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SIMULTANEOUS RECOGNITION OF TWO SPATIAL-FREQUENCY COMPONENTS

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Abstract—A two-frequency two-response paradigm was employed. The stimulus on any trial was a compound grating containing two sinusoidal components of different spatial frequencies where the contrast in one or both components could be zero. The observer gave two responses each indicating confidence that one of the two components had been presented with contrast greater than zero. If these responses are assumed to be related to outputs of spatial-frequency channels according to specified recognition linking hypotheses, then channel properties such as bandwidth, negative influences, correlation of noise, and additivity can be inferred. Possible modifications of spatial frequency channel models are discussed.

INTRODUCTION

The concept of channels selectively sensitive to spatial-frequency components or size of pattern elements (Campbell and Robson, 1968; Pantle and Sekuler, 1968; Thomas, 1970) has been employed to explain a large body of psychophysical results (See review in Graham, 1981). In particular, detection thresholds for many patterns have been quantitatively explained. More recently, spatial-frequency channel models have been extended in an effort to explain the results of near-threshold recognition studies (Nachmias and Weber, 1975; Thomas and Barker, 1977; Furchner *et al.*, 1977; Thomas and Olzak, 1979; Watson and Robson, 1979; Williams and Wilson, 1979; Wilson and Gelb, 1979). The study reported here investigates the assumptions of spatial-frequency channel models as applied to recognition data.

In this study the stimulus on any trial could be a compound grating containing two sinusoidal components, or a single sinusoidal component, or a blank. The observer made two responses following each trial. Each response indicated the observer's confidence that one of the two frequency components had been presented. (A similar procedure was employed by Nachmias, 1974.) Since each component was separately identified, the task is referred to here as a recognition task. This task differs from a detection task where the observer is allowed only one response indicating whether any pattern was detected. The purpose of the two-response procedure was to directly investigate simultaneous activity in two separate spatial frequency channels. Such an investigation is not possible using a single response detection paradigm.

METHODS

Two-frequency patterns

In every session the stimuli consisted of patterns with two components. The stimuli used in a session are indicated schematically in the upper left of Fig. 1.

Formally each of the nine patterns was the sum of two sinusoidal components of different spatial frequencies. Each of the two components was displayed at one of three contrast levels: 0, 1 or 2 where 0 represented no contrast, 1 represented a contrast level that produced a value of d' approximately equal to 1.00, and 2 represented a contrast level that produced a value of d' approximately equal to 2.00. The contrast levels at each frequency necessary to attain approximately these d' values were measured prior to the experimental sessions for each observer. (These prior measurements were detection rather than recognition judgments.) Contrast levels of the stimuli were slightly adjusted over sessions as observer sensitivity fluctuated. Cell (0,0) in Fig. 1 represents the blank stimulus. Cells (0,1), (0,2), (1,0) and (2,0) represent simple gratings in which only one sinusoidal component was present, and cells (1,1), (1,2), (2,1) and (2,2) represent compound gratings in which two sinusoidal components were present. The relative phase between the two components was constant within a session and randomized across sessions.

All eight of these simple and compound patterns were presented with equal probability, randomly intermixed with each other and with the blank stimulus. The blank stimulus was presented twice as often as any of the eight patterns (in a subsequent control experiment the blank stimulus was presented with the same probability as any pattern stimulus). The two frequencies used were constant within a session but varied across sessions. Anywhere from 500 to 1000 trials were presented each session depending on scheduling limitations of observers.

The overall experiment was designed to investigate properties of two spatial-frequency channels (or small groups of channels), one sensitive to 3 and the other

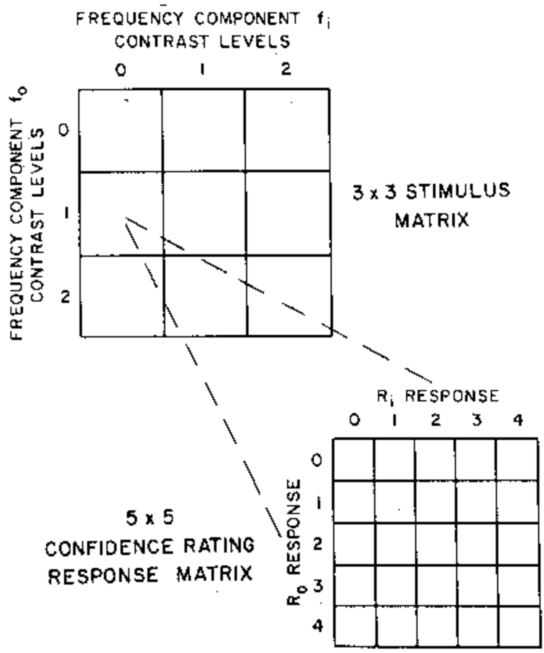

Fig. 1. Schematic illustration of the two-frequency two-response experimental paradigm. Recognition responses R_o and R_i correspond to the two stimulus frequencies f_o and f_i respectively.

to 15 c/deg. In each session, therefore, one of the stimulus frequencies was always either 3 or 15 c/deg and was called the center frequency, f_o. The other frequency used in each session was called the side frequency, f_i. The side frequency was 4, 5, 6 or 9 c/deg when the center frequency was 3 c/deg, and 5, 9, 16, 20 or 25 c/deg when the center frequency was 15 c/deg. Approximately six sessions were run for each combination of center and side frequency (except for pairs 3 and 5 c/deg and 15 and 20 c/deg where approximately twice as many sessions were run as a control for possible changes over time). The exact number of sessions for each frequency pair is listed in Table 2 in the column labelled *N*.

