Test Problems* website:

http://www.bgu.ac.il/~bargera/tntp/

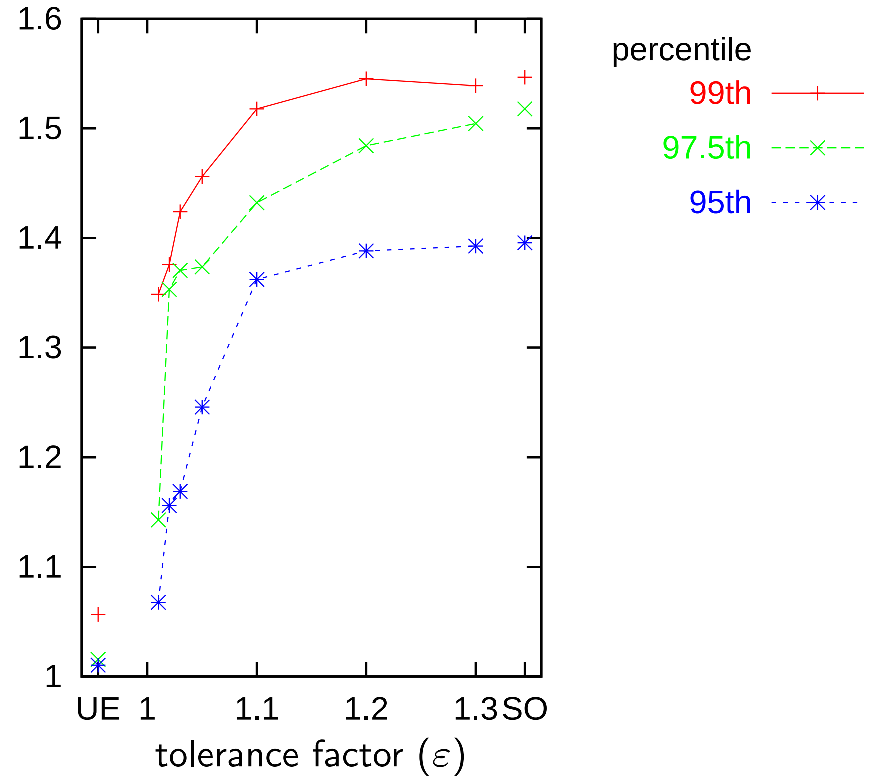
| Instance Name | $\|V\|$ | $\|A\|$ | $\|K\|$ | $\|A\| \cdot \|K\|$ |
|---|---|---|---|---|
| *Sioux Falls* | 24 | 76 | 528 | 40K |
| *Friedrichshain* | 224 | 523 | 506 | 265K |
| *Winnipeg* | 1,067 | 2,975 | 4,344 | 13M |
| *Neukölln* | 1,890 | 4,040 | 3,166 | 13M |
| *Mitte, Prenzlauerberg & Friedrichshain* | 975 | 2,184 | 9,801 | 21M |
| *Chicago Sketch* | 933 | 2,950 | 83,113 | 245M |
| *Berlin* | 12,100 | 19,570 | 49,689 | 972M |

