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PROBABILITY SUMMATION AND REGIONAL VARIATION IN CONTRAST SENSITIVITY ACROSS THE VISUAL FIELD*

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Abstract—Contrast sensitivity at different positions in the visual field has been measured at various spatial frequencies using a patch of grating suitably vignetted to give a stimulus localized in both space and spatial frequency. While contrast sensitivity along a vertical line through the fixation point falls off steadily from a maximum at the centre, sensitivity along a horizontal line displaced 42 periods of the grating above the fixation point is approximately constant, at least out to 32 periods from the mid-line. The way in which detectability increases with increasing number of cycles (2 up to 64) has been measured for gratings with short horizontal bars centred on the fixation point and for gratings with short vertical bars centred on the mid-line 42 periods above it. The relation between sensitivity and number of cycles can in each case be explained exactly assuming probability summation across space as long as the variation in sensitivity across the visual field is taken into account.

INTRODUCTION

Many studies have shown that the detectability of a periodic pattern increases as the pattern is enlarged to include a greater number of cycles. While these studies have mostly concentrated on the improvement in detectability produced by the successive addition of the first few cycles, there are indications that slow improvement continues as the number of cycles is increased further, although in typical experiments the improvement ultimately appears to cease (e.g. see Howell and Hess, 1978 and Legge, 1978, who give earlier references).

While there is general agreement that the initial improvement in detectability with increasing number of cycles largely reflects summation within the receptive fields of the detectors involved, the exact magnitude of the improvement in detectability for larger numbers of cycles is less certain, and the mechanism underlying this summation effect is not entirely clear. In this paper we report further measurements of the detectability of sine gratings with large numbers of cycles, and consider whether these results can be explained by probability summation across space, that is by assuming that an extended grating pattern will be detected if any of the independently perturbed detectors on whose receptive field the stimulus falls signals its presence.

Since the probability-summation hypothesis can only be used to predict the detectability of extended patterns if the sensitivities of the detectors whose receptive fields are in different positions are known, we have used a small patch of grating to study the way in which sensitivity varies with position in the visual field. These measurements have not only confirmed that sensitivity falls off with increasing distance from the fixation point at all spatial frequencies, but have also allowed us to delimit a substantial region of the peripheral visual field over which sensitivity is essentially constant. Measurements of the effect of increasing number of cycles on the detectability of gratings located in this region can reveal more clearly whether or not the contribution made by peripheral cycles is consistent with the probability-summation hypothesis than can measurements of the detectability of centrally fixated patterns.

Application of the probability-summation hypothesis also requires a knowledge of the way in which the detectability of a grating patch varies with contrast. We have therefore studied how the probability of correct response in a two-temporal-alternative forced-choice task varies with contrast for different grating stimuli.

METHODS AND PROCEDURES

Stimulus patterns

Two groups of patterns were used: (1) patches of sine grating with short bars containing various numbers of cycles (from 2–64) centred at the mid-point of the region of the visual field being studied, and (2) small patches, usually containing 4 cycles, located at various positions within the area occupied

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by the largest pattern used (the pattern containing 64 cycles). The whole set of patterns could be located in one of two general regions of the visual field: (1) in a vertical strip centred on the fixation point, the bars of the gratings being horizontal, or (2) in a horizontal strip centred directly above the fixation point at a distance equal to 42 periods of the gratings; in this case the gratings were vertical. These two regions of the visual field were chosen so as to have in one case as much and in the other as little, regional variation in sensitivity as possible.

The whole set of two groups of patterns could be at one of several spatial frequencies: 3, 6, 12 or 24 c/deg. Small patches at different positions were also used at 1.5 and 18 c/deg.

The edges of the patterns were not sharp (Figs 1a and 1b). In the direction parallel to the bars, the contrast varied with distance, x, as does one cycle $(-4p < x < 4p)$ of $[1 + \cos(2\pi x/8p)]/2$, a raised cosine function that has a period equal to eight times the spatial period, p, of the sine grating itself (Fig. 1A). Thus the length of the bars at half maximum contrast was four periods. In the direction perpendicular to the bars, the luminance profile was that of a sine wave weighted by an envelope. In the case of a pattern having only one cycle (measured at half maximum contrast), the envelope was a single cycle of a raised cosine with a period equal to twice that of the grating itself. The envelopes for patterns having more than one cycle had flat portions inserted in the middle of the one cycle of cosine. The solid line in Fig. 1B shows the profile for a pattern having 4 cycles; the envelope is shown as a dashed line.

The patterns were always turned on and off gradually. The temporal profile of contrast (Fig. 1C) was a Gaussian function of time with a time constant of 100 msec; the contrast was above one half of its peak value for 167 msec.

The patterns were presented as a raster display on a cathode ray tube with a P31 phosphor. This display had a mean luminance of 500 cd m^{-2} and appeared a desaturated green. The observers viewed the patterns binocularly.