The patterns were displayed on the face of an oscilloscope (Tektronix 5103N, P31 phosphor) as described by Campbell and Green, 1966. The width of the rectangular stimulus display was 4.0 deg of visual angle with a height of 3.3 deg (at a viewing distance of 150 cm). The display was set in a 12 deg circular background that was matched in color and brightness to the display. The mean luminance of the display was constant at 20 cd/m^2. Pattern duration was 760 msec (with sharp onset and offset) and marked by a tone that coincided with the stimulus presentation. The spatial phase of each stimulus was independently randomized from trial to trial to prevent observers from using position of, for example, a bright bar as a cue to pattern frequency. Observers viewed the stimulus binocularly.

Two recognition responses

On each trial one of the nine stimuli was presented, following which the observer made two responses. The center response, R_o, indicated the observer's level of confidence that the component of center frequency f_o had been presented. The side response, R_i, indicated the observer's level of confidence that the component of side frequency f_i had been presented. A five-point scale (0–4) was employed where a response of 4 indicated the highest level of certainty that a given component *had* been presented and a response of 0 indicated the highest level of certainty that the component *had not* been presented. Observers were instructed to use the entire range of the rating scale.

For clarification of the two-response procedure consider that responses to any one of the 9 stimulus patterns could be tabulated in a 5 × 5 matrix, where the rows indicated values of the center response, R_o, and the columns indicated values of the side response R_i (lower right of Fig. 1). For example, cell (3,2) in the response matrix would give the number of trials (of a given stimulus) on which the observer responded "3" for the center response R_o and "2" for the side response R_i. In the similar two-response paradigm employed by Nachmias (1974), observers responded "yes" or "no" for each of the two responses rather than with a confidence rating.

Average confidence ratings and d'

Values of *d'* were calculated for each of the eight non-blank stimuli for each session for each of the two confidence-rating responses. ROC curves were generated by computer from the confidence ratings (Green and Swets, 1966) and slopes and intercepts of those curves were estimated as if plotted on double-probability axes. Under the conditions of this experiment, results using *d'* values and results using average confidence ratings did not differ significantly. One was very close to a linear transformation of the other (Hirsch, 1977). Average confidence ratings are reported here, therefore, as they represent a less processed form of the data.

Observers

Two principal observers participated in the experiment over the course of a year. Observer L.H. was unaware of the purpose of the experiment, and J.H. was a principal investigator. All results were confirmed on two other less-experienced observers. Results from the two well-studied observers are reported in this paper. All observers had normal vision.

RESULTS

Overview

To provide a general overview of the experimental results, a representative sample of data for observer L.H. has been plotted in Fig. 2. Average confidence

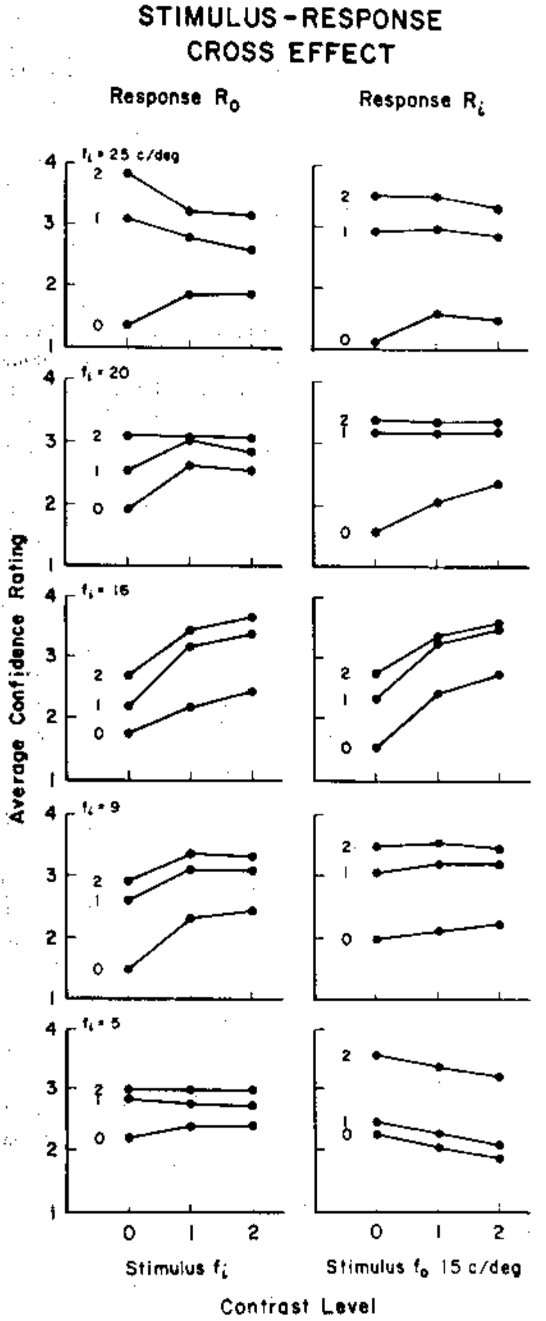

Fig. 2. Average confidence ratings are plotted as a function of stimulus contrast. Plots show cross-effects where average confidence ratings from one recognition response (R_o or R_i) are shown as a function of contrast in the other frequency component (f_i or f_o) respectively. Data are shown for one observer, L.H., for all frequency pairs around f_o = 15 c/deg. The SEM for all points is near ±0.25 and includes sources of variability that are due to combination of data across sessions.

ratings for five frequency pairs with center frequency f_o equal to 15 c/deg, were plotted against contrast levels. (For this figure the ratings were averaged across different trials of the same stimulus within a session, and the average values from different sessions with the same spatial frequencies were averaged together.)* Side frequency of the stimulus pair is indicated in the upper left of each pair of sections. For example, the top two sections of the figure show results from sessions with side frequency f_i of 25 c/deg. Average confidence ratings were given in the left half of the figure for the center response R_o and are given in the right half for the side response R_i. The three different functions in each section of the figure represent three different contrast levels of the frequency component corresponding to the indicated response. That is, the three functions in any section in the left half of the figure represent three different levels of contrast in the center frequency f_o (15 c/deg) and the three functions in any section in the right half represent three different levels of contrast in the side frequency f_i. These data demonstrate that (as expected) average confidence rating increased with increases in contrast of the corresponding frequency component. That is, in all sections the function for contrast level 2 is above that for level 1 which is above that for level 0 (no contrast, i.e. a blank field).

