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Associative Strength Theory of Recognition Memory for Pitch[1]

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Recognition memory for pitch was studied by means of a delayed comparison task, with the standard (S) tone and the comparison (C) tone separated by a variable delay interval (0–180 sec). Evidence is presented for the existence of an unsigned familiarity or similarity dimension, in addition to the signed pitch-difference dimension. Subjects relied on familiarity exclusively in the same–different judgment and used both dimensions in the higher–same–lower judgment. There appear to be two memory traces, short-term and intermediate-term, which are decaying exponentially to zero at very different rates. The decay of the short-term trace appears to be an essentially passive, temporal decay process, rather than an interference process, since the frequency-similarity and intensity of the interference tone have no effect on the rate of decay. The decay of the memory trace appears to have the same form and rate regardless of the initial level of acquisition and regardless of the frequency difference between the S and C tones.

INTRODUCTION

All psychophysical judgments involve some kind of memory, but only rarely is much attention given to the memory factors involved. This is completely justifiable, whenever it is reasonable to assume that the memory is perfect.

When memory is imperfect, but constant over all experimental conditions, it seems likely that there will be a gain in the theoretical precision of one's conclusions if the role of memory is specified in an explicit way. One's assumptions about memory in such situations can often be quite minimal (perhaps consistent with every existing theory of memory), but even a minimal account of the role of memory factors will often suggest new interpretations of the results. For example, just noting that memory

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may well be imperfect in a successive comparison experiment (regardless of interval) suggests that the jnd may be as much or more a function of the acquisition and decay properties of memory for the stimuli as of the "grain" of the internal representation of these stimuli.

When memory is imperfect and varying, as it undoubtedly is in successive comparison experiments where the durations of both of the stimuli and of the interval between them are independent variables, it is very important to include the role of memory in one's theory of the phenomena. Furthermore, it is necessary to make rather explicit assumptions about acquisition and decay of the memory trace, if perceptual and memory factors involved in the performance are to be disentangled.

Given that perceptual and memory factors in a psychophysical judgment have been explicitly distinguished, a third factor that must be considered is the decision process. The internal representatives of perceptual inputs and associated memory traces are transformed into responses, or else we cannot study them behaviorally. Under a particular set of instructions concerning what the alternative responses are, and what correspondence they are supposed to have to the alternative stimuli, subjects use some decision rule for transforming their internal representation of these stimuli into responses. This decision rule must be specified in any precise theory of a psychophysical judgment.

Furthermore, the observed dependence of responses (or response probabilities) upon payoffs and subjective probabilities suggests that the decision rules must usually have response-bias inputs as well as perceptual and memory inputs, and there is every reason to believe that these response biases may be influenced by anything that affects the subject's subjective probability of obtaining various types of perceptual and memory inputs to the decision system. Thus, we certainly cannot assume that response bias to say "higher" as opposed to "lower" in an intensity comparison task is invariant over different time intervals between the standard and comparison intensities. Such a consideration creates substantial uncertainty with regard to the interpretation of the "time-error" found in such situations.

The present paper is primarily concerned with developing and testing alternative theories of same–different (S-D) judgments of the frequency of two pure tones (the standard, S, and the comparison, C, tones) presented successively, and separated by a time interval that is filled with an interference, I, tone.

The present paper is divided into three major sections. The first section is devoted directly to the determination of what is judged in recognition memory for pitch and indirectly to the determination of the suitability in this task of the criterion decision rule of signal detection theory. The second section considers the acquisition, storage, and retrieval laws of recognition memory for pitch, discussing experiments in the context of a "molar" familiarity theory. The third part presents a more "molecular" associative strength theory which has greater generality and intuitive plausibility than familiarity theory and from which the laws of familiarity theory can be derived.

I. DECISION RULES: WHAT IS JUDGED TO DETERMINE "SAMENESS" OF PITCH?

For the last 30 years, successive comparison experiments on physically unidimensional stimuli have relied almost exclusively on higher–lower, greater–lesser type judgments. Viewed from the perspective of present-day decision theory (Luce, 1959; Tanner and Swets, 1954), the primary reason for avoiding "same" judgments was that they were subject to greater variations in response bias than were "higher–lower" judgments. Lacking any conceptual framework for the separation of response bias from sensory or memory factors, it is quite clear that one should use H-L judgments, rather than S-D judgments or H-S-L judgments. The reason for this is that "higher" and "lower" appear to have roughly equal response bias for almost all subjects under almost all circumstances, whereas the relative bias to say "same" differs greatly over subjects and conditions.

Of course, there is no guarantee that "higher" and "lower" have equal response bias in all conditions, so it is very desirable to have a decision theory for H-L judgments as well as for S-D and H-S-L judgments.

Assuming that we have confidence in our ability to develop an approximately correct decision theory for any successive comparison task, there is no longer any reason to be afraid of "same" judgments and a good reason to be interested in them. The reason is that even when the stimuli differ on a single psychological dimension, S-D judgments *may* be based on a different mechanism from that which determines H-L judgments.

There are at least three existing bits of evidence to support the plausibility of this suggestion. First, early introspective studies of successive comparison sometimes suggested that it was possible to be fairly sure that two tones were different in pitch, without having any idea which was higher or lower (Whipple, 1901).

Second, Tanner and Rivette (1964) noted that certain Indians who speak tone languages appear to be very deficient in H-L judgments. Particularly since they speak tone languages, it seems likely that they are very much less impaired or unimpaired in S-D judgments, though this has not been rigorously demonstrated, to my knowledge.

Third, it is conceptually very difficult, perhaps impossible, to extend a mechanism for H-L judgments on single dimensions to handle successive comparison (recognition memory) for dimensionally complex stimuli, such as verbal materials or nonsense forms. On the other hand, as shown below, it is quite possible to develop a theory of S-D judgments for a (locally and approximately) single dimension such as pitch, that generalizes without difficulty to multidimensional S-D judgments, provided one does not demand that the mechanism for judging similarity (or familiarity) be able to determine the direction of the difference (higher or lower).

However, these bits of evidence are hardly compelling proof that a different process mediates S-D judgments from that which mediates H-L judgments. Certainly, the most satisfactory existing theories of H-L judgments using the criterion decision rule of signal detection theory (Pierce and Gilbert, 1958; Sekey, 1963) can be easily extended to make predictions about S-D judgments and H-S-L judgments. The theory of Pierce and Gilbert extended in several ways will be referred to as "pitch-difference theory" (PDT), and its predictions will be compared to those of "familiarity theory" (FT), which claims that a different mechanism mediates "same" judgments (in either S-D or H-S-L tasks) from that which mediates H-L judgments.

Pitch-Difference Theory (PDT)

The PDT assumes that in an H-S-L task, at the time of presenting the C tone, the subject determines the pitch of the C tone, p_C, attempts to recall the S tone, recalling a pitch, p_S, and takes the signed difference between them, $h(\mathrm{C}, \mathrm{S}) = p_C - p_S$. He then says the C tone is lower than the S tone, iff $h \leqslant c_L$, he responds "same," iff $c_L < h \leqslant c_H$, and he responds "higher," iff $c_H < h$. In an S-D task, the subject does the same thing, except that he lumps H and L together, responding D in either case. Of course, there is no reason to assume that the criteria, c_L and c_H, are necessarily the same for S-D as for H-S-L tasks.

In H-L tasks, the subject sets only one criterion, which partitions the h dimension into two regions. Naturally, this criterion need not be at zero, and may be a function of many task variables (excluding C-S).

Incidentally, for the purposes of this paper, PDT is essentially equivalent to a theory which assumes that it is the pitch ratio, p_C/p_S, which is judged rather than the pitch difference $p_C - p_S$. What is crucial to PDT is that the comparison of S and C be "directional" or "noncommutative," i.e., $h(X, Y) \neq h(Y, X)$.

The above criterion decision rules for H-S-L, S-D, and H-L tasks are easily extended to yield a more extensive partitioning of the (psychological) pitch-difference dimension h, as is necessary, for example, in choosing among the following set of nine responses: "very sure it was higher" (sure-high), "pretty sure it was higher" (high), "think it was higher, but might have been the same" (high-same), "think it was the same, but might have been higher" (same-high), "same" (sure-same), "think it was the same, but might have been lower" (same-low), "think it was lower, but might have been the same" (low-same), "pretty sure it was lower" (low), "very sure it was lower" (sure-low). According to PDT, each one of these nine responses is made, iff the value of h lies in the particular region corresponding to that response. The nine regions of the h dimension are determined by eight criteria, ordered in the obvious way.

Since subjects do not have perfect perception of the pitch of the C tone and do not have perfect memory for the pitch of the S tone, it is necessary to assume that p_C and p_S are, in general, random variables. It is plausible and convenient to assume that both are normally distributed, $p_C \sim N[C', \sigma_C]$, $p_S \sim N[S', \sigma_S]$ with $\sigma_S > \sigma_C$, since σ_S is subject to both perceptual and memory error, while σ_C is subject to only perceptual error. Thus, $h = p_C - p_S \sim N[C' - S', (\sigma_C^2 + \sigma_S^2)^{1/2}]$.

Let S be the frequency of the S tone, C the frequency of the C tone, I the frequency of an interference tone in the delay interval, i_S the intensity of the S tone, i_C the intensity of the C tone, i_I the intensity of the interference tone, t_C the duration of the C tone, t_S the duration of the S tone, and t_I the interval (delay) between the S and C tones.

Certainly, in general: $\sigma_C = \sigma_C(C, t_C, i_C)$ and $\sigma_S = \sigma_S(S, t_S, i_S, I, t_I, i_I)$. The dependence of σ_C and σ_S on C and S, respectively, is one way of accounting for the change in the *jnd* with frequency. Within a limited range (100 or 200 cps), it will be assumed that σ_C and σ_S are approximately constant with respect to C and S, respectively.

The dependence of σ_C and σ_S on t_C and t_S, respectively, expresses the fact that perception of frequency and also the memory trace do not develop instantaneously after experiencing one cycle, but require considerable time to reach maximum—on the order of one sec. (Konig, 1957; Turnbull, 1944).

The dependence of σ_S on t_I allows us to express the deterioration of the memory trace for pitch with time. (Earlier studies are reviewed by Harris, 1952; also, see Konig, 1957, Bindra, Williams, and Wise, 1965, and Wickelgren, 1966). As pointed out by Harris (1952), experiments in which t_I has had little effect were always experiments in which only one S tone was used. This is extremely conductive to the development of long-term memory for the one S tone, which would not be expected to be decaying appreciably over delays on the order of seconds or minutes. This suggestion of Harris also applies to the recent study by Aiken and Lau (1966), who found no effect of delay on pitch discrimination and who also used only a single S tone.

The dependence of σ_C on i_C and σ_S on i_S is a substantial effect near the absolute threshold for detection of the tone, but apparently there is no significant effect beyond 30 db above the subject's absolute threshold level (Harris, 1952; Shower and Biddulph, 1931). It is not known whether σ_S depends on I or i_I, and results bearing on this will be reported in the present paper.

In general, $C' = C'(C, t_C, i_C)$ and $S' = S'(S, t_S, i_S, I, t_I, i_I)$. Of course, one would not have an internal representation of frequency, if C' were not a function of C and S' a function of S. Within a limited range (100 or 200 Hz), it will be assumed that C' and S' are approximately linear functions of C and S, respectively.

Some investigators claim that the mean perceived pitch (C' or S') is also affected by the time for which the tone is presented (t_C or t_S), but there is disagreement about the direction of the effect (Stevens and Davis, 1938), suggesting that the previous methods of studying this may be subject to considerable effects of response bias. Certainly, the effect can be neglected for tones longer than 1 sec. Whether this is a large effect for tones lasting less than 1 sec. remains to be established. Some evidence suggesting a very small effect will be presented later in this paper.

The dependence of S' on t_I is the familiar question of time error, with some previous studies yielding the classical negative time error (Tresselt, 1948; Wada, 1932), some showing assimilation effects (Koester and Schoenfeld, 1946), and others indicating little or no effect at all (Koester, 1945; Postman, 1946). Such a pattern of findings suggests the presence of uncontrolled and variable response biases. Aided by modern decision theory it is possible to eliminate the effects of such response biases, often making it possible to answer definitively such questions as the existence of time error.

Early reports of a large effect of the intensity of a tone (i_C or i_S) on its mean perceived pitch (C' or S') by Stevens, 1935, and Snow, 1936, have not been found by subsequent investigators (e.g., Morgan, Garner, and Galambos, 1951, and Cohen, 1961). The best guess at present is that effects of intensity on mean pitch are so small they can be neglected.

All of the possible functional dependencies so far discussed can be summarized in the following way. The mean and standard deviation of a perceived or a remembered pitch may depend on the frequency, intensity, and duration parameters of its presentation, and, in the case of the remembered pitch, may also depend on the frequency, intensity, and duration parameters of any tone interpolated in the delay interval. It is important to note what functional dependencies are implicitly assumed to be absent, namely, effects of the properties of the S tone on the pitch distribution produced by the C tone. It is also assumed that the properties of the C tone do not affect the pitch distribution of the remembered S tone. These two classes of independence assumptions are the key features of PDT. Note that we are postulating the absence of an effect of the S tone on the perceived pitch of the C tone. However, the "total" perception of the C tone, particularly h, is, of course, affected by the nature of the S tone.

These assumptions are the heart of PDT. If they are invalid, PDT is not likely to be elegantly modifiable to incorporate the effects. On the other hand, if these assumptions are valid, if one dimension is judged in successive comparison of pitch, and if that dimension is h, then PDT provides a rather elegant and comprehensive framework for the description of many other types of findings about successive comparison of pitch, regardless of the nature of these findings.

Also, it is obvious that PDT, in the form of functional equations with unspecified functions, can be applied to any type of successive comparison on one psychological dimension.

Familiarity Theory (FT)

The FT differs from PDT primarily in its assumption concerning the dimension judged in S-D and H-S-L tasks. The FT assumes that the judgment of "sameness" is based on the *familiarity* of the C tone. Familiarity is another dimension of the "total perception" of the C tone, just like pitch (p_C) and pitch-difference (h). According to FT, a C tone has a certain familiarity, which is greater, the closer the C tone is to the S tone in frequency and in time. (The intensity similarity of the C and S tones will always be assumed to be held constant at identity in the present paper.) Thus, familiarity is a generalization of the concept of similarity since familiarity is assumed to encode the similarity of two stimuli in both qualitative attributes and time of occurrence.

The FT decision rule for S-D judgments is the criterion rule: "Respond same, iff $f(\mathrm{C, S}) - c \geqslant 0$," where f is the familiarity of the C tone, and c is the criterion for a same response. The f must be considered a random variable, and it is assumed to be approximately normally distributed: $f(C, S) \sim N[\bar{f}(\mathrm{C, S}), \sigma]$. The criterion rule is easily extended to handle confidence judgments accompanying an S-D judgment in exactly the manner described by Egan (1958), Wickelgren (1966), and Wickelgren and Norman (1966).

