New Developments in Aggregation Economics*

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1 Introduction

Basic issues The notion of aggregation is pervasive in economics.[1] Many (arguably most) economic decisions are made by groups, not individuals. Firms are an obvious example; it has long be recognized that the standard model of a unique, profit-maximizing decision unit must often be extended to take into account the multi-person nature of the decision process. The same remark applies to committees, clubs, villages and other local organizations, which have also attracted much interest. Even standard micro demand analysis, although it routinely uses the tools of consumer theory, exploits data on households or families, which in general gather several individuals. Partial equilibrium analysis relies on aggregate demand or supply functions. And, quite obviously, macroeconomics concentrate on the aggregate behavior of vast classes of agents ('households', 'firms', etc.), each being routinely identified with a single decision maker.

In all these cases, aggregation issues are raised, at least implicitly. When can a multiperson entity be analyzed as a single decision maker (i.e., when is there a 'representative consumer')? What data are needed to fully summarize the situation of a group? Are there testable re-

*Forthcoming in *Annual Review of Economics.* Financial support from the NSF (grant SBR 0532398) is gratefully acknowledged.

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[1] Throughout the paper, we consider issues related to aggregation over *individuals.* The word 'aggregation' is sometimes used in a totally different context - namely, the aggregation of several commodities into some 'composite' good (say, the aggregation of various types of meat, vegetables, dairy and others into the general category of 'food'). Aggregation of commodities is not considered in this article.

strictions on aggregate behavior stemming from the utility- (or profit-) maximizing actions of each member? Can one formulate welfare evaluations at the aggregate level, and what are their implications for the individuals under consideration? To what extend is it possible to recover individual-level characteristics (preferences, resources, ...) or information on the intragroup decision process from the sole observation of aggregate behavior?

Quite often, the answer to these questions are taken for granted without much analysis; for instance, macro models typically *assume* the existence of a representative agent with little discussion of either the prerequisites for the assumption or its implications. Still, theoretical investigations of the aggregation issues just mentioned - and of many others - have been available for several decades. Moreover, the field of aggregation theory has recently attracted renewed interest. Old problems have recently been solved; existing questions have been reconsidered from a different perspective; and more generally, a new subfield has emerged, with original emphasis, techniques and results. In this paper, we survey some of these recent developments.

The structure of the paper is as follows. We first describe the notations we shall use throughout the article. In the next section, we briefly summarize the main features of 'traditional' aggregation theory, as it has been developped up to the late 70s/early 80s. Section 3 describes how some of the 'traditional' questions have recently been either solved or reinterpreted. The recent literature on the aggregate behavior of small groups ('aggregation in the small') is covered in Section 4.

Finally, one caveat is in order. The goal of this paper is simply to provide a quick overview of some recent results. For the sake of brevity, we will omit the proofs of some of the most important (and most complex) results, as well as many interesting but specific developments. The interested reader is referred to our recent book (Chiappori and Ekeland 2009b) for a more complete exposition.

Notations We first define the notations that will be used throughout the article. In what follows, the transpose of vector x is denoted x^T, and the scalar product of vectors x and y is denoted $x^T y$; i.e., if $x = (x_1, ..., x_N)$ and $y = (y_1, ..., y_N)$ then:

$$x^T y = \sum_k x_k y_k$$

Commodities We consider a group consisting of H members. Agents may consume M commodities, I of which are privately consumed by each member while the remaining $J = M - I$ are public within the group; note that, formally, the list of commodities may include leisure. Moreover, a given, 'physical' commodity may be further indexed by the period or the state of the world (or both) at which it is available. Therefore, our setting extends to intertemporal behavior (savings, investment, human capital accumulation) as well as risk sharing and group decision under uncertainty.

Let x_h^i denote the private consumption of commodity i by group member h, and X^j the group's consumption of public good j. An *allocation* is a $J + HI$-vector $(X, x_1, ..., x_H)$, where

$$X = \left(X^1, ..., X^J\right) \in \mathbb{R}^J \text{ and}$$
$$x_h = \left(x_h^1, ..., x_h^I\right) \in \mathbb{R}^I \text{ for } h = 1, ..., H.$$

and the group's aggregate demand is the vector $(X, x) \in \mathbb{R}^M$ where $x = \sum_h x_h$. For brevity, the vector (X, x) is often denoted ξ.

Utility functions We assume that each person has a utility function over allocations. We denote h's utility function by $U^h(X, x_1, ..., x_H)$. This formulation is fully general; it allows the utility of h to depend on the private consumptions of other members in a non-restricted way. This interaction may be the result of altruism (i.e., h cares about other members' well being) or paternalism (h is concerned with her partners' consumptions) ; it may also reflect other external impacts between consumptions (say, a member's smoking bothers the other members by reducing their utility, an intra group 'externality' in the usual sense). Note, in particular, that other members' consumptions of private goods may impact on h's marginal rate of substitution between her own private and public goods; in other words, we do not impose separability restrictions so far.

The utility functions $U^h, h = 1, ..., N$, are assumed continuously differentiable and strictly concave; in some cases, stronger restrictions (e.g., infinite differentiability, *strong concavity*, defined by the fact that the matrix of second derivatives is negative definite everywhere, or *strong quasi-concavity*, defined by the fact that the restriction of this matrix to the subspace orthogonal to the gradient is negative definite) will be required.

Although quite reasonable, the form just described is sometimes too general - if only because it is difficult to incorporate such preferences into a model in which agents live alone for some part of their life-cycle. Consequently, many models considers *egoistic* preferences, of the form $U^h(X, x_h)$. Finally, a fraction of the literature deals with *market economies.* In this context, preferences are strictly egoistic, and all commodities are privately consumed. In particular, interactions between group members (if any) are restricted to commodity trading. Then the general form just defined boils down to:

$$U^h\left(X, x_1, ..., x_H\right) = u^h\left(x_h\right). \tag{1}$$

Aggregate budget constraint Let p denote the price vector of private goods, P that of public goods and y the group's total income. Again for brevity, the vector (P, p) is often denoted π, so that the aggregate demand (as a function of prices and income) becomes $\xi(\pi, y)$

The group has limited resources; specifically, its purchase vector $\xi = (X, x)$ must satisfy a standard market budget constraint of the form

$$\pi^T \xi = P^T X + p^T \left(\sum_h x_h\right) \leq y$$

Throughout the book, we assume that behavior is zero homogenous in prices and income. For some computations, we may therefore normalize the group's total income to be one. Also, we sometime consider the group's *budget shares*, defined by:

$$\Psi = (\Psi_1, ..., \Psi_M) \quad \text{where } \Psi^i = \frac{\pi_i \xi^i}{y}$$

2 Standard aggregation theory: a brief overview

As a first step, it is useful to briefly reconsider some crucial aspects of aggregation theory as it has developed until the early 80s. Our goal, here, is not to provide a survey; the interested reader is referred for instance to Deaton and Muelbauer (1980) and Shafer and Sonnenschein (1982) for that purpose. Instead, we want to briefly recall the main features of this literature - and in particular the questions it asked and the answers it provided. The general notion of aggregation theory gathers a host of different and more or less related approaches. To provide an overview, it is convenient to distinguish between two core approaches: one in which the group is considered as a (mostly exchange) economy, and a more general perspective that allows for richer interactions such as public consumptions or intragroup production.

2.1 Groups as market economies

In this first subsection, we assume that *all commodities are privately consumed*; we thus rule out public goods, as well as externalities of any type, and do not consider intragroup production. A few questions have played a crucial role in the development of the various branches of the field; we shall organize our presentation around them.

2.1.1 When does a group behave as a single decision maker?

A first question relates to the conditions under which the aggregate behavior of a group can validly be described using the tools of standard consumer theory. A first version, that has been known for at least a century (since Antonelli's pathbreaking result), is the following. Assume that some total income y is distributed between H agents, who each freely spend their share on several goods. When is it the case that the group's aggregate demand for each good can be expressed as a function of y alone - i.e., does not depend on how the income has been allocated within the group? The answer is straightforward: for the property to be satisfied, it must be the case that transferring a dollar from one group member to another does not change total consumption - in other words, that the marginal propensity to consume (MPC) each good be the same for all agents, *irrespective of their income*. This can only happen under two conditions: (i) each agent's MPC is independent of the agent's income, and (ii) these constant MPCs are identical across agents. In other words, individual Engel curves must be linear or affine, and the coefficients of income must be common to all agents. When it is the case, then one can readily show that the resulting, aggregate demand is compatible with utility maximization.

Clearly, this statement sounds like an impossibility result; while a group *may* in theory behave like a single individual, the restrictions required in practice are quite stringent, and largely counterfactual. We know for a fact that most Engel curves are not linear, and that MPCs are largely heterogeneous across agents.

This negative conclusion has led to a reformulation of the question along a slightly different line, usually called 'exact non linear aggregation'. Assume, again, that total income is distributed between the agents, agent h receiving some amount y_h, and consider now aggregate *budget shares*. Can one find some aggregate statistic $\tilde{y}$, possibly dependant on the *distribution* of income within the group (and not only on total income), such that aggregate budget shares can be expressed as functions of prices and $\tilde{y}$ only? If so, are these aggregate budget shares compatible with the maximization of a single utility under budget constraint (taking $\tilde{y}$ as a 'pseudo income', i.e. as the right hand side of the constraint)?

This relaxation of the problem significantly enlarges the set of acceptable preferences. Specifically, assume that individual demands have a *Gorman* form:

$$x_h^i(p, y_h) = \sum_{k=1}^{K} a_h^{k,i}(p)\, b_h^{k,i}(y_h) \tag{2}$$

A well known result by Gorman states that the number of *independent* expressions in the right-hand side sum cannot exceed $K = 3$; moreover, the maximum rank can only be reached for specific functions $b_k(y_h)$. To keep things simple, let us consider the case $K = 2$. Adding up requires one of the two functions of income to be linear; we can therefore consider the form:

$$x_h^i(p, y_h) = a_h^{1,i}(p)\, y_h + a_h^{2,i}(p)\, b_h^i(y_h) \tag{3}$$

Note that the functions a and b must satisfy additional restrictions to be compatible with utility maximization at the individual level; these are coming from the fact that the Slutsky matrix, which can readily be derived from the previous form, must be symmetric and negative. A widely used specification is the PIGLOG form, in which

$$b_h^i(y_h) = y_h \ln(y_h)$$

In that case, individual budget shares are given by:

$$\psi_h^i(p, y_h) = p_i a_h^{1,i}(p) + p_i a_h^{2,i}(p) \ln(y_h) \tag{4}$$

This is a nicely flexible functional form, that has been widely used in empirical applications.

Coming back to (3), assume in addition that the various coefficients $a_h^{1,i}, a_h^{2,i}$ and b_h^i are independent of h. Aggregate demand becomes:

$$X^i = \sum_h x_h^i(p, y_h) = a^{1,i}(p) \sum_h y_h + a^{2,i}(p) \sum_h b^i(y_h)$$

generating the (aggregate) budget shares:

$$\begin{aligned}\Psi^i &= p_i a^{1,i}(p) + p_i a^{2,i}(p) \frac{\sum_h b^i(y_h)}{\sum_h y_h} \\ &= p_i a_h^{1,i}(p) + p_i a^{2,i}(p)\, b^i(\tilde{y}) \end{aligned} \tag{5}$$

where $\tilde{y}$ is defined by:

$$b^i(\tilde{y}) = \frac{\sum_h b^i(y_h)}{\sum_h y_h}$$

For instance, the PIGLOG specification gives:

$$\ln \tilde{y} = \frac{\sum_h y_h \ln(y_h)}{\sum_h y_h}$$

which is closely related to Theil's inequality index.[2]

[2]Specifically, Theil's index is defined by:

$$T_1 = \frac{\sum_h y_h \ln(y_h)}{\sum_h y_h} - \ln(\bar{y}) = \ln(\tilde{y}/\bar{y})$$

where $\bar{y}$ denotes average income.

