The Micro Economics of Efficient Group Behavior: Identification*

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Abstract

Consider a group consisting of S members facing a common budget constraint $p'\xi = 1$; any demand vector belonging to the budget set can be (privately or publicly) consumed by the members. Although the

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intra-group decision process is not known, it is assumed to generate Pareto efficient outcomes; neither individual consumptions nor intra-group transfers are observable. The paper analyzes when, to what extent and under which conditions it is possible to recover the underlying structure - individual preferences and the decision process - from the group's aggregate behavior. We show that the general version of the model is not identified. However, a simple exclusivity assumption (whereby each member is the exclusive consumer of at least one good) is sufficient to guarantee generic identifiability of the welfare-relevant structural concepts.

1 Introduction

Group behavior: beyond the 'black box' Consider a group consisting of S members. The group has limited resources; specifically, its global consumption vector ξ must satisfy a standard market budget constraint of the form $p'\xi = y$ (where p is a vector of prices and y is total group income). Any demand vector belonging to the global budget set thus defined can be consumed by the members. Some of the goods can be privately consumed, while others may be publicly used. The decision process within the group is not known, and is only assumed to generate Pareto efficient outcomes[1]. Finally, neither individual consumptions nor intragroup transfers are observable. In other words, the group is perceived as a 'black box'; only its aggregate behavior, summarized by the demand function $\xi(p, y)$, is recorded. The goal of the present paper is to provide answers to the general question: when is it

[1] We view efficiency as a natural assumption in many contexts, and as a natural benchmark in all cases. For instance, the analysis of household behavior often takes the 'collective' point of view, where efficiency is the basic postulate. Other models, in particular in the literature on firm behavior, are based on cooperative game theory in a symmetric information context, where efficiency is paramount (see for instance the 'insider-outsider' literature, and more generally the models involving bargaining between management and workers or unions). The analysis of intra group risk sharing, starting with Townsend's seminal paper (1994), provides other interesting examples. Finally, even in the presence of asymmetric information, first best efficiency is a natural benchmark. For instance, a large part of the empirical literature on contract theory tests models involving asymmetric information against the null of symmetric information and first best efficiency (see Chiappori and Salanie (2000) for a recent survey).

possible to recover the underlying structure - namely, individual preferences, the decision process and the resulting intragroup transfers - from the group's aggregate behavior?

In the (very) particular case where the group consists of only one member, the answer is well known: individual demand uniquely defines the underlying preferences. Not much is known in the case of a larger group. However, recent results in the literature on household behavior suggest that, surprisingly enough, when the group is 'small', the structure can be recovered under reasonably mild assumptions. For instance, in the model of household labor supply proposed by Chiappori (1988, 1992), two individuals privately consume leisure and some Hicksian composite good. The main conclusion is that the two individual preferences and the decision process can generically be recovered (up to an additive constant) from the two labor supply functions. This result has been empirically applied (among others) by Fortin and Lacroix (1997) and Chiappori, Fortin and Lacroix (2002), and extended by Chiappori (1997) to household production and by Blundell et al. (2000) to discrete participation decisions. Fong and Zhang (2001) consider a more general model where leisure can be consumed both privately and publicly. Although the two alternative uses are not independently observed, they can in general be identified under a separability restriction, provided that the consumption of another exclusive good (e.g. clothing) is observed.

Altogether, these results suggest that there is information to be gained on the contents of the 'black box'. In a companion paper (Chiappori and Ekeland 2006), we investigate the properties of aggregate behavior stemming from the efficiency assumption. We conclude that when the group is small enough, a lot of structure is imposed on collective demand by this basic assumption: there exist strong, testable, restrictions on the way the black box may operate. The main point of the present paper is complementary. We investigate to what extent, and under which conditions, it is possible to recover much (or all) of the interior structure of the black box without opening it. We first show that in the most general case, there exists a continuum of observationally equivalent models - i.e. a continuum of different structural settings generating identical observable behavior. This negative result implies that additional assumptions are required.

We then provide examples of such assumptions, and show that they are surprisingly mild. Essentially, it is sufficient that each agent in the group be the exclusive consumer of (at least) one commodity. Moreover, when a 'distribution factor' (see below) is available, this requirement can be reduced to the existence of an assignable good (i.e., a private good for which individual consumptions are observed). Under these conditions, the structure that is relevant to formulate welfare judgments is non parametrically identified in general (in a sense that is made clear below), *irrespective of the total number of commodities.* We conclude that even when decision processes or intra group transfers are not known, much can be learned about them from the sole observation of the group's aggregate behavior. This conclusion generalizes the earlier intuition of Chiappori (1988, 1992); it shows that the results obtained in these early contributions, far from being specific to the particular settings under consideration, were in fact general.