The horizontal axis of Fig. 2 indicates the contrast level in the frequency component *not* corresponding to the response. In the left half it indicates contrast of the side frequency and in the right half it represents contrast of the center frequency. Each function of Fig. 2, therefore, shows a "cross effect", the average confidence rating from one recognition response plotted against the contrast level of the *other* frequency component. When the center frequency f_o and the side frequency f_i were close together, as in the case of 15 and 16 c/deg (middle two sections in Fig. 2), each function rises sharply; that is, the average confidence rating for each recognition response rises with increases in contrast in the *other* frequency component. For example, consider any one of the three functions in the left section of the 15 and 16 c/deg plots. As the contrast of the other frequency component f_i (16 c/deg) increased, the observer responded with increasingly more confidence that the 15 c/deg component was present. That is, the observer "confused" 15 and 16 c/deg. When the frequency components were well separated, as in the case of 5 and 15 c/deg and of 15 and 25 c/deg, the average confidence rating was less affected by increases in contrast in the other frequency component. In fact, the slopes of the lines tended to be negative. That is, the confidence rating of the "cross response" tended to decrease with increases in contrast of the other frequency component. For example, as shown in the lower right section of Fig. 2, the more the contrast in the 15 c/deg component, the less confident the observer was that the 5 c/deg component was present. Similar observations held for other frequency pairs and other observers. These ob-

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* These data include sources of variation (such as across session variability due to stimulus contrast changes, absolute shifts in observer sensitivity, and possible changes in use of the rating scale) which have been partially removed in later analyses.

servations and others are quantified and described more rigorously in the following section. Brief rationales are provided to introduce the theoretical issues later presented in the Discussion.

1a. *Response covariation as center frequency contrast is changed*

Rationale. If the two recognition responses, R_o and R_i, were monotonic functions of the output from two channels (or from two small groups of channels) maximally sensitive to the two frequencies, f_o and f_i, then, on the basis of existing spatial-frequency channel models, the following pattern of results would be expected. In cases where the two spatial frequencies were close enough together to stimulate the same channel, both recognition responses should increase when the contrast of either frequency component was raised. That is, there should be positive covariation between the two responses with changing stimulus contrast. However, in cases where the two frequencies were well enough separated that they stimulated none of the same channels (given an assumption of no interaction between channels), only one recognition response should increase with changes in the contrast of one component. That is, there should be no covariation between the two responses with changing stimulus contrast. As can be seen from plots like those of Fig. 2, this prediction is qualitatively upheld for cases where the two spatial frequencies were close together. However, for cases of well separated frequencies negative covariation rather than no covariation was observed. As will be discussed later, the range of frequencies over which positive covariation was observed should be related to the bandwidths of individual channels.

Analysis. Further comparison of the results for both observers and both center frequencies (3 and 15 c/deg) requires a more compact description of the observed positive and negative response covariation than can be obtained from figures like Fig. 2. Therefore, the following computation was done to allow comparison of the frequency ranges over which positive and negative influences occurred. As the description of the computation is necessarily technical, the reader may wish to skip to the conclusion of this section.

For a single session, both responses to all nine stimuli were summarized in two 3 × 3 matrices as shown in Fig. 3. Each of the matrices has the same form as that in the upper left of Fig. 1. The right matrix represents data from the center response R_o, and the left matrix represents data from the side response R_i. The number entered in each cell of these matrices (X or Y) is the average confidence rating given in that session (k) for the corresponding stimulus (ij) and response. A general entry in the righthand matrix is $X_{ij}(k)$, the average confidence rating given in the kth session (for a particular pair of frequencies) in response to a stimulus with contrast level i in the center-frequency component (row) and contrast level j in the side-frequency component (column). Likewise, a general entry in the lefthand matrix is $Y_{ij}(k)$, where the sub and superscripts have the same meaning as for the righthand matrix except that Y indicates the confidence rating for the side response R_i. The average of the entries in a row or in a column is indicated by the bar placed over the X or Y in the margins and a dot which has replaced the subscript averaged over. The grand mean of all nine entries in the matrix is indicated by the two dots which have replaced both subscripts.

At each contrast level of the center-frequency component the average confidence rating in a given session (averaged over all levels of contrast of the side frequency) was computed for both the center, R_o, and side responses, R_i, i.e. the row means in the two matrices of Fig. 3 were computed. To minimize effects due to absolute shifts in sensitivity and possible variations in the use of the rating scale over sessions, the grand mean for each session was subtracted from each of the row means for that session to form "cor-

Contrast level of center frequency, f_o

SIDE RESPONSE R_i

Contrast level of side frequency, f_i

	0	1	2	row mean
0	$Y_{00}^{(k)}$	$Y_{01}^{(k)}$	$Y_{02}^{(k)}$	$\bar{Y}_{0.}^{(k)}$
1	$Y_{10}^{(k)}$	$Y_{11}^{(k)}$	$Y_{12}^{(k)}$	$\bar{Y}_{1.}^{(k)}$
2	$Y_{20}^{(k)}$	$Y_{21}^{(k)}$	$Y_{22}^{(k)}$	$\bar{Y}_{2.}^{(k)}$
column mean	$\bar{Y}_{.0}^{(k)}$	$\bar{Y}_{.1}^{(k)}$	$\bar{Y}_{.2}^{(k)}$	$\bar{Y}_{..}^{(k)}$