Display size and fixation marks

The exposed face of the cathode-ray tube was seen through a rectangular hole (20 × 29 cm, the long dimension being perpendicular to the bars of the grating) in a large screen (61 × 61 cm) illuminated to approximate the cathode ray tube screen in luminance and hue. To be able to have a large number of cyles of the pattern on the display while not having the period of the grating so small as to tax the resolution capability of the equipment, we chose to keep the number of cycles per centimeter on the display constant at a value of 3 throughout the experiments. Thus the available display size was 87 periods by 60 periods. The spatial frequency at the observer was varied by varying viewing distance, a viewing distance of 57 cm giving a spatial frequency of 3 c/deg.

Except in some of the earliest experiments, all of the patterns were presented within a strip parallel to the long edge of the exposed area of the cathode-ray tube and centred 3 cm (9 periods) away from the edge. Figure 1D shows the 4-cycle patch used to study variation of threshold contrast with position while Fig. 1E shows the largest pattern presented within the strip. The display and surround could be rotated between sessions to make the strip either vertical (Fig. 1D) or horizontal (Fig. 1E), at the same time

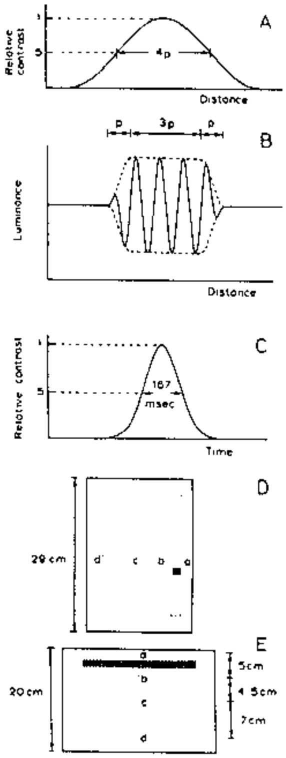

Fig. 1. (A) Variation in contrast of grating patterns parallel to their bars; *p* is the spatial period of the sinusoidal variation in luminance perpendicular to the bars. (B) An example of the instantaneous variation in luminance perpendicular to the bars of a grating patch containing 4 cycles. (C) Time-course of the contrast of all stimuli. (D) Arrangement of display in those experiments in which grating patterns with horizontal bars appeared within a vertical strip (dotted outline) centred on the fixation point. In this example a 4-cycle patch of grating is present 8 periods below the fixation point (mid-way between spots a and b). The grating patch is represented diagrammatically only; in all experiments the contrast of the pattern fell off smoothly in all directions. (E) Arrangement of display for experiments in which gratings with vertical bars were presented within a horizontal strip 42 periods above the fixation point (spot d). The grating with the largest number of cycles used (64) is shown here. The dimensions marked in D and E apply to both. The spot labelled c was in the centre of the screen; it was not used as a fixation mark in these experiments.

making the bars of the gratings either horizontal or vertical.

With the help of several dark spots (each 1.5 mm dia), the observer fixated one of two different places to make the strip where the patterns would be shown fall in the desired region of the visual field: the observer fixated either mid-way between two spots 5 cm apart centred on the mid-point of the pattern strip (spots a and b in Fig. 1D) or on a spot 14 cm away from the mid-point of the pattern strip (spot d in Fig. 1E).

In the earliest (6 c/deg) experiments, the placement of the stimuli and fixation marks on the cathode-ray tube was haphazard, although they were never nearer than 2 cm to the edge of the screen. No differential effect of placement was ever noticed.

The electronic and computing equipment has been described by Graham *et al.* (1978).

Procedures

Thresholds. The two-temporal-alternative forced-choice staircase procedure described by Graham *et al.* (1978) was used. This procedure determined the contrast at which the subject made approximately 90% correct responses. This contrast will be referred to as the threshold contrast of the pattern. The standard error for a set of 4 staircase determinations of a pattern's threshold contrast was about 0.025 log units. In any one session, all patterns presented had the same spatial frequency and were in the same general region of the visual field (either in the strip centred on the fixation point or in the strip 42 periods above it).

To measure threshold contrast as a function of number of cycles, trials of patterns having different number of cycles (but the same spatial frequency and centred at the same point in the visual field) were randomly intermixed. Four sets of staircases were run producing 4 measurements for each number of cycles.

In measuring threshold contrast as a function of position, a pattern having a small number of cycles was used. (There were 4 cycles in all cases except for the 6 c/deg patches in the centrally fixated strip for which there were only 2). Trials of the pattern at various positions (symmetric about the mid-point of the strip being used) were randomly intermixed. In general, 2 sets of staircases were run. Since 2 positions equally distant on either side of the mid-point are equivalent for many purposes, the results for 2 positions could be averaged. Thus 4 determinations of threshold contrast were produced for each distance away from the mid-point. Four were also collected for the mid-point.

