Strength/Resistance Theory of the Dynamics of Memory Storage

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The theory described in this chapter attempts to characterize the dynamics of storage in memory from the end of the learning period to the time of retrieval. In addition, it is necessary to make some nondynamic assumptions concerning elemental retrieval processes, in order to test the theory of storage dynamics.

This chapter deals with such theoretical questions as the following. First, how many memory traces are there with different storage dynamics? The theory described here is a two-trace theory (short-term memory and long-term memory), and arguments will be given to support this choice of exactly two, dynamically different memory traces.

Second, what kinds of variables should be chosen to represent the properties of memory traces in storage? The present theory characterizes a short-term memory trace (short trace) by a single real-valued property, its strength. The theory characterizes a long-term memory trace (long trace) by two real-valued properties, its strength and its resistance. Only trace strengths influence the probability of correct recognition or recall at any given retention interval. However, the resistance of a long trace affects its susceptibility to subsequent decay, interference, or disruption due to a variety of causes. No arguments will be advanced to try to prove that existing evidence requires characterization of memory traces by continuous variables, rather than a small number

From Contemporary Developments in Mathematical Psycholgy, Volume I, edited by D. H. Krantz et al. W. H. Freeman and Company. Copyright © 1974

of states. This issue has proved so complex that very likely the issue can only be 'settled' by determining which approach yields the simplest general theory of memory.

Third, how are the increments to memory contributed by multiple learning trials integrated with one another? The present theory assumes that trace strengths contributed by multiple learning trials simply add, and that the resistance of the long trace is determined entirely by the resistance of the first increment. Relearning and retention following relearning are also discussed as a natural extension of the problem of multiple learning trials.

Fourth, what is the form of the strength-retention function for the short and long traces? Determining the form of the strength-retention function means determining the way in which the rate of forgetting depends upon the level of strength and the time since the end of learning. Currently available evidence favors the assumption that the short trace decays exponentially ($s = \lambda e^{-\beta t}$) and very strongly favors the assumption of an 'exponential power' decay ($\ell = \lambda e^{-\psi t^{1-\gamma}}$) for the long trace.

Fifth, what are the storage-interference properties of both short and long traces? Neither the short trace nor the long trace shows a 'pure' time-decay function. The rates of decay are influenced by the nature of the conditions that obtain during the retention interval, but the exact nature of the storage-interference properties differs for the short and long traces.

Sixth, how are accidental and experimentally produced retrograde amnesia related to the theory?

Seventh, how valid are the retrieval assumptions made concerning 'yes-no' recognition-memory tasks? The strength-resistance theory assumes that in 'yes-no' recognition-memory tasks, a subject responds 'yes' if and only if the total strength of the relevant trace exceeds a criterion. The central property of this criterion decision rule is the assumption that it is only the total trace associated with the single test item that is judged in recognition memory. No 'competition' among several different memory traces is necessarily involved in the 'yes-no' recognition-memory task. Evidence for this assumption of 'independence from irrelevant strengths' is discussed. The effects of context on retrieval and the assumption that noise in retrieval and acquisition of memory strengths are normally distributed are briefly discussed.

Eighth, what is the relation between different retrieval tasks? The retrieval tasks to be compared are 'yes-no' recognition memory, multiple-choice recognition memory, recall from small populations of response alternatives, recall from large populations of response alternatives, and recency judgments. Although specific retrieval assumptions are formulated for each of these retrieval tasks, relatively little evidence is available to support the validity of these assumptions, except in the case of 'yes-no' recognition memory.

It is also important to take note of the limitations in the scope of the present theory. The theory is concerned with all aspects of the dynamics of the storage phase in memory and with a limited characterization of elementary retrieval processes. The theory is not concerned with the dynamics of acquisition (learning) nor with the dynamics of the retrieval phase. That is to say, the theory does not attempt to develop time functions for the initial establishment of memory traces during the study period, nor does it attempt to develop time functions for the accessing of memory traces during retrieval.

Furthermore, the theory does not attempt to account for many of the more complex combinations of retrieval processes that must be occurring in such tasks as ordered recall of a long list of events or even recognition and recall of a single item, where considerable time is allowed for the recognition or recall and subjects are permitted to generate and test a variety of alternative hypotheses. In short, the retrieval theory is far from being a complete theory of all aspects of retrieval.

Nothing is said in the current theory about the coding aspects of memory. The modalities of memory, associative or nonassociative character of storage in any modality, the nature of representation of any event within a modality, etc., are all memory-coding phenomena that are outside the scope of the present theory. Finally, along the same line, the theory is not concerned with characterizing the logical nature of what is stored in our memory. That is to say, this theory does not characterize the types of concepts, facts, principles, etc., by which human beings encode their knowledge of the world.

Finally, the theory is concerned with (associative) conceptual memory, not (nonassociative) sensory, adaptational, fatigue, or physical types of memories. In particular, the theory is not concerned with the nonassociative type of memory involved in visual or other sensory very short term memory, such as persistence of vision or audition, or afterimages.

It might be worth mentioning that recognition memory for 'single' items such as words, letters, and digits are just as much tests of associative memory as tests of the recognition or recall of paired associates. In 'single' item recognition memory, one is deciding whether that item appeared earlier in the experimental session or on some trial of the session. One is not deciding whether he has ever encountered the item before in his life, in virtually all cases experimentally investigated. In all likelihood, subjects are making judgments based on the associations between the items and some encoding of the experimental context. However, except for the assertion that both single-item recognition memory and paired-associate recognition memory should be incorporated within the same theory of memory dynamics, no more explicit characterization of the nature of encoding in either case is incorporated in the present theory.

THEORY

This section is divided into two parts. The first part presents the definitions and assumptions concerning the dynamics of memory traces. The second part presents the additional definitions and assumptions relevant to the retrieval of memory traces in 'yes-no' recognition, multiple-choice recognition, recall, and recency memory tasks. Each part contains an introductory verbal description of the theory, followed by the definitions and a formal description and discussion of the axioms of the theory and their principal consequences.

Dynamics

The theory of memory-storage dynamics described in this section is a slightly modified combination of the short-term-memory theory presented in Wickelgren (1970) and the long-term-memory theory presented in Wickelgren (1972). The theory assumes that there are two types of associative memory traces with different dynamics: the short trace and the long trace. Each trace is characterized by a real-valued *strength* that determines recognition and recall probabilities. In addition, long traces are characterized by a second real-valued quantity, *resistance*. Greater strength implies greater probability of correct recognition or recall (in general), and greater resistance implies less susceptibility to disruption by time, interference, and certain noxious agents (e.g., concussion, electroconvulsive shock, and some drugs).

Because learning (acquisition) must probably be assumed to be somewhat variable, even under controlled experimental conditions, initial strengths of memory traces are characterized by real-valued random variables assumed to have normal distributions (under certain controlled experimental conditions). In the retrieval section of the theory, there are additional assumptions concerning normally distributed noise introduced during retrieval of both strengths and resistances. However, it is assumed that the noise (variability) contributed by the storage phase is negligible in comparison with the noise contributed by the learning and retrieval phases. If this assumption is correct, then one can essentially lump acquisition and retrieval noise together and ignore it throughout the storage phase. Thus, one can have essentially a real-variable theory of memory dynamics, without the necessity of getting involved in stochastic processes. In the present theory, learning variability and retrieval variability simply sum and do not affect the basic form of the retention function.

Short-term memory is assumed to be formed essentially simultaneously with perception (within fractions of a sec) and to decay exponentially with a time constant varying from a few seconds to perhaps 10 seconds, in the

absence of conscious rehearsal. Even when there is no conscious rehearsal, there is assumed to be an (unconscious) trace-maintenance process that retards the decay of the short trace. This trace-maintenance process is assumed to be insensitive to storage load (the sum of short-trace strengths), at least up to the memory span. However, the maintenance process is sensitive to the rate of forming new short traces for events during the retention interval. Although short-term memory is associative and subject to associative interference effects in retrieval via recall tests, the rate of decay of short traces in storage is assumed to be unaffected by the similarity of new events to previously stored events within the same modality.

Currently, the long trace is also assumed to be formed simultaneously with perception. That is to say, the present assumption is that there is no need to separate an acquisition and a consolidation phase in the establishment of long traces. However, although the strength of the long trace is assumed to be formed at the time of perception or very shortly thereafter, the resistance of this trace is extremely low immediately after its formation. When resistance is low, the long trace is easily destroyed by interference, head injury, and possibly other causes. The resistance of a long trace is assumed to increase monotonically with time since learning. Thus, the older a long trace is, the more difficult it is to destroy or partially degrade the trace. Note that this theory of 'consolidation-type effects' (such as retrograde amnesia) does not require the long trace to be unavailable for use in recognition or recall tests for some period of time following learning. The trace can be available to mediate recall and recognition judgments, but still be in a state of extreme susceptibility to disruption. Furthermore, in congruence with the clinically observed range of human retrograde amnesias, the increase in trace resistance is assumed to be continuing (at a decelerating rate) for the lifetime of the memory.