The FT cannot handle H-L judgments because familiarity codes the distance of the C tone from the S tone, but does not code the direction of the distance. Both $|h|$ and f provide measures of distance of the C tone from the S tone on the frequency dimension, but these measures are different. In fact, f decreases with increasing C-S, while $|h|$ increases. Both are undoubtedly functions of the same variables, e.g., $f = f(C, t_C, i_C, I, t_I, i_I, S, t_S, i_S)$, and monotonic functions in exactly opposite ways, but there is not necessarily any simple relationship between them. Incidentally, when a random variable such as f is said to be a function of other variables, it means that the parameters of its distribution function are functions of these variables. In any event, f cannot mediate H-L judgments, so if FT is correct for S-D tasks, some other dimension must be judged for H-L tasks, possibly the h dimension of *PDT*.

PDT and H-S-L Judgments

The FT and PDT make contrasting predictions about both H-S-L and S-D tasks. The PDT must claim that an H-S-L task is performed in essentially the same way as an H-L task, with different criterion placements and, of course, somewhat different labels on some of the responses. The left half of Fig. 1 illustrates the criterion place-

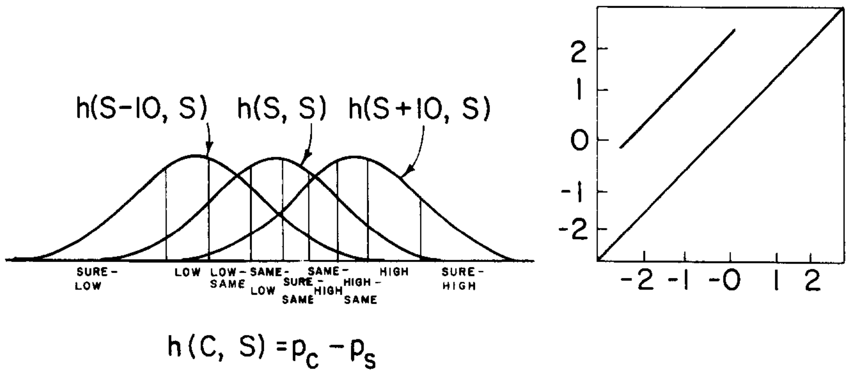

FIG. 1. Criteria and distributions for H-S-L judgments according to PDT, on left; expected type of operating characteristic, on right.

ments in an H-S-L task and the distributions of h, according to PDT, for three cases: where the C tone is 10 Hz lower than the S tone, where the C tone has the same frequency as the S tone, and where the C tone is 10 Hz higher than the S tone. Assuming these distributions are normal implies that an operating characteristic, obtained by plotting the cumulative of any one of these distributions against the cumulative of any other distribution, will be a straight line on normal–normal probability coordinates. Such an operating characteristic is shown in the right half of Fig. 1.

Over a 20-Hz range it is completely reasonable to assume that σ_C is a constant, independent of C. Assuming that the variation in the pitch of the recalled standard tone is independent of the frequency of the C tone (σ_S independent of C), the three (normal) distributions of h in Fig. 1 must all have the same standard deviation. This implies that the operating characteristic obtained by plotting any one of these distributions (cumulatively) against any other will have a slope of unity on normal–normal probability coordinates. The "distance" of the operating characteristic from the chance diagonal (d') will then provide a measure of the distance between the means of the two h distributions in units of their common standard deviation.

FT and H-S-L Judgments

The FT claims that H-S-L judgments will be done in two steps: first, an S-D judgment based on the f dimension and second, an H-L judgment based on some other dimension, for instance, the algebraic sign of the h dimension of PDT. Even assuming that the f dimension is judged first, there are still many different ways that the subject might combine these two steps into one H-S-L judgment. The simplest to analyze is the following: Suppose the subject uses f (which is always positive) to obtain a measure of the absolute value of the distance of the C tone from the S tone on the pitch dimension. Since this distance measure is a different measure of the absolute value of pitch difference than $|h|$, call it $|h'|$. The $|h'|$ is some monotonic decreasing function of f converging to zero as f approaches ∞. Thus, if f has a unimodal distribution, so does $|h'|$. Having determined $|h'|$, the subject converts $|h'|$ to $h' = \pm |h'|$ by making a single-criterion H-L judgment on the h dimension. Since there must be some significant probability of error in this H-L judgment for C tones not too different from the S tone in frequency, one can obtain bimodal distributions of h' for C tones above and below the S tone. This is true provided that the probability of an erroneous H-L judgment on the h dimension does not fall off "too" rapidly with increasing $|h'|$. In particular, it is true if h and $|h'|$ are independent. However, one must obtain either a unimodal distribution of h' for a C tone identical to the S tone, or obtain a bimodal distribution whose two modes are both inside the two modes for the case where $C \neq S$. Thus, according to FT, it is possible to obtain distributions on the h' dimension and operating characteristics that look like those in Fig. 2.

If one plots the cumulative of a bimodal distribution against any distribution all of whose modes lie within the two modes of the bimodal distribution, then the operating characteristic so obtained will be very nonlinear (S-shaped) on normal–normal probability coordinates. These operating characteristics will start only a little above or even below the chance diagonal, rise steeply above the chance diagonal in some region, and then level off. The slope of the steeply rising portion will be much greater than unity. If such strange operating characteristics are obtained for H-S-L judgments, it will be very strong evidence for familiarity theory as opposed to pitch difference theory.

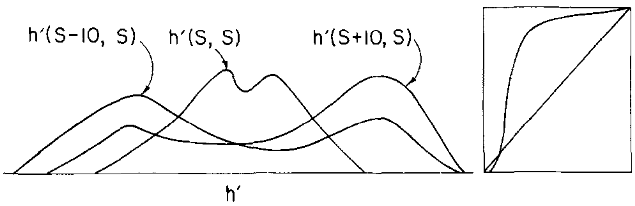

FIG. 2. Distributions for H-S-L judgments according to one dual-judgment extension of FT, on left; expected type of operating characteristic, on right.

FT and S-D Judgments

The FT assumes that S-D judgments are based on normal *f* distributions as shown in Fig. 3. Thus, FT predicts that operating characteristics for S-D judgments will be straight lines on normal-normal probability coordinates. If σ is a constant, independent of C-S, the operating characteristics will have unit slope. Nothing in FT is inconsistent with the assumptions that σ is a function of C-S, but it would certainly be computationally convenient if σ were approximately constant over C-S.

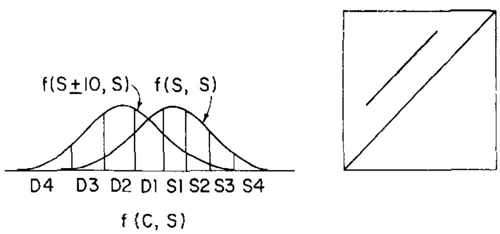

FIG. 3. Criteria and distributions for S-D judgments according to FT, on left: S = same, D = different, 4 = greatest confidence, and 1 = least confidence; expected type of operating characteristic, on right.

PDT and S-D Judgments

The PDT does not make any simple predictions about the shape or slope of operating characteristics obtained from S-D judgments. According to PDT, such judgments are made by determining the value of h, converting it to $|h|$, and making an S-D judgment on $|h|$. If h has a normal distribution, $|h|$ for C tones identical to and different from the S tone will have distributions similar to those illustrated in Fig. 4.

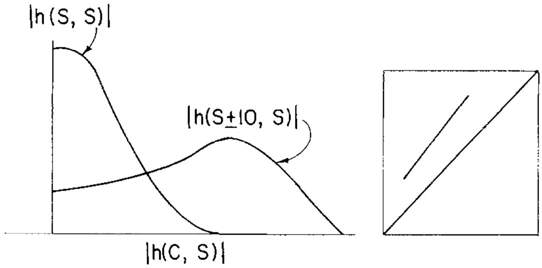

FIG. 4. Distributions for S-D judgments according to PDT, on left; expected type of operating characteristic, on right.

Plotting operating characteristics for some pairs of distributions of this type demonstrates that they would be extremely difficult to distinguish from the operating characteristics obtained from overlapping normal distributions of slightly unequal variance. In the first place, they are very close to being straight lines on normal–normal probability coordinates. In the second place, the slope of the best-fitting straight line (in the empirically obtainable section of the operating characteristic) increases very gently from 1 to 1.3 as the distance between the two h distributions increases from 0 to 2 standard deviation units, then at some point the slope begins to decrease gently to 1.2 for distances of 4 standard deviation units. Even if unit slope for operating characteristics were a definite prediction of FT, slopes of 1.2 are rather difficult to distinguish from slopes of unity in the present experiments. Thus, tests of the shape and slope of the operating characteristic for S-D judgments are not promising as ways of distinguishing between pitch-difference and familiarity theories of "same" judgments.

Experiment H-S-L

Since the most definitive task for determining the existence of a familiarity dimension in successive comparison of pitch appears to be an H-S-L task, such an experiment was performed. Much of the description of this experiment will also apply to all other experiments reported below.

Procedure. Subjects listened to an S tone lasting 3 sec, followed by an I tone lasting t_I sec, followed by a C tone lasting 1 sec, followed by 4 sec in which to make a decision about the C tone in relation to the S tone, followed by about 3 sec in which the number of the next trial was announced. Subjects used the 9-category H-S-L response scale described earlier.

Design. There were 10 different S tones, ranging from 400 to 490 Hz in 10-Hz increments. The I tone was always 930 Hz. There were 9 different values of t_I : 1, 2, 4, 6, 12, 24, 45, 90, and 180 sec. For each value of t_I there were 3 values of C-S : 0, 10, and -10 Hz. C-S $= 0$ was presented as often as C-S $= 10$ or -10 put together. Conditions were presented randomly in blocks of $9 \times 4 = 36$ trials, with 2 blocks per set. Three practice trials preceded the 72 experimental trials of every set. One set was given in a 1-hour session, with subjects taking 0, 1, or 2 sessions per day (never less than 2 hours apart). There were 5 different sets each taken 10 times, after an initial practice session, or 51 1-hour sessions for each of 3 subjects.

Subjects. Subjects were run in groups of 1, 2, or 3, as their schedules permitted. Two of the subjects (BF and DW) were M.I.T. undergraduates, and the third (MS) was the assistant who ran the experiment. All were paid, and all had participated (for 51 hours) in the identical experiment, but using S-D judgments, and confidence (1–4), prior to participating in this experiment.

Instructions. Subjects were instructed to concentrate on each tone as it was presented, and not to attempt to rehearse the S tone during the I tone. During the initial practice session, they were instructed to shift their criteria for the 9 different responses so as to use each with relatively equal frequency. However, after the first session they were told to maintain whatever criteria they had settled on and never change them for the next 50 sessions. Unless otherwise specified, these are the standard instructions for all the experiments to be reported herein. In general, subjects who have difficulty in following any of these instructions during the practice session or who say they might become excessively bored before the end of the experiment are discarded.

Stimulus control and sources of error. Because of unavailability of some equipment, limited availability of other equipment, and in the interests of efficient data collection, the stimulus control in this and all other experiments reported herein is far from what can be achieved by other methods. Nevertheless, it is more than adequate for the principal purposes of the present study. The method was to produce the three tones on three different Hewlett Packard oscillators (two model 200CD and one model 201C) using the dial on the oscillator to set the frequency, but checking the agreement of the S and C tone oscillators' dials periodically during the experiment (by human immediate-successive comparison). A part of the dial was used which permits setting within ± 1 Hz. The durations of tones were controlled by Massey-Dickinson timing units in the present experiment (absolute accuracy is $\pm 3\%$) and by Hunter timer in some other experiments (absolute accuracy is $\pm 5\%$). The intensities of the S, I, and C tones were set to be approximately equal in loudness (human immediate-successive comparison) and at a comfortable listening level, with no further specification. Signals were gaited with no attempt to eliminate transients, which appeared as slight clicks at the onset of each tone. Another source of error was introduced by the necessity of recording all the stimuli on tape and playing back over a loudspeaker, using an Ampex tape recorder (model $PR - 10$—timing accuracy $\pm 0.25\%$; or $F - 44$—timing accuracy $\pm 1\%$). The timing accuracy is of no significance for its effect on the durations of signals, but only for its effect on the frequency of signals.

The theoretical significance of duration and intensity errors of the size found in the present experiments is nil for successive comparison of pitch. However, the random error in control of stimulus frequency places certain limitations on the conclusions that can be drawn from these experiments. The principal limitation is that it is impossible to separate the external contributions to the noise from the internal contributions. Since both h and f are measured in units of the standard deviation of the total noise, greater external noise means that the values of h and f will be proportionately lower. Thus, the present experiments will underestimate the absolute sensitivity of the organism, but any substantial differences in sensitivity of the organism under different conditions will still be observed and, when observed, they will not be confounded in any way by the added external noise of the present procedure. Qualitative findings are particularly invulnerable to this external noise, and there is reason to believe that certain quantitative findings, such as the form and the rate parameters of consolidation and decay curves, are relatively unaffected by the degree of external noise found in the present experiments.

To get a check on the overall accuracy of the frequency control using the present setup, a sample was taken of the taped tones played back over the Ampex *F*-44 tape recorder and analyzed for frequency using a Hewlett Packard electronic counter. The sampling period was 1 sec and the delay interval between samples was 0.2 sec. Out of 300 such samples of the 930-Hz I tone, 298 were within the accuracy limit of the counter (± 1 count in 1 sec or ± 1 Hz or $\pm 0.1\,\%$), the other 2 were $-0.5\,\%$ and $-1\,\%$. The two deviations were widely separated in time. Thus, there is no steady drift in speed for the tape recorder, which might have complicated the interpretation of consolidation and decay curves to some extent. Furthermore, the large deviations are very infrequent. Thus, we may conclude that the methods of the present study are adequate for pitch comparison studies where the presence of an interference tone and the use of generally longer delay intervals between S and C tones increases the "difference limen" far above the 2 to 6 Hz found for shorter delays with no I tone. Incidentally, the electronic counter revealed a constant error of about $+0.3\,\%$ in the calibration accuracy and/or human error in setting the oscillators. This, of course, has no significance for the present study.

Results. The results of H-S-L are in striking confirmation of familiarity theory. The operating characteristics in Fig. 5 are highly nonlinear (except some with only a few points on or very near the chance diagonal) in exactly the way predicted by familiarity theory. The steeply rising portion of these operating characteristics and the frequency of points at or below the chance diagonal for operating characteristics whose other points are far above the chance diagonal establishes quite definitely that PDT is incorrect for H-S-L judgments, and indicates that FT is correct, at least to a first approximation. No statistical test exists to test the fit of these operating characteristics to straight lines. However, a Wilcoxon Signed-Ranks test on the slopes of the best-fitting straight lines through these points allows rejection of the unit slope hypothesis at beyond the .01 level for each of the three subjects.