CENTER RESPONSE R_o

Contrast level of side frequency, f_i

	0	1	2	row mean
0	$X_{00}^{(k)}$	$X_{01}^{(k)}$	$X_{02}^{(k)}$	$\bar{X}_{0.}^{(k)}$
1	$X_{10}^{(k)}$	$X_{11}^{(k)}$	$X_{12}^{(k)}$	$\bar{X}_{1.}^{(k)}$
2	$X_{20}^{(k)}$	$X_{21}^{(k)}$	$X_{22}^{(k)}$	$\bar{X}_{2.}^{(k)}$
column mean	$\bar{X}_{.0}^{(k)}$	$\bar{X}_{.1}^{(k)}$	$\bar{X}_{.2}^{(k)}$	$\bar{X}_{..}^{(k)}$

Fig. 3. Two 3 × 3 stimulus matrices are each filled with confidence ratings for either response R_o or R_i. Row and column marginals were computed by correcting each row and column mean by the session grand mean. (See text for definitions of notation).

ion was done to allow comparison ges over which positive and negaurred. As the description of the essarily technical, the reader may conclusion of this section.

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st level of the center-frequency comge confidence rating in a given sesver all levels of contrast of the side omputed for both the center, R_o, and R_i, i.e. the row means in the two were computed. To minimize effects hifts in sensitivity and possible variof the rating scale over sessions, the each session was subtracted from means for that session to form "corrected" row means. Measures of covariation between the corrected row means (marginals) for the center response and those for the side response were then computed for every pair of frequencies used in this study (for each observer separately). The several sessions run with a given center-frequency-side-frequency combination (for each observer) were considered together so that each measure of covariation was computed across $3N$ pairs of corrected row means (3 per session with N sessions using that particular frequency combination). That is, each measure was computed across the set of points:

$$\{\bar{X}_{i.k} - \bar{X}_{..k}, \bar{Y}_{i.k} - \bar{Y}_{..k} / k = 1, N; i = 0,2\}.$$

Figure 4 shows a representative sample of response-response scatter-plots where row marginals for the R_i response are plotted agains response for all three co quency component.

Table 1 shows the comp tion for each combinatio two columns of response changes in center frequenc product-moment correlati dard error of estimate (SE the slope of the best-fit lin for predicting the side re center response marginal.* ratio of changes in the side the center response, R_o, as frequency, f_o, was varied.

Results. Figure 5 shows t regression lines (described pal observers when the cen plotted as a function of the shows the response cova center frequency f_o was 3

ER RESPONSE R_0

rast level of side frequency, f_i

	1	2	row mean
	$X_{01}^{(k)}$	$X_{02}^{(k)}$	$\bar{X}_{0.}^{(k)}$
	$X_{11}^{(k)}$	$X_{12}^{(k)}$	$\bar{X}_{1.}^{(k)}$
	$X_{21}^{(k)}$	$X_{22}^{(k)}$	$\bar{X}_{2.}^{(k)}$
	$\bar{X}_{.1}^{(k)}$	$\bar{X}_{.2}^{(k)}$	$\bar{X}_{..}^{(k)}$

tings for either response R_o or R_i. and column mean by the session tation).

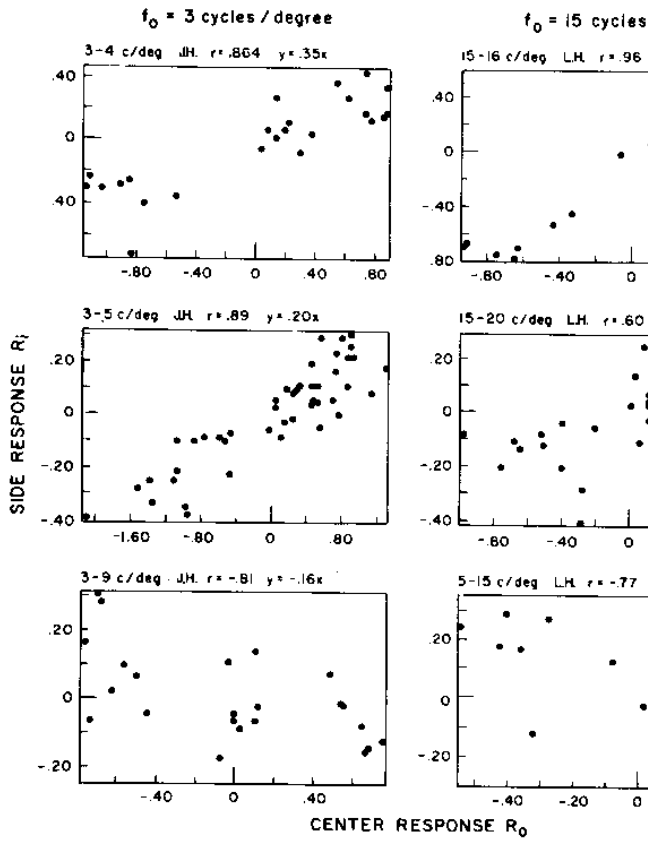

Fig. 4. Side response R_i row marginals (See Fig. 3—left) are plotted against cen marginals (Fig. 3—right) for the three contrast levels (0, 1, 2) of the center frequenc frequency pair, correlation coefficient (r), and the best-fitting least squares regressi above each plot. These selected scattergrams are shown to illustrate the analysis pro findings.

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* The intercepts of all least squares best fit lines are 0.00 by construction. Due to the analysis procedure where the grand mean was subtracted from each row and column average, the X and Y coordinates on the scattergrams were constrained.