When there are multiple learning trials separated by some appreciable period of time, the last learning trial is assumed to add an increment to the decaying strength established by previous learning trials. This assumption holds for both short and long traces. However, in the case of long-term memory, an additional assumption must be made concerning the resistance of the long trace. Currently, the theory assumes that the resistance of the entire trace (the sum of the increments contributed by all learning trials) is determined entirely by the time since the first learning trial. That is to say, each new increment to learning is entered with a resistance that increases proportionally to the time since the first learning trial for the item.

Definitions. Let s be the strength of the short trace. Let ℓ be the strength of the long trace.

Let ϕ be the decay force constant for the short trace, $\phi > 0$. Let ρ be the trace-maintenance parameter for the short trace, $1 > \rho > 0$.

Let r be the resistance of the long trace. Let f be the force of decay acting on the long trace.

Let π be the similarity of the material currently being learned to the material involved in some previously formed trace (decay-force parameter for the long trace).

Let μ be the rate of growth of long-trace resistance.

Let t be the retention interval (time from the end of the study period to the retention test).

Let α be the degree of learning in short-term memory: the short strength at $t = 0$, $\alpha \sim N[\bar{\alpha}, \sigma_\alpha]$. (That is, α is normally distributed with mean $\bar{\alpha}$ and standard deviation σ_α.)

Let λ be the degree of learning in long-term memory: the long strength at $t = 0$, $\lambda \sim N[\bar{\lambda}, \sigma_\lambda]$.

Axioms.

(A1) $\dfrac{ds}{dt} = -(1 - \rho)\phi s, \quad 0 \leq \rho \leq 1.$

(A2) A short-trace maintenance process determines ρ. This maintenance process operates in a limited-capacity relation with learning or other processing in the same modality.

Axioms 1 and 2 assert that the decay of the short trace is proportional to its strength. That is to say, short traces decay exponentially. In addition, the rate of decay of short-term memory is assumed to be influenced by the effectiveness of operation of a hypothetical short-trace maintenance process. This trace-maintenance process operates to reduce the rate of decay of short-term memory below some hypothetical maximum-decay rate (ϕ), which would obtain if the constant short-trace decay force were not balanced to some extent by the maintenance process. Trace maintenance is assumed to be unaffected by storage load, that is to say, the number or total strength of traces in short-term memory. However, the trace-maintenance process is assumed to utilize some of the same neural machinery as is involved in learning or other processing in the same modality. Thus, when an individual is learning new material, his short-trace maintenance process will operate less effectively, increasing the rate of decay for previously acquired short traces.

To some extent this is assumed to be a modality-specific limited-capacity mechanism, so that the maintenance process operates more effectively when the interpolated information processing is in a modality different from that in which the traces are stored. This implies a certain degree of similarity dependence to the storage-interference processes in short-term memory, but it is assumed that, within a modality, there is no further dependence upon the 'fine grain' similarity of interpolated learning to original learning. Thus, for example, in the retention of verbal material during interpolated verbal learning, the phonetic or semantic similarity of the interpolated material to the original material should be irrelevant. However, listening to tones or looking at pictures during the retention interval should allow the short-trace maintenance process to operate more effectively for verbal short traces, producing a comparatively low decay rate in this case.

(A3) $\frac{d\ell}{dt} = -\frac{f}{r} = -\frac{\pi\ell}{\mu t^{\gamma}}.$

(A4) $\mu = \mu_0(1 - e^{-\lambda}).$

Axiom 3 expresses the principal assumptions concerning the basic form of the strength-retention function for the long trace as formed by a single learning trial. The rate of decay of the long trace is determined by the ratio of two abstract quantities: the force of decay acting on the trace and the resistance of the trace. The force of decay is assumed to be interference from learning occurring during the retention interval. The magnitude of this force is jointly determined on the one hand by the similarity of interpolated learning to original learning (π), and on the other hand by the remaining strength of the long trace (ℓ). Thus the theory assumes that the decay of long traces is not a passive temporal process, but rather is due to storage interference resulting from new learning. Furthermore, the degree of storage interference is proportional to the similarity between the traces currently being established and the previously learned trace. Underlying the abstract assumption of similarity-dependent storage interference may be the intuitive structural assumption that similar traces involve strengthening associations to internal representatives that are somewhat overlapping. If internal representatives have limited connection capacity, then a very likely mechanism for living within this limited capacity would be one that strengthens a new association to an internal representative and weakens all previously established associations to that internal representative.

Making storage interference proportional to trace strength provides a natural vehicle for keeping all trace strengths nonnegative. At the same time, it is reasonable to suppose that total trace strength is made up of many 'molecular' components, with each component being equally subject to the storage-interference force. Such a process requires that the total force on a trace be considered proportional to the strength of the trace.

The resistance of a trace is assumed to increase monotonically as a power function of its age. The exponent of the power function (γ) is assumed to be a universal constant for all subjects, all materials, and all conditions. This constant is currently assumed to be in the vicinity of $\gamma = 0.75$. The increasing resistance of a memory trace makes it increasingly less vulnerable to storage interference.

If the rate of growth of trace resistance (μ) is relatively constant for all traces that were originally learned above some minimum criterion of strength, then the resistance of all such memories provides information about the relative time of occurrence of different past events for which one has established memory traces. If the resistance of a memory trace is a retrievable property of that trace, then recency judgments might be based on this resistance property. Thus, in addition to expressing the increasing invulnerability of memory traces of increasing age, the concept of resistance might also provide a mechanism for at least one type of 'biological clock.' Using trace resistance as a biological clock to make relative recency judgments for different events will be reliable only if the rate of increase of trace resistance for different events is close to being a constant. However, the rate of growth of trace resistance must depend to some extent on the strength of the initial trace established during the learning period. Otherwise, traces that had zero degree of initial learning would be hypothesized to have the same growth of trace resistance initiated for them as traces established with substantial initial degrees of learning. It seems absurd to imagine that a trace would have a substantial degree of resistance when it had never been learned in the first place or had been learned only at a very, very low level. Thus, one probably must assume that μ is in some way a function of the initial degree of learning in long-term memory (λ). A reasonable choice for such a function, that would satisfy all the constraints previously mentioned, would be to have μ exponentially approach a limit (μ_0) with increasing λ. This is precisely the assumption stated in Axiom 4 concerning the relationship between the rate of growth of trace resistance and the level of acquisition in long-term memory.

Note, however, that Axiom 4 provides only a very limited degree of coupling from trace strength to trace resistance, compared with the much more extensive coupling provided in Axiom 3 from trace resistance to trace strength. At all delays, the resistance of the trace affects the rate of change of long-trace strength, but only the initial strength of the long trace influences the growth of trace resistance. The retrieval assumptions specify that only strength determines recall and recognition judgments and that, under some circumstances, only resistance determines recency judgments. Such a theory produces the anomalous (and probably absurd) prediction that subjects could judge normally the recency of an event that they had completely forgotten, provided that it was initially well learned but had been subjected to extensive storage interference. There are other reasons for thinking that an extensive coupling from trace strength to trace resistance may be needed in some future formulation of this theory, and Axiom 4 should only be viewed as a first step in this direction.

One might note that the rate of decay of the short trace as expressed in Axiom 1 could be considered to result from a force applied to the short trace divided by the resistance to this force provided by the short-term memory trace-maintenance process. This formulation is somewhat clumsier than that expressed in Axiom 1, but it does highlight the systematic differences in the assumptions concerning basic trace dynamics for storage in short- and long-term memory. In both cases, the force on the trace is proportional to trace strength. However, the force on the long trace is proportional to the similarity of interpolated learning to original learning, while the force on the short trace is independent of such similarity. The resistance of the long trace is assumed to grow monotonically with its age. By contrast, the resistance of the short trace is in no way dependent upon its age. Rather, the resistance of the short trace depends on the effectiveness with which the short-trace maintenance process operates in conjunction with any requirements for new learning or processing during the retention interval. In the present formulation, resistance is not really a property of the short trace in the same way that resistance is a property of the long trace. If one formulates the decay or storage dynamics for short-term memory in terms of forces and resistances, resistance is a property of the short-term memory *system*, not of each individual trace. This provides another reason, besides simplicity, for not formulating Axioms 1 and 3 in a parallel manner.