Psychometric functions. For the determination of a psychometric function, one pattern (of fixed spatial frequency, number of cycles, and position) was used throughout a session. Each trial was a two-temporal-alternative forced-choice trial like those in the staircases but, rather than using a staircase procedure, trials of various different contrasts were randomly intermixed as in the method of constant stimuli. Generally 7 or 8 different contrast levels spaced 0.075 log units apart were used with 180 trials at each level.

Observers. The authors, who have normal vision when corrected, were the observers in this experiment. The forced-choice procedure, the random intermixing of patterns, and the quantitative nature of the comparisons of interest, protected against possible influence of observers' expectations.

RESULTS AND INTERPRETATION

Threshold contrast as a function of position and number of cycles

Figure 2 shows how the threshold contrast of a small patch of grating varies with its position (given on the horizontal axis as number of periods away from the mid-point of the strip being measured). Results for both observers and several frequencies are shown in each panel. In the vertical strip centred on the fixation point (Fig. 2, left panel) threshold contrast rises quite quickly as the patch is moved away from the fixation point and the change is rather similar for all frequencies tested. On the other hand, in the strip running horizontally 42 periods above the fixation point (Fig. 2, right panel) the threshold contrast is much more uniform, perhaps even decreasing slightly at positions away from the mid-point.

Figure 3 shows threshold contrast of patches of grating containing various numbers of cycles. Threshold contrast is plotted against the number of cycles (shown on the horizontal axis). All patches were centred at the mid-point of the strip. Note that a grating 64 cycles wide fills the whole of the region in which threshold contrast for a small patch of grating was measured (out to 32 periods on either side of the mid-point). In the centrally fixated strip (left panel), sensitivity increases as the number of cycles increases but levels off for large numbers of cycles. This levelling off is quite similar at 3, 6, and 12 c/deg. The left-most portion of the function for 24 c/deg is somewhat different for reasons discussed later. In the strip 42 periods above the fixation point (right panel), threshold contrast continues to fall out to 64 cycles, the largest number used.

The curves in Fig. 3 show the increases in sensitivity to be expected on the basis of a summation rule to be described and discussed later.

In each panel of both Figs 2 and 3, all the results for different spatial frequencies are approximately vertical translations of each other. However, if the threshold contrasts are replotted as functions of actual visual angle rather than of number of cycles, the curves for the centrally fixated strip are no longer vertical translations of each other. Figure 4 illustrates this point. Some of the results in the left panel of Fig. 2 giving threshold contrast as a function of position (those for subject JGR, 3 and 24 c/deg) are replotted in Fig. 4A along with additional results from the same subject. The horizontal axis again gives

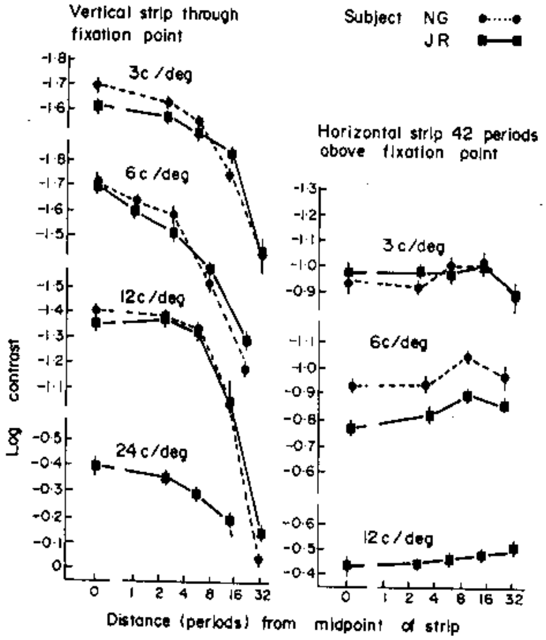

Fig. 2. Left panel: log threshold contrast for a 4-cycle patch of grating with horizontal bars as a function of its vertical distance from the fixation point. Points plotted are averages of measurements made with patches at the same distances above and below the fixation point. Distances are given in terms of the spatial period of the grating at each spatial frequency and are plotted on a logarithmic scale. Right panel: log threshold contrast for a 4-cycle patch of grating with vertical bars located within a horizontal strip 42 periods above the fixation point as a function of distance in periods away from the mid-line. Points plotted are averages of measurement made with patches at the same distances to left and right of the mid-line. Note that there is a separate contrast scale for each spatial frequency but that the same scales are used for both left and right panels.

position as number of periods away from the fixation point although in this plot a linear scale is used and distances above and below the fixation point are shown separately. All the functions are vertical translations of each other. Fig. 4B shows the same results as Fig. 4A, but the horizontal axis gives position as distance (in degrees of visual angle on a linear scale) away from the fixation point. Threshold contrast can be seen to rise much faster for high spatial frequencies than for low; the curves are certainly not just shifted vertically.