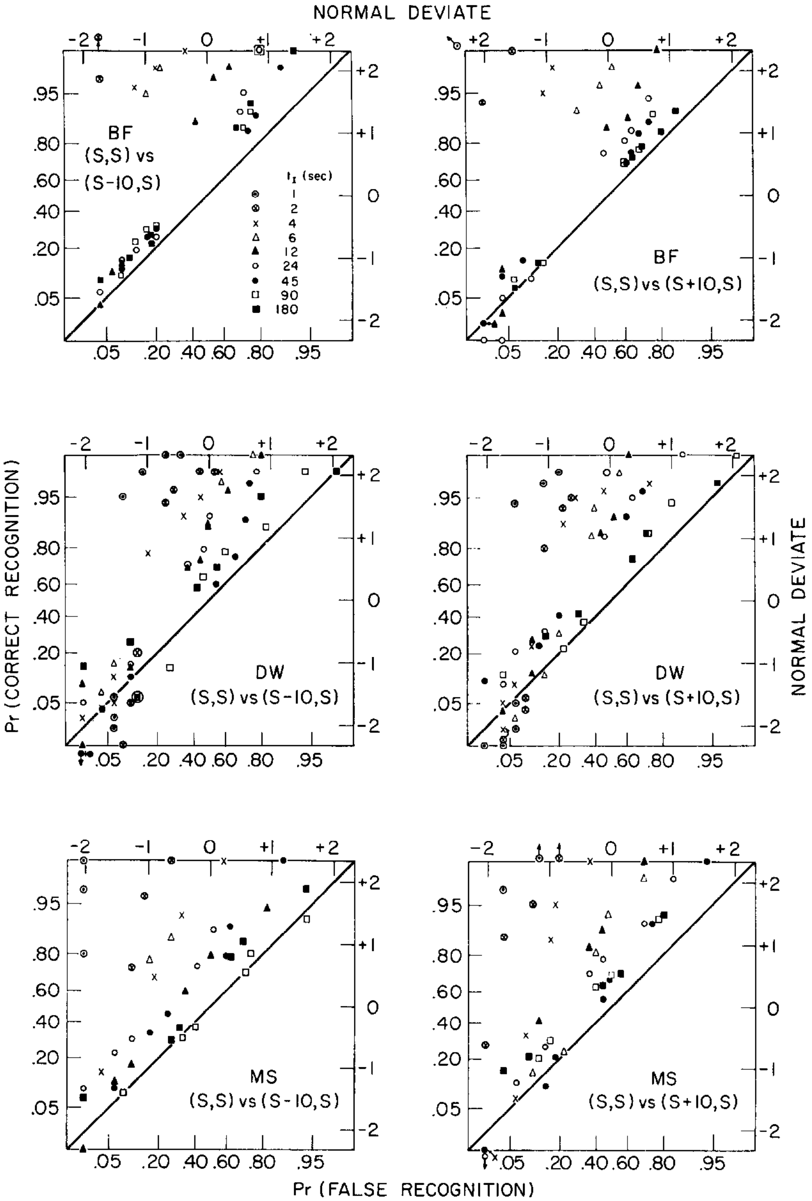

FIG. 5. Operating characteristics for H-S-L judgments from subjects BF, DW, and MS, with condition (S, S) plotted against condition (S-10, S) on the left and condition (S, S) plotted against condition (S $\pm$ 10, S) on the right.

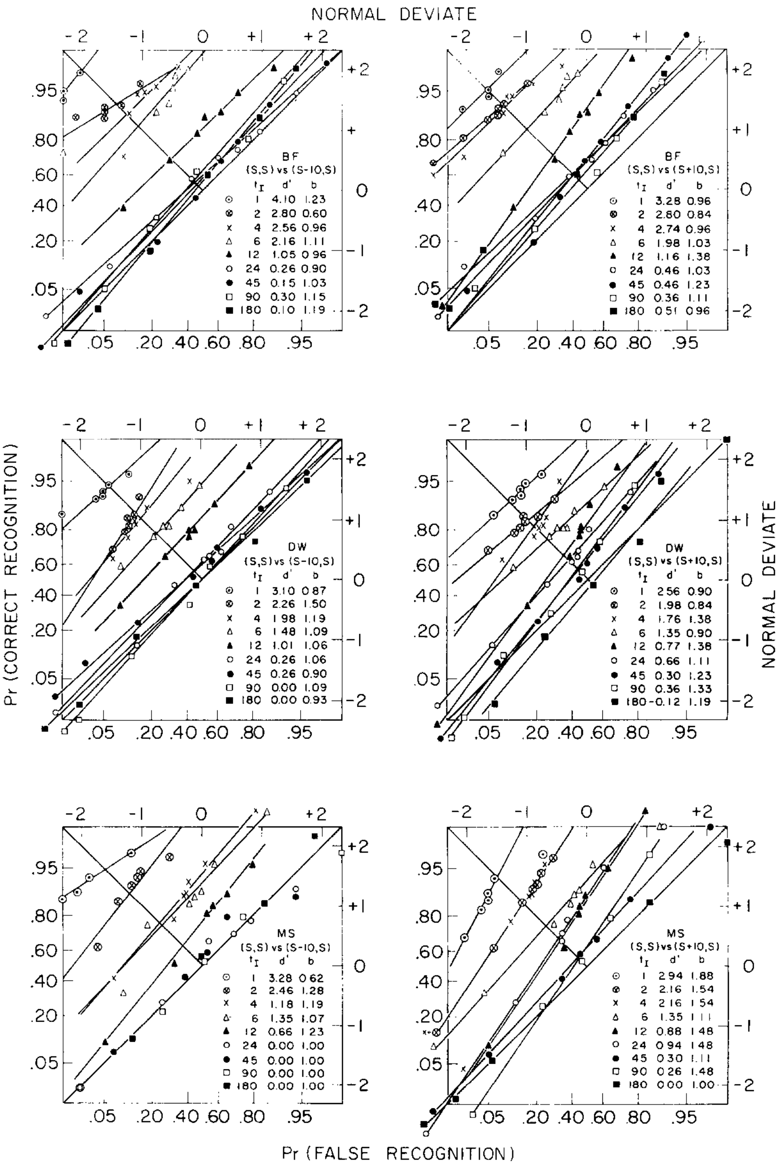

FIG. 6. Operating characteristics for S-D judgments from subjects BF, DW, and MS for exactly the same conditions as in Fig. 5.

Operating characteristics obtained for the same conditions using S-D judgments are shown in Fig. 6. Slopes (b) of the best-fitting straight line are also given in Fig. 6, along with the d' value determined by entering the tables of Elliott (in Swets, 1964, pp. 651–684) with the point at which the best-fitting straight line intersects the negative diagonal. The operating characteristics for the S-D task are much better fit by straight lines on normal–normal coordinates than the operating characteristics obtained for the H-S-L task, and there are no S-shaped curves that start below the chance diagonal and rise steeply to a point far above the chance diagonal.

Slopes of the (visually) best-fitting straight lines are not too far from unity, athough there is a slight tendency in all three subjects for slopes to be above unity. The tendency is insignificant for two of the subjects and significant at the .01 level for MS using a Wilcoxon Signed-Ranks test. Furthermore, there is no significant correlation of slope with d', indicating that the standard deviations of the strength distributions in S-D judgments are constant, independent of their mean. The present results contrast sharply with the strong positive correlation between mean and standard deviation (negative correlation between d' and slope) observed in most signal detection experiments (e.g., Nachmias and Steinman, 1963; Swets, Tanner, and Birdsall, 1961).

If the dual judgment theory of H-S-L judgments is correct, as it seems to be, then subjects are using f to determine $|h'|$ and converting this to $\pm h'$ by an H-L judgment on h. If we make the further assumption that they place criteria for H-S-L responses symmetrically about the middle-same region on the h' dimension, then it should be possible to obtain from the H-S-L data the same operating characteristics as we obtained in the S-D task by folding the h' axis and combining response categories at equal distances from the "sure-same" category. Thus, same-low is combined with same-high, high-same is combined with low-same, low is combined with high, and sure-low is combined with sure-high. According to the dual judgment theory, this folding undoes the H-L judgment that the subjects made, leaving only the S-D judgment on the familiarity dimension of FT.

Operating characteristics obtained from this folding of the H-S-L data are shown in Fig. 7. Like the operating characteristics for the S-D task, these operating characteristics are well fit by straight lines on a normal–normal plot. Comparing the 18 operating characteristics for this folding of the H-S-L data with the corresponding 18 operating characteristics for the S-D data yields no significant differences in d' for any of the three subjects, and no significant differences in slope for two of the three subjects (Wilcoxon Signed-Ranks test). BF's operating characteristics had consistently steeper slope in the reflected H-S-L data than in the S-D data, but the differences were not large. BF also had slightly, but not significantly, lower d' values in the reflected H-S-L data than in the S-D data. This pattern of results for BF suggests some small violation of the assumption that the criteria on h' were symmetrically placed rather than any inconsistency with the basic features of FT and the dual judgment theory of

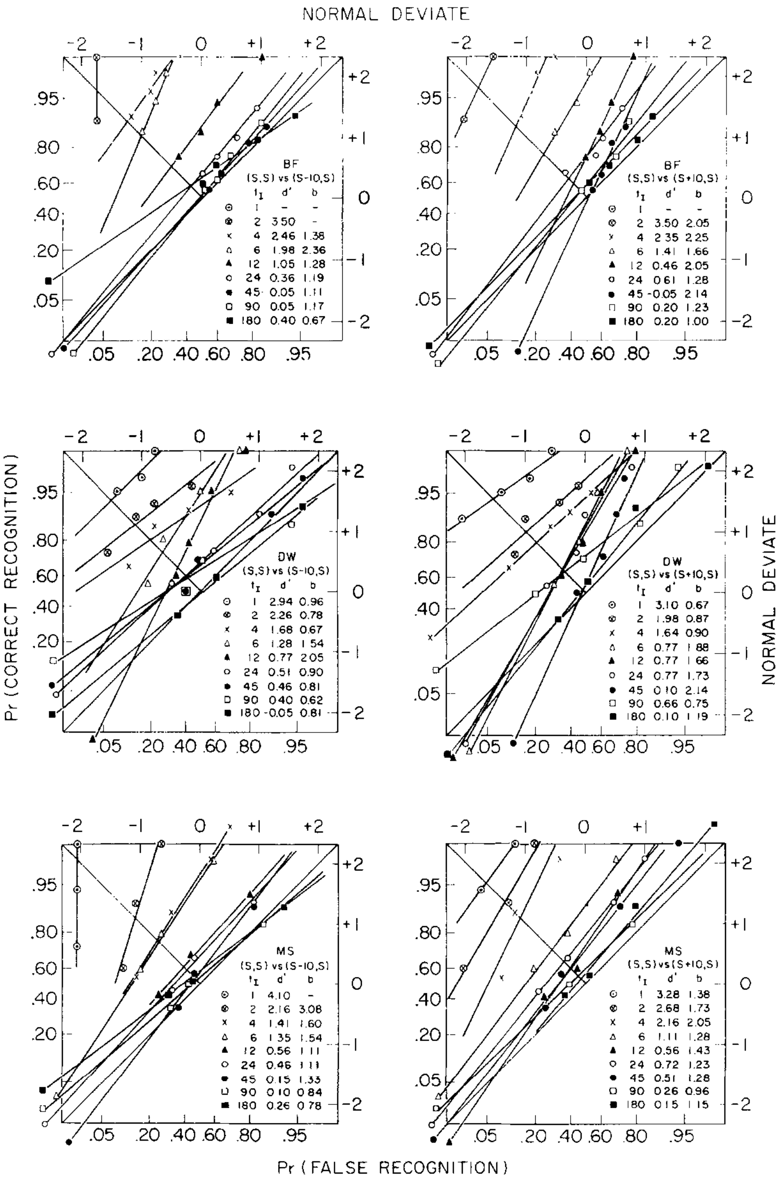

FIG. 7. Operating characteristics obtained by folding the H-S-L data for subjects BF, DW and MS about the "sure-same" category.

H-S-L judgments. The results of these experiments are in very close agreement with the predictions of FT both for S-D tasks and for the first phase of H-S-L tasks.

SI, CI Comparison Theories

Before leaving the question of what is judged in our S-D and H-S-L tasks, let us consider the possibility that the subject performs these tasks by comparing the S tone to the I tone (either the pitch difference or the pitch ratio), remembers this difference or ratio over the I tone, and then compares this difference or ratio to the difference or ratio of the C and I tones. There are three difference comparison processes being assumed by this theory: the SI comparison, the CI comparison, and the SI, CI comparison (which is the comparison of the results of the other two comparisons), and there are a variety of possible assumptions we can make about the nature of these comparisons.

As before, it makes no difference for the purposes of this paper whether $h(\mathrm{S}, \mathrm{I}) = p_S - p_I$ or p_S/p_I, whether $h(\mathrm{C}, \mathrm{I}) = p_C - p_I$ or p_C/p_I, and whether $H[h(\mathrm{S}, \mathrm{I}), h(\mathrm{C}, \mathrm{I})] = h(\mathrm{S}, \mathrm{I}) - h(\mathrm{C}, \mathrm{I})$ or $h(\mathrm{S}, \mathrm{I})/h(\mathrm{C}, \mathrm{I})$. Ratios of normally distributed random variables have Cauchy distributions. But since Cauchy distributions are unimodal and not too asymmetric for values of p_S and p_C far from zero, the differences between Cauchy and normal distributions will not be experimentally detectable, at present. Furthermore, the h functions could even be commutative (similarity) functions. The primary question of interest in this paper is whether H is commutative or not, and the results of the H-S-L experiment are as definitive in rejecting a directional SI, CI comparison as they are in rejecting a directional S, C comparison.

Is it possible to decide between a nondirectional S, C comparison (as assumed by FT) and a nondirectional SI, CI comparison? A verbalized SI comparison (e.g., "a little over an octave") is ruled out by the rapid and substantial decay observed with increasing t_I, but a nonverbal comparison of this type is harder to disprove. However, the results of an experiment varying the distance of the I tone from the S tone make even a nonverbal SI, CI comparison seem very unlikely, and the specific arguments against the SI, CI comparison theory will be presented later in the paper in conjunction with these results. One fact might be mentioned now, namely that performance is much poorer, and the decay rate much greater with the I tone present than with it absent. This certainly suggests that the I tone is not used by the subjects to bridge the delay interval, but it does not prove it.