Besides the invariance assumptions already mentioned in previous paragraphs, there is one more invariance assumption that is implicitly contained in Axioms 1 through 4 and that bears explicit mention. The decay in storage for both short and long traces is assumed to be independent of the number of previously established traces (the storage load). Thus, proactive interference is assumed to affect learning and/or retrieval, not storage, of memory traces. The contrary view of proactive interference proposed by Posner and Konick (1966) as the 'acid bath' theory of proactive interference in short-term memory has recently been disconfirmed by Hawkins, Pardo, and Cox (1972). In agreement with this result, neither the number of previously learned memory traces nor the similarity of these traces one to another is assumed to affect the subsequent decay of either short or long traces.

(A5) The short- and long-strength increments from multiple learning trials are additive.

(A6) The resistance of a long trace transfers completely to subsequent increments.

The first four axioms describe the storage dynamics for traces established by a single learning trial. When the same event occurs on several different learning trials separated by intervening mental processing (e.g., other learning or test trials), then some assumptions must be made concerning the integration of learning across the multiple learning trials.

Axiom 5 makes what is probably the simplest assertion concerning the combination of trace strengths established by multiple learning trials, namely, that these strengths are strictly additive. It should be noted that this is not equivalent to the assertion that all that a second, or third, or later learning trial does is to increment the strength established by the first learning trial. Axiom 5 asserts that subsequent learning trials do increment the originally established strengths, but it is quite likely that other traces different from the trace established on the first learning trial may also be established on subsequent learning trials. For example, on the second learning trial, a subject may establish a memory trace that this item or pair of items has been presented twice in the experiment. Nothing in Axiom 5 should be interpreted to detract from this very likely possibility.

In the case of short traces, Axiom 5 is sufficient to describe completely the interaction of multiple learning trials. However in the case of long traces, it is necessary to make some assumption concerning the resistance either of the entire memory trace or of each increment to the memory trace contributed by any of the learning trials. Probably the most natural assumption would be that each learning trial contributes its own increment to trace strength and has its own resistance (determined by the time since that particular learning trial). However, there are two simpler alternative assumptions that are extreme in opposite directions. One extreme assumption would be that the resistance of a long trace is determined by the time since the *last* learning trial. According to this assumption, multiple learning trials increase the degree of learning, but each new learning trial resets the trace resistance value back to zero. Finally, the extreme assumption incorporated in Axiom 6 is that there is a single resistance for the entire trace, and it is the resistance determined by the *first* learning trial. That is to say, each additional learning trial contributes an increment to the strength of the long trace and that increment automatically acquires the resistance of the previously established long trace. Clearly, such an assumption provides a natural advantage for spaced over massed practice, in terms of the rate of forgetting following the last learning trial, when a fixed number of learning trials has been employed in both instances (not necessarily when learning to a criterion).

Theorems. The most basic equation to derive from the axioms of storage dynamics is the form of the strength-retention function for both short and long traces, under constant conditions during the retention interval. A strength-retention function specifies the value of strength as a function of time since the end of learning. Under constant conditions during the storage period, the forms of the strength-retention functions for both short and long traces are quite simple.

Axiom 1 specifies that the short trace decays according to a simple exponential:

$$s = \alpha e^{-\beta t}, \quad \text{where } \beta = (1 - \rho)\phi. \tag{1}$$

The acquisition parameter (α) specifies the initial degree of learning established after either a single learning trial or a series of learning trials. The decay-rate parameter (β) depends on the degree to which the trace-maintenance process can operate during the retention interval but, if the operation of the trace-maintenance process is constant over time, then the β parameter is constant for different delays since learning. Thus, the form of the short-term-memory strength-retention function should be linear on a semilog plot (log strength plotted against linear time): $\log s = \log \alpha - \beta t$.

The form of the long-term strength-retention function, as specified by Axiom 3, is only slightly more complicated, being an 'exponential power' function:

$$\ell = \lambda e^{-\psi t^{1-\gamma}}, \quad \text{where } \psi = \frac{\pi}{\mu(1-\gamma)} \tag{2}$$

$$\ell = \lambda e^{-\psi t^{0.25}}, \quad \text{if } \gamma = 0.75. \tag{3}$$

The acquisition parameter λ specifies the initial degree of learning in long-term memory established either by a single learning trial or at the end of a series of learning trials. The decay-rate parameter (ψ) is directly proportional to the similarity of interpolated learning to original learning and inversely proportional to the rate of growth of trace resistance multiplied by $(1 - \gamma)$, where γ is the exponent of growth of trace resistance. Assuming that the exponent of the growth of trace resistance is a universal constant (in the vicinity of 0.75, as specified in Eq. 3), then the decay rate of long-term memory is simply determined by the similarity of original to interpolated learning divided by the rate of growth of trace resistance. The form of the long-term strength-retention function as specified in Equation 2 should be linear on a plot of log strength against time to the $(1 - \gamma)$th power. If γ equals 0.75, then the form of long-term strength-retention function should be linear on a plot of log strength against the 0.25 power of time.

The long-term strength-retention functions specified in Equations 2 or 3 are a specific quantitative formulation of Jost's Second Law (in Hovland, 1951, p. 649) that the rate of forgetting is constantly decreasing with increasing age of the memory trace. According to the present theory, the absolute loss of trace strength per unit time is decreasing for two reasons: (a) because strength is decreasing and the force of decay is proportional to memory strength, and (b) because resistance to decay is increasing with time since learning.

Retrieval

The theory of memory retrieval described in this section is equivalent to the theory described by Norman and Wickelgren (1969), Wickelgren and Norman (1966), and Wickelgren (1968a) for recognition memory, multiple choice, and recall from a small population of alternatives. However, this section deviates from these previous strength theories in opting for a high-threshold rule for recall from a large population of alternatives (following Bahrick, 1965); it also extends previous strength-theory formulations to provide a theory of the retrieval-decision processes involved in certain nonassociative recency judgments. The statistical decision theory used in this analysis of memory retrieval is in the tradition of Thurstonian Scaling and Signal Detection Theory (see Green & Swets, 1966), and the analysis of recall from a large number of alternatives follows Bahrick (1965).

Definitions. Let d be the total strength of the short and long traces, $d = s + \ell$.

Let c_i be the criterion for a response more extreme than i, $c \sim N[\bar{c}, \sigma_c]$.
Let k be the recall threshold, $k \sim N[\bar{k}, \sigma_k]$.

Axioms.

(A7) Only the sum of short and long traces ($d = s + \ell$) can be retrieved.

The meaning of Axiom 7 is that it is impossible for subjects to separately judge the strength of short and long traces for any purpose. Thus, according to Axiom 7 it would be impossible for subjects to judge the recency of an item by the strength of the short trace alone, ignoring a substantial long-trace component. Also, according to Axiom 7, it would be impossible for subjects to judge whether an item had appeared in the last list by judging only the short component. Both short- and long-component strengths must be retrieved as a sum as input to any judgment process based on trace strength. According to Axiom 7, in order for an experiment to be concerned solely with short-term memory, it must normally be necessary for the long traces of both correct and incorrect items for the last trial to be equal. In this case, the difference between the strengths for correct and incorrect items would tap only short-term memory.

(A8) In recognition, a subject chooses a response more extreme than i, iff the total strength of the bidirectional trace exceeds a criterion: $(d - c_i) \geq 0$. (Criterion Decision Rule.)

Axiom 8 specifies that recognition-memory judgments use the criterion decision rule, according to which a subject responds 'yes' to a test item, iff the total (short and long) strength of a memory trace for that item exceeds the criterion for a 'yes' response. In addition, when subjects are asked to employ confidence judgments, in addition to their 'yes-no' responses, the responses are considered to be ordered on a continuum from the most confident 'yes' to the least confident 'yes' and the least confident 'no' to the most confident 'no.' Each response in this rating scale is assumed to be defined by criteria (c_i), which are ordered in the obvious way on the strength dimension, paralleling the ordering of the responses in the rating scale.

Probably the most critical component of the criterion decision rule for recognition memory is the assumption that only a single memory trace for the test item needs to be judged in an elementary recognition-memory task. According to this decision rule, the strengths of all other items in memory are irrelevant to recognition-memory judgments, since they play no role in the recognition-memory retrieval-decision process. Obviously, this does not have to be true for all 'real life' recognition-memory judgments. Subjects can and frequently do recall a number of additional events or contextual cues to assist them in making their recognition-memory judgments. Such recognition-memory judgments are considered to be 'complex.' Axiom 8 asserts that there exists an elementary recognition-memory judgment process that follows the simple criterion decision rule. It is of course an open experimental question whether one can control recognition-memory judgments to follow a simple criterion decision rule, even if such an elementary process does exist.

The term 'bidirectional' simply means that both forward and backward associations are included.

(A9) In multiple-choice recognition memory or recall from a small population of alternatives, a subject chooses the alternative with the maximum total strength of the bidirectional trace: max $\{d_j - c_j\}$. (Maximum Decision Rule.)