Table 1. Covariation of center and side confidence ratings as contrast was changed in center frequency component f_o (either 3 or 15 c/deg) or in side frequency component f_i

Observer	Frequency component		Response covariation statistics* Center frequency			Side frequency		
	f_o	f_i	r	SEE	slope	r	SEE	slope
L.H.	3	4	0.93	0.13	0.56	0.94	0.69	0.67
	3	5	0.89	0.08	0.20	0.35	0.13	0.07
	3	6	0.60	0.14	0.11	0.56	0.15	0.13
	3	9	−0.14	0.11	−0.02	−0.42	0.15	−0.14
J.H.	3	4	0.86	0.15	0.35	0.92	0.13	0.46
	3	9	−0.62	0.11	−0.16	−0.21	0.11	−0.04
L.H.	15	5	−0.77	0.10	−0.37	−0.13	0.09	−0.03
	15	9	0.36	0.11	0.08	0.84	0.15	0.35
	15	16	0.96	0.13	0.91	0.97	0.12	0.91
	15	20	0.60	0.13	0.22	0.66	0.15	0.22
	15	25	0.15	0.12	0.03	−0.81	0.10	−0.15
J.H	15	16	0.95	0.16	0.89	0.96	0.11	0.70
	15	20	0.92	0.11	0.40	0.22	0.10	0.04

* Linear analysis by method of least squares.

from high positive values when the side and center frequencies are close together to low and negative values when the side and center frequencies are well separated. The results in Figs 5 and 6 for the two observers are similar, and the righthand sides of the curves for the two center frequencies are similar when plotted against logarithmic frequency.

Negative values for the measures of covariation between responses occurred when the combination of frequencies was 3 and 9 c/deg or 5 and 15 c/deg. That is, for those frequency pairs, the center response tended to increase while the side response tended to decrease when the contrast of the center-frequency component was increased.

1b. *Response covariation as a function of* f_i *contrast*

The rightmost three columns of Table 1 show the response covariation statistics when the contrast in the side frequency was varied. That is, measures of covariation between corrected *column* means (marginals) for the center and side responses (see Fig. 3) over contrast levels of frequency component, f_i, were computed. The results were similar to those for contrast changes in the center frequency as shown on Figs 5 and 6. Positive covariations occurred for close-together frequency pairs and negative covariation occurred for the well separated frequency pairs 3 and 9, 5 and 15, and 5 and 25 c/deg.

2. *Response covariation over repetitions of the same stimulus*

Rationale. If the magnitude of a recognition response in this experiment was a function of the magnitude of the output from a spatial frequency channel (or small group of channels), then the variability in a recognition response over repetitions of a single stimulus could be caused by inherent variability or noisiness in the response of the channel (or small group of channels). In this case, correlation between two recognition responses over different presentations of the same stimulus might reflect the extent to which (if at all) the sources of noise in the two channels (or two small groups of channels) were correlated. (This information is not available from Fig. 2 which shows each recognition response averaged across all presentations of a given stimulus.)

Analysis. For each of the nine stimuli from each session, a Pearson product-moment correlation coefficient was calculated between the values of side responses and the values of center responses. The percent of all sessions where the computed correlation coefficient was significantly different from zero at the 0.05 level of significance (two-tailed) is listed on Table 2. (Since the values of each response were discrete, assumptions of the statistical tests of Pearson product-moment correlation coefficients were not strictly met: however, the results were also confirmed by chi-squared tests.)

Results. As can be seen from the table, most of the correlations were significant for the blank stimulus (0,0); and more of the correlations were significant than the 5% expected by chance for the pattern stimuli. No systematic trend in the percent of significant correlations across different frequency pairs was observed. All of these significant correlations tended to be positive with those for the blank higher than those for the patterned stimulus. The median correlation for the blank stimulus was usually between 0.2 and 0.4, and for the stimuli containing the highest contrasts the median correlation was near zero.

In the main experiment reported here blanks occurred twice as often as any one of the eight patterned stimuli. In a later control experiment blanks occurred with the same probability as any one of the eight patterned stimuli so that the probabilities of presenting particular contrast levels in the two fre-

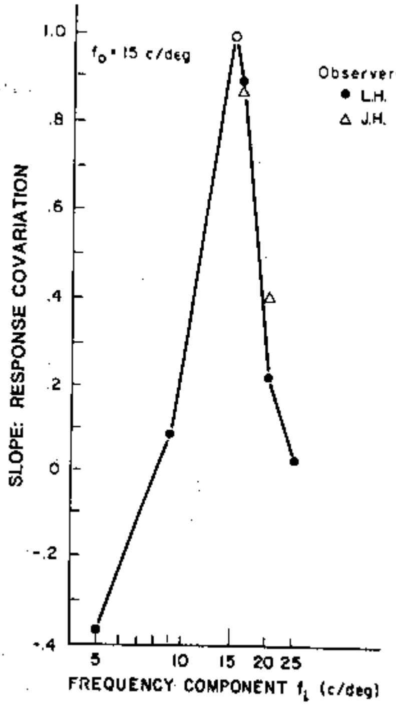

Fig. 5. R_i vs R_o covariation slopes (as from Fig. 4) plotted against the side frequency component f_i for each frequency pair where f_o was 15 c/deg. The open circle (○) indicates a theoretical point where f_o and f_i are equal.

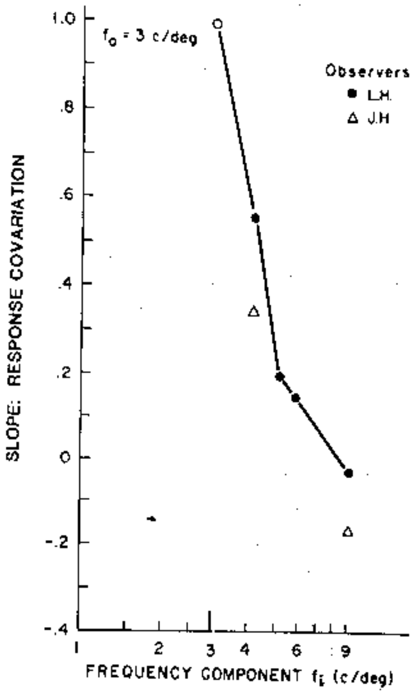

Fig. 6. Same as Fig. 5 with f_o equal to 3 c/deg.

quency components were independent. In this later experiment, there was a much smaller proportion of significant correlations for the blank stimulus. However significant response correlations for all stimuli still tended to occur more frequently than expected by chance.