# Unfairness Percentiles

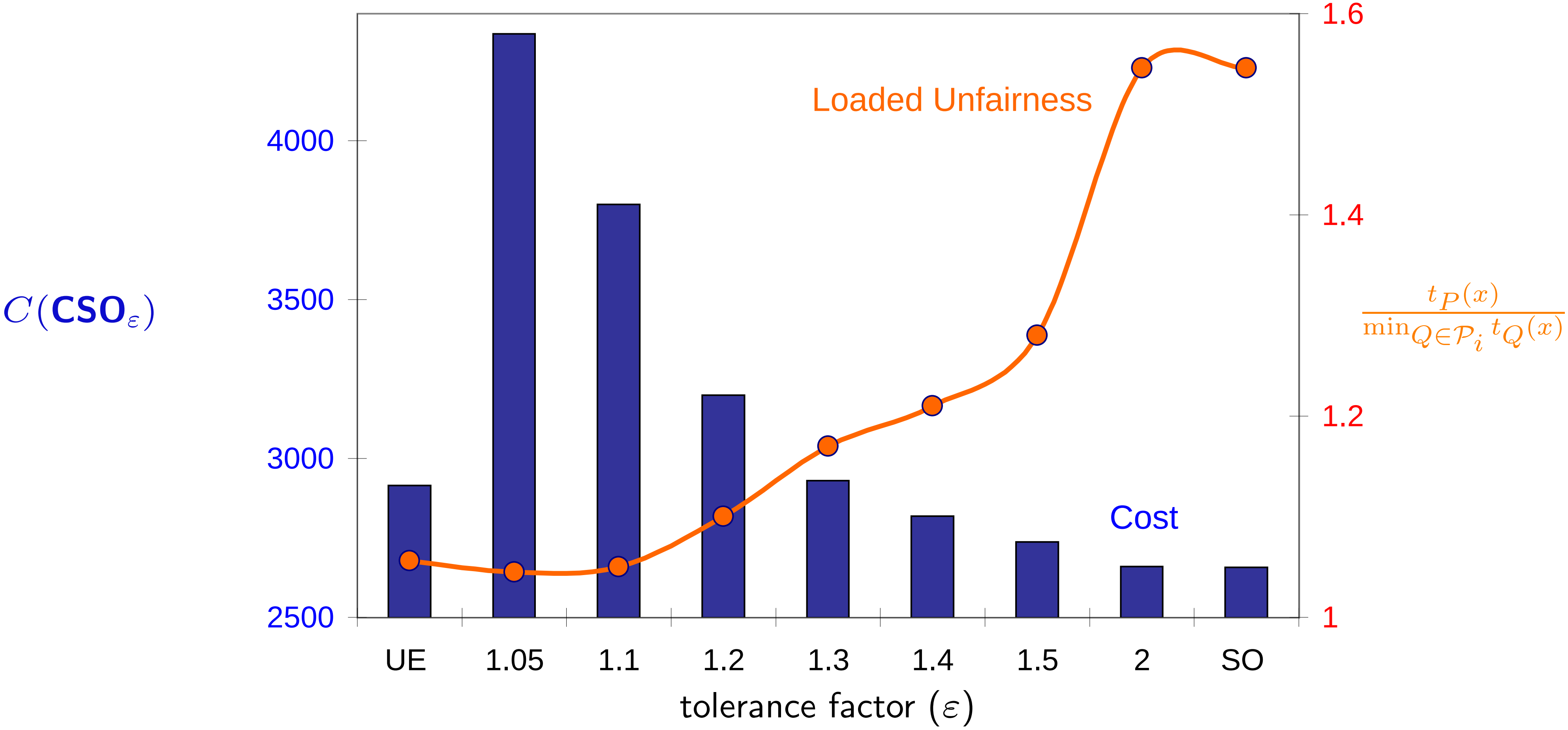
normal unfairness: controlled directly



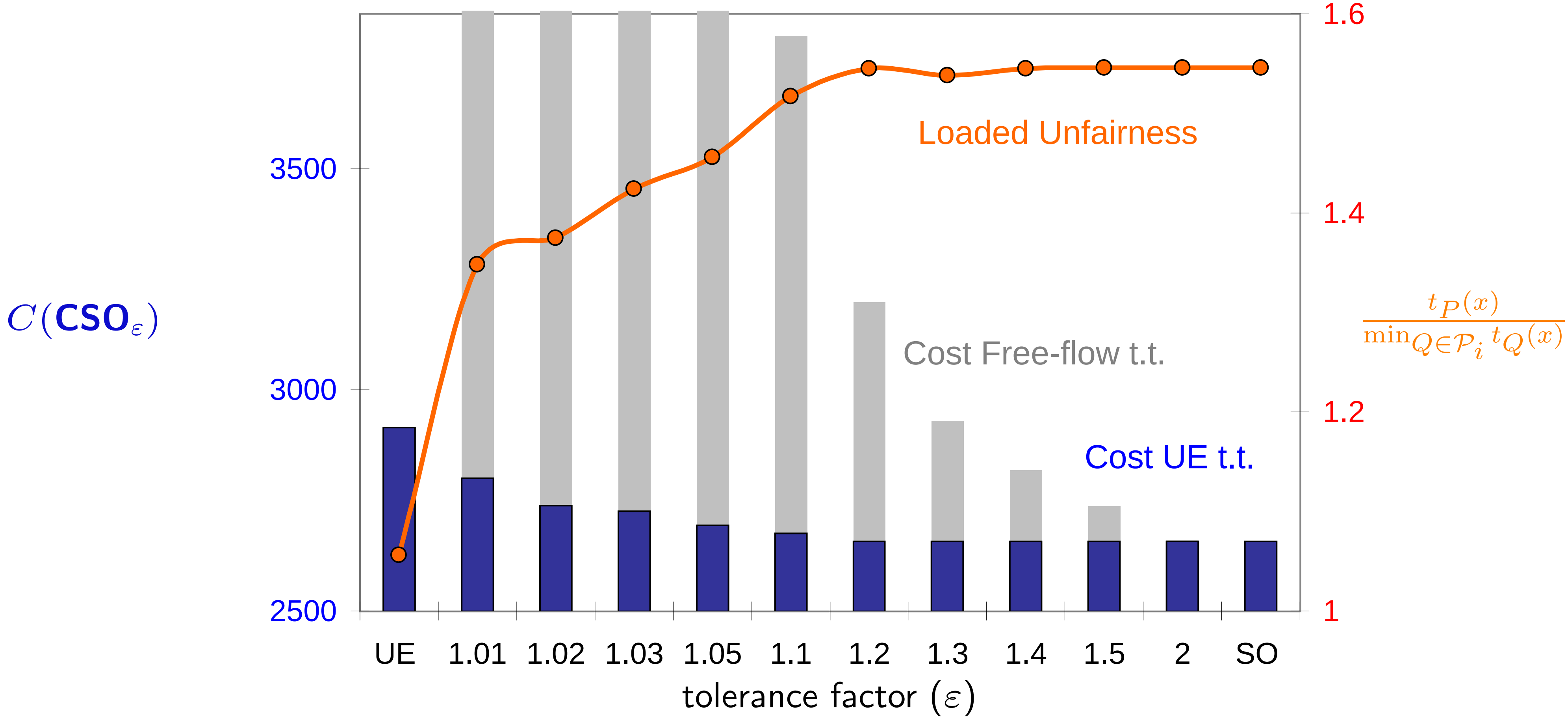

loaded unfairness: influenced



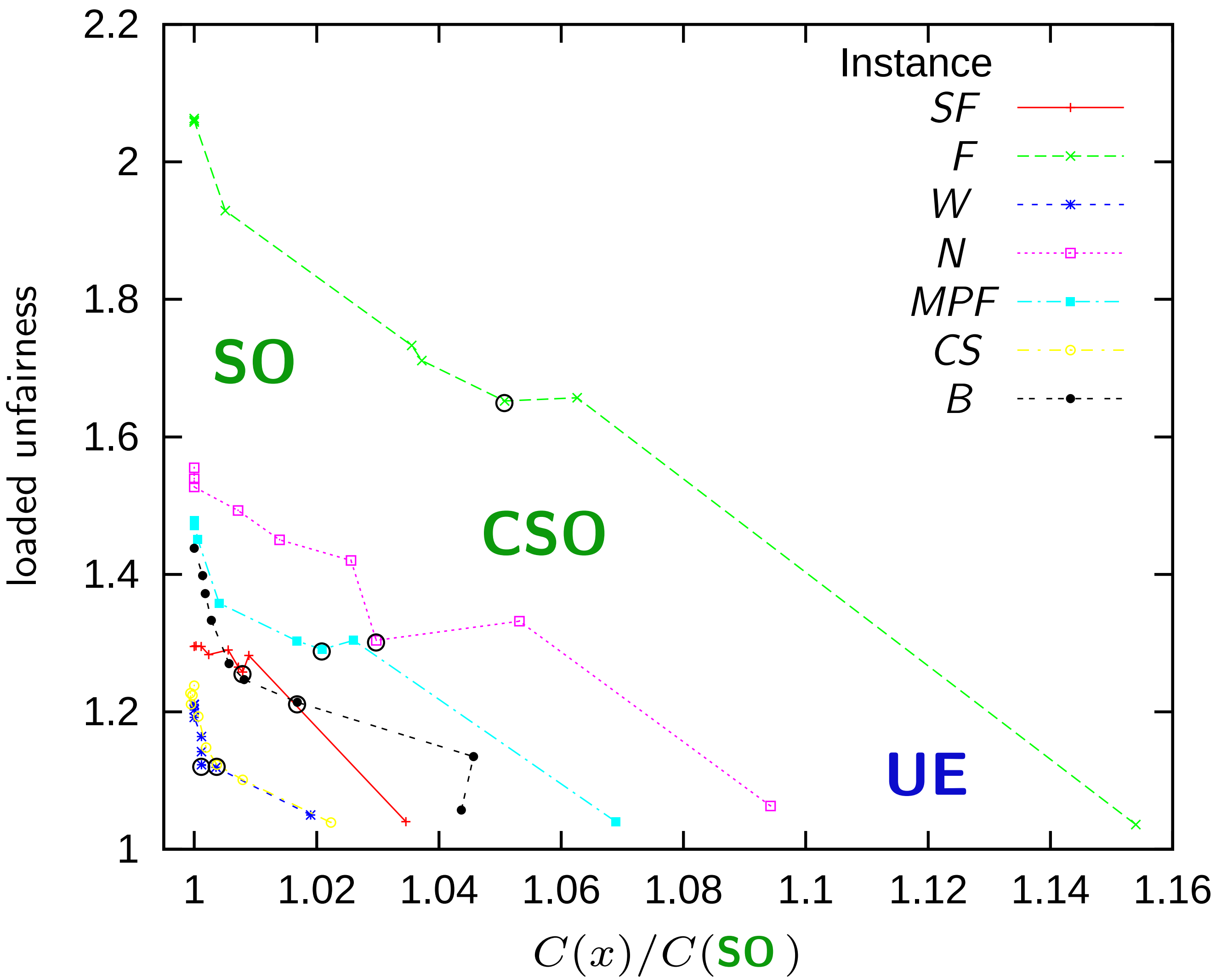
# Free-flow Normal Lengths: High Cost



# UE Travel Times as Normal Lengths: Good Choice



# CSO allows us to control the tradeoff between efficiency and unfairness




Solutions marked with '○' denote $\textbf{CSO}_{1.02}$

# Summary

- Optimization Approach to Route Guidance
  - Conventional route guidance methods focus on the individual
  - **SO** not implementable and **UE** not efficient
  - **CSO** is a better alternative: efficient and fair
- Software to compute **CSO**
- Used **CSO** as more realistic benchmark to compute Price of Anarchy
- Worst-case ratio for **UE**/**CSO**
  - "Free-flow Travel Times" as normal lengths: **UE**/$\textbf{CSO}_0 \approx 1$
  - "UE Travel Times" as normal lengths: **UE**/**SO** $=$ **UE**/**CSO**