We see that, as required, aggregate market shares Ψ_i in (5) only depend on individual incomes through the aggregate indicator $\tilde{y}$. Moreover, they have exactly the same form as the individual market shares in (3); therefore they can be derived from the maximization of a utility of the same form, using the aggregate indicator $\tilde{y}$ as the group's 'pseudo income'. It follows that, in terms of budget shares, the group behaves like a single agent, endowed with the same utility as each individual in the group and a pseudo income equal to $\tilde{y}$.

An important outcome of this analysis is that the same functional form can be used at both the individual and the aggregate level. This may be useful when individual data do not display enough price variation. Indeed, a strong motivation for earlier research was the design of a demand function that would allow the estimation of Engel curves from cross-sectional data and of price effects from aggregate time series.

One can see from this example that one drawback of Antonelli's restrictions - the lack of realism of individual demands - is largely alleviated by the non linear aggregation approach. In fact, state of the art estimations of individual demands routinely use functional forms such as the Quadratic Almost Ideal, which is a particular Gorman form - it relies on a quadratic ('rank 3') version in which

$$\psi_h^i(p, y_h) = p_i a_h^{1,i}(p) + p_i a_h^{2,i}(p) \ln(y_h) + p_i a_h^{3,i}(p) \ln^2(y_h)$$

However, the second problem remains: aggregation is possible only under very strong restrictions regarding heterogeneity of preferences (in our example, all agents must have identical preferences, although this can be slightly relaxed). In short: a representative consumer may exist for acceptably general individual demands - but only when agents have 'essentially identical' preferences.

A last remark is that one should be very cautious with welfare judgments suggested by the representative consumer's utility. Indeed, several authors have provided examples showing that a given reform may increase the utility of the representative consumer while decreasing the welfare of *all* individuals in the economy.[3]

2.1.2 Is some structure preserved by (large scale) aggregation?

A second line of research adopts the opposite viewpoint. Instead of imposing a priori some desired structural properties of aggregate demand (e.g., the existence of a representative consumer) and trying to find sufficient conditions on individual preferences for these properties to be satisfied, these approaches consider general preferences, and ask which

[3] See for instance Jerison (1982) and Dow and da Costa Werlang (1988).

structure (if any) aggregate demand must have, given that each individual in the economy maximizes a well-behaved utility under budget constraint. We have known for a long time that utility maximization generates a lot of structure for *individual* demands (namely, symmetry and negativeness of the Slutsky matrix); the question is whether (some of) this structure is preserved by aggregation.

The problem has been initially raised by Sonnenschein in a seminal article (1972). Technically, Sonnenschein stated two versions of the problem. In both cases, individuals each maximize utility under a budget constraint; but in the first version each individual h receives some nominal income y_h, whereas in the second they each receive a fixed *endowment* ω_h. Technically, individual maximization programs are therefore of the form:

$$\begin{array}{c} \max_{x_h} U^h (x_h) \quad \text{subject to} \\ p^T x_h = y_h \end{array} \tag{6}$$

in the first case, and

$$\begin{array}{c} \max_{x_h} U^h (x_h) \quad \text{subject to} \\ p^T x_h = p^T \omega_h \end{array} \tag{7}$$

in the second. The first case corresponds to the *market demand* problem; what is considered in the second is the agent's *excess demand*, defined as the difference between the agent's desired consumption bundle and her initial endowment.

In both cases, aggregate demand depend on prices and initial endowments. The main question, however, is the characterization of aggregate demand *as a function of prices.* It can therefore be stated as follows: *Consider a given function $x(p)$. When is it possible to find H smooth, increasing, strongly concave utility functions U^h and*

- *(market demand) H scalars $(y^1, ..., y^H)$ such that*

  $$x(p) = \sum_h x_h(p)$$

  where $x_h(p)$ solves

  $$\begin{array}{c} \max_{x_h} U^h (x_h) \quad \text{subject to} \\ p^T x_h \leq y^h \end{array} \tag{8}$$

- *(excess demand) H vectors $(\omega_1, ..., \omega_H)$ in $\mathbb{R}^N$ such that*

  $$x(p) = \sum_h x_h(p)$$

where $x_h(p)$ *solves*

$$\begin{array}{c}\max_{x_h} U^h(x_h) \quad \text{subject to} \\ p^T x_h \leq p^T \omega_h\end{array} \tag{9}$$

The statement of the excess demand case can actually be slightly simplified by introducing *individual excess demands* $z_h = x_h - \omega_h$. For any given set of direct utilities $\left(U^1, ..., U^H\right)$ and initial endowments $(\omega_1, ..., \omega_H)$, one can define the utilities $\left(\tilde{U}^1, ..., \tilde{U}^H\right)$ by $\tilde{U}^h(z) = U^h(z + \omega_h)$, $h = 1, ..., H$. With this notation, program (9) can be rewritten as:

$$\begin{array}{c}\max_z \tilde{U}^h(z) \quad \text{subject to} \\ p^T z \leq 0\end{array} \tag{10}$$

There exists obvious properties that the aggregate market or excess demand will satisfy. One is continuity (or differentiability in our context). Another is adding-up (sometimes called Walras Law); namely, it must be the case that $p^T x(p) = \sum_h y^h$ for market demand, and $p^T z(p) = \sum_h p^T (x_h(p) - \omega_h) = 0$ for excess demand. Finally, excess demand functions are zero-homogeneous in prices. The question is whether these properties are sufficient, or whether the underlying structure generates stronger properties at the aggregate level.

In practice, a large fraction of the subsequent literature has been devoted to the case of a 'large economy' - i.e., one in which the number of agents exceeds the number of commodities. Sonnenschein's conjecture was that in large economies, the obvious properties just listed fully characterize aggregate demands: individual structure is therefore lost by aggregation, at least if the latter takes place on a sufficiently large scale.

Within months after Sonnenschein's initial statement, the excess demand case was independently solved by Mantel and Debreu (this literature is actually often referred to as 'Debreu-Mantel-Sonnenschein' or DMS). Specifically, Mantel (1974) established that any smooth function satisfying homogeneity and adding up could be decomposed as the aggregate excess demand of an economy with at least $H = 2M$ agents. Debreu (1974) showed that the result was valid for $H = M$, and Mantel (1976) proved that one could, in addition, assume that all utilities were homothetic.

Recently, Chiappori and Ekeland (1999) have provided a short proof of a slightly stronger result. Their approach is based on the properties of individual excess demand functions. Surprisingly enough given their theoretical importance, individual excess demands had not, until recently, been studied in detail. Their key result is the following. Suppose that $V(p)$ is a smooth function defined on some neighborhood $\mathcal{O}$ of $\bar{p}$, with

$D_pV(\bar{p}) \neq 0$. Assume that it is quasi-convex, positively homogeneous of degree zero (which implies that $p^T.D_pV(p) = 0$), that $D^2_{pp}V(p)$ has rank $(N-1)$ and that the restriction of $D^2_{pp}V(p)$ to Span $\{p, D_pV(p)\}^{\perp}$ is positive definite. Take any C^2 function $\lambda(p) > 0$, homogeneous of degree (-1) on $\mathcal{O}$, and set:

$$z(p) = -\frac{1}{\lambda(p)} D_pV(p) \tag{11}$$

so that $p^T z(p) = 0$. Then $z(p)$ is the excess demand function of some consumer; i.e., there exists a strictly quasi-concave function $U(z)$, defined and C^2 in a neighborhood $\mathcal{N}$ of $z(\bar{p})$, such that V is the indirect utility associated with U. It follows, in particular, that if $z(p)$ is an individual excess demand, then for any zero homogeneous scalar function $\zeta(p)$, $\zeta(p) z(p)$ is also an an individual excess demand.

Consider now some compact subset K of the positive orthant. For $H \geq M$, take some family $V^h(p), 1 \leq h \leq H$ such that at every $p \in K$, the set of linear combination of the $DV^h(p)$ with non negative coefficients spans $T_pS^{\mathbf{N}-1}$ (the tangent space $T_pS^{\mathbf{N}-1}$ at p to the N-dimensional simplex, to which the price vector can be normalized to belong). Then for any C^2 map $x(p)$ defined on K , homogeneous of degree zero and satisfying the Walras law $p^T x(p) = 0$, one can find excess demand functions $z_h(p), 1 \leq h \leq H$, such that the decomposition $x(p) = \sum_h z_h(p)$ holds on K and the indirect utility associated to z_h is V^h. This version of the result is slightly stronger than Debreu's because the indirect utilities can be defined independently of the excess demand at stake; i.e., the *same* $V^1(p), ..., V^H(p)$ can be used in the decomposition of any given function.[4]

The DMS result was quite influential in the profession. Its theoretical implications are far from trivial; for instance, it immediately implies that for any compact subset of the positive orthant, one can always find economies with exactly one equilibrium within the compact subset, such that this equilibrium is not stable by the Walrasian tatonnement. From a more epistemological perspective, it has also be widely interpreted as a negative result: if aggregate demand can be anything, it was argued, then general equilibrium theory has no testable implication (except maybe the existence of an equilibrium), at least when applied to a large enough economy. As we shall see, this somewhat excessive claim

[4]This property may sound surprising. It reflects the fact that, in sharp contrast with market demands, there exists a continuum of individual excess demands that correspond to the same indirect utility (of course, they involve different initial endowments in general).

has however been drastically reconsidered by the recent literature on the topic.

Finally, the case of a 'small' economy (i.e., one with less agents than goods) has been considered by Diewert (1977) and Geanakoplos and Polemarchakis (1980). These authors have showed that, in such a setting, additional conditions (beyond the obvious ones) have to be fulfilled for a given function to be the aggregate excess demand of such an economy. Their approach relies on a local linearization of the problem; in particular, while the conditions they provide are indeed necessary, neither article provides sufficiency results. We shall come back to these results below.

2.1.3 Can aggregation *create* structure?

The main conclusion of the Sonnenschein program is that assembling a sufficiently large number of sufficiently diverse utility-maximisers may result in a collective demand with bizarre - in fact arbitrary - properties; in short, aggregation 'in the large' tends to destroy any structure that may exist at the individual level (and in small groups, see below). An interesting question, however, relies on an opposite perspective: is it the case that the aggregation of sufficiently diverse individual demands results in an object that is *more* regular that its component? In other words, can aggregation *create* structure?

This line of research has been pionneered by Hildenbrand. In a series of papers originating in 1983 and culminating in his 1994 book, he investigated the Law of Demand (see Hicks, 1956), henceforth referred to as LD. Denoting by $X(p)$ a demand function, it is said to satisfy LD if the inequality:

$$(X(p) - X(q))^T (p - q) \leq 0$$

holds for all price systems p and q. If $X(p)$ is differentiable, it is equivalent to the Jacobian matrix:

$$D_p X = \left(\frac{\partial X^i}{\partial p_j} \right)_{1 \leq i,j \leq I}$$

being negative semi-definite at every p. Roughly speaking, LD means that consumption and prices move in opposite directions. It implies that the demand curve for every good is downwards sloping, but it is of course much more. For instance, if aggregate demand satisfies LD, then the equilibrium is unique.

It is well-known (Giffen goods) that individual demand functions need not satisfy LD. For an individual having nominal income y, Marshallian demand is a function $x(y, p)$, and we have:

$$D_p x = S(p, y) - (D_y x) x'$$

where the D_p and D_y denote partial derivatives, The first term, $S(p,y)$, is the Slutsky matrix, which is negative definite. The second term, $(D_y x) x'$, describes the income effect, that is, the change in wealth due to the change in prices; if preferences are homothetic, it is positive semi-definite, so LD is satisfied. Apart from this very special case, the income effect bites, and LD needs not be satisfied at the individual level.