3. *Effect of two frequencies on one response: a test for additivity*

Rationale. The "two-frequency two-response" paradigm used in this study allows direct observation of the effects of each of two stimulus frequencies, f_o and f_i, on a single recognition response. The following analysis tests the hypothesis that the effects of the two frequency components were additive. If the magnitude of a recognition response was a linear function of the magnitude of the output from one spatial-frequency channel, then additive effects on the recognition response imply additive effects on the channel output. Although the results of this study are represented in terms of the average confidence ratings, the results of the following analyses were the same when d' values were computed from confidence ratings and applied

Table 2. Correlation of R_o and R_i confidence ratings

	$f_o f_i$	N	Stimulus (0,0)	(0,1)	(0,2)	(1,0)	(1,1)	(1,2)	(2,0)	(2,1)	(2,2)	Mean across stimuli
L.H.	3–4	7	0.86	0.56	0.43	0.43	0.43	0.43	0.71	0	0.29	0.462
	3–5	15	0.93	0.40	0.20	0.53	0.27	0.13	0.13	0.20	0.13	0.324
	3–6	6	1.0	0	0.20	0.40	0.60	0	0.20	0.20	0.20	0.289
	3–9	8	0.88	0.12	0.12	0.25	0	0.12	0.12	0.12	0.12	0.206
J.H.	3–4	8	1.0	0.12	0.25	0.12	0.12	0.25	0.38	0.12	0.12	0.275
	3–9	8	1.0	0.20	0.25	0.50	0.50	0.38	0.12	0.25	0.12	0.402
L.H.	15–5	6	1.0	0.17	0.17	0.50	0	0.17	0.17	0.17	0	0.261
	15–9	5	01.0	0	0	0.60	0	0.20	0.60	0	0	0.267
	15–16	7	1.0	0.43	0.57	0.57	0.57	0.29	0.43	0.57	0.43	0.540
	15–20	11	1.0	0.09	0.18	0.27	0.09	0.18	0.27	0.27	0.18	0.281
	15–25	5	1.0	0.40	0	0.60	0.20	0.20	0.60	0.20	0.40	0.400
J.H.	15–16	5	1.0	0	0.40	0.20	0.20	0	0.60	0	0.60	0.333
	15–20	6	1.0	0.33	0.33	0.33	0.33	0.17	0.50	0.17	0	0.351

Table values are the percent of cases that were significantly correlated at $\alpha = 0.05$.
Stimuli are labelled by the matrix notation shown in Figure 1.
Entries in column labeled N indicate number of sessions.

instead. (Values of d' are frequently assumed to be a linear function of sensory, e.g. channel, output.)

Analysis. An analysis of variance model was employed where the effect of the two stimulus frequencies on the center response was additive if:

$$(X_{ij}^{(k)} - \bar{X}_{..}^{(k)}) = (\bar{X}_{i.}^{(k)} - \bar{X}_{..}^{(k)}) + (\bar{X}_{.j}^{(k)} - \bar{X}_{..}^{(k)})$$

$$(A) \quad = \quad (B) \quad + \quad (C)$$

where the symbols are as described above, and A, B and C represent the three terms respectively in the equation. For the side response, the equation for additivity is identical to that for the center response except that X symbols are replaced by Y symbols. If the effects of the two frequencies on each recognition response were additive, the functions in plots like Fig. 2 would be parallel (except for variability).

The analysis of variance was performed by computing the multiple regression of term A against terms B and C in the above equation and also in the analogous equation for the side response.* A multiple R^2 from this regression indicates how much of the variation in the data was accounted for by the additive model and is presented in Table 3.

Results. The column labeled R_o gives the results for the center response. The column labeled R_l gives the results for the side response. For all frequency pairs studied and for both responses and for both observers, a large proportion (the median proportion is 95%) of the total variation in the mean confidence ratings could be accounted for by the additive model in the above equation. The residual variation can be attributed to non-additivity and/or noise. In this analysis, non-additivity and noise were not distinguished. Further analyses (Hirsch, 1977) showed that there was some non-additivity (significant interactions in the analyses of variance). However, a model in which the effects of the two stimulus frequencies on a single recognition response are additive appears to account for a large percentage of the variation in the mean confidence ratings (Table 3). The observed additivity confirms that the curves in plots like Fig. 2 are approximately parallel.

Summary of results

(1) As the contrast of one frequency component was varied, positive covariation between the two recognition responses was observed when the two frequencies were separated by a small frequency difference, and negative covariation was observed when the two frequencies were separated by a large frequency difference. The frequency separations over which the positive and negative covariations occurred were similar for the two observers and also for the two center frequencies (3 and 15 c/deg) when frequency was plotted on a logarithmic axis (Table 1, Figs 5 and 6).

Table 3. Percent of total variation (R_2) in each category of response ratings accounted for by an additive model*

Observer	Frequency component		Response category		
	f_o	f_l	R_o	R_l	N*
L.H.	3	4	0.97	0.93	7
	3	5	0.98	0.97	15
	3	6	0.98	0.98	6
	3	9	0.97	0.88	8
J.H.	3	4	0.98	0.97	8
	3	9	0.92	0.94	8
L.H	15	5	0.92	0.98	6
	15	9	0.93	0.95	5
	15	16	0.96	0.97	7
	15	20	0.88	0.89	11
	15	25	0.88	0.97	5
J.H.	15	16	0.94	0.96	5
	15	20	0.94	0.93	6

* Entries under the N column indicate number of sessions.