II. MEMORY LAWS OF FAMILIARITY THEORY

In Section I, nothing was said about the properties of familiarity, except how it is used by the decision system to make S-D and H-S-L judgments. In Section II, we present the acquisition and decay properties of f, along with the dependence of the determination of familiarity in retrieval on the duration of the comparison tone.

The following five topics are discussed: First, is f a single memory trace or a combination of two or more separate memory traces with different decay rates? Along with this question is an attempt to determine the form of the decay function and the approximate rates of decay of the traces involved. Second, is this a passive temporal decay process, or is a storage or retrieval interference interpretation more plausible? Third, what is the approximate form of the acquisition function and the approximate rate of acquisition of the memory trace(s) with respect to the duration of the S tone? In the section on acquisition a key assumption of familiarity theory regarding the interaction between acquisition and decay of the memory trace is tested. Fourth, what is the approximate form of the familiarity generalization gradient around the S tone, and is the decay rate invariant with distance from the S tone on the pitch dimension? Fifth, how is trace retrieval affected by the duration of the C tone?

Even while discussing the memory laws of the system that performs recognition memory (successive comparison) for pitch, the term "familiarity theory" is retained for the molar theory being developed. This is done to emphasize that it is familiarity (similarity) which is judged, not pitch difference. However, familiarity theory could equally well have been called "strength theory," since the present theory of recognition memory for pitch is completely compatible with the most basic assumptions of the strength theory for recognition memory of verbal materials developed by Norman (1966), Norman and Wickelgren (1965), Wickelgren (1967), and Wickelgren and Norman (1966). With obviously multidimensional stimuli such as letters or digits, there does not appear to be any alternative, at present, to the assumption that it is familiarity (similarity) which is judged in recognition memory. Thus, in verbal memory, there is much less need to emphasize the primacy of the assumption concerning what is judged. With pitch memory this is much more important. However, the more molecular theory of pitch memory developed in Section III is referred to as "associative strength theory" to emphasize the memory assumptions and the close relationship between the present theory of pitch memory and the previous strength theories of verbal memory.

Sources of Noise

Before discussing the memory laws of familiarity theory, it is necessary to clarify what is being assumed about the sources of noise in pitch memory experiments. This is especially important when the concern is with testing memory assumptions because the relative contribution of three different sources of noise, acquisition, storage, and retrieval, has a qualitative effect on the predictions made by a given set of memory assumptions.

First, the critical quantity for the criterion decision rule is $f - c$, so variation in c is added to variation in f. To provide a simple visual representation of the criterion decision rule (and only for this reason), all the variation has been assumed to be in f, including the variation in the criterion c. This criterion variation is a part of the noise component which will be referred to as retrieval variance. Retrieval variance includes both variation in the criteria and in the transmission of the familiarity value from the memory system to the decision system. It can be assumed to be largely independent of the mean familiarity value, $\bar{f}$, though no great complexity is introduced by assuming retrieval noise to be a function of $\bar{f}$. If retrieval variance is the only significant source of variance and it is independent of $\bar{f}$, then it is possible to have a real-variable memory theory and just add a random variable with zero mean and unit variance to the real-valued memory strength before entering it into the criterion decision rule.

There are, of course, several other possible noise components in the operation of the memory system, namely acquisition variance, and storage (decay) variance, to say nothing of variance external to the organism due to imperfect stimulus control. External variance can be absorbed in acquisition variance by assuming that the acquisition parameter, α, is a random variable.

Storage variance is expressed similarly by assuming that the decay parameter, β, is a random variable. Storage variance greatly complicates computation with FT. Thus, we tried to minimize this source of noise by experimental procedures which control rehearsal by the subject—the likely cause of storage variance.

Acquisition variance complicates FT less than storage variance, but it certainly would be convenient to be able to assume it to be small in relation to retrieval variance. This will be assumed. Tones seem to be pretty "attention-grabbing," and the use of carefully instructed, conscientious subjects should help to reduce both acquisition and storage variance. In any event, the assumption that acquisition and storage variance are small in relation to retrieval variance can be tested in a variety of ways. If retrieval variance is independent of $\bar{f}$ and large in relation to acquisition and storage variance, then operating characteristics should have slopes close to unity, and these slopes should not be negatively correlated with d'. As mentioned previously, the operating characteristics for S-D judgments are consistent with these predictions to a rather close approximation. Thus, we may proceed to develop a real-variable theory of the memory system, being content to add noise (with zero mean and unit standard deviation) to the memory trace only at the time of retrieval, though for other applications of familiarity theory, it will undoubtedly be necessary to assume additional sources of noise in order to account for operating characteristics with slopes very different from unity.

Decay and Single-Trace vs Dual-Trace Theories

Let $\bar{f}_x(t_I)$ be the mean familiarity of a C tone x Hz from the S tone and presented after an I tone of t_I sec(x = C-S). Let us consider the possibility that $\bar{f}_x(t_I)$ is the sum of a short-term trace and an intermediate-term trace,

$$\bar{f}_x(t_I) = s_x(t_I) + i_x(t_I) = \alpha_x e^{-\beta t_I} + \lambda_x e^{-\gamma t_I}, \quad \gamma \ll \beta.$$

Alternatively, there may be only a single (short-term) trace, $\bar{f}_x(t_I) = \alpha_x e^{-\beta t_I}$.

An operating characteristic only allows us to measure the *difference* between two mean familiarity values in units of the standard deviation of the total noise, so the prediction of the single-trace theory that we actually test is that:

$$d'_{10}(t_I) = \bar{f}_0(t_I) - \bar{f}_{10}(t_I) = \alpha'_{10}e^{-\beta t_I}, \quad \text{where} \quad \alpha'_{10} = \alpha_0 - \alpha_{10} .$$

The prediction of the dual-trace theory is that:

$$d'_{10}(t_I) = \alpha'_{10}e^{-\beta t_I} + \lambda'_{10}e^{-\gamma t_I}, \quad \text{where} \quad \lambda'_{10} = \lambda_0 - \lambda_{10} .$$

Thus, the single-trace theory predicts that a plot of log d'_{10} against t_I will be linear with a slope of $-\beta$ and an intercept on the log d'_{10} axis of log α'_{10}. The dual-trace theory predicts a systematic deviation from this simple linear relation on a semilog plot in the direction of too little decay at longer values of t_I (i.e., a curve that is concave up). In the special case of the dual-trace theory where γ is much smaller than β and the values of t_I are not too large, γ may be assumed to be effectively zero and then the dual-trace theory predicts that the total trace will decay exponentially to a positive asymptote λ'_{10}.

A semilog plot of d'_{10} against t_I for the S-D judgment data of the previous experiment for each of the three subjects is shown in Fig. 8. In obtaining these d'_{10} values, the data for $x = 10$ and $x = -10$ were lumped together to reduce variability, and

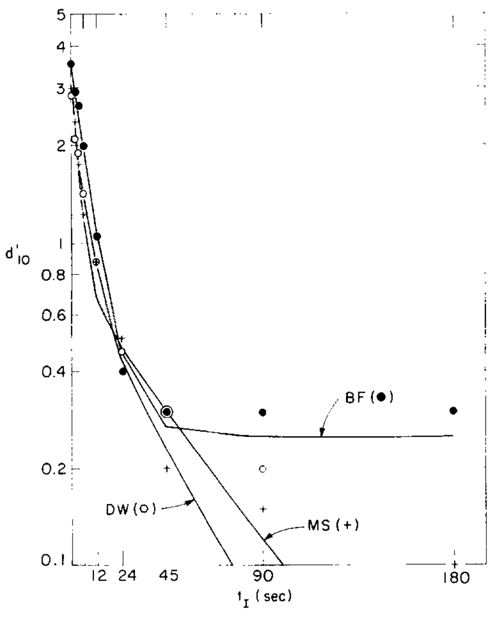

FIG. 8. Decay functions: semilog plots of d'_{10} against delay for delays from 1 to 180 sec. Points with $d'_{10} < .1$ were not plotted.

because to a first approximation $d'_{10} = d'_{-10}$ for each subject. Fig. 8 certainly suggests that the dual-trace theory is correct as all three curves show the type of deviation from linearity predicted by the dual-trace theory. However, the magnitude of the intermediate-term trace is in all cases rather small, and, of course, semilog plots grossly overemphasize small effects of this type.

To determine whether inclusion of the intermediate-term trace substantially improves the fit to the data, chi-square goodness-of-fit tests were performed using parameter estimates obtained by a minimum chi-square procedure. The procedure was the same as that described in Wickelgren and Norman (1966), with two exceptions. First, ordinary chi-square tests were used, and second, the pair of correct and false recognition probabilities whose sum was closest to unity was used for each delay condition, rather than always using the pair of probabilities at the "same-different" cutoff. The obtained parameter estimates and chi-square values for both single-trace and dual-trace theories are shown in Table 1. Predicted values of d'_{10} obtained from the dual-trace theory are plotted as lines in Fig. 8.

TABLE 1

PARAMETER ESTIMATES AND MINIMUM CHI-SQUARE VALUES FOR THE DECAY EXPERIMENT WITH DELAYS FROM 1 TO 180 SEC

Theory	*S*	Parameters α'_{10}	β	λ'_{10}	γ	χ^2	*df*	*p*
Single-trace								
$d'_{10} = \alpha'_{10}e^{-\beta t_I}$	BF	3.5	.09			33.6	7	$<.001$
	DW	2.7	.095			19.7	7	$<.001$
	MS	3.1	.14			52.6	7	$<.001$
Dual-trace								
$d'_{10} = \alpha'_{10}e^{-\beta t_I}$	BF	3.5	.115	.25	0	10.0	5	$>.05$
$+ \lambda'_{10}e^{-\gamma t_I}$	DW	2.1	.16	.70	.025	9.8	5	$>.05$
	MS	3.0	.29	.75	.02	6.6	5	$>.20$

Obviously, the dual-trace theory fits substantially better than the single-trace theory. Furthermore, the absolute goodness-of-fit of the dual-trace theory is extremely good, whether this is judged by the magnitude of the error in predicting d'_{10} or by the probability of obtaining errors of such size with an *N* of 200, given that the dual-trace theory is correct. Thus, the memory trace for pitch appears to be composed of a large short-term trace which decays exponentially to zero with a time constant of 3 to 9 sec and a smaller intermediate-term trace which decays exponentially to zero with a time constant of 40 sec or more. However, it does not seem likely that the intermediate-term trace lasts for weeks or months, and thus it would be confusing to call it a long-term trace.

The foregoing discussion of single-trace vs dual-trace theories made the implicit assumption that the decay of a particular trace has to be exponential in form. In other words, the rate of decay of a single trace is constant for all values of t_I, $d\bar{f}/dt_I = -\beta\bar{f}$. Although a large number of decay processes are exponential, this might not be true in the present instance. The good fit to the data of the sum of two exponentials suggests two traces, but it does not logically disprove certain more complex single-trace theories. For example, one could assert that there was only one trace, but the rate of decay was a (monotonically decreasing) function of t_I, $d\bar{f}/dt_I = -[\beta(t_I)]\bar{f}$. For some function $\beta(t_I)$, this single-trace theory could be made to fit the data of the present experiment. It is a rather inelegant theory, but that does not prove it untrue. However, in the section on Acquisition and the Interaction with Decay, evidence contrary to this complex single-trace theory is presented.

It should be noted that the present experiment cannot be handled by the class of more complex single-trace theories suggested by Melton (1963) for verbal memory. This class of single-trace theories assumes that the rate of decay is a function of the nature of the material to be learned, the nature of the prior and subsequent material, and the number of repetitions (analogous to t_S in the present study). This class of single-trace theories does not work because all the acquisition variables are held constant in the present experiment, and the slope of the retention curve is not constant.

If one is so committed to a single-trace theory that he is willing to assume the rate of decay to be a function of *both* acquisition variables (t_S in the present study) and t_I, then nothing in the present paper can contradict this incredibly complicated single-trace theory. However, parsimony is clearly on the side of the dual-trace theory.

Moreover, the dissociation of short-term memory and longer-term memory in patients with temporal lobe lesions provides extremely convincing evidence for the existence of at least two memory traces (see Milner, 1966). Patient HM, in particular, has no deficit in short-term memory, but virtually no ability to form new intermediate-term and long-term memory traces. HM was tested in the present tone memory task and was the only subject ever run in this situation who showed no intermediate-term trace (Wickelgren, 1968). Thus, it seems very likely that there are two traces mediating pitch memory in the present experiments, a short-term trace with a decay constant in the vicinity of 5 sec and an intermediate-term trace with a decay constant exceeding 40 sec (perhaps as long as several minutes or hours).

Just what this longer-term trace is remains to be seen. It could be a true intermediate-term trace in the same system of auditory internal representatives in which the short-term trace resides. On the other hand, the intermediate-term trace could be a short-term trace in a verbal memory system (verbalization of tone height relative to an average S tone), the rehearsal of which is not much diminished by the task of concentrating on the I tone. The former possibility would be of genuine interest; the latter would just be a nuisance (although the verbal judgment of relative tone height does require some type of intermediate-term or long-term memory for the pitch of the S tones used in the experiment).

The effect of t_I on familiarity (measured by d'_{10}) for smaller values of t_I was assessed in another experiment, whose description is identical to the description of the previous experiment with the following exceptions: first, $t_S = 2$ sec; second, the values of t_I were 0.25, 0.5, 0.75, 1, 2, 4, and 8 sec; third, the number of replications of each false recognition condition was 90 for $C - S = 10$ and 90 for $C - S = -10$; the number of replications of each correct recognition condition ($C - S = 0$) was 180.

A semilog plot of d'_{10} against t_I is shown in Fig. 9. The theoretical lines are for the best-fitting dual-trace theory. Parameter estimates and chi-square values for the dual-trace theory are shown in Table 2. Note that the best-fitting theory has nonzero intermediate-term memory in every case. To a first approximation, the dual-trace exponential-decay theory provides a good description of the data for subjects AB, RM, and JH in Fig. 9. For RM the dual-trace exponential-decay theory fits extremely well. For the other two subjects the fit of the theory is not as good as in the previous experiment, but the poorer fit appears to be largely a matter of random variation.