Hildenbrand's idea is that it can nevertheless be satisfied at the macroeconomic level, because of special properties of the income distribution. This idea is best explained from the example in his 1983 paper. Suppose all individuals have identical preferences, and the income distribution has a differentiable density $\mu(y)$ on $[0,\ \bar{y}]$ with $\mu(\bar{y}) = 0$. The aggregate demand is:

$$X(p) = \int_0^{\bar{y}} x(p,y)\,\mu(y)\,dy$$

The aggregate Slutsky matrix is obviously negative definite, and the aggregate income effect is:

$$\begin{aligned} \sum \int_0^{\bar{y}} \frac{\partial x^i}{\partial y} x^i \mu(y)\,dy &= \sum \int_0^{\bar{y}} \frac{\partial}{\partial y} \left(x^i\right)^2 \mu(y)\,dy \\ &= -\sum \int \left(x^i\right)^2 \frac{d\mu}{dy} dy \end{aligned}$$

where we have integrated by parts. The two boundary terms vanish, the first one because $x^i(0) = 0$ and the second one because $\mu(\bar{y}) = 0$, and we are left with the integral term. And now a striking result emerges: if $\frac{d\mu}{\partial y} \leq 0$, that is if the density is decreasing, then the aggregate income effect is positive definite, and the collective demand $X(p)$ satisfies LD, even though the individual demands do not.Of course, it is unrealistic to assume that individuals have identical preferences, but this example vindicates the idea that particular properties of the wealth distribution can result in LD. As shown in Chiappori (1985), one can also obtain LD by putting conditions both on the form of individual demand and the wealth distribution, the Hildenbrand result, where all preferences are identical, being just a polar case. The question is then to find characteristics of consumption and wealth distributions which (a) are empirically verifiable, and (b) will generate LD.

Jerison (1982, 1999) showed that LD holds if there is increasing demand dispersion, that is, if the cloud of consumption vectors for individuals of a given income level is increasingly dispersed.as the level rises. In other words, the Engel curves spread out at higher income levels. Grandmont (1992) decomposes a population into subclasses which satisfy a specific parametric model of demand, and then shows that sufficient heterogeneity of the parameter distribution, as measured by the flatness of the corresponding density, leads to LD. Kneip (1999) introduces a non-parametric notion of demand heterogeneity, with the same result. Hildenbrand takes a different approach, and checks directly, on British and French family expenditure data, that the aggregate income effect is positive definite (Härdle et al., 1991; Hildenbrand, 1994). There are a number of econometric problems to overcome. For instance, since such surveys do not follow an individual through time, one cannot infer from the data how a small change in income would affect the average consumption of individuals with at a given income level. However, the surveys give the average consumption of individuals with slightly higher or slightlly smaller income, and this should be a reasonable stand-in, provided the other characteristics of the population do not change dramatically across income classes. This being said, the econometric conclusions do seem to always provide empirical support to the Law of Demand.

2.2 Groups as complex economies

A second line of research has considered aggregation problems from a totally different perspective. On the one hand, it essentially deals with small groups (typically household or families); on the other hand, instead of focussing on market economies, it considers potentially more complex interactions, involving possibly public consumption and intragroup production. The initial literature almost exclusively concentrates on one question - namely, *Which assumptions would guarantee that the group under consideration behaves like a single individual?* We shall briefly describe the two main contributions to this literature: Samuelson's aggregate welfare index and Becker and Bergtrom's transferable utility (TU) setting.

2.2.1 Samuelson's index

Assume that all individuals agree on some global index, the arguments of which are the various individual utilities, that will be maximized by the group. Technically, there exists some strictly increasing W such that the group, by common agreement, maximizes:

$$W\left(U^1\left(X, x_1, ..., x_H\right), ..., U^H\left(X, x_1, ..., x_H\right)\right) \tag{12}$$

under the budget constraint

$$P^T X + p^T\left(\sum_h x_h\right) = y \tag{13}$$

Household production could be introduced at little cost; this task is left to the reader.

It is straightforward to see that this formulation boils down to a standard utility maximization problem. Indeed, define the group utility $\mathcal{U}^G$ by:

$$\mathcal{U}^G(X,x) = \max_{\sum_h x_h = x} W\left(U^1(X, x_1, ..., x_H), ..., U^H(X, x_1, ..., x_H)\right)$$

Then $\mathcal{U}^G$ is the utility of a representative consumer for the group: for any consumption vector $(X, x_1, ..., x_H)$ that maximizes (12) under (13), the vector (X, x) where $x = \sum_h x_h$ maximizes $\mathcal{U}^G$ under the budget constraint $P^T X + p^T x = y$. Note that this conclusion is just a restatement of an old result by Hicks, sometimes referred to as the 'composite good theorem': in this setting, for any i the 'commodities' $x_1^i, ..., x_H^i$ are always purchased at the same price p_i.

Simple as it may seem, this approach has some interesting properties. Note, first, that the relationship between $\mathcal{U}^G$ on the one hand and $\left(W, U^1, ..., U^H\right)$ on the other hand is *not* one-to-one: there exists many (in fact a continuum) of different structures $\left(W, U^1, ..., U^H\right)$ that generate the same representative utility U^G. This is exactly the spirit of Hick's theorem: without variations in the respective prices of the $(x_1, ..., x_H)$, individual utilities simply cannot be recovered. It follows that in this approach, the group is doomed to be a black box: its aggregate behavior can certainly be studied (using standard consumer theory), but its inner mechanisms (individual utilities and the index W) are necessarily unrecoverable. Ironically, we shall see below that Samuelson's index is a particular case of a more general representation (the so-called 'collective approach'), which only postulates that the group decisions are Pareto efficient. In this general family, simple exclusion restrictions are generically sufficient for individual utilities to be identified; the Samuelson index case is among the very few exceptions for which individual utilities can never be recovered.

A second remark is that, like all models admitting a representative consumer, the Samuelson index case satisfies *income pooling*; that is, the group's behavior only depends on total income, not on its allocation between the group members. In this setting, thus, paying a benefit to one member instead of another (say, to the husband instead of the wife) *cannot possibly* have any impact on the outcomes. As we shall see, there is strong empirical evidence against this prediction.

Finally, there is a relationship between the Samuelson index and the market economy approach described above, although the link is somewhat subtle. Assume for a moment that agents are egoistic, only consume private goods and there are no externalities, so that we are back in the setting studied in subsection 2.1. Now, the maximization of $W\left(U^1,...,U^H\right)$ under budget constraint generates a consumption plan that is Pareto efficient; for otherwise an alternative allocation would increase all the U^h and strictly increase at least one of them, but this would strictly increase W, a contradiction. By the second welfare theorem, this efficient allocation can be decentralized; i.e., there exists an income distribution within the group such that this allocation obtains as an equilibrium; in practice, if each agent h receives a specific income y_h (with $\sum_h y_h = y$) and consumes it at her will, the resulting consumption plan maximizes W under budget constraint.[5] Clearly, this argument can be applied for any specific value of the price vector $p = (p_1, ..., p_M)$. The crucial remark, however, is that the income distribution that decentralizes the optimal allocation at prices p may (and generally will) depend on p in an arbitrary way. In particular, there is no reason to expect that it will be either constant - as in the market demand case - or a linear function of p - as in the excess demand case. In other words, in Samuelson's index story, there exists some income allocation $y(p) = \left(y^1(p), ..., y^H(p)\right)$ such that individual behave as if they were maximizing their own utility under a budget constraint - solving a program of the form:

$$\begin{array}{c} \max_{x_h} U^h(x_h) \quad \text{subject to} \\ p^T x_h = y^h(p) \end{array} \tag{14}$$

But the market economy approach imposes an *additional* restriction on the income allocation - specifically, that is takes one of the following two forms:

- Constant nominal income: $y^h(p) = y^h \in \mathbb{R}$ for all p, h, or
- Proportional income: $y^h(p) = p^T.\omega^h$, where $\omega^h \in \mathbb{R}^N$.

[5] The precise, formal argument goes as follows. Consider an economy with H customers $U^1, ..., U^H$ and $(M+1)$ commodities - the M physical commodities plus money. There exists an initial (total) endowment of the $(M+1)$th commodity (money) equal to y; regarding the other commodities, the initial endowment is nil, but they can be *produced* from the $(M+1)$th according to the linear production technology

$$y = \sum_k p_k x^k.$$

In this economy, any Pareto efficient allocation can, by the second welfare theorem, be decentralized as an equilibrium. Given the linear technology, equilibrium prices must be proportional to $(p_1, ..., p_M)$ - and we can always normalize them to be equal to that vector. An equilibrium is uniquely characterized by the allocation of initial endowments $(y_1, ..., y_H)$ - which we interprete as an income distribution within the group.

These conditions may be satisfied for very special functional forms for individual utilities and the index. For instance, one can readily check that if utilities are Cobb-Douglas and W is linear:

$$W\left(U^1,...,U^H\right)=\sum_h \lambda_h U^h$$

then the optimal consumption plan can be decentralized by allocating to agent h a fixed nominal income equal to $y_h=\lambda_h y$. Most of the time, however, the conditions are not satisfied; the relationship that exists, for given individual utilities $\left(U^1,...,U^H\right)$, between the index W and the allocation $y(p)=\left(y^1(p),...,y^H(p)\right)$ is in fact quite complex, and in general highly non linear.

2.2.2 The Transferable Utility (TU) case

An alternative situation in which the group's behavior boils down to a single utility maximization is when individual utilities exhibit a *transferable utility* (TU) property.[6] This happens when one can find, for each agent h, a particular cardinalization such that, *for all values of prices and income,* the Pareto frontier is an hyperplane of equation:

$$\sum_h U^h=K$$

for some K that depends on prices and income. In other words, it must be the case that for some well chosen cardinalization of individual preferences, agents are able to transfer utility between them at a constant 'exchange rate' (which can be normalized to 1).

When all goods are private, TU obtains only for quasi linear utilities:

$$U^h\left(x_h\right)=x_h^1+u^h\left(x_h^2,...,x_h^M\right)$$

Here, the marginal utility of an additional dollar spent on private consumption of commodity 1 is always constant (and can be normalized to 1). This form has very strong (and unrealistic) implications; for instance, individual demands for all commodities but the first have a zero income elasticity. Things become much more interesting when public goods are considered. Bergstrom and Cornes (1983) have proved that the TU property obtains if and only if individual utilities can be (possibly after an increasing transform and a renaming of the private goods) put into a 'Generalized Quasi Linear' (GQL) form:

$$U^h\left(X,x_h\right)=u^h\left(x_h^2,...,x_h^M,X\right)+G\left(X\right)x_h^1 \tag{15}$$

[6] See for instance Browning, Chiappori and Weiss (2010).

where $G(X) > 0$ for all X. Note that the G function must be identical for all members, whereas the u functions can be individual-specific. In words, the transferable utility assumption implies that, for some well chosen cardinalization of individual preferences, the marginal utility of an additional dollar spent on private consumption of commodity 1 is always the same for all members (although it needs not be constant - it may vary with the vector of public goods).

While this form remains constrained, the restrictions are much less stringent than the quasi-linear case.[7] Interestingly, and similarly to the market economy case, the main restriction affects the level of heterogeneity that is allowed between individual preferences: it must be the case that the function G, which determines the marginal utility of private commodity 1, be the same for all agents in the group.

Under TU, the sole assumption of Pareto efficiency is sufficient to generate a representative consumer, at least when all agents consume a positive quantity of the first private commodity. Indeed, one can readily show that efficiency then requires that the group maximizes *the sum of individual utilities*. In particular, the level of all public and private consumptions (but that of the first private good) is the same for all efficient outcomes. Thus under transferable utility and assuming efficiency, group members will agree on almost all consumption choices; the only conflict will be in how to divide the private good x^1 which is often referred to as 'money' but may be interpreted more broadly as a medium of exchange. Lastly, if we define:

$$U^G(X, x) = \max_{\sum_h x_h = x} \sum_{h=1}^{H} u^h\left(x_h^2, ..., x_h^M, X\right) + G(X)\, x^1$$

then the group's aggregate demand (X, x) maximizes U^G under budget constraint, and U^G is therefore the utility of the group's representative consumer.