(2) Correlation was observed between the two recognition responses across repeated presentations of the same stimulus (Table 2).

(3) Effects of the two frequency components on each recognition response were nearly additive. That is, the increment in a confidence rating caused by an increment in the contrast of one frequency component was nearly independent of the contrast level of the other frequency component (Table 3).

DISCUSSION

Spatial-frequency channel models have quantitatively accounted for the detection thresholds of many patterns, including compound gratings (e.g. Sachs *et al.*, 1971; Graham *et al.*, 1978; Quick *et al.*, 1978; Bergen *et al.*, 1979). Measurement of a detection threshold requires that the observer indicate whether any pattern (rather than a blank field) is present on a given trial. This study investigated the ability of these models to account for recognition data where the observer was asked to indicate whether each of *two* particular patterns was present on a given trial.

Detection model

The spatial-frequency channel model frequently used (first by Sachs *et al.*, 1971) to account for detection thresholds contains four main assumptions. The first three specify general properties of channels, and the fourth is a "linking hypothesis" that links channel outputs and observer responses.

(1) Spatial information in the human visual system is analyzed by multiple channels. Each channel is sensitive to patterns that contain spatial frequencies in some restricted range. The frequency ranges for differ-

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* Although the formal analysis was multiple regression, the coefficients were constrained so that an analysis of variance was effectively employed. For a concise discussion of the basic statistical principals applied to these data the reader is referred to: Winner B. J. (1962) *Statistical Principals in Experimental Design.* McGraw Hill, New York (1st edn)/1971 (2nd edn), and Nie N., Hull C. H., Jenkins J., Steinbrenner K. and Bent D. (1975) *Statistical Package for the Social Sciences,* SPSS. McGraw Hill, New York.

ent channels can overlap. The output of a single channel can be represented as a positive number and is a monotonically increasing function of contrast.

(2) There is variability over trials in the output of any channel to any pattern, and the variabilities in different channel outputs are independent. That is the probability that two channel outputs are greater than criterion on a single trial is the product of the probabilities that each channel output is greater than criterion. In other words, noise sources in the channels are independent.

(3) Channels do not directly excite or inhibit each other. For example, if channel 1 is excited by pattern A but not by pattern B and channel 2 is excited by pattern B but not by pattern A, then the average (averaged over trials) output of channel 1 to a combination of A and B is equal to its average response to A alone, not smaller (or larger) as it would be if it were inhibited (or excited) by channel 2.

(4) If any single channel output is sufficiently large, the observer indicates a grating was present (or a certain degree of confidence that a grating was present). That is, a grating is detected if and only if at least one of the channel outputs exceeds a criterion level.

Recognition model

To apply the above model to experimental results reported here, the first three assumptions can remain unchanged and the fourth assumption must be replaced by a linking hypothesis appropriate for recognition. Four possible versions of a recognition linking hypothesis are distinguished, each a more specific version of the one before.

(4a) The magnitude of a recognition response in this experiment (e.g. a center response) is determined by a small group of channels sensitive to the corresponding spatial frequency (e.g. the center frequency). The recognition response is a monotonically increasing function of the output from each channel in this group.

(4b) Like 4a except for the additional constraint that no channel contributing to one recognition response in a session also contributes to the other.

(4c) The magnitude of a recognition response (e.g. a center response is a monotically increasing function of the output magnitude of the *one* channel most sensitive to the corresponding frequency (e.g. the center frequency).

(4d) Like 4c except "monotonic" is replaced by "linear".

All four versions of this recognition linking hypothesis imply that the output of each spatial-frequency channel is "labelled" or "tagged" with the identity of the spatial frequency to which the channel is most sensitive. That is, information about which channel produced which response is preserved. The first three assumptions of the detection model plus any (or all) of the above four recognition linking hypotheses are referred to here as the recognition model.

Predictions of recognition model compared to results

1a. *Range of frequencies over which positive covariation occurred as contrast was varied.* According to the recognition model (with any version of the recognition linking hypotheses, 4a–4d) increments of the contrast for either of two close spatial frequencies should increase the magnitude of both recognition responses. That is, channels associated with each recognition response are assumed to be sensitive to both frequencies. (Overlapping frequency ranges are specified in assumption 1). For similar, i.e. close spatial frequencies, therefore, covariation (measured between the two recognition responses as stimulus contrast was varied) is expected to be positive. This prediction was supported by the data (Figs 2, 5 and 6).

Also, according to assumption 1 and versions 4b–4d of the recognition linking hypothesis, information about underlying channel bandwidths can be obtained from the positive covariation between responses. That is, the extent of overlap between two channels would be reflected by covariation between the two responses. Thus, Figs 5 and 6 may provide information about channel bandwidths for recognition of patterns at 15 and 3 c/deg respectively. If all other properties of channels at 15 and 3 c/deg are identical, the near coincidence of the functions in Figs 5 and 6 suggests that channel bandwidth was nearly constant on a logarithmic frequency axis. That is, bandwidth was proportional to center frequency.

1b. *Negative covariation as stimulus contrast was varied.* The recognition model with the first three assumptions and even the most general linking hypothesis 4a does not allow negative influences like those found in this study, i.e. the negatively sloped lines observed in Fig. 2 and reflected as negative covariations shown in Figs 5 and 6. In particular, assumption 1 states that the direct influence of any stimulus on a channel response is positive; assumption 3 states there are no interactions between channels that might produce decreases in some channel outputs when others increase; and assumption 4a defines each recognition response as a monotically *increasing* function of the response in any channel sensitive to the corresponding frequency. As the contrast of any stimulus is raised, therefore, the magnitude of any recognition response is expected to increase (or stay at zero).