The maximum decision rule for multiple-choice recognition and recall from a small population of alternatives is probably the most natural assumption for these retrieval processes. When one has a choice among a number of alternatives, it seems reasonable (within continuous strength theory) to make that choice be the one with the maximum strength in memory. The only other property of Axiom 9 is that a response bias (c_j) is assumed to be attached to every alternative response (j) in addition to the memory strength (d_j) associated with that response. However, one assumes that in a variety of situations these response biases are approximately equal, or else that pooling across different types of responses permits one to ignore effects of response bias.

(A10) In recall from a large population of alternatives, the subject chooses an alternative only if the total strength of its unidirectional trace exceeds the recall threshold, $d - k \geq 0$. (High-threshold Decision Rule.)

Basically, Axiom 10 applies to situations where it seems unreasonable to imagine the subject could in any way compare the strengths of all the possible alternatives in the recall task. Such situations include prominently those in which the set of response alternatives might be any word in a subject's vocabulary. The term 'unidirectional' means that only the forward association can contribute to recall from a large population.

At present, the theory is deliberately vague concerning the transition region between a small population of alternatives and a large population of alternatives. Presumably, the resolution of this issue will have something to do with limitations on a subject's ability to maintain a response set. To the extent that a subject can maintain a small set of responses in a readily available state, recall would be assumed to follow the maximum decision rule. To the extent that a subject cannot maintain the entire relevant set of possible response alternatives, recall would be assumed to follow the high-threshold decision rule.

(A11) In nonassociative recency judgments, a subject chooses a response more extreme than i, iff $(r - c_i) \geq 0$.

Axiom 11 asserts that subjects can retrieve trace resistance for the purpose of determining how long ago the event occurred that established the memory trace. Traces with high resistance are judged to have been established a long time ago. The thrust of this axiom is that we can have a direct feeling regarding the recency of different events. Clearly this process must be assumed to have considerable noise involved in it, because our ability to determine whether an event occurred one hour ago or an hour and fifteen minutes ago is rather poor.

Furthermore, many recency judgments are based directly or indirectly on associations to time (including calendar) concepts. Thus, we often determine how long ago an event occurred by looking at our watch at the time some event occurred and later remembering that time by association. Very long-term recency judgments, on the order of years, are undoubtedly based largely on direct or indirect associations to time concepts. Thus, we remember how long ago it was that some event took place because we remember the context in which it occurred including some context that is associated to a more or less definite date. However, there are frequently situations in which it seems that there is no possibility of determining recency by direct or indirect associations to time concepts, and yet we are able to make judgments of recency that are considerably in excess of chance (even though these judgments may be terribly poor in relation to what can be achieved by examining a tape-recorder type of memory). Because it seems necessary to postulate trace resistance in order to account for a variety of other phenomena, it is parsimonious to assume that the same trace resistance is judged in these 'nonassociative' recency judgments. Because recencies fall on a single-ordered rating scale, the most natural decision assumption is the criterion decision rule.

Theorems. The most important theorem derived from the retrieval assumptions concerns the form of the operating characteristic for recognition memory:

$$z[P(\text{yes} \mid d)] = \frac{\sigma}{\sigma_d} z[P(\text{yes} \mid 0)] + \frac{d}{\sigma_d}. \tag{4}$$

Equation 4 specifies that the z transform (tails normal deviate transform) of a recognition-memory rating probability for an item with memory strength d is a linear function of the z transform of the corresponding recognition probability for a condition with memory strength zero. The slope of this function on normal-normal probability coordinates (z coordinates) is $m = \sigma/\sigma_d$, where σ is the standard deviation of the strength distribution for the condition with zero mean strength and σ_d is the standard deviation for the strength distribution with mean strength d.

The x and y intercepts (d_x and d_y) of this linear function provide estimates of the mean difference between the strength distributions in units of the standard deviation of either the zero mean-strength distribution or the strength distribution whose mean equals d, namely, $d_y = d/\sigma_d$, $d_x = d/\sigma$. The x intercept (d_x) is generally referred to in signal detection theory as d'.

In determining strength-retention functions, it is important to measure strengths at all different delays using the same unit, usually the standard deviation of the zero mean-strength condition (new items). This would indicate plotting the condition at each different delay vs the new-item condition and choosing the x intercept (d_x) from all such operating characteristics. Although d_x is usually the unbiased estimate of the strength differences one wishes to measure, d_x usually has high variance because slope is quite variable. A considerably lower variance, but biased, estimate is provided by determining the intersection of the operating characteristic with the negative diagonal and looking up this point in the tables of Elliott (1964). This estimate of total strength difference d is $d_s = d/\frac{1}{2}(\sigma_d + \sigma)$. Derivation of the form of the operating characteristic and equations for the slope and intercept parameters can be found in Green and Swets (1966), Wickelgren and Norman (1966), and Wickelgren (1968a).

When one has a large number of different operating characteristics obtained from the same subject at different delays since learning, it is possible to obtain an estimate of d for each condition which has both the unbiased property of d_x and the low-variance property of d_s. To achieve this estimate (d_a), we assume that the variance of the strength distribution of new items is σ^2 and the variance of the strength distribution for old items tested t sec after learning is $[\sigma^2 + \sigma_0^2 f^2(t)]$, where $f(t)$ is the strength-retention function for the decay of the total memory trace. In general, this assumption states that the variance in the strength of old items contains a fixed component common to the variance in the strength of new items (perhaps due largely to retrieval noise) plus a variable component which is proportional to the magnitude of the mean strength (perhaps largely due to acquisition noise).

From the above formulation of the relationship between the means and variances of strength distributions, it is possible to derive rather simple equations that permit low variance and unbiased estimates of d over a set of operating characteristics. Let d_a be the new (unbiased and low-variance) estimate of d_x.

$$d_a = \frac{d_s}{1 - Kd_s^2}, \quad \text{where } K = \frac{(1-m)}{(1+m)} \cdot \frac{1}{d_s^2} \text{ and } m = \frac{1 - Kd_s^2}{1 + Kd_s^2}.$$

In practice, one determines values of slope (m) and d_s for each of the operating characteristics in the set being considered and determines the average value of K for the set of operating characteristics. Then one can convert the low-variance estimate d_s for each operating characteristic into an unbiased and low-variance estimate d_a by means of the previously specified formula.

In addition, whenever one has but one point on an operating characteristic, making it impossible to determine d_s in the usual way, it is possible to estimate d_s and then d_a for that one-point operating characteristic by means of the following equation: $z_d = m(d_a + z_0)$ where z_d is the z transform of the probability for the (old item) strength distribution with mean d, and z_0 is the z transform of the probability for the (new item) strength distribution with mean zero. This equation involves two unknowns, m and d_a, but the equation can be converted into an equation involving the two variables d_s and K:

$$\frac{d_s}{1 + Kd_s^2} + \frac{1 - Kd_s^2}{1 + Kd_s^2} z_0 - z_d = 0.$$

Since one will already have determined the value of K from the set of operating characteristics that have two or more points each, the above equation involves only one unknown, d_s. The above equation can be solved by numerical methods for the value d_s, which can then be converted into d_a using preceding formulas.

Provided that certain simplifying assumptions are valid, it is possible to determine estimates of strength differences (d) from probabilities of correct choice in multiple-choice experiments and recall from a small population of alternatives. The assumptions involved in this process and the method of deriving such strength estimates are discussed in detail in Norman and Wickelgren (1969) and Wickelgren (1968a). Earlier equivalent formulations of multiple-choice decision processes in a psychophysical context are discussed in Green and Swets (1966) and Swets (1964).

Estimates of d can be derived from probabilities of correct recall from a large number of alternatives, provided two parameters are estimated: k and K. The parameter k is the distance of the recall threshold from the mean of the strength distribution for new items in units of the standard deviation for new items. The parameter K expresses the relationship between a slope (m) and d_s, as specified earlier. If one has run recognition-memory experiments using the same materials, contexts, and conditions as were used for recall memory, then the value of K derived from the recognition experiments might well be used in the analysis of the recall data. If not, then K will have to be an estimated parameter for the recall data, just as k must necessarily be. Presumably, the value of k will be somewhere between 1.5 and 4 standard deviations above the mean of the strength distribution for new items.

The scaling of recency and the determination of degrees of discriminability between different recency distributions is accomplished in the same way as for recognition memory.

PHENOMENA

Dynamics

Exponential decay of the short trace. The earliest success of strength theory was the obtaining of exponential decay in certain short-term memory tasks (Wickelgren & Norman, 1966). The type of task in which exponential decay of the short trace is consistently observed includes 'two-phase' tasks in which subjects are given a list of items to remember, followed by a probe item that may be a cue for a recognition judgment of that item or for recall of the succeeding item in the previous list. Besides Wickelgren and Norman (1966), the most extensive and definitive test of the exponential decay hypothesis for short-term memory is found in a study of recognition memory for single letters at different rates of presentation from one letter per sec up to four letters per sec (Wickelgren, 1970).