The TU framework is extremely convenient for many economic problems, and is therefore widely used.[8] Still, it comes at a cost. Since TU is compatible with the existence of a representative customer, the resulting behavior satisfies income pooling; as mentioned above, empirical evidence does not support this property. Moreover, the representation of group behavior it provides is highly peculiar: this is a world in which, under efficiency, group members cannot possibly disagree about anything except the allocation of one private good. Applied to household

[7] See Chiappori (2010) for a precise characterization of these restrictions.

[8] According to Bergstrom (1989), it lies at the core of Becker's celebrated 'Rotten Kid' theorem. See Browning, Chiappori and Weiss (2010) for a precise discussion.

economics, this implies that parents must always agree on all public expenditures, from housing to health care and from the brand of the new car to the level of education to be provided to each child. Such a representation may sometimes be convenient; it many contexts, however, it is grossly counterfactual, and omits some of the most interesting issues of group behavior - namely how shifts in the members' respective powers affect the group's decisions and aggregate behavior. These are issues on which new approaches - and especially the collective model that we shall describe later on - put a lot of emphasis, thus requiring a more general framework.

3 Aggregation in market economies: new results, new perspectives

The market economy approach was very actively pursued in the 70s and the 80s. Since then, new advances have been realized. First, the market aggregate demand problem, which had been open since Sonnenschein's 1972 formulation, was solved; several extensions, dealing primarily with the case of 'small' groups, have subsequently been developed. Secondly, the standard interpretation of the DMS results - that general equilibrium theory has no empirical content - has been challenged, and a more subtle interpretation has emerged.

3.1 Aggregate market demand

While Sonnenschein's first problem - the excess demand case - was solved within months, the second remained open for twenty five years, and was solved only in 1997.[9] The existence of a decomposition has so far only be proved locally (i.e., in some open neighborhood of a regular point), and only for analytic functions; moreover, the proof relies on one of the most impressive results of XXth century mathematics, the Cartan-Kähler theorem (see Kähler 1934, Cartan 1945 and Bryant et al. 1991 for a modern presentation).

We shall not provide here the entire proof; the interested reader is referred to Chiappori and Ekeland (1999 and 2009b). Instead, we shall briefly present the mathematical nature of the problem, and try to explain why the market demand problem turned out to be way more difficult than its apparently similar counterpart, the excess demand one.

We start by assuming that $y^h = 1$; this simplifies the notations without reducing the generality of the proof, which can readily be extended to any vector $y = \left(y^1, ..., y^H\right)$. Also, we assume that $H \geq N$. We consider some mapping $x(p)$ from $\mathbb{R}^N$ to itself, that has to be decomposed

[9] However, Andreu (1982) provided a solution for finite data sets.

into the sum of H individual market demand functions:

$$x(p) = x_1(p) + ... + x_H(p) \tag{16}$$

such that for all h, $x_h(p)$ solves

$$\begin{array}{c} \max U^h(x) \\ p^T x = 1 \end{array} \tag{17}$$

where U^h is 'smooth' (in a sense that will be discussed below), strongly convex and strictly increasing.

As usual, the indirect utility of agent h is defined as the value of program (17). By the envelope theorem,

$$D_p V^h = -\lambda^h x_h(p) \tag{18}$$

where λ^h is the Lagrange multiplier of the budget constraint in (17). The problem thus becomes: Finding H smooth, decreasing, quasiconvex functions $V^1, ..., V^H$ such that the function $x(p)$ can be written as (the opposite of) a convex combination of the gradients of the V^h:

$$x(p) = -\sum_h \frac{1}{\lambda^h} D_p V^h \tag{19}$$

where the V^h satisfy the additional restriction:

$$p^T D_p V^h = -\lambda_h \tag{20}$$

Note that the market demand problem is similar to the excess demand one except for one feature - namely, the individual budget constraint is $p^T x = 1$ instead of $p^T z = 0$, so that the condition on indirect utilities is (20) instead of $p^T D_p V^h = 0$. This apparently minor variation results in a considerably more difficult problem. As mentioned above, an obvious but crucial property of a constraint like $p^T z = 0$ is that if it is satisfied by some function z, then it is also satisfied by $k.z$ for any scalar function k - and the proof of the result heavily exploits this fact. No such property exists in the market demand context.

Decomposing a given function into a linear or convex combination of gradients is a standard problem (often referred to as the Darboux problem; see for instance Ekeland and Nirenberg 2002). Here, however, two additional complexities appear. One is that the V functions must be quasi convex; the other is that they must satisfy (20). Unlike the excess demand case, these complexities cannot be overcome by simple manipulations; they require the full strength of the Cartan-Kähler approach. The same tools can actually be applied to the case of 'small' economies, in which necessary and sufficient conditions can be derived; see for instance Ekeland and Djitte (2006).

Finally, it should be stressed that the class of mathematical problems just described - decomposing a given function into a convex combination of gradients, possibly under additional constraints - lies at the core of most, if not all, modern aggregation theory. It appears not only in the market problem, but also in the much more general approach that will be presented in the next Section.

3.2 Is general equilibrium theory testable?

A widely accepted interpretation of the DMS results is that they shed light on a severe weakness of general equilibrium theory, namely its inability to generate empirically falsifiable predictions. A prominent illustration of this stand is provided for instance by Kenneth Arrow, who listed among the main developments of utility theory the result that "*in the aggregate, the hypothesis of rational behavior has in general no implications*", concluding that "*if agents are different in unspecifiable ways, then [...] very few, if any, inferences can be made*" (Arrow 1991, p. 201).

This view has however be recently challenged as overly pessimistic. New results show that general equilibrium theory can actually generate strong testable predictions, even for large economies. The main idea, initially introduced by Brown and Matzkin (1996), Snyder (1999), Brown and Shannon (2000) and Kubler (2002) and reformulated from a differential perspective by Chiappori et al. (2002, 2004), can be summarized as follows. The DMS approach concentrates on the properties of aggregate excess (or market) demand as a function of prices. However, this viewpoint is not the most adequate for assessing the testability of general equilibrium theory. As far as testable predictions are concerned, the structure of aggregate excess demand is not the relevant issue, if only because excess demand is, in principle, *not* observable, except at equilibrium prices — where, by definition, it vanishes. However, prices are not the only variables that can be observed to vary. Price movements reflect fluctuations of fundamentals, and the relationship between these fundamentals and the resulting equilibrium prices is the natural object for empirical observation. One of the goals of general equilibrium theory is precisely to characterize the properties of this relationship. As it turns out, this characterization generates strong testable restrictions.

To illustrate this view, Brown and Matzkin (1996) consider the simplest possible structure, namely an exchange economy. Here, for given preferences, the economy is fully described by the initial endowments, which are in principle observable; and general equilibrium theory precisely describes the link between endowments and equilibrium prices by characterizing the structure of the equilibrium manifold. Brown and Matzkin derive a set of necessary and sufficient conditions under the form of linear equalities and inequalities that have to be satisfied by any finite data set consisting of endowments and equilibrium prices. They show that these relationships are indeed restrictive. Dealing with the same problem, Chiappori et al. (2002, 2004) adopt a differentiable viewpoint; their necessary and sufficient conditions take the somewhat more familiar form of a system of partial differential equations, reminiscent of Slutsky conditions. In particular, these conditions can readily be imposed on a parametric estimation of the equilibrium manifold, therefore can be tested using standard econometric tools. They also show that these restrictions, if fulfilled, are sufficient to generically recover the underlying economy - including individual preferences. These results, however, require that individual endowments be observable; indeed, when only aggregate endowments are observable, a non testability result can be proved.

The conclusion that emerges from this literature is that, in contrast with prior views, general equilibrium theory does generate strong, empirically testable predictions. The subtlety, however, is that tests can only be performed if data are available *at the micro* (here individual) *level*. One of the most interesting insights of new aggregation theory may be there - in the general sense that testability seems to be paramount when micro data are available, but does not seem to survive (except maybe under very stringent auxiliary assumptions) in a 'macro' context, when only aggregates can be observed.

4 Aggregation in the small: the microeconomics of efficient group behavior

A major development in aggregation theory has been the emergence of the so called 'collective' models of group behavior. Unlike the market economy literature developed in the 70s and 80s, these models mostly concentrate on 'small' groups (formally defined as groups where the number of agents is small relative to the number of commodities); therefore some structure is preserved by aggregation. And unlike Samuelson's or Becker's approaches, they do not try to force aggregate behavior into the 'unitary' structure of consumer theory; on the contrary, they explicitly acknowledge that groups cannot be expected to behave as single individual. The emphasis is actually put on what precisely distinguishes groups from individuals - that is, the existence of a (possibly complex) *decision process*, and more specifically the notion of *power*. Central to the collective approach is the view that *power matters* - that in any variation in the allocation of power between members will systematically result in changes in the aggregate behavior of the group, and that these changes constitute an extremely interesting object for economic analysis. In collective models, paying a benefit to the wife instead of the husband makes a difference - and this difference is a major topic of interest.

This perspective opens a host of new questions: How, and under which assumptions, should the decision process be modeled? How can we formally represent the abstract notion of power? Should the group remain a black box, or is there something one can say about its structure (utilities, decision process) from the sole observation of its aggregate behavior? Are empirical predictions possible, and of what kind? In what follows, we describe the answers provided by the main line of research in this direction. We first present the formal model. We then provide a full characterization of the aggregate demand functions stemming from this framework. Finally, we discuss issues related to identification; we show that generically, a set of simple exclusion restrictions (one per group member) are sufficient to fully recover welfare allocation between members.

4.1 Efficiency and power

The collective approach essentially relies on one basic assumption, namely efficiency. Whatever the decision process may be, it is assumed that it leads to efficient outcomes, in the usual (Pareto) sense that no alternative would have been preferred by all group members. Innocuous as it may seem, this assumption still excludes several existing models of group (often household) behavior based for instance on non cooperative game theory; it also rules out asymmetric information or agency problems. As such, it is particularly relevant for modeling long term interactions between members that know each other well (families being a typical example). More generally, it can be seen as a benchmark formulation, that will be extended in the future. Also, it can be stressed that it encompasses and generalizes both the market economy approach (since, in the latter setting, equilibria are Pareto efficient) and the 'unitary' perspective a la Becker/Samuelson.

Formally, we thus assume the following:

Axiom 1 (Efficiency) *The outcome of the group decision process is Pareto efficient; the consumption* $(x_1, ...x_H, X)$ *chosen by the group is such that no other vector* $\left(\bar{x}_1, ...\bar{x}_H, \bar{X}\right)$ *feasible at the same prices and incomes could make all members better off, one of them strictly so.*

The set of Pareto efficient allocations can be characterized in a number of equivalent ways. First, for any vector (π, y) of prices and income in $\mathbb{R}^{M+1}$, there must exist numbers $\bar{u}_2, ..., \bar{u}_H$ and vectors $X, x_1, ..., x_H$, which may depend on (π, y), such that $(X, x_1, ..., x_H)$ solves:

$$\begin{array}{c} \max_{X,x_1,...,x_H} U^1 (X, x_1, ..., x_H) \quad \text{subject to} \\ U^h (X, x_1, ..., x_H) \geq \bar{u}_h, \; h = 2, ..., H \\ \pi^T \xi = y \end{array} \qquad (P_0)$$

where, again, $\pi = (P, p)$ and $\xi = (X, \sum_h x_h)$.

Second, if μ^h denotes the Lagrange multiplier of the hth constraint, the axiom can be restated as follows: there exists $H - 1$ scalar functions $\mu^h (\pi, y) \geq 0, \; 2 \leq h \leq H$ such that $(X, x_1, ..., x_H)$ solves:

$$\begin{array}{c} \max_{X,x_1,...,x_H} \sum_h \mu^h U^h (X, x_1, ..., x_H) \quad \text{subject to} \\ \pi^T \xi = y \end{array} \qquad (P_1)$$

where $\mu^1 = 1$. The equivalence between efficiency and the maximization of a weighted sum of utilities is well known; the μ^h are the *Pareto weights* of the program. Clearly, Pareto weights are defined only up to some normalization. In program P_1, the first weight is normalized to be 1. Clearly, other normalizations are possible, since the maximization problem does not change when the maximand is multiplied by an arbitrary, non negative scalar function. For instance, one may define $\bar{\mu}^h = \mu^h / (\sum_r \mu^r)$; then $\sum \bar{\mu}^h = 1$.