Careful examination of Fig. 9 reveals only one somewhat systematic deviation from

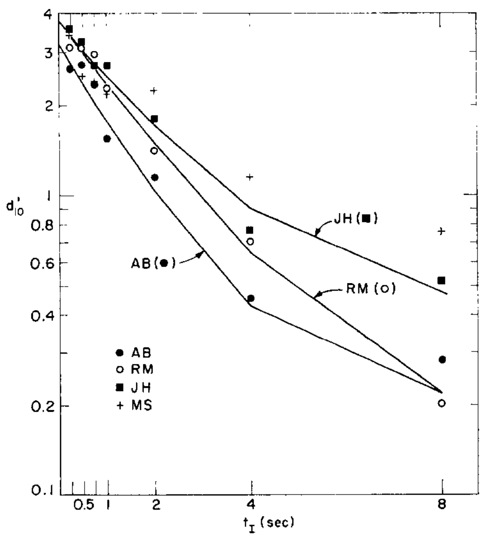

FIG. 9. Decay functions: semilog plots of d'_{10} against delay for delays from 0.25 to 8 sec. Subjects AB, RM, and JH were concentrating on the I tone during the delay, while subject MS was trying to rehearse the S tone during the delay and ignore the I tone.

the dual-trace exponential-decay theory, namely, too little decay over the first second. It could be that some of the perceptual, acquisitional or rehearsal activity present during the S tone persists in a rapidly decaying manner over the first second of the I tone. In any event, the effect is small, even during the first second of the I tone.

TABLE 2

PARAMETER ESTIMATES AND MINIMUM CHI-SQUARE VALUES FOR THE DECAY EXPERIMENT WITH DELAYS FROM 0.25 TO 8 SEC.

		Parameters						
Theory	S	α'_{10}	β	λ'_{10}	γ	χ^2	df	p
Dual-trace								
$d'_{10} = \alpha'_{10}e^{-\beta t_I}$	AB	3.0	.64	.2	0	25.5	3	$<.001$
$+ \lambda'_{10}e^{-\gamma t_I}$	RM	3.6	.52	.25	.05	7.0	3	$>.05$
	JH	3.4	.47	.39	0	13.7	3	$<.01$

The sort of "random" irregularity that can occur as a result of the subject trying to rehearse the S tone during the I tone is illustrated by the results for subject MS in Fig. 9. Subject MS was taking exactly the same experiment as the other subjects in Fig. 9, except that she was attempting to rehearse the S tone and ignore the I tone during the delay interval. The subjects shown in Fig. 9 (and 8) were following the standard instructions of concentrating on the I tone while it was being presented and trying not to think of the S tone. Although there is clearly decay of familiarity with increasing t_I and the decay is not systematically greater or less than that shown by the same subject in the first experiment discussed in this section, the decay curve is quite irregular. The best fit achievable for MS with the dual-trace theory had $\chi^2 = 51$ on 3 *df*, which is rather poor. In the previous experiment under instructions to concentrate on the I tone during its presentation, MS had $\chi^2 = 6.64$ on 5 *df*, for the dual-trace theory, which is extremely good. Clearly, the extent to which the subject follows the rehearsal instructions can have a marked effect on the goodness of fit of the dual-trace exponential-decay theory. However, the effect of S tone rehearsal seems to be largely a matter of increased irregularity, rather than any systematic deviation from the sum of two exponentials.

The experiment with delays from 0.25 to 8 sec indicates, in accord with intuition, that the noise in the decay curve due to uncontrolled rehearsal is greatest at theshorter values of delay ($t_I < 2$ sec). It is presumably for this reason that the previous experiment covering a wider range of delays gave a somewhat better fit to the theory than did the present experiment concentrating on shorter delays.

Temporal Decay vs Interference

Thus far, decay has been discussed as a function of the delay interval between the S and C tones, but since the delay interval is always filled with another tone, the decline in memory is interpretable either as a passive decay in time, or as a direct consequence of the interfering effects of the I tone. The interfering effects of the I tone could be either or both of two types. First, the I tone could be setting up a competing memory trace which in some way interferes with the retrieval of the trace for the S tone (retrieval interference). Second, the I tone could be responsible for an active destruction of the trace for the S tone (storage interference). However, in either case one would expect these interference effects to be greater the closer the I tone is to the S tone in frequency. Furthermore, one would expect the amount of interference to be greater the greater the intensity of the I tone. On the other hand, if the frequency similarity and intensity of the I tone have no effect on the decay function, then interpretation in terms of temporal decay seems quite plausible. Hence, the following experiments were performed.

Similarity of I tone to S tone. In this experiment, $t_S = 1$ sec, $t_I = 2$ sec, $t_C = 1$ sec, $S - C = 0$ or ± 10 Hz. The S tones were randomly selected in 10 Hz steps over the interval from 400 to 590 Hz, and I tones were ± 15, ± 20, ± 40, ± 100, ± 200, $+3000$, or $+8000$ Hz from the S tone. The $3 \times 12 = 36$ conditions were randomized in blocks of 36 trials each, with 8 blocks in a 1-hour session. A 5-minute break occurred after the fourth block. There were 4 practice trials at the beginning of the experiment and 4 after the break. There were two instructional conditions. One group of 23 subjects was given the standard instructions to attend to the I tone and not try to rehearse the S tone during the delay interval. The second group of 26 subjects was given the instruction to try to ignore the I tone and to rehearse the S tone during the delay interval. Subjects were M.I.T. students recruited from psychology courses and paid for their services.

Plotting the $S - C = 0$ condition against both the $S - C = 10$ and $S - C = -10$ conditions for each of the 12 I tone similarity conditions yields 24 operating characteristics for each of the two instructional groups. These group operating characteristics were quite well fit by straight lines on normal–normal coordinates, and the slopes were reasonably close to unity, although generally above it. However, the shape and slope of these group operating characteristics are of no present interest. What we want to know is whether I tone similarity affects the d'_{10} measure of accuracy of recognition memory and, secondarily, whether rehearsal instructions have any effect on performance. The d'_{10} values for each of the 24 operating characteristics for each of the two instructional groups are given in Table 3.

Instructing subjects to try to rehearse the S tone appears to have a small facilitative effect on recognition memory, but the effect is very small for unpracticed subjects under the conditions of the present experiment. This result and the results obtained for the one practiced subject (MS), reported in the previous section, suggest that attempts to rehearse the S tone in the presence of the I tone may have both facilitative and interfering effects on the memory trace. These two effects appear to be roughly equal for 2-sec delays, but there is undoubtedly an interaction with the length of the delay interval.

TABLE 3

EFFECTS ON PITCH MEMORY OF INTERFERENCE TONE SIMILARITY TO STANDARD TONE

	d'_{10}				d'_{-10}		
S-C	I-S	Attend to I Tone	Rehearse S Tone	S-C	I-S	Attend to I tone	Rehearse S tone
0 vs 10	15	1.25	1.48	0 vs −10	15	.12	.23
	−15	.47	.51		−15	1.19	1.32
	20	1.94	1.86		20	.61	.91
	−20	.88	.77		−20	1.25	1.16
	40	.66	.69		40	.80	.80
	−40	.80	1.08		−40	.94	1.14
	100	1.16	1.19		100	.66	.72
	−100	.77	.85		−100	.74	.85
	200	.98	1.25		200	.56	.80
	−200	1.01	1.14		−200	.74	.66
	3000	1.41	1.22		3000	1.01	1.05
	8000	.77	.85		8000	.82	.88

Under either instructional condition, there appears to be no effect of I tone similarity on recognition memory performance at and beyond a difference of 40 Hz between the S and I tones. When the I tone is only 15 or 20 Hz from the S tone, there is a facilitation of performance when the C tone is on the same side of the S tone as the I tone, and an approximately equal decrement in performance when the C tone is on the opposite side of the S tone. In all the other experiments reported in this paper, the I tone is at least 300 Hz from the S and C tones. The present results indicate quite clearly that the decay occurring for I tones at this distance from the S tone is not a function of the similarity of the I and S tones. This suggests that the decline in recognition memory performance with increasing delay is a temporal decay of trace strength, not a storage or retrieval interference phenomenon.

Even the results for I tones within 20 Hz of the S tone do not suggest interference. Rather, they suggest the use of pitch-difference judgments on the S vs I and I vs C tones. However, the absence of any effect of the similarity of the I tone to the S and C tones outside of a 40-Hz range around the S tone indicates that under these conditions subjects are *not* using SI, CI comparisons to make their S-D judgment. If they were, one would certainly expect the accuracy of the judgment to depend on the distance of the I tone from the S and C tones.

Intensity of I tone. In this experiment the conditions were exactly the same as in the previous experiment, except that only one I tone (930 Hz) was used, and its intensity relative to the S tone was varied in 7 steps: 0, ± 5, $+10$, or ± 20 db from the intensity of the S tone. Intensity was measured without regard for the recording and playback response characteristics of the tape recorder, so these numbers should be taken to have only ordinal sound-intensity properties. What was determined was that the different I tone intensities differed substantially in subjective loudness. Twenty-three subjects were run in a 1-hour session in which each condition was replicated 14 times in random order. The d'_{10} values for $S - C = 0$ plotted against $S - C = \pm 10$ for each of the I tone intensity conditions are shown in Table 4.

TABLE 4

EFFECTS ON PITCH MEMORY OF INTERFERENCE TONE INTENSITY

Intensity of I tone relative to S tone (db)	d'_{10} Best subjects	Intermediate subjects	Poorest subjects
−20	1.68	1.20	.66
−10	1.41	1.28	.56
−5	1.68	1.41	.85
0	1.72	1.28	.77
5	1.68	1.11	.40
10	1.76	1.22	.46
20	1.68	1.11	.61

Clearly, there is no effect of I tone intensity, within the range studied in the present experiment. If the I tone was exerting any interfering effect on the memory trace for the S tone or the retrieval of the S tone, we would have expected the magnitude of that interfering effect to be dependent on the frequency-similarity and intensity of the I tone with respect to the S tone. Since no such effects were observed, we must conclude that the memory trace for the S tone is decaying strictly as a function of time.

Acquisition and the Interaction with Decay

During (and perhaps after) presentation of the S tone, memory traces are established. The process by which these traces are established will be referred to as an acquisition process.

Since the analysis of trace decay indicated the existence of two memory traces with very different decay rates, we must consider the acquisition of the total trace to occur by virtue of the acquisition of each component trace. The two component traces need not have the same rate of acquisition, just as they do not have the same rate of decay, and, although it is not logically necessary, one might expect the trace with the slower rate of decay to have a slower rate of acquisition.

The present section has two major purposes. First, we shall attempt to determine an approximate characterization of the acquisition functions, $\alpha_x(t_S)$ and $\lambda_x(t_S)$, i.e., degree of learning of the pitch of a tone as a function of tonal duration.

Second, we shall test a basic assumption of familiarity theory, regarding the interaction between acquisition and decay. The assumption is that the strength of each trace (short-term and intermediate-term) can be expressed as the product of two functions, the acquisition function ($\alpha_x(t_S)$ and $\lambda_x(t_S)$) and the decay function ($B(t_I)$ and $G(t_I)$). Formally, the assumption is that:

$$s_x(t_S, t_I) = \alpha_x(t_S) \cdot B(t_I),$$

and (1)

$$i_x(t_S, t_I) = \lambda_x(t_S) \cdot G(t_I).$$

What is asserted is that the decay of the memory trace has the same form and rate, regardless of the initial level of acquisition, and, conversely, that the acquisition of the memory trace has the same form and rate, regardless of the delay at which retention is tested. This very special interaction assumption is made, not only because it is elegant, but also because of an underlying intuitive assumption that the acquisition process finishes at the time that the decay process begins. If this intuitive assumption is false, then much more complex interactions between acquisition and decay are possible.

It is important to note that this assumption of the "multiplicative interaction of acquisition and decay" is far more central to familiarity theory than any particular assumption about the forms of the various acquisition and decay functions. There are some reasons for preferring exponential decay functions, and, fortunately, the previous data on d'_{10} as a function of delay were well fit by the sum of two exponential decay functions. However, the acquisition function could have almost any form, provided it is bounded above and below.

Perhaps the first issue to consider in the determination of the acquisition function for each trace is whether acquisition starts immediately upon presentation of the S tone or begins only after some fixed lag. The second issue is whether the acquisition process ends immediately upon presentation of the I tone or only after a lag. Of course, the answer to both questions must be that there are lags at both ends, the real issues are whether the lags are identical and, if not, whether the discrepancy is substantial. Note that, as a first approximation, we are assuming both "turnon" and "turnoff" of the acquisition process to be abrupt. One particularly simple possibility is that the acquisition of both traces is tied directly to activation of the internal representative of the S tone, acquisition occurring only when that internal representative is activated.

If the S tone is followed by a silent period, the representative of the S tone may remain in a state of heightened activity for some time following termination of the S tone. But if the S tone is followed immediately by an I tone, the representative of the S tone may be inhibited, terminating its activity and the associated acquisition process. Subjects may be able to counteract this in part by attempts to rehearse the S tone. However, if they are instructed not to rehearse the S tone and to concentrate instead on the I tone, we might be successful in sharply limiting the persistence of the acquisition process. If these rehearsal instructions in conjunction with an I tone are reasonably successful in truncating the acquisition process, we might find the initial and terminal lags to be virtually identical.

The exact form of the acquisition functions will not be determined in the present paper. However, we must assume that the traces are bounded. The present paper will make approximate estimates of the upper bounds, and will try to determine the approximate form and rates of the approaches to these bounds. In particular, we will attempt to determine whether the approach to the upper bound is linear or exponential, for both the short-term and intermediate-term traces. These possibilities are expressed formally as follows: Bounded linear acquisition (L):

$$\begin{aligned} \alpha_x(t_S) &= \alpha_x t_S\,, \quad \text{for} \quad t_S < t_s \\ &= \alpha_x t_s = \alpha, \quad \text{for} \quad t_S \geqslant t_s\,, \\ \lambda_x(t_S) &= \lambda_x t_S\,, \quad \text{for} \quad t_S < t_i \\ &= \lambda_x t_i = \lambda, \quad \text{for} \quad t_S \geqslant t_i\,; \end{aligned}$$

Bounded exponential acquisition (E):

$$\begin{aligned} \alpha_x(t_S) &= \alpha_x(1 - e^{-\theta t_S}) \\ \lambda_x(t_S) &= \lambda_x(1 - e^{-\psi t_S}). \end{aligned}$$

Since familiarity is tested after a delay, each acquisition function must be multiplied by the decay function for that trace to get the degree of familiarity expected under a particular combination of acquisition and decay conditions. Also, one can only test the difference (d_x') between the mean familiarity values under two different conditions of $x = C - S$. Thus, the predictive equation one actually tests for the theory that assumes bounded exponential acquisition of both traces (EE) is as follows:

$$\begin{aligned} d_x'(t_S\,, t_I) &= (\alpha_0 - \alpha_x)(1 - e^{-\theta t_S})\, e^{-\beta t_I} + (\lambda_0 - \lambda_x)(1 - e^{-\psi t_S})\, e^{-\gamma t_I} \\ &= \alpha_x'(1 - e^{-\theta t_S})\, e^{-\beta t_I} + \lambda_x'(1 - e^{-\psi t_S})\, e^{-\gamma t_I} \end{aligned} \tag{2}$$

For the short delays to be considered in the present section, we can assume $\gamma t_I \doteq 0$, yielding:

$$d'_x(t_S, t_I) \doteq \alpha'_x(1 - e^{-\theta t_S})\, e^{-\beta t_I} + \lambda'_x(1 - e^{-\psi t_S}). \tag{3}$$

Testing whether the acquisition function for each trace is closer to linear or closer to exponential in form, will be done both by graphical analysis and by minimum chi-square parameter estimation and goodness-of-fit tests. In addition, the possibility of unequal initial and terminal lags will be considered by substituting $t_S - \delta$ for t_S in the equations for d'_x. Finally, one attractive possibility to be considered is that acquisition of the short-term trace is bounded from above by the occurrence during the S tone of the same exponential decay process that occurs during the I tone. This possibility, which was suggested in an earlier paper (Wickelgren, 1966) makes the strong prediction that the acquisition function will exponentially approach a limit from below at the same rate as the decay function exponentially approaches zero from above (i.e., $\theta = \beta$).