An example of the excellent fit provided by the exponential decay hypothesis to the data for all three rates of presentation used in this latter study is shown in Figure 1. The results shown in Figure 1 are averaged across all six subjects who participated in this experiment. To avoid the possibility of distorting the form of the retention function via the averaging process, the logs of the strengths for each subject at each rate of presentation were averaged, rather than averaging the strength values themselves or pooling the choice data across all subjects, etc.

In addition, there is a monotonic (approximately linear) relation between the decay rate and the presentation rate. This relation can be derived from a relatively simple quantitative formulation of Axiom 2, which concerns the short-trace maintenance process (see Wickelgren, 1970). The hypothesized short-trace maintenance process is discussed in more detail in the following section concerned with storage interference in short-term memory.

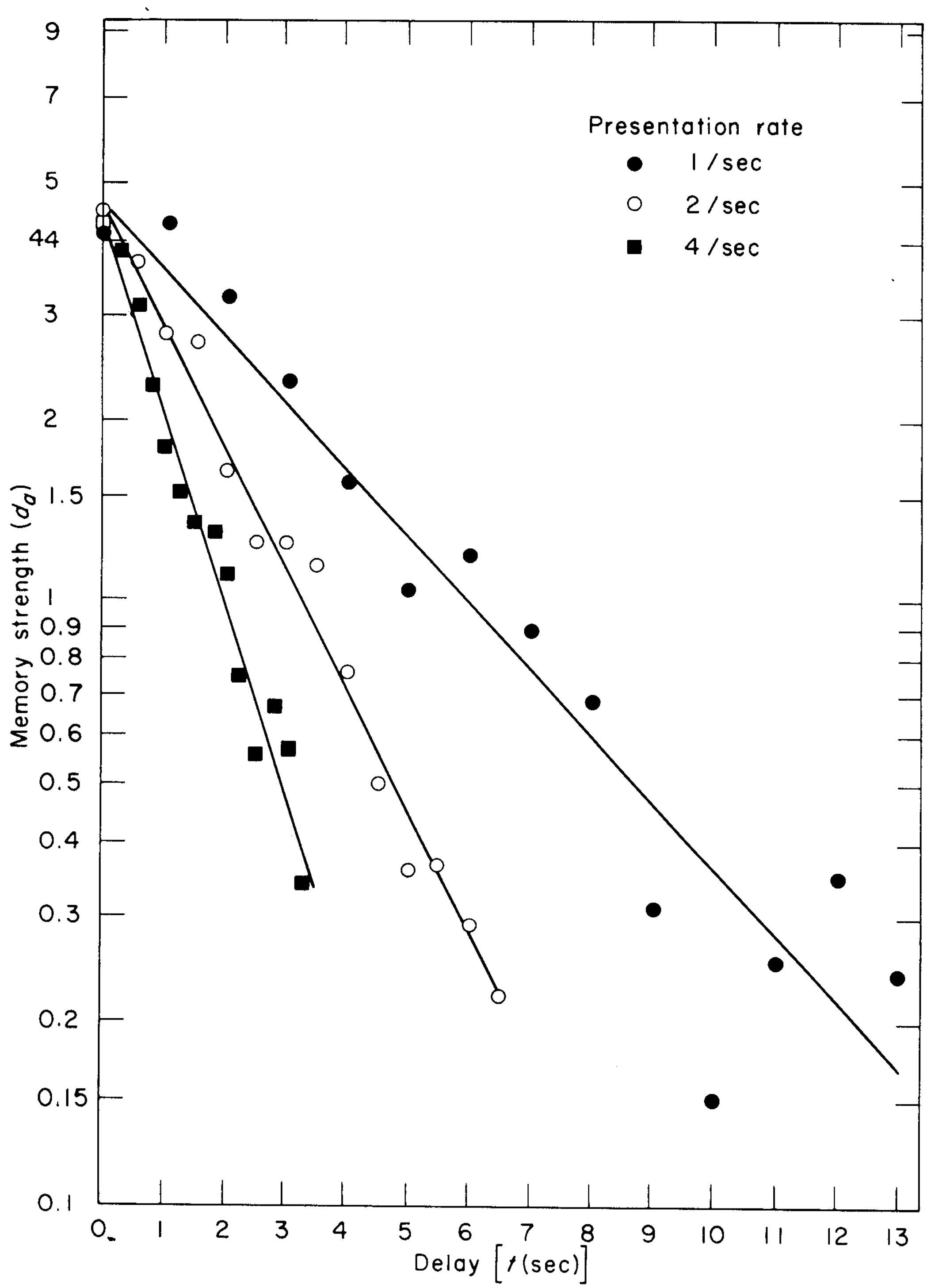

FIGURE 1.
Average strength retention functions on a semilog plot for probe recognition of single letters presented at rates from 1 letter/sec to 4 letters/sec. The straight line represents the best-fitting exponential decay function.

The excellent fit of the exponential decay hypothesis contrasts sharply with the very systematic deviation (shown in Fig. 2 for these same short-term retention data) from the exponential power retention function ($\ell = \lambda e^{-\psi t^{1-\gamma}}$) found by Wickelgren (1972) to provide a good fit to long-term retention data. In addition to these two studies, several other two-phase recognition-memory studies also provide somewhat noisier support for the hypothesis of exponential decay (Wickelgren, 1967, 1968b; Wickelgren & Norman, 1971). Finally, Norman (1966) found approximately the same decay process in two-phase probe-recall tasks as in two-phase probe-recognition tasks.

Unfortunately, at present, the confirmation of exponential decay of the short trace is confined exclusively to two-phase paradigms. The other paradigm that is suitable for assessing the strength retention function for the short trace—namely, the three-phase paradigm used originally by Brown (1958) and Peterson and Peterson (1959)—has consistently yielded strength-retention functions that deviate from simple exponential decay. This deviation is considered by almost everyone to result from the presence of a long-trace component in addition to the short-trace component in these tasks. Wickelgren (1969) and Wickelgren and Berian (1971) provided some support for this hypothesis by finding that when the long component was subtracted from the total memory trace, in three-phase tasks involving both memory for pitch and memory for verbal materials, then the remaining (short?) component can be well fit by an exponential decay. Nevertheless, the paradigm-invariance of the exponential decay of the short trace requires extensive further demonstration.

However, it should be noted that the present theory well explains why the three-phase paradigms that have been used should have exhibited a long component, whereas the two-phase memory paradigms previously discussed did not. The reason derives directly from retrieval Axiom 7 that only the sum of short and long traces can be judged, in combination with observations regarding the frequency of repeating the same item across different trials in the experiment. All of the two-phase studies that are well fit by exponential decay involve a small population of items from which the selection occurs for the list on each trial, with rather little time elapsing between the occurrences of the same item from one trial to the next. Assume that the traces for all trials are strengthened associations from some set of cues (concepts, representatives, etc.) that are approximately identical over a long sequence of trials to the item (letter, digit, etc.) representatives. For example, a concept such as 'presented on this trial' is potentially common to all trials. Under such circumstances the long traces for correct and incorrect items on a given trial must be approximately equal, because long traces from previous trials will not have decayed very much. Because the measure of strength (d) is an interval scale measuring strength differences between correct and incorrect items, only the short component will be present in these two-phase paradigms.