A more geometric interpretation is the following. For any given utility functions U^1,...,U^H and any price-income bundle, the budget constraint defines a Pareto set for the group; under the assumptions stated (concave utilities, convex production set) the Pareto set is moreover convex. From the Efficiency Axiom, the final outcome will be located on the frontier of the Pareto set. Under standard smoothness assumptions, this frontier is a $(H - 1)$-dimensional manifold, indexed by the vector $\mu = (1, \mu^2, ..., \mu^H)$.

An important remark is that the vector μ, normalized for instance by $\sum \mu_h = 1$, *summarizes the decision process,* since it determines the final location of the demand vector on this frontier. In that sense, it describes the distribution of power within the group. If one of the weights, μ^h, is equal to one for every (π, y), then the group behaves as though h is the effective dictator. For intermediate values, the group behaves as though each person h has some decision power, and the person's weight μ^h can be seen as an indicator of this power. This 'power' interpretation must be used with some care, since the Pareto coefficient μ^h depend on the particular cardinalization adopted for individual preferences; if U^h is replaced with $G(U^h)$ for some increasing mapping G, the set of Pareto efficient allocation does not change, but the parametrization through the vector μ has to be modified accordingly. It follows that interpersonal comparisons of Pareto weights are meaningless; for instance, the fact that $\mu^h > \mu^r$ does *not* imply that 'h has more power than r'. However, the *variations of* μ^h are significant, in the sense that for any fixed cardinalization, a policy change that increases μ^h while leaving μ^r constant unambiguously ameliorates the position of h relative to r.

If the μ^h are constant, then Program (P) boils down to the maximization of a unique utility under production and budget constraint. We then get a variant of the Samuelson index model, and the group behaves as if it was a single decision maker. In general, however, the weights μ^h depend on prices and income, since these variables may in principle influence the distribution of 'power' within the group, hence the location of the final choice over the Pareto frontier. The maximand in P is therefore price-dependent; the standard properties of unitary models do not apply in this context. However, the dependence on prices and income has a specific form, which will be exploited in what follows.

Three additional remarks can be made. First, since we postulate throughout the absence of monetary illusion: the μ^h are taken to be zero-homogeneous in (π, y). Second, following Browning and Chiappori (1998), we often add some structure by assuming that the μ^h are continuously differentiable. Third, if we assume that all commodities are privately consumed and there are no externalities, then by the second welfare theorem any Pareto efficient allocation can be decentralized as an equilibrium - and we are back to the framework studied in Section 2. Indeed, the market economy approach is a (very) special case of the collective model.[10]

4.2 Aggregate demand of an efficient group: a characterization

The characterization problem can be stated as follows. Take a group that satisfy the assumptions made above, and which make Pareto efficient decisions under the constraints defined by its production technology and its budget. What restrictions (if any) on the aggregate demand function characterize the efficient behavior of the group, and how do these restrictions vary with the size of the group? In other words, is it possible to derive conditions that are sufficient for some demand function to stem from a Pareto efficient decision process within a 'well behaved' group? Technically, consider a demand function $\xi = (X, x)$ of $(\pi, y) = (P, p, y)$

[10] The collective approach also encompasses several models of household behavior that have been developed in the literature, including models based on cooperative bargaining (Manser and Brown 1980, McElroy and Horney 1981) or on equilibrium (Grossbard-Schechtman and Neuman, 2003).

that satisfies two standard conditions, namely homogeneity and adding up (i.e. $\pi^T \xi = y$ for all π, y), and that is sufficiently 'smooth' in a sense that will be defined later. Are there necessary and sufficient conditions on ξ that stems from the theoretical structure under consideration - i.e., from the fact that it is the Pareto efficient demand of a H-person group?

4.2.1 The SNR$(H-1)$ condition

We start with a set of necessary conditions that characterize group demand in the most general framework. In what follows, utilities are of the unrestricted form $U^h (X, x_1, ..., x_H)$ - we simply assume that U^h is increasing and strongly concave; moreover, intragroup production could be introduced at no cost. We maintain the homogeneity assumption; therefore we normalize y to be 1. The budget constraint is:

$$\pi^T \xi = 1$$

and aggregate demand is now a function $\xi(\pi)$ of prices only.

Household utility As discussed above, Pareto efficiency requires that the group demand solves the program (P_1) above. We define the function $\mathbb{U}^H$, from $\mathbb{R}^M \times \mathbb{S}$ to $\mathbb{R}$, where $\mathbb{S}$ denotes the H-dimensional simplex, by

$$\begin{array}{c}\mathbb{U}^H (\xi, \mu) = \mathbb{U}^H \left(X, x, \mu^1, ..., \mu^H\right) = \max_{X, x_1, ..., x_H} \sum_h \mu^h U^h (X, x_1, ..., x_H) \\ \text{subject to } x = x_1 + ... + x_H\end{array} \tag{21}$$

In words, $\mathbb{U}^H$ denotes the maximum value of the weighted sum $\sum_h \mu^h U^h$ when aggregate group demand is ξ; in that sense, $\mathbb{U}^H$ can be interpreted as the group's utility function, and (P_1) is equivalent to maximizing U^H under the budget constraint:

$$\begin{array}{c}\max \mathbb{U}^H (\xi, \mu) \\ \text{subject to } \pi^T \xi = 1\end{array} \tag{22}$$

In what follows, let $\tilde{\xi}(\pi, \mu)$ denote the solution to (22).

It is crucial to remark that $\mathbb{U}^H$ also depends on the vector of Pareto weights $\mu = \left(\mu^1, ..., \mu^H\right) \in \mathbb{S}$. In particular, $\mathbb{U}^H$ is *not* a standard utility function: since the μ^h are generally price- and income-dependent, so is $\mathbb{U}^H$. In practice, $\tilde{\xi}$, considered as a function of π only (for some *fixed* μ), is a standard demand function; as such, it satisfies Slutsky symmetry and negativeness. However, $\tilde{\xi}$ is *not* observable, because one cannot vary π while keeping μ constant. What the econometrician observes (or may recover), i.e. the demand function ξ, is related to $\tilde{\xi}$ by:

$$\xi(\pi) = \tilde{\xi}(\pi, \mu(\pi)) \tag{23}$$

Slutsky matrix We now define the Slutsky matrix associated to ξ by by

$$S(\pi) = (D_{\boldsymbol{\pi}}\xi)\left(I - \pi\xi^T\right)$$

This is the standard definition of a Slutsky matrix, adapted to take into account the normalization $y = 1$.[11] Note, incidentally, that $S(\pi)v = 0$ for all vectors $v \in \text{Span}\{\pi\}$; indeed,

$$S(\pi)\pi = (D_{\boldsymbol{\pi}}\xi)\left(\pi - \pi\xi^T\pi\right) = 0 \text{ since } \xi^T\pi = 1$$

Now, from (23), we see that:

$$\begin{aligned} S(\pi) &= \left(D_\pi\tilde{\xi} + D_\mu\tilde{\xi}.D_\pi\mu^T\right)\left(I - \pi\xi^T\right) \\ &= \left(D_\pi\tilde{\xi}\right)\left(I - \pi\xi^T\right) + D_\mu\tilde{\xi}.D_\pi\mu^T\left(I - \pi\xi^T\right) \\ &= \Sigma(\pi) + R(\pi) \end{aligned}$$

where

$$\begin{aligned} \Sigma(\pi) &= \left(D_\pi\tilde{\xi}\right)\left(I - \pi\xi^T\right) = \left(D_\pi\tilde{\xi}\right)\left(I - \pi\tilde{\xi}^T\right) \quad \text{and} \\ R(\pi) &= D_\mu\tilde{\xi}.D_\pi\mu^T\left(I - \pi\xi^T\right) \end{aligned}$$

$\Sigma(\pi)$ is the Slustky matrix corresponding to the function $\tilde{\xi}(.,\mu)$, as computed at $\mu(\pi)$; as such, it is symmetric, negative semi-definite and satisfies $v^T\Sigma(\pi)v = 0$ for all vectors $v \in \text{Span}\{\pi\}$. Moreover, the rank of $R(\pi)$ cannot exceed that of $(D_\pi\mu)$, which is at most $H-1$. We can therefore state the basic result from Browning-Chiappori 1998:

Proposition 2 *(**The SNR**$(H-1)$ **condition**). If the C^1 function $\xi(\pi)$ solves problem (P), then the Slutsky matrix $S(\pi) = (D_{\boldsymbol{\pi}}\xi)\left(I - \pi\xi^T\right)$ can be decomposed as:*

$$S(\pi) = \Sigma(\pi) + R(\pi) \tag{24}$$

where:

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[11] Homogeneity implies by Euler relation that

$$D_\pi\xi.\pi + yD_y\xi = 0$$

The Slustky matrix is defined as

$$S(\pi, y) = D_\pi\xi + D_y\xi.\xi'$$

therefore

$$S(\pi, y) = D_\pi\xi + \left(-\frac{1}{y}D_\pi\xi.\pi\right).\xi'$$

and for $y = 1$ the result obtains.

- *the matrix* $\Sigma(\pi)$ *is symmetric and satisfies* $v^T\Sigma(\pi)v = 0$ *for all vectors* $v \in \mathrm{Span}\{\pi\}$, $v^T\Sigma(\pi)v < 0$ *for all vectors* $v \notin \mathrm{Span}\{\pi\}$
- *the matrix* $R(\pi)$ *is of rank at most* $H-1$.

Equivalently, there exists a subspace $\mathcal{E}(\pi)$ *of dimension at least* $M-H$ *such that the restriction of* $S(\pi)$ *to* $\mathcal{E}(\pi)$ *is symmetric, definite negative, in the sense that* $v^TS(\pi)w = w^TS(\pi)v$ *and* $v^TS(\pi)v < 0$ *for all vectors* $v, w \in \mathcal{E}(\pi)$.

Here, SNR($H-1$) stands for 'Symmetric Negative plus Rank ($H-1$)'. As discussed above, a very appealing property of these conditions is that they stem from the most general version of the collective model; i.e., they do not require much beyond efficiency and differentiability. Also, note that the SNR($H-1$) property nicely generalizes the standard Slutsky symmetry of the unitary model. Indeed, when $H=1$ (the unitary setting), then $R(\pi)$ is the null matrix and $S(\pi) = \Sigma(\pi)$ is symmetric. In the general case where $H \geq 1$, then $S(\pi)$ needs not be symmetric, and $R(\pi)$ represents the 'deviation from symmetry'; then the rank of this deviation is at most the number of members minus one.

On a more technical side, note that the $(H \times M)$ matrix $D_\pi\mu^T\left(I - \pi\xi^T\right)$ can be written as:

$$D_\pi\mu^T.\left(I - \pi\xi^T\right) = \begin{pmatrix} v_1^T \\ v_2^T \\ \vdots \\ v_H^T \end{pmatrix}$$

where the vectors $v_1, ..., v_H \in \mathbb{R}^N$ are linearly dependent.[12] It follows that:

$$\begin{aligned} R(\pi) &= D_\mu\tilde{\xi}.\begin{pmatrix} v_1^T \\ v_2^T \\ \vdots \\ v_H^T \end{pmatrix} \\ &= \sum_h D_{\mu^h}\tilde{\xi}.v_h^T = \sum_h u_h.v_h^T \end{aligned}$$

where

$$u_h = D_{\mu^h}\tilde{\xi}.$$

[12] Obviously, the vectors v_h vary with π, and should be written $v_h(\pi)$. To simplify notations, we shall omit the reference to π whenever it can be done without ambiguity.