Identifiability and identification From a methodological perspective, it may be useful to define more precisely what is meant by 'recovering the underlying structure'. The structure, in our case, is defined by the (strictly convex) preferences of individuals in the group and the decision process. Because of the efficiency assumption, for any particular cardinalization of individual utilities the decision process is fully summarized by the Pareto weights corresponding to the outcome at stake. The structure, thus, consists in a set of individual preferences (with a particular cardinalization) and

Pareto weights (with some normalization - e.g., the sum of Pareto weights is taken to be one).

This structure is not observable; what can be recorded is the group's aggregate demand function $\xi(p, y)$. In practice, the 'observation' of $\xi(p, y)$ is a complex process, that entails specific difficulties. For instance, one never observes a (continuous) function, but only a finite number of values on the function's graph. These values are measured with some errors, which raises problems of statistical inference. In some cases, the data are cross-sectional, in the sense that different groups are observed in different situations; specific assumptions have to be made on the nature and the form of (observed and unobserved) heterogeneity between the groups. Even when the same group is observed in different contexts (say, from panel data), other assumptions are needed on the dynamics of the situation, e.g. on the way past behavior influences present choices. All these issues, which lay at the core of what is usually called the *identification* problem, are outside the scope of the paper.

Our interest, here, is in what has been called the *identifiability* problem, which can be defined as follows: when is it the case that the (hypothetically) perfect knowledge of a smooth *demand function* $\xi(p, y)$ uniquely defines the underlying structure within a given class? Formally, for any given structure, the maximization of the (Pareto) weighted sum of utilities generates a unique demand function. This defines a mapping from the set of structures to the set of demand functions. Identifiability obtains if this mapping is *injective*, in the sense that two different structures can never generate the same demand function. In other words, non identifiability does not result from the econometrician's inability to exactly recover the form of demand functions - say, because only noisy estimates of the parameters can be obtained, or even because the functional form itself (and the stochastic structure added to it) have been arbitrarily chosen. These econometric questions have, at least to some extent, econometric or statistical answers. For instance, confidence intervals can be computed for the parameters (and become negligible when the sample size grows); the relevance of the functional form can be checked using specification tests; etc. The non identifiability problem has a different nature: even if a *perfect* fit to *ideal* data was feasible, it would still be impossible to recover the underlying structure from observed behavior.[2]

In the case of *individual* behavior, as analyzed by standard consumer the-

[2]The distinction between identification and identifiability can be traced back to Koopmans's (1949) seminal paper (we thank Martin Browning for suggesting this reference). A difference is that Koopmans's defines a 'structure' as 'a combination of a specific set of structural equations and a specific distribution function of the latent variables' - a 'model' being defined as a 'set of structures'. Koopmans clearly distinguishes two types of identification problems, namely those linked with 'statistical inference' and those due to 'identifiablity'.

ory, identifiability is an old but crucial result. Indeed, it has been known for more than a century that an individual demand function uniquely identifies the underlying preferences. Usual as this property may have become, it remains one of the strongest results in microeconomic theory. It implies, for instance, that assessments about individual well-being can unambiguously be made from the sole observation of demand behavior - a fact that opens the way to all of applied welfare economics. The present work can be seen as an attempt at generalizing this classical identifiability property to efficient groups of arbitrary sizes.

Identifiability is a necessary condition for identification. If different structures are observationally equivalent, there is no hope that observed behavior will help to distinguish between them - only ad hoc functional form restrictions can do that. Since observationally equivalent models may have very different welfare implications, non identifiability severely limits our ability to formulate reliable normative judgments: any normative recommendation based on a particular structural model is unreliable, since it is ultimately based on the purely arbitrary choice of one underlying structural model among many.

Note, finally, that identifiability is only a necessary first step for identification (in the standard, econometric sense). Whether an *identifiable* model is econometrically *identified* depends on the stochastic structure representing the various statistical issues (measurement errors, unobserved heterogeneity,...) discussed above. After all, the abundant empirical literature on consumer behavior, while dealing with a model that is always identifiable, has convinced us that identification crucially depends on the nature of available data.

Parametric versus non-parametric identifiability The identifiability problem may be approached from a parametric or a non parametric perspective. In the parametric approach, a particular functional form is chosen for the structural model, and a reduced form for the demand function is derived. In particular, the derivation highlights the links between the parameters of the structural model and the coefficient of the demand function that will be taken to data. Identification, in this context, is equivalent to the uniqueness of the set of parameters of the structural model corresponding to any specified values for the (estimated) coefficients of the reduced form. Note that in such a context, uniqueness or identifiability are *conditional on the functional form*; i.e. it obtains (at best) within a specific and narrow set of candidate functions, namely those compatible with the functional form chosen at the outset.