Any of several assumptions could be changed to account for the observed negative influences. For example, channels might be inhibited directly by stimuli containing far away frequencies (a change in assumption 1), or inhibited by channels responding to far-away frequencies (a change in assumption 3), or recognition responses might show "response biases" which allow the response of the observer to be positively influenced by channels sensitive to the corresponding frequency but negatively influenced by channels sensitive to well-separated frequencies (a change in the recognition linking assumptions 4a–4d).

Further development of these alternatives is not possible on the basis of the existing results alone.

2. *Correlation over stimulus repetitions.* The recognition model consisting of the first three assumptions above and all versions except 4a of the linking hypotheses predicts that the two recognition responses are independent over repetitions of the same stimulus. According to 4b, 4c or 4d, two completely separate sets of channels determined the two responses and, according to assumption 2, the variabilities in different channel outputs are independent.

This prediction was not confirmed by the data (Table 2). Variabilities of the two responses (channel outputs) were correlated. The model might be repaired by modification of assumption 2, by modification of the linking assumption (4b), or both. Additional study, however, is necessary for such theoretical development.

3. *Additivity of effects of two spatial frequencies on one recognition response.* A model consisting of the first three assumptions with the most specific version of the linking hypothesis 4d, can be employed to predict expected channel properties. If, as assumption 4d specifies, the magnitude of a recognition response is a *linear* function of the magnitude of a channel output, then the approximate additivity of the effects of two frequencies on a recognition response implies the approximate additivity of the effects of those two frequencies on the appropriate channel output. (As indicated in the results section, this additivity was observed when recognition responses were measured either as average confidence ratings or as d' values. If a recognition response depended on the output of only one channel, the usual interpretations of d' values allow the assumption that d' is a linear function of the channel's response).

Summary. Two results from the recognition experiment are not consistent with the recognition model described above: (1) correlation between recognition responses for repetitions of one stimulus, and (2) negative influences of well-separated frequency components. Correlation between the two recognition responses over repetition of the same stimulus could be accounted for by allowing correlation between channel response variabilities (a change in assumption 2) or by specifying a more complicated linking hypothesis (a change in assumptions 4a–4d). Negative influences of well-separated frequencies on recognition responses could be accounted for by direct inhibition of a channel output by some spatial frequencies (a change in assumption 1), inhibitory interaction among channels (a change in assumption 3), or by a more complicated linking hypothesis (a change in assumptions 4a–4d).

The recognition results were consistent with the prediction of positive covariation between the two responses for similar (near) spatial frequencies, and with models where bandwidths of channels at different spatial frequencies are in approximate proportion to the preferred spatial frequency (approximately equal on a logarithmic frequency axis). Further, additivity of the influences from each spatial frequency component on one response was observed using the most restrictive version of the linking hypothesis, and suggests that the effect of two frequencies on a single channel output was approximately additive.

Comparison of recognition and detection results

The first three assumptions with the detection linking hypothesis (assumption 4) have generally accounted for the detection thresholds of compound gratings. As was described in the last section, however, these same three assumptions with a linking hypothesis appropriate to the two-response recognition experiment (assumption 4a–4d) cannot completely account for the recognition results reported here. However, the modifications necessary to account for the recognition results may be consistent with detection results as well.

Although independent variables and lack of negative influences across channels have usually been assumed in explaining detection thresholds, these assumptions have been more for convenience than out of necessity. Relaxing these assumptions would account for most of the recognition results and may only change the predicted detection thresholds of compound gratings by a small amount. The changed predictions might be consistent with the experimentally-measured detection thresholds. Other experiments have suggested that negative influences may be necessary to completely account for the detection thresholds of gratings (DeValois, 1977; Olzak and Thomas, 1981), and Thomas *et al.* (1969) found negative spatial interactions for both detection and recognition of rectangular stimuli.

The approximate additivity and the suggestion of equal bandwidths on a logarithmic frequency axis, derived above by application of the recognition model to the recognition results, are consistent with properties derived from detection results. Complete additivity of the effects of two frequencies on the response of each channel was assumed in the original explanations of detection results (Sachs *et al.*, 1971; Quick and Reichert, 1975). More recently, with increasing evidence for probability summation across space, departures from strict additivity have been assumed in explaining detection results (e.g. Quick *et al.*, 1978; Bergen *et al.*, 1979; Graham, 1980; Robson and Graham, 1981). These departures from complete additivity may be small enough, however, to be consistent with the approximate additivity derived from the recognition results.

Bandwidth of a channel as a function of preferred frequency has not been extensively studied in detection experiments. Available evidence, however, shows bandwidth as increasing with preferred frequency fast enough that bandwidth should be approximately equal on a logarithmic frequency scale in the range studied here. For example, Quick *et al.* (1978) showed the bandwidth for the channels at 10 and 14 c/deg was twice that for 5 c/deg.

The detection model presented above (assumptions 1–4) assumes uncorrelated variability in the responses of different channels, that is, probability summation among channels. This uncorrelated variability makes the detection threshold of a compound gratings somewhat lower than the threshold of the most detectable components. However, a different version of the detection model (Quick, 1974), does not postulate uncorrelated variability in the responses of different channels but rather deterministic nonlinear pooling of outputs. (Many of these issues are discussed in Robson and Graham, 1981.) The fact that two recognition responses in the present experiment were at least partially uncorrelated is difficult to explain by any natural extension of this determininistic nonpooling model.

CONCLUSION

Four basic assumptions for spatial frequency models as applied to detection data were identified and four progressively restricted versions of the linking hypothesis to expand the model to recognition data were presented. If it is assumed that recognition responses were related to activity in spatial-frequency channels as specified in the recognition linking hypotheses 4b–4d, then channel properties such as spatial selectivity, covariation, noise, and additivity (as specified in assumptions 1–3) were evaluated in this investigation. The data indicated that the four basic assumptions were not completely satisfactory to account for the observations. Both the previous detection results and the current two-response recognition results for compound gratings may be consistent with any of several modifications to the spatial-frequency-channel model.

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