Also,

$$R(\pi).\pi = D_{\mu}\tilde{\xi}.D_{\pi}\mu^{T}\left(I - \pi\xi^{T}\right).\pi = 0$$

since $\left(I - \pi\xi^{T}\right).\pi = \pi - \pi\left(\xi^{T}\pi\right) = \pi - \pi = 0$. Therefore, $v_h^T.\pi = 0$ for $h = 1, ..., H$. By the same token,

$$S(\pi).\pi = \Sigma(\pi).\pi = 0$$

Let $\mathcal{E}(\pi)$ denote the subspace orthogonal to $\{\pi, v_1, ..., v_H\}$; its dimension is at least $M - H$. Then the space $\mathbb{R}^M$ can be decomposed as:

$$\mathbb{R}^M = \text{Span}\{\pi\} \oplus \text{Span}\{v_1, ..., v_H\} \oplus \mathcal{E}(\pi)$$

and we know that for any two vectors v, w in $\mathcal{E}(\pi)$:

$$\begin{aligned} & v^T S(\pi) w = v^T \Sigma(\pi) w = w^T \Sigma(\pi) v = w^T S(\pi) v \\ \text{and } & v^T S(\pi) v = v^T \Sigma(\pi) v < 0 \end{aligned}$$

which shows that the restriction of S to $\mathcal{E}(\pi)$ is symmetric, definite negative, as stated in the last part of Proposition 2.

Geometric interpretation A geometric interpretation of SNR($H-1$) is the following. Remember, first, that for any given H-uple of utilities, the budget constraint defines the Pareto frontier as a function of the price-income bundle; then μ determines the location of the final outcome on the frontier. Under smoothness assumptions, the Pareto frontier is actually a manifold of dimension $H-1$. Assume, now, that prices and income are changed. This has two consequences. For one thing, the Pareto frontier will move. Keeping μ constant, this would change demand in a way described by the Σ matrix. Note, however, that this change will *not* violate Slutsky symmetry; that is, it is not different from the traditional, unitary effect. The second effect is that μ will also change; this will introduce an additional move of demand *along* the (new) frontier. This change (as summarized by the R matrix) *does* violate Slutsky symmetry. But moves along a $(H-1)$-dimensional manifold are quite restricted. For instance, the set of price-income bundles that lead to the *same* μ is likely to be quite large in general; indeed, under our smoothness assumption, it is a $(M-H-1)$-dimensional manifold. Considering the linear tangent hyperspace, this means that there is a whole linear manifold of codimension $(H-1)$ such that, if the (infinitesimal) change in prices and income belongs to that hyperplane, then no deviation from Slutsky symmetry can be observed. In other words, *the SNR($H-1$) condition is a direct consequence of the fact that, in a H-person household, the Pareto frontier is of dimension $H-1$, whatever the number of commodities.*

Testing for SNR(H-1) How can a property like SNR$(H-1)$ be tested ? The basic idea is that a matrix S is SNR$(H-1)$ if and only if the antisymmetric matrix $\mathcal{M} = S - S^T$ is of rank at most $2(H-1)$ (remember that a matrix $\mathcal{M}$ is antisymmetric if $\mathcal{M}^T = -\mathcal{M}$). A more precise statement is the following :

Lemma 3 *Let S be some SNR$(H-1)$ matrix. :*

$$S = \Sigma + \sum_{h=1}^{H-1} u_h . v_h^T$$

where the vectors $(u_1, ..., u_H)$ are linearly dependent and the vectors $(v_1, ..., v_H)$ are linearly dependent. Then the matrix $\mathcal{M} = S - S^T$ is of rank at most $2(H-1)$, and $\text{Im}(\mathcal{M})$ *(the subspace spanned by the columns of $\mathcal{M}$) is spanned by the vectors $(u_1, ..., u_H, v_1, ..., v_H)$.*

Therefore, testing for the collective model amounts to testing for the rank of matrix $\mathcal{M} = (S - S^T)$. The collective model predicts this rank should be at most $2(H-1)$, while it would be zero in the unitary case (note that antisymmetry implies that the rank of $\mathcal{M}$ must be an even integer).

Several tests of SNR$(H-1)$ have been empirically performed (Browning and Chiappori 1998; Dauphin and Fortin, 2001; Dauphin 2003; Dauphin et al. 2008; Kapan 2009). They conclude that standard symmetry of the Slutsky matrix is strongly rejected for multi person families, although quite interestingly it fails to be rejected for singles.; moreover, SNR(1) is not rejected for couples. Finally, one can use these approaches to assess the number of actual decision makers in the family (see Dauphin et al. 2008 and Kapan 2009).

4.2.2 Sufficiency of the SNR$(H-1)$ condition

The conditions SNR(H-1) has been known to be necessary for some time. A more difficult question is sufficiency. Take a 'smooth' demand function $\xi(\pi)$ that satisfies homogeneity, adding-up and SNR(H-1). Can it be constructed as the aggregate demand of a Pareto-efficient group? Formally, thus, the sufficiency problem can be stated as follows: *Is it possible to find (i) functions $(x_1(\pi), ..., x_H(\pi), X(\pi))$, (ii) increasing, concave utility functions, $U^1(x_1, ..., x_H, X), ..., U^H(x_1, ..., x_H, X)$, (iii) a production function f, and (iv) a vector function $\mu(\pi)$ in the $H-$dimensional simplex, such that $(\xi(\pi), x_1(\pi), ..., x_H(\pi), X(\pi))$ solves program (P_1)?*

In other words, we are looking for an equivalent, in the collective setting, to the *integrability* theorem in the unitary case, whereby Slutsky conditions (with homogeneity and adding up) are sufficient for the existence of a well-behaved utility function generating the demand function under consideration.

We start with a simple methodological point, namely that in order to prove sufficiency, one only has to prove the existence of just *one* set of utility and production functions, however simple. In particular, it suffices to prove sufficiency for the set of egoistic preferences of the form $U^h(x_h, X)$.

As it turns out, any demand that is (locally) compatible with the collective approach is compatible with the collective approach with egoistic preferences. Two caveats must however be made. First, the proof require some degree of 'smoothness' of the demand; in practice, we shall assume that the function ξ is continuously differentiable. Secondly, the construction of individual utilities and Pareto weights is only local; i.e., we prove sufficiency in an open neighborhood of any 'regular' point (in a sense that will be precisely defined). The global construction is still an open problem.

Our first task is to describe the basic mathematical structure of the identification problem. We start with introductory examples which show how the structure obtains in two specific but intuitive cases - namely, commodities are either all public or all private. We then address the general setting.

Two introductory examples We now describe the mathematical structure of the problem in more detail. Our main conclusion will be that some known function (aggregate demand, aggregate inverse demand or a function derived from these) must be written as a convex combination of gradients. In other words, the key structure is the same as for the aggregate excess or market demand of a market economy, as discussed in the previous sections - despite the fact that the model is much more general.

We start with two simple examples that illustrate the main result in an intuitive way. For expository convenience, we disregard distribution factors for the moment.

Public goods only We first consider a version of the model in which all commodities are publicly consumed (therefore $\xi = X$ and $\pi = P$). Keeping the normalization $y = 1$, program (P_1) above can be written as:

$$\begin{cases} \max_X \sum_h \mu^h(P)\, U^h(X) \\ P^T X = 1 \end{cases} \tag{25}$$

Let $X^*(P)$ denote its solution. Assuming an interior solution, first order conditions give:

$$\sum_h \mu^h(P) D_X U^h(X) = \lambda(P).P \tag{26}$$

where λ denotes the Lagrange multiplier of the budget constraint; note that λ is a scalar function of P.

Next, we assume that *the Jacobian matrix $D_P X$ is of full rank* on some open set. It follows that the function $X(P)$ is invertible, and we can define the inverse demand function $P(X)$. Then (26) becomes:

$$\sum_h \frac{\mu^h(P(X))}{\lambda(P(X))} D_X U^h(X) = P(X)$$

In this equation, the right hand side is the known (inverse) demand function, while all functions in the left hand side are unknown - and we want to prove their existence. The specific structure, here, is that the *inverse demand function must be a linear combination of gradients of increasing, concave functions; moreover, the coefficients of the combination must be nonnegative.* Note that when $H = 1$, this equation boils down to a well known result, namely that the inverse demand function stemming from the maximization of a unique utility under budget constraint must be proportional to the gradient of the utility function.

Finally, assume, conversely, that some given, C^1 demand $X(P)$, satisfying $P^T.X(P) = 1$, is regular in the sense just defined in some neighborhood and such that the inverse demand $P(X)$ can be written as

$$P(X) = \sum_h \bar{\mu}^h(X) D_X U^h(X)$$

where the $\bar{\mu}^h$ are positive and the U^h are increasing and strongly concave. Define $\tilde{\mu}^h(P) = \bar{\mu}^h(X(P))$ for $h = 1, ..., H$, and consider the program:

$$\begin{cases} \max_X \sum_h^h \tilde{\mu}(P) U^h(X) \\ \qquad P^T X = 1 \end{cases} \tag{27}$$

Since the maximand is strongly concave, the first order conditions are sufficient for a global optimum; hence $X(P)$ is the aggregate demand of the group thus defined.

Private goods only The previous argument may seem specific to the public good structure in which it was constructed. As it turns out, the underlying intuition is more general. To see why, let us briefly discuss an alternative polar case in which all commodities are privately consumed and individual utilities belong to the egoistic family. This case that has been repeatedly studied in the literature, starting with Chiappori (1988, 1992). Now the demand function $\xi(\pi)$ is in fact $x(p)$; the program is therefore:

$$\begin{array}{c} \max_{x_1,\ldots,x_H} \sum_h \mu^h U^h(x_h) \quad \text{subject to} \\ p^T\left(\sum_h x_h\right) = 1 \end{array} \tag{28}$$

where y has again been normalized to 1. Let $(x_1^*, \ldots, x_H^*)$ denote the solution to this program.

The notion of *sharing rule* provides an equivalent but often more tractable version of this program. It relies on the following result:

Proposition 4 *There exists H scalar functions $\rho^1, \ldots, \rho^H$ of p, with $\sum_h \rho^h(p) = 1$, such that for any $h = 1, \ldots, H$, x_h^* solves*

$$\begin{array}{c} \max_{x_h} U^h(x_h) \quad \textit{subject to} \\ p^T x_h = \rho^h(p) \end{array} \tag{P_h}$$

Proof. *Define $\rho^h = p^T x_h^*$, and assume that x_h^* does not solve (P_h). Then there exists some $\bar{x}_h$ such that $p^T \bar{x}_h = p^T x_h^*$ and $U^h(\bar{x}_h) > U^h(x_h^*)$. But then the allocation $(x_1^*, \ldots, \bar{x}_h, \ldots, x_H^*)$ is feasible and Pareto dominates $(x_1^*, \ldots, x_H^*)$, a contradiction.* ■

This is just a particular application of the second welfare theorem. Consider the group as a small, convex economy, in which all commodities $1, \ldots, N$ can be produced from a single input, money, according to the linear production technology $p^T\left(\sum_h x_h\right) = 1$. Then any Pareto efficient allocation can be decentralized as an equilibrium; moreover, the linear technology requires that the prices within the economy be proportional to market prices p, hence the result.

In other words, when commodities are all private, an efficient allocation can always be seen as stemming from a two stage decision process.[13] At stage 1, members decide on the allocation of total income $y = 1$ between the members; member h receives ρ^h. At stage 2, agents each chose their vector of private consumption subject to their own budget constraint.

The vector $\left(\rho^1, \ldots, \rho^H\right)$ is the group's *sharing rule*. In a private good context, the intragroup decision process is fully summarized by the sharing rule; in particular, there is a one-to-one mapping between

[13] Needless to say, we are not assuming that the actual decision process is in two stages. The result simply states that any efficient group behaves *as if* it was following a process of this type.

(normalized) Pareto weights and the sharing rule. This mapping is moreover monotonic in the following sense: if we increase the Pareto weight of member i while keeping the other weights constant (possibly before renormalization), then the new sharing rule allocates more resources to i than the initial one. A nice property of the sharing rule is that it does not depend on the particular cardinalization of individual utilities (it is expressed 'in dollars'). The price to pay for this superior tractability is that sharing rules are less general, being defined for private goods only - although we shall extend the concept to a more general setting below.