Experiment S. Two sec after a ready signal, subjects heard an S tone lasting for t_S sec, followed by an I tone (930 Hz) lasting for t_I sec, followed by a C tone lasting 1 sec, followed by a 4-sec decision period in which they were to decide whether the C tone was the same as, or different from the S tone, followed by the next trial. The S tones were randomly selected in the range from 400 to 490 Hz in 10 Hz steps. $S - C = 0$ (presented as often as $S - C = \pm 10$ put together) or ± 10 Hz; $t_S = 0.1$, 0.2, 0.4, 0.8, or 1.6 sec; $t_I = 0.5$, 1, 2, or 4 sec. Conditions were randomly ordered in blocks of $4 \times 5 \times 4 = 80$ trials, with 3 blocks in a 1-hour session. The second and third blocks were preceded by a 5-min rest period, followed by 3 practice trials. The first block was preceded by 4 practice trials. There were 3 different sessions recorded on tape and given 10 times to each of 3 subjects. An initial practice session was also given. Thus, the N for each $S - C = 0$ condition was 180, and the N for each combined $S - C = \pm 10$ condition was also 180. The paid subjects were students recruited through the M.I.T. student employment office. Subjects were instructed to concentrate on the I tone when it was present and not to try to rehearse the S tone.

Experiment I. The procedure was identical to the previous experiment, with the following exceptions: $t_S = 1$, 2, 4, or 8 sec; $t_I = 0$, 0.5, 1, 2, 4, or 8 sec. The S tones were randomly selected at 10-Hz steps from the range 400–590 Hz. Conditions were randomized in blocks of $4 \times 4 \times 6 = 96$ trials, with 2 blocks per session, 5 different sessions recorded on tape, and each session given 5 times. There was an initial practice session, a 5-min break between blocks, and 4 practice trials before each block. The N for each of the 3 subjects for each $S - C = 0$ condition was 100, as was the N for each combined $S - C = \pm 10$ condition.

Results. A graphical analysis of the acquisition process is achieved by plotting d'_{10} against t_S, considering t_I to be a parameter. This is done in Fig. 10, for Experiment S.

In Fig. 10, $+10$ and -10 are separated to demonstrate an interesting asymmetry between them that occurs only for S tones of very short duration (0.4 sec or less), namely that 1-sec C tones which are 10 Hz above the S tone have greater strength than 1-sec C tones which are 10 Hz below the S tone. This suggests that the maximum of the familiarity generalization gradient may be slightly displaced in the direction of higher pitch for very short tones in the range from 400 to 490 Hz. However, the effect is rather small, even in Experiment S, and it was ignored in the goodness-of-fit tests.

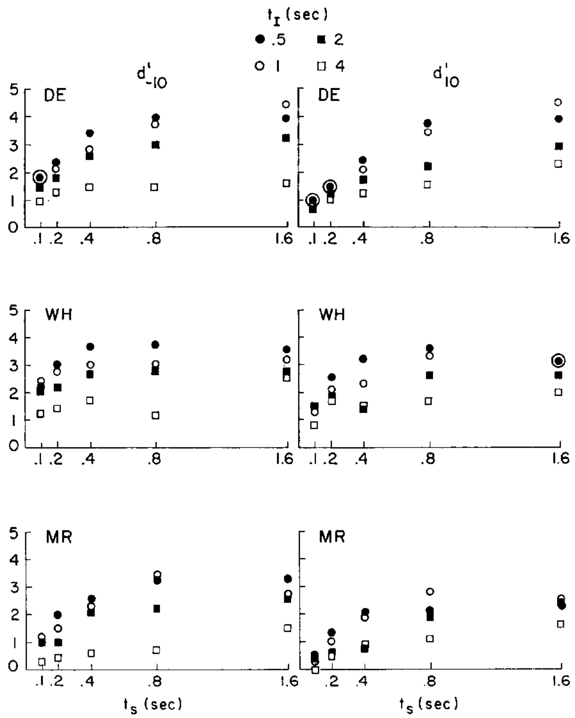

FIG. 10. Acquisition functions: plots of d'_{10} and d'_{-10} against t_S with t_I as the parameter.

Semilog plots of d'_{10} against t_I are shown in Fig. 11 for each subject in Experiment S, and in Fig. 12 for each subject in Experiment I. The lines shown in these figures are the theoretical predictions of the best-fitting theory (EE), which assumed bounded exponential consolidation of both short-term and intermediate-term traces, with equal initial and terminal lag ($\delta = 0$). Table 5 gives the parameter estimates and chi-square values for the goodness-of-fit of theory EE to the data.

Figures 11 and 12 do not indicate any systematic deviation of the data from the exponential consolidation and decay theory EE (Eq. 3). However, there appears to

TABLE 5

PARAMETER ESTIMATES AND MINIMUM CHI-SQUARE VALUES FOR DUAL-TRACE THEORY EE APPLIED TO THE ACQUISITION AND DECAY EXPERIMENTS S AND I

Experiment	S	Parameters α'_{10}	β	θ	λ'_{10}	ψ	χ^2	df	p
S	DE	3.2	.30	6.	3.0	.4	84	15	$<.001$
	WH	3.0	.15	11.	2.5	.11	110	15	$<.001$
	MR	3.0	.30	4.2	.5	.81	94	15	$<.001$
I	VS	2.2	.32	1.4	1.4	1.0	83	19	$<.001$
	JN	2.3	.56	9.	1.7	.9	159	19	$<.001$
	MF	1.35	.66	5.5	1.5	.03	68	19	$<.001$

have been considerable noise in these experiments, as some of the points show reversals that are unsystematic and could not be predicted by any reasonable theory. One guess as to the source of the noise is the unpredictable variation in t_S from trial to trial. Perhaps future experiments should hold t_S (and t_I) constant over blocks of trials so that subjects can develop a better set.

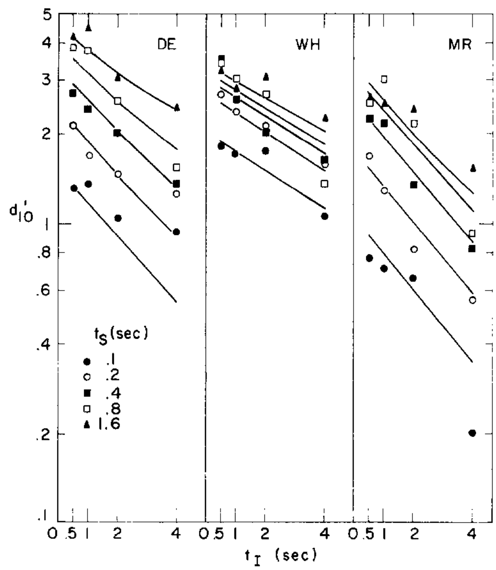

FIG. 11. Decay functions: semilog plots of d'_{10} against delay with t_S as the parameter, for acquisition and decay experiment S.

In any event, the noise results in substantially larger chi-square values, as shown in Table 5, and limits, to some extent, the conclusions we can draw from the experiments. Firstly, we cannot determine precisely the form and rate of the short-term and intermediate-term acquisition functions. Secondly, the hypothesis of multiplicative interaction of acquisition and decay cannot be as strongly confirmed as it could be if there were less noise in the data, though a definite rejection of the hypothesis is still quite possible. Despite these limitations, the experiments do provide answers to some of our questions about acquisition and its interaction with decay.

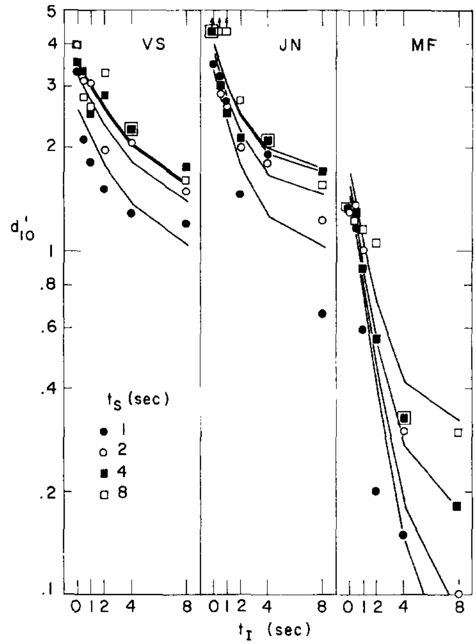

FIG. 12. Decay functions: semilog plots of d'_{10} against delay with t_S as the parameter, for acquisition and decay experiment I; points with $d'_{10} < .1$ were not plotted; unmeasurably high d'_{10} values have arrows pointing upward.

In the first place, there is no systematic deviation from the assumption of multiplicative interaction of acquisition and decay. The evidence for this assertion is that the empirical decay functions for each level of acquisition do not differ systematically in form or rate of decay from the predicted form and rate of decay for each level of acquisition. Similarly, the form and rate of acquisition for each delay interval is not systematically different from the predicted form and rate of acquisition for that delay interval.

In the second place, there does not appear to be any systematic deviation from the predicted form of the acquisition function, namely, the sum of two exponential approaches to limits. The evidence for this is that the curves in Fig. 10 have a convex-up shape, and the predicted lines in Figs. 11 and 12 do not lie systematically above or below the empirical points for any value of the acquisition parameter t_S. However, the goodness-of-fit tests make it clear that it is primarily the short-term trace whose acquisition function must be assumed to have a convex-up form, rather than a linear form. Because the intermediate-term trace is so much smaller than the short-term trace at short delay intervals and because neither Experiment S or I employed the long delay intervals that would be necessary for adequate assessment of the intermediate-term trace, essentially nothing can be concluded about the form of the acquisition function for the intermediate-term trace. However, it should be noted that a better fit is achieved in every case by assuming some amount of intermediate-term memory, confirming the previous decision in favor of a dual-trace theory.

In the third place, there does not appear to be an appreciable asymmetry between the initial and terminal lag in the present experiments. No improvement in fit can be realized by assuming $\delta > 0$. Of course, it should be kept in mind that asymmetries on the order of 50 msec or less would probably not be detected by Experiment S, and only asymmetries of 100 msec or more are logically ruled out by Experiment S.

In the fourth place, the estimated values of θ for each subject are much larger than the estimated values of β, suggesting that the decay-bounded short-term acquisition assumption proposed by Wickelgren (1966) is incorrect. However, in Section III, a more molecular analysis of the acquisition process is developed which gives convex-up curves and which is compatible with the assumption of decay-bounded acquisition.

In the fifth place, the data suggest that the more complex single-trace theory mentioned in the above section on trace decay is also invalid. This single-trace theory assumed that rate of decay was a function of delay $d\bar{f}/dt_I = -[\beta(t_I)]\bar{f}$. The argument against this theory is as follows:

Let

$$\bar{f}_x(t_I = 0, t_S) = \alpha_x(t_S) = \alpha_x .$$

Then

$$\int_0^{\bar{f}_x(t)} \frac{1}{\bar{f}_x(t_I)} d\bar{f}_x(t_I) = -\int_0^t \beta(t_I)\, dt_I$$

$$\log \bar{f}_x(t) - \log \alpha_x = -\int_0^t \beta(t_I)\, dt_I$$

$$\frac{\bar{f}_x(t)}{\alpha_x} = \exp\left(-\int_0^t \beta(t_I)\, dt_I\right)$$

$$d'_x(t) = \alpha'_x \exp\left(-\int_0^t \beta(t_I)\, dt_I\right)$$

where $\alpha'_x = \alpha_0 - \alpha_x$. Thus, for $t_1 < t_2$ and $d'_x(t_1) > d'_x(t_2)$

$$\log d'_x(t_1) - \log d'_x(t_2) = \int_{t_1}^{t_2} \beta(t_I)\, dt_I .$$

Notice that according to this single-trace theory, $\log d'_x(t_1) - \log d'_x(t_2)$ has the same value regardless of the level of acquisition, $\alpha'_x(t_S)$. This is equivalent to the assertion that the decay curves in Figs. 11 and 12 will be parallel. In Experiment S where there is much less opportunity for the intermediate-term trace to have an effect, the theoretical predictions of the dual-trace theory are also virtually parallel lines on a semilog plot. However, in Experiment I, the dual-trace theory predicts decay curves that deviate to a noticeable degree from parallelism on a semilog plot. The data in Fig. 12 show much closer agreement with the predicted lines of the dual-trace theory than with the more complex single-trace theory that predicts parallel (although curved) decay functions for different values of t_S. The data for each subject in Experiment I show a systematic deviation from parallel decay functions in the direction predicted by the dual-trace theory.

Despite the noise in the data, the deviation from the predicted equality of $\log d'_{10}(0) - \log d'_{10}(8)$ for each of the four values of t_S in Experiment I is significant at the .05 level using the one-tailed test, which is certainly appropriate in this case. The statistical test was to compute the probability of obtaining by chance a rank ordering of the $\log d'_{10}(0) - \log d'_{10}(8)$ values for each value of t_S as extreme as, or more extreme in the direction predicted by the dual-trace theory than the rank ordering actually obtained for each subject. JN's very high accuracy at short delays forced the use of $\log d'_{10}(.5) - \log d'_{10}(8)$ and only three t_S values, 1, 2, and 4 sec. Thus, for JN there were only six possible rank orderings, and JN demonstrated the most extreme one, for a probability of .167. There were four orderings as extreme as, or more extreme than, that demonstrated by VS, so for him the probability was $4/24 = .167$, also. MF had the most extreme rank ordering out of 24 for a probability of .04. Transforming each of these probabilities to chi-square on 2 *df* and adding the chi-square values (Fisher's method of combining independent tests of significance) gives an overall chi-square on 6 *df* of 13.6 ($p < .05$). Thus, we may reject even the more complex single-trace theory which lets rate of decay be a function of delay.