Similarly, if threshold contrast (plotted as a function of grating size in numbers of cycles in Fig. 3) is replotted as a function of grating size in degrees of visual angle, then the curves for the centrally fixated strip also no longer appear as vertical translations of each other.

It is not surprising that the decrease in threshold contrast with increasing number of cycles is similar for all frequencies in the centrally fixated strip (Fig. 3, left) as it is presumably a direct consequence of the fact that, for all frequencies, local sensitivity falls off in the same way with increasing distance when distance is expressed in numbers of periods away from the fixation point (Fig. 2 left).

For all the spatial frequencies tested, even the lowest one (1.5 c/deg), threshold contrast was lowest at, or certainly very near, the fixation point. Wilson and Giese (1977) also reached this conclusion on the basis of measurements with patterns containing spatial frequency gradients. Limb and Rubinstein (1977) reached an apparently contradictory conclusion on the basis of their results with line-plus-line patterns, but they did not consider the possibility of probability summation among different spatial frequency channels in their analysis (see Wilson, 1978, for further discussion of their calculations).

The extent of spatial summation

While our measurements of the threshold contrast of centrally fixated grating patterns (Fig. 3, left) indicate that detectability increases continuously as the number of cycles is increased to large values, they make it clear that once the grating has about eight cycles any further effect is only small (no more than 0.08 log units decrease in threshold contrast as the number of cycles is increased from 8 to 64). On the other hand, when the gratings are viewed peripherally, so that the sensitivity is more or less constant over the area of the pattern, the continuous decrease

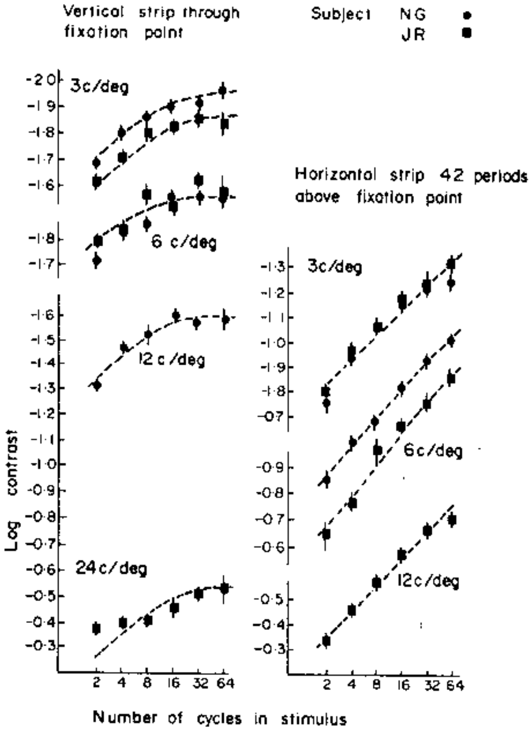

Fig. 3. Log threshold contrast for gratings with different numbers of cycles (logarithmic scale). On the left are results for gratings with horizontal bars located within a vertical strip centred on the fixation point; on the right are results for gratings with vertical bars presented within a horizontal strip whose centre fell 42 periods vertically above the fixation point. In both cases all gratings were centred in the strip within which they could appear. The dashed curves show the predictions of the simple probability-summation model discussed in the text.

in threshold contrast as the number of cycles is increased from 8 to 64 is considerably greater, being on average about 0.27 log units (Fig. 3, right). Thus, while it may not be well established for central viewing it must be accepted that in the periphery some kind of summation process takes place over at least something approaching 64 cycles of our patterns.

While it is not possible to rule out absolutely the idea that this summation may be occurring within the receptive fields of individual detectors, it is stretching credulity rather far to suppose that the visual system contains detectors with receptive fields having as many as 64 pairs of excitatory and inhibitory regions. Moreover it would be necessary to suppose not only that there were large numbers of detectors of this kind (to account for summation as number of cycles is increased at different spatial frequencies) but also that there were other detectors with smaller receptive fields, and hence broader spatial frequency bandwidths, which are necessary to account for other observations e.g. sine-plus-sine experiments like those of Sachs *et al.* (1971), King-Smith and Kulikowski (1975) and Quick and Reichert (1975). It therefore seems more reasonable to suppose that the great extent of the observed spatial summation results from the combination of signals from many detectors with smaller, spatially distributed receptive fields. Although it is possible to envisage other ways in which these signals might be combined to give the same effect (e.g. see Quick, 1974; Mostafavi and Sakrison, 1976; Graham and Rogowitz, 1977) one of the simplest and most frequently suggested combination rules is the "inclusive or". This combination rule, with the assumption that the response of each detector is variable, leads to the hypothesis of "probability summation across space" (e.g. King-Smith and Kulikowski, 1975; Legge, 1978).