Let $W^h(p)$ denote the value of program (P_h): it is called the *collective indirect utility*. It is defined as the utility reached by agent h, taking into account the intragroup decision process. If V^h denote the standard, 'individual' indirect utility of member h, we have that

$$W^h(p) = V^h\left(p, \rho^h(p)\right) \tag{29}$$

By the envelope theorem applied to program (P_h):

$$D_p W^h = \lambda^h \left(x_h - D_p \rho^h\right)$$

where λ^h is the Lagrange multiplier of the budget constraint, i.e. the marginal utility of money of h. Therefore:

$$\sum_h \frac{D_p W^h}{\lambda^h} = \sum_h \left(x_h - D_p \rho^h\right)$$

hence

$$\sum_h \frac{D_p W^h}{\lambda^h} = x(p)$$

since $\sum_h \rho^h(p) = 1$ implies $\sum_h D_p \rho^h(p) = 0$.

We can rewrite this equation in a slightly different way. Define $\tilde{W}(p) = -W(p)$; if $\alpha^h = 1/\lambda^h$, we have that:

$$-x(p) = \sum_h \alpha^h D_p^h \tilde{W}^h \tag{30}$$

This time, it is the direct group demand function which is equal to a linear combination of gradients. Note that when $H = 1$, this equation boils down to the well known Roy's identity, which states that a demand function stemming from the maximization of a unique utility under budget constraint must be proportional to the gradient of the indirect utility (indeed, when $H = 1$ then $\rho(p) = 1$ and W is the standard indirect utility).

Conversely, assume that some smooth function $x(p)$, satisfying the Walras Law, also satisfies (30) in the neighborhood of some $\bar{p}$ for some positive α^h and some strictly decreasing, strongly convex $\tilde{W}^h$ (so that the W^h are strictly increasing and strongly concave). We now show that x can be decomposed as the aggregate demand of a group in which all commodities are privately consumed.

For each h, define a function $\rho^h(p)$ by

$$\rho^h(p) = p^T.\left[D_p\rho^h - \alpha^h(p)\, D_p\tilde{W}^h\right] \tag{31}$$

This is a linear first-order partial differential equation for $\rho^h(p)$. Note that the sum $\rho(p) = \sum \rho^h(p)$ satisfies a similar equation:

$$\rho = p^T D_p\rho + p^T x = p^T D_p\rho + 1 \tag{32}$$

which has the obvious solution $\rho(p) = 1$.

Equation (31) can be solved by the method of characteristics[14]. It follows that $\rho^h(p)$ can be prescribed arbitrarily on the affine hyperplane H defined as the set of p where $\bar{p}^T(p-\bar{p}) = 0$ (technically speaking, this is a non-characteristic hypersurface, at least in some neighborhood of $\bar{p}$). We choose $\rho^h(p) = 1/S$ on H. It follows that $\rho = \sum \rho^h = 1$ on H, and since ρ satisfies equation (32), it follows that $\sum \rho^h(p) = 1$ everywhere. As a consequence, we have:

$$\sum D_p\rho^h = 0$$

Now define:

$$x_h(p) = D_p\rho^h - \alpha^h(p)\, D_p\tilde{W}^h \tag{33}$$

We have:

$$p^T x_h(p) = \rho^h(p) \quad 1 \le h \le S$$
$$\sum_h x_h(p) = x(p)$$

[14] In the case at hand, the method of characteristics consists in considering the flow:

$$\frac{dp}{dt} = p$$

in R^N, the solutions of which are given by $p(t) = p(0)\,e^t$, and to note that the function $\bar{\rho}^h(t) := \rho^h(p(t))$ solves the differential equation

$$\bar{\rho}^h(t) = \frac{d\bar{\rho}^h}{dt}(t) - \alpha_s(p(t))\;\; p(t)^T.D_p\tilde{W}^h(p(t))$$

on R. This determines the solution $\bar{\rho}^h(p)$ on each trajectory of the flow. See for instance Bryant et al (1991) for details.

We now have to show that the $x_h(p)$ solve the consumer's problem. For each h, consider the function:

$$U^h(x) = \min_p \left\{ \tilde{W}^h(p) \mid p^T x \le \rho^h(p) \right\} \tag{34}$$

Note that, by the envelope theorem, U^h is differentiable and strictly increasing, and $D_x U^h(x_h(p))$ is proportional to p. But equation (33) is the optimality condition for this problem. Since $\tilde{W}^h$ is strongly convex, this condition is sufficient, so that:

$$U^h(x_h(p)) = \tilde{W}^h(p) \tag{35}$$

Now set:

$$\bar{W}^h(p) = \sup_x \left\{ U^h(x) \mid p^T x \le \rho^h(p) \right\} \tag{36}$$

We have $\bar{W}^h(p) \ge U^h(x_h(p)) = \tilde{W}^h(p)$. On the other hand, for every x such that $p^T x \le \rho^h(p)$, we have $U^h(x) \le \tilde{W}^h(p)$. Taking the supremum with respect to all such x, we get $\bar{W}^h(p) \le \tilde{W}^h(p)$. Finally $\bar{W}^h = \tilde{W}^h$, and equation (36) becomes:

$$\tilde{W}^h(p) = \max_x \left\{ U^h(x) \mid p^T x \le \rho^h(p) \right\} = U^h(x_h(p))$$

which tells us that $x_h(p)$ solves the consumer's problem for the utilities $U^h(x)$ and the sharing rule $\rho^h(p)$.

It remains to show that the U^h are quasi-concave, at least in some neighborhood of $\bar{p}$. To do this, pick x_1 and x_2 and a number a such that $U^h(x_1) \ge a$ and $U^h(x_2) \ge a$. We have:

$$U^h\left(\frac{x_1 + x_2}{2}\right) = \min_p \left\{ \tilde{W}^h(p) \mid p^T \left(\frac{x_1 + x_2}{2}\right) \le \rho^h(p) \right\}$$

Now, if $\frac{1}{2} p^T x_1 + \frac{1}{2} p^T x_2 \le \rho^h(p)$, then we must have $p^T x_i \le \rho^h(p)$ for $i = 1$ or $i = 2$. Hence:

$$\left\{ p \mid p^T \left(\frac{x_1 + x_2}{2}\right) \le \rho^h(p) \right\} \subset \left\{ p \mid p^T x_i \le \rho^h(p) \right\} \cup \left\{ p \mid p^T x_2 \le \rho^h(p) \right\}$$

$$U^h\left(\frac{x_1 + x_2}{2}\right) \ge \min_{i=1,2} \left\{ \tilde{W}^h(p) \mid p^T x_i \le \rho^h(p) \right\} = \min_{i=1,2} U^h(x_i) = a$$

So the U^h are differentiable and quasi-concave. It is well known that the same preferences can be represented by concave functions, which concludes the proof.

The general case In the two polar examples just considered - all goods are privately consumed, and all goods are publicly consumed - the sufficiency problem can thus be reformulated as follows: *when can a given map from $\mathbb{R}^N$ to $\mathbb{R}^N$ be written as a linear combination of H gradients of increasing, strongly concave functions from $\mathbb{R}^N$ to $\mathbb{R}$?* Specifically, we have seen that in both cases, this condition was necessary; and that, furthermore, the condition was also sufficient, in the sense that whenever it was fulfilled one could construct a group for which the function at stake was indeed the aggregate demand.

We now show that this gradient structure is in fact general, and that it fully characterizes the collective conditions. As explained above, it is sufficient to consider egoistic preferences without intragroup production. Therefore, we study the program:

$$\begin{cases} \max_{x_1,\dots x_H,X} \sum \mu^h(p,P)\,U^h(x_h,X) \\ p^T(x_1+\dots+x_H)+P^TX=1 \end{cases} \tag{P'}$$

Let $x_1(p,P),\dots,x_H(p,P),X(p,P)$ denote its solution. The household demand function is then $\xi(p,P)=(x(p,P),X(p,P))$ where $x=\sum_h x_h$.

In what follows, we repeatedly use the duality between private and public consumption, a standard tool in public economics. Assuming that the Jacobian matrix D_PX is of full rank, we consider the following change in variables:

$$\begin{aligned} \psi:\mathbb{R}^N &\to \mathbb{R}^N \\ (p,P) &\to (p,X) \end{aligned} \tag{37}$$

The economic motivation for such a change in variables is clear. A basic insight underlying the duality between private and public goods is that, broadly speaking, quantities play for public goods the role of prices for private goods and conversely. Intuitively: in the case of private goods, all agents face the same price but consume different quantities, which add up to the group's demand; with public goods, agents consume the same quantity, but face different (Lindahl) prices, which add up to the market price if the allocation is efficient. This suggest that whenever the direct demand function $x(p)$ is a relevant concept for private consumption, then the inverse demand function $P(X)$ should be used for public goods. The change of variable ψ allows to implement this intuition.

In particular, instead of considering the demand function (x,X) as a function of (p,P), we shall often consider (x,P) as a function of (p,X) (then the public prices P are implicitly determined by the condition that demand for public goods must be equal to X while private prices are equal to p). While these two viewpoints are clearly equivalent (one can switch form the first to the second and back using the change ψ), the computations are much easier (and more natural) in the second setting.

Conditional sharing rule It is convenient, at this point, to introduce the notion of a *conditional sharing rule*, which directly generalizes the sharing rule introduced earlier in the case of private goods. It stems from the following result:

Lemma 5 *For any given (p, P), let $\left(\bar{x}_1, ..., \bar{x}_H, \bar{X}\right)$ denote a solution to (P'). Define $\rho^h = p^T \bar{x}_h$ for $h = 1, ..., H$. Then for $h = 1, ..., H$, $\bar{x}_h$ solves*

$$\begin{array}{c} \max_{x_h} U^h(x_h, X) \\ p^T x_h \leq \rho^h \end{array} \qquad (P_s)$$

Proof. *Assume not, then there exists some $\tilde{x}_h$ such that $p^T \tilde{x}_h \leq \rho^h$ and $U^h(\tilde{x}_h, X) > U^h(\bar{x}_h, X)$. But then the allocation $\left(\bar{x}_1, ..., \tilde{x}_h, ...\bar{x}_H, \bar{X}\right)$ is feasible and Pareto dominates $\left(\bar{x}_1, ..., \bar{x}_H, \bar{X}\right)$, a contradiction.* ■

In words, an efficient allocation can be seen as stemming from a two stage decision process. At stage 1, members decide on the public purchases X, and on the allocation of the remaining income $y - P^T X$ between the members; member h receives ρ^h. At stage 2, agents each chose their vector of private consumption, subject to their own budget constraint and taking the level of public consumption as given. The vector $\rho = (\rho_1, ..., \rho_H)$ is the conditional sharing rule; it generalizes the notion of sharing rule developed in collective models with private goods only because it is defined conditionally on the level of public consumption previously chosen. Of course, if all commodities are private ($K = 0$) then the conditional sharing rule boils down to the previous notion. In all cases, the conditional sharing rules satisfy the budget constraint

$$\sum_h \rho^h = 1 - P^T X. \qquad (38)$$

As above, the conditional sharing rule can be expressed either as a function of (p, P) or, using the change in variable ψ, as a function of (p, X). We define the conditional indirect utility of member h as the value of program (P_h); hence

$$V^h(p, X, \rho) = \max \left\{ U^h(x_h, X) \text{ subject to } p^T x_h = \rho \right\} \qquad (39)$$

which can be interpreted as the utility reached by member h when consuming X and being allocated an amount ρ for her private expenditures. Obviously, V^h is zero homogeneous in (p, ρ).