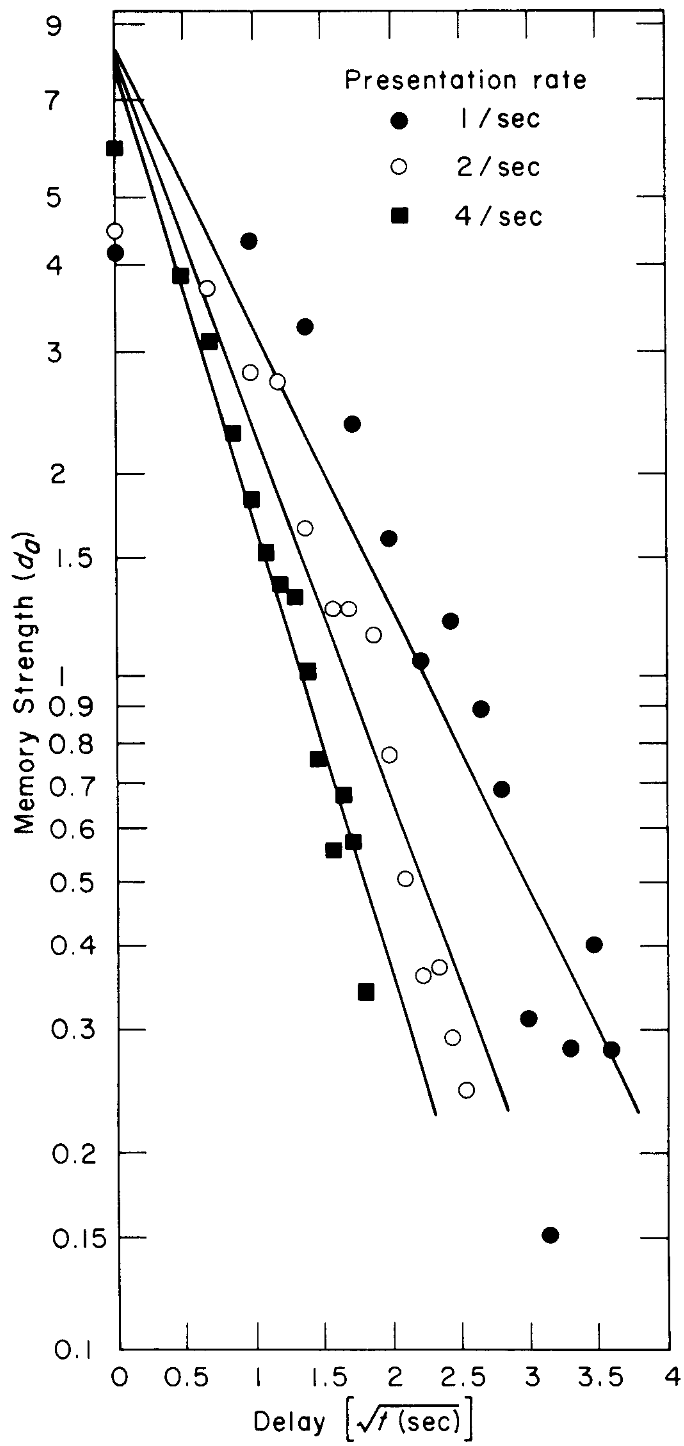

FIGURE 2.
Same data as in Figure 1 on a plot of log d_a vs $t^{0.5}$, the type of plot which is approximately linear for the long trace.

By contrast, in all of the three-phase experiments that have demonstrated substantial long components, the frequency of repeating the same item from trial to trial is very much lower than in the two-phase experiments. Obviously, further experiments should be done to show that it is the frequency of repeating the same item that accounts for the difference, not the different paradigms.

Storage interference in short-term memory. Bower and Bostrom (1968) and Wickelgren (1967) have shown that storage interference produced by subsequent learning is independent of the similarity of this subsequent learning to original learning in short-term memory. This conclusion derives from the finding that the ability to correctly recognize a previously presented A–B pair in contrast to a nonpresented A–D pair is identical whether interpolated learning included an A–C pair or consisted entirely of X–C pairs. Of course, interpolated A–C learning causes more retrieval interference than does X–C learning on a recall test of short-term memory to the probe A. However, on a recognition test this difference disappears, indicating that the effect of the interpolated A–C learning is entirely on retrieval and not on storage. This contrasts sharply with results discussed in a subsequent section, which indicate that storage interference (as measured by recognition tests) is similarity-dependent in long-term memory.

However, the lack of similarity-dependent storage interference for the short trace should not lead one to conclude that storage in short-term memory is characterized by a passive temporal-decay process with storage interference playing no role. The short trace is characterized by its own particular type of similarity-independent storage interference. Although the 'fine grain' (A–B, A–C) similarity of interpolated to original learning seems to be unimportant in short-term memory, a grosser similarity of the modality employed in original and interpolated tasks is important to the rate of decay of the short trace. Reitman (1969) has shown that retention over a 15 sec interval (presumably including some short-trace component) is substantially superior when the interpolated task consists of detection of tones in white noise, compared to an interpolated task that consists of detection of a target verbal item against a background of nontarget verbal items. Reitman argues that conscious rehearsal was eliminated from all of her conditions and, if it was, the results argue for the modality dependence of an unconscious trace-maintenance process as specified in the present theory. Despite Reitman's moderately strong case for the absence of rehearsal in her task, caution must be exercised in the interpretation of these findings. Nevertheless, whether the trace-maintenance process in this case was conscious or unconscious, the results argue that similarity of modality for interpolated vs original material does play a critical role in the short-trace storage process.

In addition to the effects of gross modality similarity between original and interpolated learning, storage in short-term memory is affected by the amount and the difficulty of information-processing activity in the retention interval, whether the interpolated activity involves learning or not. Marcer (1968) has shown that recall of a CCC trigram becomes poorer the faster subjects perform a simple subtraction task (counting backwards by threes or fours from a three-digit number) during the retention interval. Posner and Rossman (1965), Posner and Konick (1966) and Merikle (1968) have all shown that more difficult interpolated information-processing tasks produce poorer recall. All of these experiments employed the Brown-Peterson three-phase design, but Wickelgren (1970) showed that increased rate of presentation of items in a two-phase probe-recognition design increased the rate of forgetting for any item in the list. These studies demonstrate that forgetting in short-term memory does not simply depend upon the time delay, but is influenced by the nature of interpolated activity during retention interval.

However, Wickelgren (1970) also demonstrated that the storage interference in short-term memory is not a simple 'knockout' process, where each interpolated item substitutes for some item previously presented in a buffer store of limited capacity. That is to say, amount of forgetting was not directly proportional to the number of interpolated items. Rather, forgetting was intermediate between what would be expected from a pure time-decay process and what would be expected from a pure item-interference process. Similar tendencies also exist in the probe-recall data of Waugh and Norman (1965) and Norman (1966), though the effects were smaller in the recall studies, presumably due to complicating effects of retrieval interference in the recall tasks.

There has been some suggestion that the requirement of *learning* interpolated material increases the rate of decay for the short trace, compared with doing other types of information processing during the retention interval. The strongest evidence for this is the faster decay rates observed with the two-phase probe tasks (where interpolated material is being learned), compared with the three-phase tasks (where the interpolated material has virtually never required learning). However the probable presence of a substantial long-trace component in all of these previous three-phase tasks makes it impossible to conclude anything definite with respect to the effects of interpolated learning vs nonlearning tasks on the short-trace decay rate. In addition, there might also be some other 'paradigm' difference.

All in all, some assumption of a short-trace maintenance process seems necessary to account for the effects of the modality and difficulty of the interpolated tasks in short-term retention. However, more experimental work is obviously needed to determine the specific nature of this hypothetical short-trace maintenance process and the conditions that affect its operation. Hopefully, the theory can be strengthened in some simple ways in order to capture more of the details of this short-trace maintenance process.

Exponential power decay of the long trace. Wickelgren (1972) studied retention intervals from 1 min to over 2 years using both continuous recognition and study-test recognition-memory paradigms under a variety of experimental conditions for a variety of types of verbal material. This study found that a single (long-term) memory trace with the exponential power form specified in Equation 2 provided a good fit to retention functions from delays of 1 min to delays of over 2 years. Wickelgren (1972) found the good fit with $\gamma = 0.5$, but noted that the optimal assumed exponent of growth of trace resistance (γ) was probably somewhat greater than 0.5. Subsequent work on a greater variety and range of experiments indicates that the correct value of the growth of trace resistance is probably closer to $\gamma = 0.75$. The exact value of the exponent is not determined at present because any exponent in the region of $\gamma = 0.5$ to 0.8 appears to provide a reasonably good fit to the data. However, the basic exponential power decay form for the long trace is rather well established by comparison with a variety of alternative simple hypotheses regarding the form of the long-term strength-retention function. Alternative hypotheses ruled out by Wickelgren (1972) were linear decay ($\ell = \lambda - \psi t$), exponential decay ($\ell = \lambda e^{-\psi t}$), logarithmic decay ($\ell = \lambda - \psi \log t$), and power function decay ($\ell = \lambda t^{-\psi}$).

Retrograde amnesia. Following severe head injury of the concussion variety, human beings frequently cannot remember events that occurred for some time prior to the injury. Clinical studies of retrograde amnesia indicate that this memory loss is temporally defined rather than being defined on the basis of trace strength, trace importance, personal-impersonal, etc. (Russell, 1959; Whitty, 1962). In retrograde amnesia following concussion, it is the most recent memories that are lost, not the strongest nor the weakest, nor the most or the least important, or personal vs general factual memory, etc. The period of time over which the RA extends appears to be almost continuously variable, from RAs that extend over only a few seconds prior to the injury to RAs that extend over periods of many minutes, hours, days, weeks, months, years, or even tens of years. The overwhelmingly predominant tendency is for short RAs of only a few seconds or tens of seconds, but the existence of RAs of almost any length led Russell (1959) to argue that something about the character of the memory trace is continually changing over the entire lifetime of the memory.