Threshold contrast predicted on the basis of probability summation across space

The probability summation hypothesis involves two basic assumptions. Firstly, it is assumed that a stimulus will be detected by the observer whenever any one or more of the detectors whose receptive fields are in the appropriate part of the visual field signal the occurrence of a stimulus (i.e. the "inclusive or" rule is assumed). Secondly, it is assumed that the probability that a particular detector will signal the

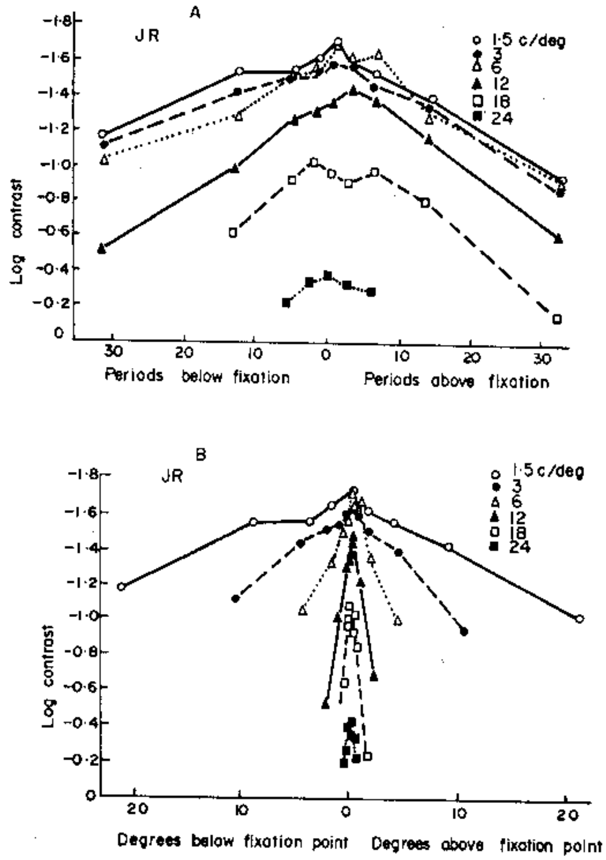

Fig. 4. Log threshold contrast for 4-cycle patches of grating with horizontal bars as a function of distance above and below the fixation point (Fig. 1D). In A the results are plotted as a function of eccentricity normalised with respect to the spatial frequency of the gratings while in B the distances are given simply in degrees of visual angle. Note that the curves are approximately symmetric for displacements above and below the fixation point and that, when distance from the fixation point is expressed as numbers of periods, the effect of changing the spatial frequency is well described as a vertical displacement parallel to the log contrast axis.

occurrence of the stimulus on any particular trial is independent of the probability that any other detector will. Then, in order to predict the probability of the observer detecting a given stimulus, it is in general necessary to know with what probability each of the independent detectors involved will signal the occurrence of the stimulus. Prediction of threshold contrast for a given stimulus requires that the way in which the probability of response of each detector varies with constrast should also be known, while prediction of the way in which threshold contrast varies as a function of some parameter of the stimulus also requires a knowledge of the way in which changing that parameter changes the response of each detector.

To apply the probability-summation hypothesis to predicting the effect of increasing the number of cycles of a grating on its threshold contrast, we must make some simplifying assumptions. First, we assume that the probability, P_i, that detector i will signal the occurrence of a stimulus is related to stimulus contrast, C, by an equation of the form suggested by Brindley (see Brindley, 1960, p. 192) and modified by Quick (1974):

$$P_i = 1 - 2^{-(S_i C)^q} \tag{1}$$

where q, a constant which determines the steepness of the probability of detection function, is the same for each detector, and S_i is the sensitivity of the i'th detector to the stimulus. It follows (see, for example, Graham *et al.*, 1978) that the probability that the stimulus will be detected by one or more of a group, j, of such detectors is

$$P_j = 1 - 2^{-(S_j C)^q} \tag{2}$$

where

$$S_j = \left(\sum_i S_i^q\right)^{1/q} \tag{3}$$

the summation of the sensitivities of the individual detectors being performed across all the detectors in the group. This has the same form as the relation for each of the individual detectors. Thus, in considering the detection of an extended grating, we can suppose the multiplicity of detectors actually involved to be replaced by a smaller number of composite detectors each equivalent to a local group. Convenient composite detectors to consider are those equivalent to groups of detectors whose receptive fields are centred within adjacent non-overlapping strips of the visual field one period of the grating wide.

Our second simplifying assumption is that the sensitivity of the composite detector for the one-period-wide strip located in the middle of a small four-periods-wide patch of grating (like the patches whose threshold was measured) can, to a first approximation, be estimated by assuming that the patch is detected by four such composite detectors acting independently. Then, by interpolation, we can estimate the sensitivities of all composite detectors in the region of the visual field which the extended gratings occupy. The sensitivity, S, for an extended grating can subsequently be calculated by summing the sensitivities of the composite detectors in the same way as the sensitivities of the individual detectors were summed to calculate the sensitivities of the composite detectors.