Generalization

In all the other experiments reported in this paper, subjects were required to discriminate between C tones identical to the S tone or else 10 Hz higher or lower than the S tone. However, familiarity theory assumes that the same basic laws of acquisition and decay of the memory trace apply, regardless of the frequency difference between C tones identical to the S tone and those different from it. In addition, it seems reasonable to require familiarity theory to assume that the rates of acquisition and decay are identical at all distances from the S tone on the pitch dimension. Only the asymptotic familiarity value approached during the acquisition process is assumed to be a monotonically decreasing function of distance from the S tone on the pitch dimension. The invariance of the acquisition and decay processes with distance from the S tone has already been assumed implicitly in the formulation:

$$d'_x(t_S, t_I) = \alpha'_x(1 - e^{-\theta t_S})\, e^{-\beta t_I} + \lambda'_x(1 - e^{-\psi t_S})\, e^{-\gamma t_I}$$

In this section, we will test the invariance of the decay parameter, β, for $d'_{10}(t_I)$, $d'_{20}(t_I)$, $d'_{30}(t_I)$, $d'_{40}(t_I)$, and $d'_{50}(t_I)$, and at the same time determine the approximate form of the generalization gradient of total familiarity, $\bar{f}_x(t_I)$, for each of three values of t_I. No attempt will be made to analyze the total familiarity generalization gradient into short-term and intermediate-term components. In fact, the values of t_S (1 sec) and t_I (1, 2, and 4 sec) will be chosen so as to allow us to assume, as a first approximation, that the intermediate-term trace is small in comparison to the short-term trace. Thus, the equation for the present experiment reduces to:

$$d'_x(t_I) = a(x)\, e^{-\beta t_I}, \qquad \text{where} \quad a(x) = \alpha'_x(1 - e^{-\theta t_S}).$$

This experiment was performed on a group of 24 subjects run for a single 1-hour session. To diminish somewhat the evils of averaging, the results are reported separately for the best nine subjects averaged together, the middle seven subjects, and poorest eight subjects. In this experiment the S tone was chosen in 10 Hz steps in the range from 400 to 590 Hz, the I tone was 930 Hz, the C tone was 0, ± 10, ± 20, ± 30, ± 40, or ± 50 Hz from the S tone, $t_S = 1$ sec, $t_I = 1, 2$, or 4 sec, $t_C = 1$ sec, and the time for an S-D decision (with confidence on a scale from 1 to 4) was about 4 sec. A session consisted of 8 blocks of $3 \times 11 = 33$ trials each, with a 5-min break halfway through and 4 practice trials at the beginning of each half of the session. Each of the 11 $S - C$ values at each of the 3 delays was tested once in a block.

The results are shown in Figs. 13 and 14. Inspection of the constancy of the slopes of the best fitting straight lines through the d'_x values plotted as a function of t_I in Fig. 13 provides a test of the assumption that the rate of decay in short-term memory is invariant with distance from the S tone. Although there are a couple of deviant points, the results are remarkably consistent with this assumption.

The familiarity generalization gradients $\bar{f}_x(t_I)$ in Fig. 14 were drawn making the arbitrary assumption that $\bar{f}_{50}(t_I) = 0$. Since the measurement of strength is on an interval scale with an arbitrary zero for each value of t_I, these generalization gradients are as good as can be obtained by this procedure. If, in some absolute sense, $\bar{f}_{50}(1) = \bar{f}_{50}(2) = \bar{f}_{50}(4)$ to a first approximation, then comparison of the heights of the $\bar{f}_x$ values for different t_I values has meaning. Otherwise, only the comparison of different $\bar{f}_x$ values for the same t_I values have meaning, although any comparison of a *difference* between two $\bar{f}_x$ values for one t_I value with a difference of two $\bar{f}_x$ values for another t_I value has meaning.

Keeping these limitations in mind, the following conclusions can be drawn from the data in Fig. 14. First, at the shortest delay (1 sec) good subjects differ from poor subjects, not in the total drop in familiarity from $x = 0$ to $x = 50$, but rather in the steepness of the drop from $x = 0$ to $x = 10$ or 20. Second, good subjects do have a larger drop in strength from $x = 0$ to $x = 50$ than poor subjects at the longest delay

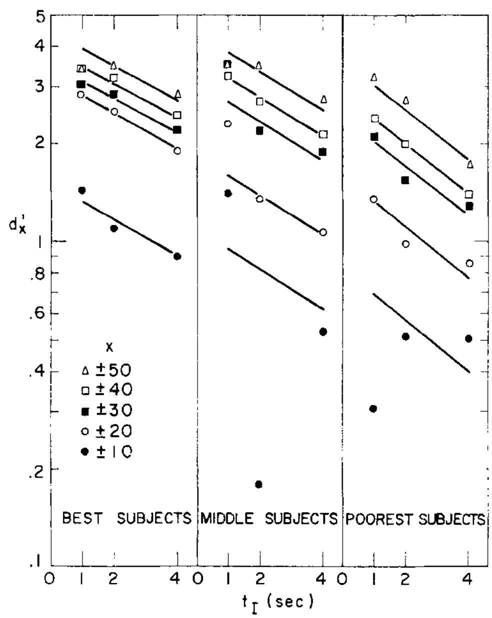

FIG. 13. Invariance of decay function with distance from the S tone (x); plots of d'_x against delay with x as the parameter.

(4 sec) suggesting that good subjects have a slower rate of delay than poor subjects. The degree of correlation between the steepness of the initial generalization gradient and the decay rate cannot be determined from this averaged data.

Retrieval

We have already considered the principal question concerning retrieval, namely, "What dimension is judged in the decision process?" Assumptions about the dimension being judged and the decision rule by which values on this dimension are combined with response biases (criteria) form the basic framework within which the memory assumptions can be formed. Nevertheless, there are some remaining characteristics of the retrieval process which have been ignored thus far. These characteristics concern the exact manner in which presentation of the C tone produces an output of a familiarity value from the memory system to the decision system. At what time does this output occur in relation to the onset of the C tone and how do the mean and variance of this familiarity output depend on the duration and intensity of the C tones?

The only question which will be answered in this section concerns the dependence of the familiarity output on the duration of the C tone. Two factors seem likely to be of primary importance in this relationship. First, the perception of the pitch of the C

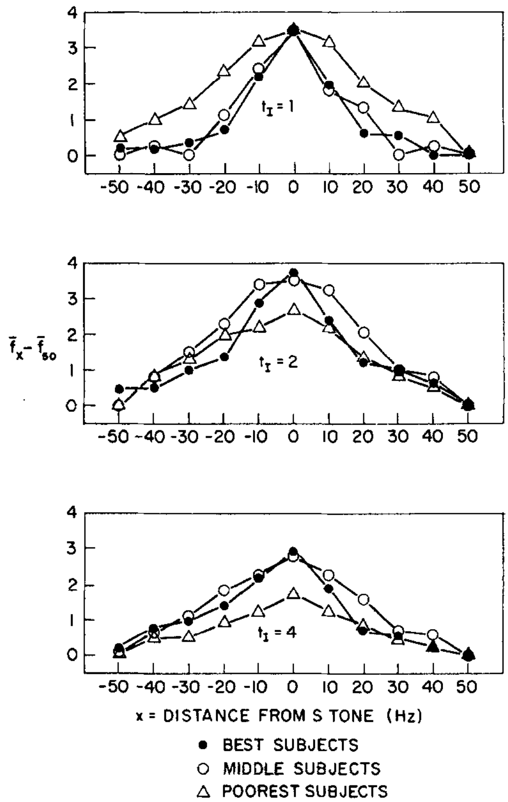

FIG. 14. Familiarity generalization gradients: plots of $\bar{f}_x - \bar{f}_{50}$ for different values of t_I and different groups of subjects.

tone requires time, and retrieval of the correct familiarity value associated with the pitch of the C tone will become more accurate as t_C increases from 0 sec to some value on the order of 1 sec. Second, however, we might very well expect the familiarity generalization gradient established by the S tone to be decaying over the duration of the C tone. Thus, if t_C gets too long, and the subject delays retrieval (output from the memory system to the decision system) longer for longer C tones (although not necessarily always delaying until the end of the C tone), we might expect performance to decline for t_C greater than some critical value. This value would be the point at which the beneficial effects of a longer C tone begin to be outweighed by the decay produced by a longer retrieval delay. The following experiment was performed to investigate these matters.

The S tones ranged from 400 to 490 Hz in 10-Hz steps, the I tone was 930 Hz, $S - C = 0$, or ± 10 Hz with 0 occurring as often as $+10$ and -10 combined, $t_S = 2$ sec, $t_I = 1$ sec, $t_C = 0.25$, 0.5, 0.75, 1, 2, 4, or 8 sec, and the time for the S-D decision (with confidence on a scale from 1 to 4) was about 4 sec. There were $7 \times 4 = 28$ trials/block, 9 blocks to a set, a 5-min break after the fifth block, and 4 practice trials at the beginning of each set and after the break. Three subjects (AB, RM, and JH) took the two different sets five time each, after a practice session. The $d'_{10}(t_C)$ values are shown in Table 6, where the results for $+10$ and -10 have been lumped together.

Note that performance improves with increasing t_C up to a maximum which is

TABLE 6

EFFECTS OF C TONE DURATION ON RETRIEVAL OF THE MEMORY TRACE

	d'_{10}						
Subject/t_C(sec)	0.25	0.5	0.75	1	2	4	8
AB	1.41	1.48	1.76	1.83	1.92	1.80	1.68
RM	2.26	2.46	2.51	2.74	2.60	2.51	2.51
JH	1.90	2.40	2.59	2.56	2.26	2.56	2.16

0.5, 1, or 2 sec for the three different subjects. Beyond the optimal t_C value, d'_{10} declines, although the decline is far less than would be predicted on the hypothesis that the subject always waits until the *end* of the C tone for retrieval. Using previous estimates of the rate of decay of the memory trace for these same subjects (see Fig. 9 and Table 2), we are forced to conclude that subjects are not waiting until the end of the C tone for retrieval, at least not for the 4- and 8-sec C tones. However, in this experiment subjects never knew in advance how long the C tone would be. It is likely that they *tended* to delay retrieval until the end of the tone, since many of the C tones were quite short. However, they probably cut this short when it appeared clear that the tone would be a long one. Thus, it is likely that they waited longer on the average for longer tones than for shorter ones. Alternatively, retrieval may have occurred at exactly the same time for C tones longer than, say, 1 sec, but retrieval was simply noisier when it had to be accomplished while a tone was being presented.

As formulated thus far, familiarity theory cannot accomodate the effects of C tone duration. However, if we ignore the relatively small decline on d'_{10} beyond the optimum t_C and assume that retrieval always takes place at the end of the C tone, $t_R = t_C$, for $t_C < 1$ sec and at $t_R = 1$ sec for $t_C \geqslant 1$ sec, then familiarity theory can be easily extended to handle the primary effect of C tone duration, which is to yield better performance with increasing t_C up to a maximum of around 1 sec. The rate at which increasing t_C produces a more complete retrieval of trace strength will be represented by η. One plausible way to modify familiarity theory to include the effects of C tone duration is as follows:

$$d'_x(t_S, t_I, t_R) = \alpha'_x(1 - e^{-\theta t_S})\, e^{-\beta(t_I + t_R)}(1 - e^{-\eta t_R}) + \lambda'_x(1 - e^{-\psi t_S})\, e^{-\gamma(t_I + t_R)}(1 - e^{-\eta t_R}), \tag{4}$$

where

$$t_R = \begin{cases} t_C & \text{for} \quad t_C < 1 \text{ sec} \\ 1 & \text{for} \quad t_C \geqslant 1 \text{ sec.} \end{cases}$$

So far, we have assumed that the time of retrieval after the onset of the C tone is a very simple function of t_C. This accounts for the major effect of t_C which is to increase d'_x from $t_C = 0$ to $t_C = 1$ sec. In principle, it is possible to account for the decline in d'_{10} past the optimum t_C by a more complex function $t_R(t_C)$. However, the present data are not adequate to justify consideration of any such more complex functions.

III. ASSOCIATIVE STRENGTH THEORY

In Section I, familiarity is shown to be the psychological dimension judged in recognition memory for pitch. In Section II, certain laws are established regarding the acquisition, storage, and retrieval of the familiarity of a tone. However, throughout the second section, familiarity is treated as the basic memory trace which was acquired and forgotten. It seems intuitively more satisfactory to regard familiarity as arising from the *interaction* between the memory trace established by the S tone and the perceptual trace established by the C tone. In the present section, a theory is developed which yields the laws of familiarity theory as a consequence of more "molecular" assumptions about the acquisition and decay of *associations* between internal representatives and the interaction of perception and memory in retrieval.

In addition, we shall briefly consider the problems of specifying the interaction between perception and memory in acquisition, the possibility of an (ugly) interaction between retrieval and acquisition of the C tone, and the possibility of nonzero latency between the formation and the availability of a memory trace.

Interaction between Perception and Memory in Retrieval

The most basic intuitive concept of associative strength theory is that of a frequency representative. A frequency representative is an element (neuron) of the system that responds maximally to a pure tone of a certain frequency. The frequency representatives (at some level of the auditory system) can obviously be ordered on a single dimension in terms of the frequency to which they respond maximally.

The spacing of these frequency representatives is a more difficult question. In general, we want to space them to reflect some psychologically significant aspect of their functioning (although the aspect of choice might be different for different tasks).

For example, it is quite clear that we cannot assume each frequency representative to be so sharply tuned to its optimal frequency that it responds vigorously to this frequency and not at all to the optimal frequency for the representative of any other frequency. In short, we must assume that a pure tone sets up an "activation gradient" on the ordered set of frequency representatives. Presumably all these activation gradients are monotonically decreasing in both directions from the frequency representative of any tone. We might choose to space the frequency representatives so that all the activation gradients had the same functional form, if this were possible. Existing evidence suggests that this would probably require us to squeeze frequency representatives closer together at high values of frequency, creating a pitch scale which is some (not necessarily linear) function of the physical frequency scale.

However, over the small frequency ranges explored in the present study, it is reasonable to assume the amount of "squeezing" to be negligible and thus to space frequency representatives on the pitch scale in direct proportion to their physical frequency. This will be done. Furthermore, we shall assume that the activation gradients around all center frequencies are identical, except for a pure translation on the pitch (frequency) scale, for a limited range of frequencies.

To avoid any possibility of misunderstanding, it should be emphasized that the assumption that the pitch scale is unidimensional and proportional to frequency is an approximation, valid at best for a limited range of frequencies spanning less than an octave. The fact that the absolute *jnd* increases with frequency (although some data, such as, Shower and Biddulph (1931), suggest a plateau between 60 and 1000 Hz) indicates that over the entire range of frequencies, it would be absurd to use a pitch scale that was proportional to frequency. Even the unidimensionality assumption is probably invalid over ranges exceeding one octave, at least in one sense (see Shepard, 1964). However, these considerations are of little or no importance for the present study.