Precisely this type of continuous change is postulated in the strength-resistance theory of memory. Furthermore, because trace strength is not the basis for the selectivity observed in retrograde amnesia, some property other than trace strength must be postulated to be continually changing over the lifetime of the memory. The increasing trace resistance that appears to be necessary in order to account for the form of the long strength-retention function provides precisely the type of memory property needed to explain the clinical phenomenon of retrograde amnesia. According to this hypothesis, the same property of the memory trace that makes it more resistant to normal forces of forgetting also makes it more resistant to loss in retrograde amnesia due to concussion.

It should be observed that retrograde amnesia is frequently only a temporary phenomenon, with substantial recovery of the memory occurring over a period of time following the injury. As the memory recovers, it appears to recover in a temporally defined manner as well, so that the oldest memories are recovered first, followed by the next oldest, and then the next oldest, . . . , up to the most recent. Frequently there is a residual amnesia of a few seconds or minutes that is never cleared up, but occasionally the residual amnesia extends for an even longer period of time. Similar temporally defined and shrinking retrograde amnesias have been found following electroconvulsive shock in humans (Williams, 1966).

Studies of retrograde amnesia in animals following electroconvulsive shock, anaesthetics, anoxia, convulsant drugs, hypothermia, concussion, spreading depression of the cortex, local brain stimulation, etc., have found retrograde amnesias in animals that are also temporally defined in a somewhat similar manner to that found for human beings (Weiskrantz, 1966). Just as in the case of retrograde amnesia in human beings, the period of time covered by the amnesia appears to be quite variable over the different conditions studied, ranging from a few seconds up to several hours or even days. Superficially, this can be taken to provide support for a long trace that is continually increasing in resistance over its lifetime. However, because the variability and extent of RA in animals under these various conditions is not well understood, it would be premature to draw any firm conclusion on this matter at the present time.

The present theory does not account in any way for the conditions under which the retrograde amnesia should be temporary or permanent, but it does account for the basic temporally defined character of this retrograde amnesia and the recovery therefrom. The accounting for retrograde amnesia is largely qualitative in comparison to the accounting for the form of the long-trace retention function. However, it should be noted that a function specifying that resistance approaches a limit will fail to fit the retrograde amnesia data, just as it will fail to fit the data from normal long-term retention. The increase in the resistance property of the long trace must be assumed to be going on continuously from the moment of formation (or within a few seconds or tens of seconds thereafter) to the lifetime of the memory (possibly years or tens of years).

Hemispheric Conflict and the Forgetting of Lateralized Engrams

Recently, Goldowitz, Burešova and Bureš (1973) have developed a hemispheric conflict technique for studying the nature of the long-term retention function in animals. The technique involves training animals to make one of two choices on one day with one hemisphere under spreading depression. Then at various retention intervals, the relative strengths of both traces are assessed with both hemispheres functioning normally in the choice situation. The results of their study indicate that habits learned on two different days cannot be equated at all retention intervals by the same relative numbers of initial learning trials for the two conflicting habits. Rather, as the retention interval increases, equal relative habit strength on the retention tests requires less and less difference in the initial degree of learning of the two habits.

This is precisely what one expects from the present strength-resistance theory, because memory traces have a type of temporal encoding ('know their age'). According to the present theory, the retention functions for habits learned on different days will intersect at progressively longer delays, the smaller the initial difference in degree of learning. (Of course, the initially learned habit must have had a greater degree of learning than the subsequently learned habit if there is to be any crossing of the retention functions.) The qualitative convergence of theoretical conclusions from such methodologically disparate studies as human verbal memory and choice memory in rats is quite satisfying. However, because of the use of a different dependent variable, it is not possible at the present time to determine whether or not there is precise quantitative agreement between the animal and human studies.

Similarity-Dependent Storage Interference in Long-Term Memory

As discussed in Wickelgren (1972), over a dozen studies use recognition tests that demonstrate that increasing the 'fine grain' similarity of interpolated to original learning increases the storage interference produced by the interpolated learning. For example, AB–AC, AB–CB, or AB–AB_r interpolated learning designs produce greater retroactive interference than does an AB–CD design. In contrast to the 15 or more positive confirmations of this law, there are no published contradictions to it known to me. Thus, we may regard similarity-dependent storage interference of the long trace as extremely well established.

Spacing of Multiple Learning Trials

It is widely believed to be well established that forgetting following spaced learning trials (distributed practice) is slower than forgetting following massed learning trials (massed practice). In actual fact, numerous experiments by Underwood and his colleagues have frequently failed to find this superiority in retention following distributed vs massed practice.

Surprisingly enough, the failures to obtain superior retention following distributed practice are as confirming of the present theory as are the successes in achieving superior retention following distributed practice. The reason for this is that the failures to obtain superior retention following distributed practice (e.g., Underwood & Richardson, 1955) have always used learning to a criterion, with the frequent result that the total time between the first and last learning trials was not much different for the massed and distributed practice conditions. Under these circumstances, the present theory would not expect any difference between massed and distributed practice in the rate of forgetting following the last learning trial. The general lack of any consistent difference under these conditions actually supports the present theory.

By contrast, the successful examples, where distributed practice produces significantly slower forgetting than massed practice (e.g., Keppel, 1964), have generally employed a fixed number of learning trials and, in the case of the Keppel experiment, an extremely long spacing between some of the trials (e.g., an entire day). Creating a vast difference between the time of the first and last learning trials is precisely what the present theory requires in order to exhibit a large difference in the rate of forgetting following the last learning trial.

These results provide largely qualitative support for the present theory, but one experiment described in Wickelgren (1972) did provide a very limited degree of quantitative support for Axiom 6 that the resistance of long traces transfers completely to subsequent increments (making the resistance determined entirely by the time since the first learning trial).

Further implications of this theory for the comparison of retention following learning trials with different types of spacing need to be investigated. For instance, the present theory asserts that two massed learning sessions separated by a long interval would be equivalent in retention properties to a series of learning trials that were evenly spaced throughout the entire period from first to last learning trials, provided that the time between the first and last learning trials was equated. Rates of forgetting in the two cases should be identical, though it might be necessary to vary the number of learning trials in the two instances in order to achieve equal degrees of learning.

Relearning. According to the present theory, retention following relearning of previously learned material should be characterized by a slower rate of decay than that following original learning or the learning of comparable new material, no matter how much time has elapsed since original learning and no matter how little memory strength remains. The reason for this is that resistance continues to increase while strength decreases during retention interval. At the time of relearning, according to Axiom 6, the resistance of the original trace transfers completely to the increments during relearning. To my knowledge, there is no available evidence concerning retention functions following relearning. However, speed of relearning is invariably faster than original learning or comparable new learning, even under conditions where recall or recognition performance is extremely low (see Nelson, 1971). Furthermore, Nelson (1971) found that savings in relearning was the same for items that were recognized as for items that were not recognized in a retention test prior to the relearning. This suggests that some other factor besides strength is influencing the relearning performance. The resistance property postulated by the present theory could serve this function, under the assumption that forgetting between learning trials is a significant factor retarding original learning but not relearning (because of high resistance).

Some of the probable differences between relearning and original learning can probably be explained by virtue of the trace-resistance property. However, it seems likely that when relevant data become available concerning relearning and retention following relearning it will be necessary to make some modification of the present theory in the direction of postulating a greater coupling from strength to resistance, in addition to the presently postulated coupling from resistance to strength. Undoubtedly, as strength decreases to a very low level, resistance does not continue to increase and may even begin to decrease. It seems counterintuitive to imagine that a memory which has been largely forgotten should be characterized by slower forgetting following relearning, the longer one waits after the memory has been essentially forgotten. However, since my intuition is not based on any relevant data, the theory may turn out to be right and my intuition to be wrong.

Evidence for two traces. Many classes of evidence that have been alleged to support a two-trace theory of memory as opposed to a single-trace theory of memory are not at all convincing (see Gruneberg, 1970). As Gruneberg emphasizes, the rapid forgetting characteristically observed during the first few seconds of a retention interval does not imply that the memory trace decaying over that period of time is different from the memory trace decaying over retention intervals from minutes to hours to days to years. It has already been repeatedly pointed out that long-term-memory traces are continually decreasing in their rate of forgetting on any absolute scale (Jost's Second Law). Furthermore, the difference between the rate of decay at retention intervals of minutes vs retention intervals of years (both considered to depend only on the long trace) is greater than the difference in decay rate of retention intervals of seconds (considered to depend on the short trace) and retention intervals of minutes or tens of minutes. From a strictly *qualitative* point of view, the extremely rapid forgetting at short retention intervals is consistent with comparable findings for long-term memory at all different comparisons of shorter vs longer retention intervals. Arguments that derive from the rapid forgetting property of the short trace are also not definitive.

However, there are at least three classes of evidence available at present that strongly support the assumption of two dynamically different memory traces, rather than a single memory trace.