That is,

$$S = \left(\sum_j S_j^q\right)^{1/q} \tag{4}$$

where the summation is extended over all the composite detectors for adjacent non-overlapping strips within the grating area. The threshold contrast, C_t, for different numbers of cycles of the grating can then be predicted by performing the summation over the different areas and setting $S \cdot C_t$ constant.

In the special case where the sensitivity of the composite detectors does not vary with position (all S_j's the same) the sensitivity for a grating with n cycles will be

$$S = (nS_j^q)^{1/q} = n^{-1/q}S_j \tag{5}$$

and hence the threshold contrast will be related to number of cycles by

$$C_t \propto n^{-1/q} \tag{6}$$

Thus, if the logarithm of threshold contrast is plotted against the logarithm of the number of cycles, we will obtain, in this special case of uniform sensitivity, a straight line with a slope of $-1/q$. Brindley (1960, p. 192) has discussed the significance of analogous relationships for discs of different diameters presented for different durations.

Predictions in Fig. 3

The lines in Fig. 3 show predictions of threshold contrast as a function of the number of cycles in the grating, calculated as described above. For 6 c/deg in the centrally fixated strip and 3 c/deg in the strip 42 periods above the fixation point, the results for the two subjects were so close that only one predicted curve is shown in Fig. 3. This was calculated from the average of the results for two subjects shown in Fig. 2.

Calculations were made using various values of the parameter q, but the predictions shown are those for $q = 3.5$. (The almost straight line predicted for the region 42 periods above the fovea has a slope of roughly $-1/3.5$). Values for q of 3 or 4 produced predictions that were similar but seemed to fit most of the results less well. Those for $q = 3$ predicted too great a fall in threshold contrast as number of cycles was increased and those for $q = 4$ predicted too little.

The vertical position of the predicted curves was chosen by eye for best fit to the central and right-hand experimental points for which the theory is most secure. While the vertical position could have been determined absolutely using the measurements of threshold contrast as a function of eccentricity, this did not seem appropriate since there were usually small shifts of threshold contrast from experiment to experiment. Thus the results to be predicted often showed slightly more or less sensitivity than the results used in making the prediction. (Since one of the stimuli was the same in the number-of-cycles series as in the position series, comparison in Figs 2 and 3 of the observed values for that stimulus, usually the one containing 4 cycles positioned at the mid-point, gives some idea of the magnitude of the variation).

In any case some small discrepancy between the two series is to be expected on account of the different uncertainty conditions. In the position series, the observer is uncertain as to which position the stimulus will occupy on the next trial; in the number-of-cycles series, the observer is uncertain as to the number of cycles. It is not clear how much effect this difference might produce but it would almost certainly be rather small (see Graham *et al.*, 1978, for an example in a very similar context).

Comparison of predicted and observed sensitivity as a function of the number of cycles in a grating

The fit of the predictions based on the simplified treatment of the probability-summation hypothesis to the observations is, in general, rather good (Fig. 3). There are, however, two places where there may be discrepancies. In the case of the highest spatial frequency (24 c/deg) in the centrally fixated strip, predicted threshold contrast is clearly higher than the observed threshold contrast for small numbers of cycles. In this case it is likely that, because of the relative insensitivity of the visual system at this high frequency, the detectability of the small patches will be significantly increased by low-frequency components introduced by truncation of the grating. Since this effect is related to the edges of the patterns it will become relatively unimportant in determining the detectability of gratings with many cycles. As a similar discrepancy is not obvious at lower frequencies it may be presumed that, at these lower frequencies, the threshold contrast of even the smallest patches (with two periods) is being determined by detectors whose optimum spatial frequency is close to that of the grating. It may also be presumed that the mechanism underlying the increase in sensitivity with grating size is the same for all gratings with two cycles or more.

In the cases where sensitivity is nearly uniform across the whole region (in the strip 42 periods above the fixation point), there is some indication that the observer may not be quite as sensitive as predicted to the largest number of cycles. Of the 5 measured functions, two of them (NG at 3 c/deg; JGR at 12 c/deg) fall below the predicted curve at 64 cycles. There is no obvious *a priori* reason for expecting a real effect of this kind and, in any case, threshold contrast clearly decreased as predicted on the basis of probability summation across space over at least 32 cycles.

Value of the summation exponent

From the present results the best estimate of the value of the exponent in equation (4) is about 3.5. A value of 3 is probably too low and a value of 4 is probably too high. The value necessary to predict summation between two different spatial-frequency components is in this same range (Graham *et al.*, 1978) as also is the value Wilson and Bergen (1978) and Quick *et al.* (1978) use in all their predictions. Mostafavi and Sakrison (1976), however, finding that a value of 6 was necessary to explain their results using a different kind of pattern, suggested that this discrepancy might be due to the existence of a slightly more complicated non-linear operation than that given in equation (4). This would result in threshold contrast for patterns with lower amplitudes at threshold (like ours) being correctly predicted using lower exponents than would be required for patterns with higher amplitudes at threshold (like theirs). Before accepting the necessity of this more complicated non-linear operation, further experimental results should probably be collected and carefully analyzed.