Associative strength theory assumes that the S and C tones set up activation gradients centered around the pitch values S' and C'. The activation gradient is thought of as a perceptual trace with tonic and/or phasic components. The perceptual trace is assumed to die out rapidly (tens or hundreds of msec) after the end of the S tone, at least under conditions where the subject is instructed not to try to rehearse the S tone and where he is given an I tone to rehearse instead.

Thus, we must assume that it is not this active perceptual trace that persists through the I tone to be used in conjunction with the C tone to determine an S-D judgment. Rather, we must assume that a passive memory trace, called a "strength gradient," is also set up by the S tone, and it is this strength gradient which persists through the delay interval. The strength gradient represents the strength of association from each pitch (frequency representative) to the familiarity representative.

It is assumed that whenever a pitch is activated, its strength of association to the familiarity representative is incremented. However, in this section we will not attempt to derive the acquisition of the strength gradient for the S tone from the (perceptual) activation gradient set up by the S tone. We will simply assume that at the end of the S tone there is a strength gradient of a certain form, with parameters which depend on the frequency, intensity, and duration of the S tone. Actually, for the experiments reported in this paper, it is necessary to assume that the total memory strength gradient is the sum of two component strength gradients, with the short-term component being much larger than the intermediate-term component. Over the course of the I tone, the short-term and intermediate-term strength gradients established by the S tone decay exponentially to zero at different rates.

Retrieval is assumed to be accomplished by the interaction of the strength gradient of the S tone with the activation gradient of the C tone to produce a greater or lesser degree of activation of the familiarity representative. The key idea is that, when the C tone activates pitches whose strengths of association to the familiarity representative are in a state of (temporary) facilitation, there is a greater degree of activation of the familiarity representative. The S-D decision is based on the degree of activation of the familiarity representative in relation to a criterion. The criterion must also be assumed to be a function of t_S, t_I, and t_C, though we have no interest at present in the nature of this function. For the present, we shall assume that the interaction between the strength gradient of the S tone and the activation gradient of the C tone has the form of a crosscorrelation between these two functions, integrated over the time used for retrieval ($t = 0$ to $t = t_R$), but weighted by a decay factor $e^{-\omega(t_R - t_C)}$ which expresses the limitations of temporal summation of familiarity.

It seems impossible to motivate any particular form for the activation and strength (generalization) gradients on intuitive grounds. However, some constraints on these functions can be motivated intuitively, and fortunately, these constraints are sufficient for making all the derivations from the theory in which we are interested at present.

The period of time during which retrieval takes place is the time during which the familiarity value is determined. The familiarity value is then sent to the decision system to be compared to the criterion for an S-D judgment. At present, we shall assume that for short values of t_C (less than 1 sec) the retrieval period is the period from the beginning to the end of the C tone ($t_R = t_C$). For C tones longer than 1 sec in duration, we shall assume that the time of retrieval is a fixed 1 sec after the onset of the C tone ($t_R = 1$ sec). A more complicated dependence of t_R on t_C would fit the facts of the last experiment reported in the second section better than this simple function, but the present function is an adequate first approximation. One presumes that t_R is under the voluntary control of the subject to some extent, and that more explicit instructions about time of retrieval (than those of the present experiments) will be required to achieve simple functions $t_R(t_C)$.

The basic definitions and assumptions of this version of associative strength theory are listed below:

DEFINITIONS.

$a(y, C, i_C, t_C)$ is the activation of pitch y at time t_C by presentation of a pure tone of frequency C and intensity i_C from time 0 to time t_C.

$m(y, S, i_S, t_S, t_I + t_C)$ is the strength of association from pitch y to the familiarity representative at the time $t_I + t_C$ sec after presentation of a pure tone of frequency S, intensity i_S, and duration t_S.

$f(x, i_S, t_S, t_I, i_C, t_R)$ is the familiarity (degree of activation of the familiarity representative) of a C tone x Hz different from the previously presented S tone ($x = C - S$).

ASSUMPTIONS.

(Factorization of the height and the shape of the activitation gradient).

$$a(y, C, i_C, t_C) = A(i_C, t_C)\, g(y - C).$$

(Factorization of the height and the shape of the memory strength gradient).

$$m(y, S, i_S, t_S, t_I + t_C) = M(i_S, t_S, t_I + t_C)\, g(y - S).$$

(Determination of familiarity (similiarity) by crosscorrelation).

$$f(x, i_S, t_S, t_I, i_C, t_R) = \int_0^{t_R} \int_{-\infty}^{\infty} a(y, C, i_C, t_C)\, m(y, S, i_S, t_S, t_I + t_C)\, e^{-\omega(t_R - t_C)}\, dy\, dt_C.$$

(Dual-trace assumption).

$$M(i_S, t_S, t_I + t_C) = s(i_S, t_S, t_I + t_C) + i(i_S, t_S, t_I + t_C).$$

(Properties of the generalization gradient).

$g(w) > g(z)$ for all $| w | < | z | < | z_m |$ and $| g(w)|$ tends to zero at a higher order of magnitude than $1/w$ as $w \to \infty$ or $-\infty$.

(Tonic and phasic components of the height of the activation gradient).

$$A(i_C, t_C) = A_1(i_C) + [A_2(i_C)][e^{-\varphi t_C}].$$

(Multiplicative interaction between acquisition and decay).

$$s(i_S, t_S, t_I + t_C) = \alpha(i_S, t_S) \cdot B(t_I + t_C),$$
$$i(i_S, t_S, t_I + t_C) = \lambda(i_S, t_S) \cdot G(t_I).$$

(Exponential acquisition functions).

$$\alpha(i_S, t_S) = [\alpha(i_S)][1 - e^{-[\theta(i_S)]t_S}],$$
$$\lambda(i_S, t_S) = [\lambda(i_S)][1 - e^{-[\psi(i_S)]t_S}].$$

(Exponential decay functions).

$$B(t_I + t_C) = e^{-\beta(t_I + t_C)},$$
$$G(t_I + t_C) = e^{-\gamma(t_I + t_C)}.$$

(Criterion decision rule).

$$\text{Respond "same" iff } \bar{f} - c + \mathbf{X} \geqslant 0,$$
$$\text{where } \mathbf{X} \sim N[0, \sigma(\bar{f})] \doteqdot N[0, 1].$$

The equation for the mean familiarity value, $\bar{f}$, is easily derived from the above assumptions:
Let

$$G'(x) = G'(C - S) = \int_{-\infty}^{\infty} g(y - C)\, g(y - S)\, dy.$$

then,

$$\begin{aligned} \bar{f} = & \int_0^{t_R} [A_1 + A_2\, e^{-\varphi t_C}][\alpha(1 - e^{-\theta t_S})\, e^{-\beta(t_I + t_C)} \\ & + \lambda(1 - e^{-\psi t_S})\, e^{-\gamma(t_I + t_C)}]\, e^{-\omega(t_R - t_C)} G(x)\, dt_C \\ = & \; G'(x) \left[\left(\frac{A_1 \alpha}{\omega - \beta} \right) (1 - e^{-\theta t_S})\, e^{-\beta t_I} (e^{-\beta t_R} - e^{-\omega t_R}) \right. \\ & + \left(\frac{A_2 \alpha}{\omega - \beta - \varphi} \right) (1 - e^{-\theta t_S})\, e^{-\beta t_I} (e^{-(\beta + \varphi) t_R} - e^{-\omega t_R}) \\ & + \left(\frac{A_1 \lambda}{\omega - \gamma} \right) (1 - e^{-\psi t_S})\, e^{-\gamma t_I} (e^{-\gamma t_R} - e^{-\omega t_R}) \\ & \left. + \left(\frac{A_2 \lambda}{\omega - \gamma - \varphi} \right) (1 - e^{-\psi t_S})\, e^{-\gamma t_I} (e^{-(\gamma + \varphi) t_R} - e^{-\omega t_R}) \right]. \end{aligned} \tag{5}$$

When t_C is constant, then $t_R(t_C)$ is constant, and the equation for $\bar{f}$ has exactly the same form as that specified in the molar familiarity theory (e.g., Eq. 5 has exactly the same form as Eq. 2). Thus, this version of associative strength theory is consistent with the results of all of the previous experiments, except the one varying t_C. Furthermore, if we can determine the dependence of t_R on t_C and the time taken by the subject to make his decision, then associative strength theory makes testable predictions about the dependence of familiarity on retrieval time. These predictions will be similar, though not identical, to those given by molar Eq. 4.

The dependence on retrieval time in Eq. 5 is for an activation function with both tonic and phasic components. If the activation function is purely tonic ($A_2 = 0$) or purely phasic ($A_1 = 0$), then Eq. 5 is greatly simplified, and one can derive an analytic expression for the optimal t_R value (making the reasonable approximations that $\gamma \doteq 0$ for small t_R and $\varphi + \beta \doteq \varphi$ since $\beta \ll \varphi$).

Interaction between Perception and Acquisition

Perhaps the most plausible way to account for the development of both the short-term and the intermediate-term memory traces from the perceptual trace is to assume that the increment in trace strength (either short-term or intermediate-term) at any given time during the presentation of the S tone is directly proportional to the degree of activation. The increments added at each moment in time are integrated over the entire duration of the S tone, but each increment is also subject to exponential decay with the passage of time during the S tone. This intuitive picture of the relation between perception and acquisition can be formulated as follows:

$$\alpha = A_3 \int_0^{t_S} (A_1 + A_2 e^{-\varphi t}) e^{-\theta(t_S - t)} dt,$$

$$= \frac{A_3 A_1}{\theta} (1 - e^{-\theta t_S}) + \frac{A_3 A_2}{\varphi - \theta} (e^{-\theta t_S} - e^{-\varphi t_S}), \qquad (6)$$

where, in general, $A_1(i_S)$ and $A_2(i_S)$, so $\alpha(t_S, i_S)$. The formulation for the intermediate-term trace is completely analogous to the above and need not be presented explicitly.

In the above formulation, we have not assumed the rate of decay of the memory trace during the S tone to be the same as the rate of decay of the memory trace during the I tone, i.e., θ is not necessarily equal to β. However, the formulation in Eq. 6 will permit us to assume $\theta = \beta$ and still be consistent with the data presented in the acquisition and decay section of this paper. The assumption of both phasic and tonic components of the activation function gives one an acquisition function that is not too different from an exponential approach to a limit, even assuming that $\theta = \beta$. This is certainly more satisfying than being forced to assume $\theta \neq \beta$.

Note also that if the activation function is assumed to have only a tonic component

($A_2 = 0$), then Eq. 6 has exactly the same form as the corresponding assumption of the previous associative strength theory, which did not attempt to derive acquisition from perception. However, in this case, we must assume $\theta \neq \beta$.

Interaction between Retrieval and Acquisition of the C Tone

Presentation of the C tone obviously can permit the acquisition of a memory trace gradient centered around the C tone. If the memory trace for the C tone builds up during the retrieval process which determines familiarity value and if this memory trace begins to affect the determination of familiarity value, then there are problems. In principle, such an interaction between retrieval and acquisition can be handled by associative strength theory. What is required is a change in the assumption about the determination of familiarity value in retrieval.

However, there is a very good reason for thinking this interaction does not occur. The reason is that such an interaction between retrieval and acquisition would be very deleterious for retrieval. Thus, if it is possible for the subject to perceive the C tone and use it in retrieval, but not acquire a memory trace for it, the subject undoubtedly does just that under the conditions of the present experiments. We expect this because memory for the C tone is never tested. It would be interesting to see if determining the familiarity of a tone delayed the onset of the acquisition process for that tone, in experiments where memory for that tone would be tested later.

Latency between the Formation and the Availability of the Memory Trace

The important events which cause the formation of a memory trace appear to begin simultaneously with the perception of the S tone. Perception of pitch must always be lagging the onset of a tone by an amount which is approximately constant, if the intensity is approximately constant. This lag is the time required for completion of the sequence of transducer and neural events that occur prior to activation of the pitch representatives determining recognition memory performance. Such a perceptual lag would affect the S, I, and C tones in exactly the same manner, so the effect can be ignored.

Nevertheless, the onset of the trace acquisition process might have lagged perception by a different amount than the offset. This apparently did not happen, and it is most parsimonious to assume that acquisition (at least of the short-term trace) begins, and ends, simultaneously with perception. However, this does not mean that the memory trace is available with zero latency following the events which caused its formation. The trace which is *available* for retrieval may be building up with some latency following the critical acquisitional events that are responsible for trace formation. Such a process is illustrated in Fig. 15.

Incidentally, if the latency is substantial ($>$0.5 sec) then we have another reason for assuming no interaction between retrieval and acqusition of the C tone.

There is some inconsistently suggestive evidence for a nonzero latency between formation and availability for retrieval in the results obtained with t_I values less than 1 sec, but the proper experiments have not been performed to investigate the matter. Thus, we cannot draw any conclusions about any such latency period between trace formation and availability. All we can do is to consider the possibility.

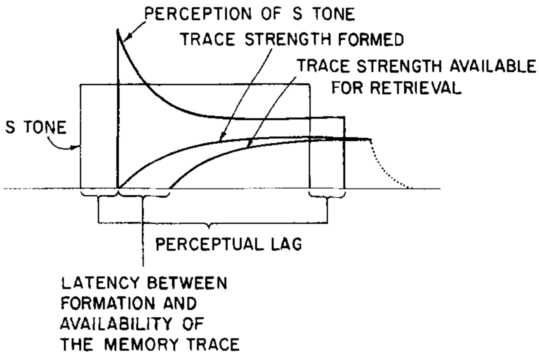

FIG. 15. Diagram illustrating the concepts of perceptual lag and the latency between the formation of a memory trace and its availability for retrieval.

CONCLUSION

Associative strength theory, considered as a theoretical language, allows one to formulate specific theories encompassing a large number of psychologically interesting processes and making predictions about a broad range of phenomena. There are certain key assumptions which must be true in order for *any* of these strength theories to be true. The key assumptions are primarily concerned with retrieval, principally the assumption that it is a familiarity (similarity) dimension which is judged and the assumption that the judgment process is described by the criterion decision rule. The H-S-L and S-D experiments in the first part of the paper indicated the validity of these assumptions.

The particular version of associative strength theory presented in this paper accounts for all the phenomena definitely established by the present experiments and makes predictions about a number of other experiments. However, no assertion is made that this version is correct in every detail. On the contrary, a number of possibilities for modification were proposed, to be considered if the facts warrant them. Nevertheless, there is considerable evidence to support a number of the basic assumptions of associative strength theory, and there is good reason to think that the assumptions which are still in doubt can be suitably modified, if necessary, without requiring modification of the rest of associative strength theory.

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