First, I have consistently found that the long-trace retention function, which works well for delays greater than a minute (recent evidence indicates that it works well down to a delay of about 20 sec), will not work when extrapolated down to retention intervals within the first 20 sec following learning. In order to fit strength-retention functions quantitatively within the first 20 sec following learning, it appears to be necessary to assume a short-trace component in addition to the long-trace component. However, it must be emphasized that there are an infinity of possible quantitative theories, and it would be very premature to contend that no single-trace theory of simple form could handle all strength-retention functions from zero sec delays to the lifetime of the memory trace. Nevertheless, the fact that these retention functions can be accounted for by a simple two-trace theory constitutes support for the two-trace theory until such time as someone can demonstrate a plausible single-trace theory that also accounts for this data.

Second, perhaps the single most convincing qualitative evidence for the distinction between short- and long-term memory is the difference in the storage-interference properties of the two traces. As has already been pointed out, experiments considered to be tapping primarily the short trace have demonstrated that storage interference is independent of the fine-grain similarity of interpolated to original learning. By contrast, storage interference for the long trace is similarity dependent. The similarity independence of storage interference in short-term memory requires considerable further replication, but if it continues to be obtained, this is probably definitive qualitative evidence for a distinction between short and long traces.

Third, the existence of certain patients (with bilateral mesial temporal and hippocampal lesions) who have severe deficits in the ability to consolidate new long-term memory, but relatively little impairment in the ability to establish new short-term memories, is an impressive piece of evidence in support of the two-trace theory (Scoville & Milner, 1957; Milner, 1966; Wickelgren, 1968b). Recently, a reverse impairment of auditory verbal short-term memory with no impairment of long-term memory has been observed by Warrington and Shallice (1969) and Warrington, Logue, and Pratt (1971) in several different patients. In this latter case, however, the memory impairment seems to be specific to the verbal modality, not a global impairment of all short-term memory (e.g., short-term memory is normal for visually presented material).

Retrieval

Independence from irrelevant associations. As pointed out in the theory section, perhaps the most important component of the criterion decision rule for recognition memory is the assumption that only a single strength needs to be judged to determine a recognition rating response. Thus, if a subject is to judge whether an A–B pair was presented in a previous list, he is assumed to judge only the strength of the A–B association, without being influenced by the strengths of any 'competing' A–C association. One way to test this is to determine whether strengthening an A–C association has any effect on the ability to distinguish a correct A–B association from an incorrect A–D association. The results of two published experiments on recognition in short-term memory support the assumption of independence from irrelevant association (Bower & Bostrom, 1968; Wickelgren, 1967).

As previously mentioned, for A–B recognition to be equally good following A–C interpolated learning as following C–D interpolated learning, storage interference in short-term memory must be similarity independent, in addition to independence from irrelevant associations holding during retrieval via the recognition test. Thus, the same studies are simultaneously evidence for both phenomena. The fact that the same findings are not observed for long-term memory is attributed to the presence of similarity-dependent storage interference in the case of the long trace, not to any lack of independence from irrelevant associations in retrieval. However, this is obviously a matter of comparative plausibility, not logical necessity. It is the independence from irrelevant associations that largely forms the factual basis for the assertion that recognition is a more direct test of the strength of memory traces in storage than is recall. Recall is obviously subject to retrieval interference effects in the form of specific A–B, A–C response competition, which the previously cited results indicate did not affect recognition.

To conclude from the above that recognition tests are free of all possible retrieval interference factors and directly indicate loss in storage would be premature. I hãve conducted some unpublished studies on the effects of changing background context on recall and recognition that indicate that contextual change is much less important in the case of recognition memory than in the case of recall memory. In some cases, contextual change may have no effect at all on memory assesed by recognition tests.

However, in other situations, DaPolito, Barker, and Wiant (1971), Light and Carter-Sobell (1970), and Tulving and Thomson (1971) have all shown that changes in associative context from learning to retrieval can have a significant effect on recognition-memory performance. If the probability of spontaneously occurring contextual change alters systematically with delay since learning, then this may be influencing the form of the retention function in addition to, or instead of, other causes of forgetting observed on recognition-memory tests. Alternatively, effects of contextual change in recognition memory under these conditions may simply serve to multiply the memory trace by the same constant at all delays (in which case it would be absorbed in the acquisition parameters). Resolution of the problem concerning 'spontaneous' contextual changes in the forgetting observed by recognition-memory tests is an important theoretical issue in the interpretation and evaluation of strength-resistance theory.

Normally distributed noise. Probably one of the least important assumptions of the present theory is that the noise involved in the acquisition and retrieval processes is normally distributed. The assumption that the noise in acquisition and retrieval is normally distributed can be assessed by determining the fit of operating characteristics to straight lines on normal-normal probability coordinates. Determining the goodness of fit for operating characteristics generated by the rating method in recognition memory can now be done quite precisely using the method of maximum likelihood as described by Dorfman and Alf (1969). To my knowledge, this has not yet been done for recognition-memory data. When it is done, I would not be surprised to see that, in at least some cases, the assumption of normally distributed noise could be rejected.

What has been done is to determine by 'eyeball' methods that the normal distribution assumption is at least approximately accurate. There are at least two interesting, theoretically predicted exceptions to this statement (Norman & Wickelgren, 1965; Wickelgren, 1969), but in general the assumption has proved quite satisfactory. Because the level of accuracy aspired to by the present theory of memory dynamics is about 1.5 significant figures, all that really needs to hold is that the strength distributions are unimodal. As long as the strength distributions are unimodal, a characterization by the best-fitting normal approximation will undoubtedly be adequate. The only danger in using normal approximations where they may not be strictly valid comes from using the tails of the distributions. It is dangerous to use a single operating characteristic to determine a d_s difference greater than four standard deviation units. In cases where one wishes to assess differences in means of strength distributions that exceed four standard deviation units, it is desirable to construct discriminability scales, using the method suggested by Creelman (1967). In this method, one compares condition 1 with condition 2 and condition 2 with condition 3 in order to derive the discriminability difference between condition 1 and condition 3. In using this method, it is important to keep in mind that slopes must be taken into account, so that all strength discriminability values are scaled in terms of the same unit standard deviation.

Relation between different retrieval tasks. For some reason, Tanner and Swets (1954), in their original paper introducing signal-detection theory to psychology, considered that the prediction from one type of decision task to another was an essential initial criterion for the evaluation of statistical decision theory. Comparison from 'yes-no' to multiple-choice (forced choice) or recall paradigms is riddled with theoretically uninteresting complications (for a discussion of these, see Wickelgren, 1968a). Thus, it is probably fortunate that the application of statistical decision theory to the study of recognition memory has largely ignored this problem of predicting across different retrieval-decision tasks.

There have been a few attempts, and surprisingly enough they have been relatively successful. Green and Moses (1966) showed that the *d*s derived from 'yes-no' recognition-memory performance could be used to predict two-alternative multiple-choice recognition-memory performance, and Kintsch (1968) showed that the statistical decision assumptions made successful predictions across 'yes-no,' two-, four-, and eight-alternative multiple-choice paradigms. Norman and Wickelgren (1969) were not quite so successful in making predictions across 'yes-no' recognition, multiple-choice recognition, and recall tasks, but all that was necessary to achieve successful prediction was the estimation of an additional parameter relating the retrieval noise for 'yes-no' recognition to the retrieval noise for multiple choice and recall. Frankly, because of all the complications discussed in Wickelgren (1968a), I have little confidence that much has been demonstrated by these few attempts to predict absolute levels of performance across different retrieval-decision tasks.

A more promising approach is probably to compare the forms and rates of retention functions obtained with different retrieval-decision tasks. Because of the greater simplicity of the criterion decision rule for recognition memory than any of the other decision rules, recognition memory has been used to determine almost all of the retention functions obtained in previous research designed to test strength theories of memory. This made good sense as an initial strategy, but it is probably now time to begin studying retention functions by recall and multiple choice. One early study of verbal short-term memory by Norman (1966) found that the form and rate of decay for the short trace was approximately the same whether probe recall or probe recognition was used. Extensive work on this problem needs to be done to confirm both the retrieval-decision rules for multiple choice and recall and also to confirm the assumption that the same dynamical types of memory traces are tapped by multiple-choice and recall memory tasks as are tapped by 'yes-no' recognition-memory tasks.

Recency judgments. Wickelgren (1972) has shown that the discriminability of the recencies of two memory traces is a negatively accelerated, monotonically increasing function of the time delay between the two memory traces. In that experiment, the function could not be well fit by a power function with an exponent similar to that used in fitting the strength-retention functions for recognition memory. However, certain flaws in the design of that recency-memory experiment may have led to precisely the type of small, consistent deviation from a power-function increase in recency discriminability that was observed. Thus, it would be premature to reject the extremely attractive possibility that the same property of the memory trace underlies recency judgments and the increased resistance to forgetting.

ACKNOWLEDGMENT

This work was supported by grant MH 17958 from NIMH.

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