Agreement with current estimates of the bandwidth of spatial channels

If the existence of pooling of signals from detectors with spatially distributed receptive fields is accepted, Graham (1977) and Wilson and Bergen (1978) have shown that the thresholds for a wide variety of periodic and aperiodic patterns are consistent with a model in which there is only one bandwidth of channel at each frequency. Thus it seems reasonable to consider such a model very seriously. When the number of cycles in a stimulus is relatively large, the predictions of this model are those given already (lines in Fig. 3). But are the sensitivities observed for small numbers of cycles consistent, at least qualitatively, with the model? To answer this the bandwidth of the channels according to this model must be known.

From sine-plus-sine results, Quick and Mullins (1978) have computed that the full bandwidth at 1/e peak amplitude (assuming a Gaussian channel sensitivity function) is approximately equal to 0.6 of the centre frequency of the channel for centre frequencies between 5 and 10 c/deg. For centre frequencies above 10 c/deg, the bandwidth stays constant at 6 c/deg. From line-plus-line results, Wilson (1978) has computed a slightly greater full bandwidth, at 1/e height, of between 0.70 and 0.85 of the centre frequency. Mostafavi and Sakrison (1976), Wilson (1978), Wilson and Bergen (1978) use even larger bandwidths. Whether or not these bandwidth estimates are in satisfactory agreement with each other is an important question, but for our purposes here, it suffices to know that even the narrowest of them implies only a few lobes in the receptive fields. The receptive fields will have excitatory centres, inhibitory surrounds, and perhaps slight secondary excitatory lobes but no more lobes. Such receptive fields are completely covered by grating patches containing $1\frac{1}{2}$ to $2\frac{1}{2}$ cycles. Thus, 2, or certainly 4, cycles is a relatively large number of cycles for this model, and the predictions in Fig. 4 should account for all the increase in sensitivity as number of cycles is increased above 2 or 4 (assuming the spatial frequency is low enough to avoid the problem described above for 24 c/deg). In short, the predictions should agree with all the observed results, as they do.

Psychometric functions

Up to this point, we have shown that the observed spatial summation is successfully accounted for by equation (4) which is the prediction of a model assuming probability summation across space. Although, as we have noted, other models can lead to the same functional relationship, if the probability-summation model is in fact the correct one, the variability in responses of individual detectors should show up in psychometric functions measured for various stimuli. The form of psychometric function expected in a two-alternative forced-choice experiment can be derived from the probability of detection function for a group of detectors (equation 2) by assuming that the probability of obtaining a correct response is one on trials when the stimulus is detected and one half on trials when it is not. This leads to the probability of correct response, P_c, being related to stimulus contrast, C, by

$$P_c = 1 - \tfrac{1}{2}\cdot 2^{-(SC)^q} \qquad (7)$$

Figure 5 shows two observed psychometric functions. The pattern was a small patch containing 4 cycles of a 3 c/deg grating placed either at the fixation point or at the most peripheral position used in this study (42 periods above and 32 periods to the left of fixation point). The solid curves show equation (5) with the exponent q equal to 4 (the steeper curve) or 3

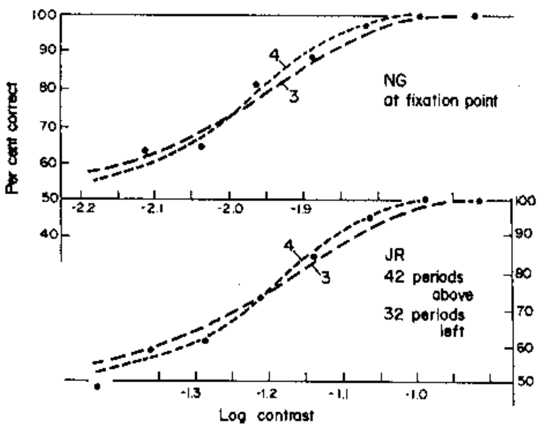

Fig. 5. Typical psychometric functions for 4-cycle patches of grating of spatial frequency 3 c/deg with vertical bars either at the fixation point (upper panel) or in the periphery (lower panel). The curves have the form of equation (7) with the exponent $q = 3$ or 4. The results of experiments on the effect of increasing number of cycles on the detectability of a grating were consistent with the hypothesis of probability summation across space assuming a value of about 3.5 for this exponent.

(the shallower curve). As the plotted proportions come from 180 trials each, the expected standard errors are 0.037, 0.022 and 0.016 for population proportions of 0.5, 0.9, and 0.95 respectively.