Collective indirect utility Following Chiappori (2006), we introduce the following, key definition, which again generalizes that introduced in the private good case:

Definition 6 *The* collective indirect utility *of agent h is defined by:*

$$W^h(p,X) = V^h\left(p, X, \rho^h(p,X)\right)$$

In words, W^h denotes the utility level reached by agent h, at prices p and with total income y, in an efficient allocation such that the household demand for public goods is X, taking into account the conditional sharing rule at stake. Note that W^h depends not only on the preferences of agent h (through the conditional indirect utility V^h) but also on the decision process (through the conditional sharing rule ρ^h). Hence W^h summarizes the impact on h of the interactions taking place within the group. As such, it is the main concept required for welfare analysis: *knowing the W^h allows to assess the impact of any reform (i.e. any change in prices and incomes) on the welfare of each group member.* Also, note that in the case of public consumption only, W^h is simply equal to the direct utility U^h. Finally, remember that we are using the normalization $y = 1$. Without it, W^h would be a function of (p, X, y).

One can then prove (Ekeland and Chiappori 2009a) the following result: there exists scalar functions $\left(\gamma^1, ..., \gamma^h\right)$ such that

$$\begin{array}{c} \sum_h \gamma^h D_p W^h = -x - D_p A \\ \sum_h \gamma^h D_X W^h = P - D_X A \end{array} \tag{40}$$

where $A(p,X) = P(p,X)^T .X$ denote the group's total expenditures on public goods. We thus see that in the general case under consideration, the sufficiency problem can be expressed as follows: *find a family of differentiable functions $W^h(p,X)$ on $\mathbb{R}^N$, each defined up to some increasing transform, such that the vector $\begin{pmatrix} -x - D_p A \\ P - D_X A \end{pmatrix}$ can be expressed as a linear combination of the gradients of the W^h.*

The main result We can now state the main result:

Theorem 7 *Suppose a positive, C^1 function $\xi(\pi)$ satisfies the Walras law $\pi^T \xi(\pi) = 1$ and condition SR($H-1$) in some neighborhood of $\bar{\pi}$:*

$$S(\pi) = (D_{\boldsymbol{\pi}} \boldsymbol{\xi}) \left(I - \pi \xi^T\right) = \Sigma(\pi) + \sum_{h=1}^{H-1} a_h(\pi)\, b_h^T(\pi) \tag{41}$$

where $\Sigma(\pi)$ is symmetric, negative, and the vectors $\xi(\pi)$, $a_h(\pi)$ and $b_h(\pi)$ are linearly independent. Then there are positive functions $\lambda_h(\pi)$

and increasing, strongly concave functions $V^h(\pi), 1 \leq h \leq H$, *both defined on some neighborhood* $\mathcal{N}$ *of* $\bar{\pi}$, *such that the decomposition:*

$$\xi(\pi) = \sum_{h=1}^{H} \lambda_h(\pi) D_\pi V^h(\pi) \tag{42}$$

holds true on $\mathcal{N}$.

Proof. *See Chiappori and Ekeland 2009b.* ■

In words: the SNR$(H-1)$ is a necessary and sufficient characterization of the aggregate demand of an efficient group. Some remarks are in order at that point:

- SNR$(H-1)$ remains necessary and sufficient even when one assumes either that all goods are publicly consumed or that all goods are privately consumed. In other words, the private versus public nature of intragroup consumption is not testable without additional assumptions.

- the SNR$(H-1)$ condition is restrictive if and only if the number of commodities is larger than the number of agents. Indeed, in the opposite case one can always write the decomposition (24) with $\Sigma(\pi) = 0$ and $R(\pi) = S(\pi)$. Quite interestingly, we confirm in this general framework an intuition already generated in the very specific case of a market economy - namely, that the individualistic foundations of the model induce some structure on the group's aggregate demand if and only if the group is small enough - technically, has less agents than commodities.

- Note, however, that the key ingredient for this testability is Pareto efficiency. In that sense, the exclusive emphasis put by the DMS literature on competitive equilibria in market economy seems ex post misleading. Equilibria are but a specific form of Pareto efficient allocations in a specific context (characterized by egoistic preferences, the absence of public goods and external effects), and the market economy literature imposes in addition highly specific types of intragroup allocation of income. The results just described imply that, perhaps surprisingly, none of these restrictions makes any difference for the basic conclusion.

4.3 Aggregate demand of an efficient group: identification

Broadly speaking, the identification question can be stated as follows: when is it possible to recover the underlying structure - namely, individual preferences, the decision process and the resulting intragroup transfers - from the sole observation of the group's aggregate behavior?

Recent results in the literature on household behavior suggest that, surprisingly enough, when the group is 'small', the structure can be recovered under reasonably mild assumptions. For instance, in the model of household labor supply proposed by Chiappori (1988, 1992), two individuals privately consume leisure and some Hicksian composite good. The main conclusion is that the two individual preferences and the decision process can generically be recovered (up to an additive constant) from the two labor supply functions. This result has been empirically applied (among others) by Fortin and Lacroix (1997) and Chiappori, Fortin and Lacroix (2002), and extended by Chiappori (1997) to household production and by Blundell et al. (2000) to discrete participation decisions. Fong and Zhang (2001) consider a more general model where leisure can be consumed both privately and publicly. Although the two alternative uses are not independently observed, they can in general be identified under a separability restriction, provided that the consumption of another exclusive good (e.g. clothing) is observed.

Altogether, these results suggest that multi-person groups need not remain 'black boxes', the structure of which cannot be investigated without precise information on intragroup decision processes. On the contrary, the group's aggregate behavior, as summarized by its demand function, contains potentially rich information on its structure - i.e., individual preferences and the distribution of powers between its members. We now substantiate this claim.

Define a *structure* as a set of individual utilities and Pareto weights (normalized for instance by the condition that their sum is one). Moreover, two structures $\left(U^1,...,U^H;\mu_1,...,\mu_H\right)$ and $\left(\bar{U}^1,...,\bar{U}^H;\bar{\mu}_1,...,\bar{\mu}_H\right)$ are *equivalent* if (i) for each h, there exists some increasing mapping F^h such that $U^h = F^h\left(\bar{U}^h\right)$, and (ii) for any (π, y), $(\mu_1,...,\mu_H)$ and $(\bar{\mu}_1,...,\bar{\mu}_H)$ correspond to parametrizations of the same Pareto efficient allocation for the respective cardinalizations of individual preferences; two structures are different if they are not equivalent.

A first result is the following:

Proposition 8 *In the most general version of the model, there exists a continuum of different structures that generate the same aggregate demand function. Moreover, the result remains valid even when either all commodities are privately consumed or all commodities are publicly consumed.*

Proof. *See Chiappori and Ekeland (2009 a and b).* ■

In the most general case, thus, there exists a continuum of observationally equivalent models - i.e. a continuum of structurally different settings generating identical observable behavior. This negative result implies that additional assumptions are required.

As it turns out, such assumptions are surprisingly mild; essentially, it is sufficient that each agent in the group be excluded from the consumption of (at least) one commodity. We start with the case in which all commodities are publicly consumed. Then:

Proposition 9 *In the collective model with H agents and public consumption only, if member 1 does not consume at least one good, then generically the utility of member 1 is exactly (ordinally) identifiable from household demand. If each member is excluded from consumption of at least one specific good, then generically individual preferences are exactly (ordinally) identifiable from household demand; and for any cardinalization of individual utilities, the Pareto weights are exactly identifiable.*
Proof. *See Chiappori and Ekeland (2009 a and b).* ■

This result, in particular, has been applied to collective formulation of household behavior. A large literature has been devoted to the analysis of labor supply, following the initial contribution of Chiappori (1988, 1992). The idea is to consider the household as a two-person group making Pareto efficient decisions on consumption and labor supply; let L^h denote the leisure of member h, and w_h the corresponding wage. Various versions of the model can be considered; in each of them Proposition 9 applies, leading to full identifiability of the model (see Chiappori and Ekeland 2009a).

The general case (in which some goods are consumed privately and some publicly) is slightly more complex:

Proposition 10 *In the general, collective model with two agents, if each member is excluded from consumption of at least one specific good, then generically the* indirect collective utility *of each member is exactly (ordinally) identifiable from household demand. For any cardinalization of indirect collective utilities, the Pareto weights are exactly identifiable.*
Proof. *See Chiappori and Ekeland (2009 a and b).* ■

Here, what is identified is the structure that is relevant to formulate welfare judgments (namely, the indirect collective utility W^h of each agent h). Remember that W^h is not identical to the standard indirect utility function V^h; the difference, indeed, is that W^h captures both the preferences of agent h (through V^h) and the decision process (which governs the way private commodities are allocated). In particular, identifying W^h is not equivalent to identifying V^h (hence U^h). If, for instance, all commodities are private, we have that:

$$W^h(p) = V^h\left(p, \rho^h(p)\right) = V^h\left(\frac{p}{\rho^h(p)}, 1\right) \tag{43}$$

and it is easy to prove that the knowledge of W^h is not sufficient to independently identify *both* ρ^h and V^h: for any W^h, there exists a continuum of pairs $\left(\rho^h, V^h\right)$ such that (43) is satisfied.[15] In contrast with the public good case, the knowledge of the collective indirect utilities is therefore not sufficient, in the presence of private consumption, to identify individual preferences and the decision process (as summarized by the sharing rule). However, the indeterminacy is welfare irrelevant: and welfare conclusion reached with one particular solution would remain valid for all the others (this is exactly the scope of the indirect collective utilities).

Finally, the previous identification result is only generic. One can find cases in which it does not obtain, but these cases are not robust to small perturbations.[16] Among these pathological contexts is the Samuelson index case, in which the group behaves as a single consumer. Intuitively, the basic condition (that some function must be decomposed as a linear combination of gradients) is then degenerate: the function is in fact proportional to a *single* gradient, which can itself be decomposed into a continuum of different sums. In other words, when a group behaves as a single consumer, then individual preferences are not identifiable. Ironically, a large fraction of the literature devoted to household behavior tends to assume a unitary setting, in which the group is described as a unique decision maker. Our conclusions show that this approach, while analytically convenient, entails a huge cost, since it precludes the (non parametric) identification of individual consumption and welfare. In a general sense, *non unitary models are indispensable for addressing issues related to intragroup allocation.*

5 Conclusion

The 'old' literature on aggregation was mostly concentrated on two issues. One was related to the structure of the aggregate (market or excess) demand of a large market economy; the other dealt with the

[15] For instance, pick up some arbitrary $\phi(p)$ mapping $\mathbb{R}^N$ into $\mathbb{R}$, and define an alternative solution $\left(\bar{\rho}^s, \bar{V}^s\right)$ by:

$$\bar{\rho}^s(p) = \phi(p)\,\rho^s(p) \quad \text{and} \quad \bar{V}^s(p,1) = V^s(\phi(p)\,p, 1)$$

Then (43) is satisfied for the alternative solution.

[16] Technically, demands for which identification does not obtain must satisfy a specific partial differential equation; see Chiappori and Ekeland (2009a).

conditions under which a small group would behave as a single decision-maker. The research programs represented by these issues have mostly been completed. The questions raised in Sonnenschein's seminal 1972 paper have been answered (some quite recently); and Hildenbrand's contributions have illuminated how aggregation of *sufficiently heterogeneous* individual behaviors could in fact create structure. On the other hand, the 'unitary' representation of small groups (mostly families) has been the basis of a considerable theoretical and empirical literature.

'Modern' approaches have recently triggered a deep reconsideration of these views. The claim that General Equilibrium theory could not generate testable predictions has been challenged; the consensus is now that testable implications exist, but they typically require micro data. What is dubious is that testable restrictions could be generated if only aggregate data are available, at least without very strong (and microempirically unrealistic) restrictions.

More importantly, the emphasis has shifted from aggregation 'in the large' to aggregation 'in the small'. Recent approaches have taken seriously the idea that the aggregate behavior of a (small) group exhibits specific features, that cannot in general and should not in any case be reduced to an individual decision process. These features actually raise fascinating issues about power relationships within groups and their impact on aggregate behavior; and a set of new results suggest that much can be learned about the former from a careful investigation of the latter. From this perspective, the macro fiction of a representative consumer no longer seems too attractive.

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