To investigate further the fit of equation (5) to the observed results we used a computer program developed by Watson (1979) to find the values of S and q which give the best fit of equation (7) to the observed results using a maximum likelihood criterion. The program also calculated a goodness-of-fit statistic which will be distributed as chi-squared if equation (7) is the correct description of the underlying psychometric function and if responses on different trials are independent. Since the values of two parameters are estimated by maximum likelihood methods, the number of degrees of freedom of that chi-squared distribution will equal the number of contrast levels minus three.

Maximum likelihood estimates of the exponent for results in Fig. 5 gave values of 3.3 for the foveal function and 4.3 for the peripheral one (with statistic values of 3.1 and 3.8 respectively indicating very good fits for chi-squared of 3 and 5 d.f. respectively). Four other psychometric functions were collected during the course of these experiments: two replications of the peripheral one in Fig. 5 were run, yielding exponent values of 3.5 and 3.6 (with statistic values of 1.5 and 11.7); and two psychometric functions were collected with the 3 c/deg patch at the midpoint of the horizontal strip 42 cycles above the fixation point yielding exponent values of 5.4 and 3.6 (with statistic values of 4.0 and 4.7).

We have found similar values for the exponent producing best fit for results collected during the course of other similar experiments using two-temporal-alternative forced-choice trials with randomly intermixed contrast levels. A collection of 16 psychometric functions extracted from staircase data for small central and peripheral patches of 2 and 6 c/deg (Graham *et al.*, 1978) gave a median value for the exponent of 3.25 with the lower quartile at 2.3 and the upper at 4.7. Three other psychometric functions collected for the same stimuli gave exponents of 3.2, 3.3 and 4.1. A number of functions for full field 1 and 10 c/deg gratings gave a median exponent of 3.2 with the lower quartile at 2.6 and the upper quartile at 3.7. For five other full field gratings of various frequencies, the exponents obtained were 4.1, 3.7, 6.2, 3.5 and 3.8. Over all 45 of these psychometric functions, the median value of best-fitting exponent was 3.3 with the lower quartile at 2.7 and the upper quartile at 4.2.

In short, as is consistent with a model in which the spatial summation of equations (3) and (4) is due to variability in the responses of detectors with receptive fields at different spatial positions (that is, to probability summation across space), measured psychometric functions can be described by equation (7) in which the value of the exponent agrees (within the precision of the experiments) with that necessary in equation (2) to explain the increase in sensitivity as number of cycles is increased. That value is approximately 3.5.

Although this agreement supports the idea that the observed spatial summation is due to probability summation across space, this conclusion should be accepted with some caution. Not only is there a great deal of variability in the experimental estimates of the exponent fitting different measured psychometric functions and also in the value producing acceptable fits to any one function, but the agreement could be fortuitous. In fact, the rather exact agreement is a little puzzling on several counts. Firstly, as Hallett

(1969) has discussed, one might expect that variation in sensitivity with time would make measured psychometric functions, collected over the course of an hour or two, somewhat shallower than "instantaneous" ones, and it is the exponent of the instantaneous one that should determine the increase in sensitivity as number of cycles is increased. Secondly, some disagreement between the values of the exponent is to be expected if the trial-to-trial variation in the responses of detectors with receptive fields at different positions is partially correlated. The magnitude of this disagreement should not be large, however, unless there is a high degree of coherence extending over substantial distances (see Graham *et al.*, 1978, for a more detailed discussion of the effects of correlation). Thirdly, the assumption used in the derivations of equations (6) and (7) that the observer either detects a stimulus or does not, represents a "high-threshold" theory of detection. Such a high-threshold theory may well be inappropriate in detail, and is certainly incomplete, although it appears to have good predictive power. Fourthly, the precise *form* of psychometric function chosen to describe our measurements (equation 7) may not in fact describe them exactly. Unfortunately prediction of the threshold contrast of gratings with large numbers of cycles requires knowledge of the shape of the psychometric function at its foot. This cannot realistically be determined experimentally with much precision. Wilson and Bergen (1978) discuss some of the consequences of assuming that the form of the psychometric function we have used is only an approximation to a true, underlying lognormal function. Thus, there are several reasons for interpreting with caution the agreement between the value of q from measured psychometric functions and that from increased sensitivity as number of cycles is increased. Furthermore, a recent careful study of summation between the moving components of a flickering grating (Watson *et al.*, 1979) has shown that, for that situation, the psychometric function q was greater than the increased–sensitivity q. Perhaps for our situation, however, the disturbing factors are either not important or maybe they work against each other and so do not destroy the agreement between the two values of q. In any case one can say that the evidence of this study is not inconsistent with the hypothesis that probability summation across space is the source of the observed spatial summation for gratings with two or more cycles.

Similar evidence for compound gratings containing two different spatial frequencies (Sachs *et al.*, 1971; Graham *et al.*, 1978), and for gratings exposed for different durations (Watson 1978; Legge, 1978), is consistent with the notion that probability summation among different spatial frequency channels and over time is responsible for the observed summation across spatial frequencies and over time.

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