Throughout this paper, our approach, on the contrary, is explicitly *non-parametric*. That is, we try to find conditions that guarantee uniqueness within the general class of smooth, strictly convex preferences and differentiable Pareto weights. One can readily provide examples in which identifiability obtains in a parametric sense, but not in the non-parametric setting (it is then functional form dependent)[3].

In practice, parametric models are often convenient. In particular, we do *not* suggest that parametric estimations should not be used, or even that it should be resorted to with some reluctance. Postulating a specific functional form is a standard, well established and often extremely fruitful methodology. We do however submit that the status of the conclusions drawn from parametric estimations crucially depend on whether or not the underlying model is *non-parametrically identifiable*. If it is, then the reliability of the parametric estimates (and, consequently, of the conclusions drawn from it) is directly related to the quality of the empirical fit, as assessed by standard econometric tests. If the econometrician can convince himself (and the scien-

[3] See for instance Blundell, Chiappori and Meghir (2006).

tific community) that the model provides a pretty faithful representation of the real phenomenon, then the same level of trust could in principle be put into the conclusions derived from it. The case is much weaker in the absence of non parametric identifiability. A good empirical fit is no longer sufficient: by definition, many different structural models, with potentially divergent normative implications, have exactly the same fit (since they generate the same reduced forms), hence are exactly as well supported by the data as the initial one.

Of course, this discussion should not be interpreted too strictly. In the end, identifying assumptions are (almost) always needed. The absence of non parametrically identifiability, thus, should not necessarily be viewed as a major weakness. We believe, however, that it justifies a more cautious interpretation of the estimates. More importantly, we submit, as a basic, methodological rule, that an explicit analysis of non parametric identifiability is a necessary first step in any consistent empirical strategy - if only to suggest the most adequate identifying assumptions. Applying this approach to collective models is indeed the main purpose of this paper.

Distribution factors An important tool to achieve identification is the presence of *distribution factors*; see Bourguignon, Browning and Chiappori (1995). These are defined as variables that can affect group behavior only through their impact on the decision process. Think, for instance, of the choices as resulting from a bargaining process. Typically, the outcomes will depend on the members' respective bargaining positions; hence, any factor of the group's environment that may influence these positions (EEPs in McElroy's (1990) terminology) potentially affects the outcome. Such effects are of course paramount, and their relevance is not restricted to bargaining in any particular sense. In general, group behavior depends not only on preferences and budget constraint, but also on the members' respective 'power' in the decision process. Any variable that changes the powers may have an impact on observed collective behavior.

In many cases, distribution factors are readily observable. An example is provided by the literature on household behavior. In their study of household labor supply, Chiappori, Fortin and Lacroix (2002) use the state of the marriage market, as proxied by the sex ratio by age, race and state, and the legislation on divorce, as particular distribution factors affecting the intrahousehold decision process, and thereby its outcome, i.e. labor supplies.

They find, indeed, that factors more favorable to women significantly decrease (resp. increase) female (resp. male) labor supply. Using similar tools, Oreffice (2005) concludes that the legalization of abortion had a significant impact on intrahousehold allocation of power. In a similar context, Rubalcava and Thomas (2000) use the generosity of single parent benefits and reach identical conclusions. Thomas, Contreras, and Frankenberg(1997), using an Indonesian survey, show that the distribution of wealth by gender at marriage - another candidate distribution factor - has a significant impact on children health in those areas where wealth remains under the contributor's control[4]. Duflo (2000) has derived related conclusions from a careful analysis of a reform of the South African social pension program that extended the benefits to a large, previously not covered black population. She finds that the recipient's gender - a typical distribution factor - is of considerable importance for the consequences of the transfers on children's health.

Whenever the aggregate group demand is observable as a function of prices *and* distribution factors, one can expect that identification may be easier to obtain. This is actually known to be the case in particular situations. For instance, Chiappori, Fortin and Lacroix (2002) show how the

[4]See also Galasso (1999) for a similar investigation.

use of distribution factors allows a simpler and more robust estimation of a collective model of labor supply. In the present paper, we generalize these results by providing a general analysis of the estimation of collective models in different contexts, with and without distribution factors.

The results Our main conclusions can be summarized as follows:

- In its most general formulation, the model is not identifiable. Any given aggregate demand that is compatible with efficiency can be derived either from a model with private consumption only, or from a model with public consumption only. Moreover, even when it is assumed that all consumptions are private (or that they are all public, or that some commodities are privately and other publicly consumed), in the absence of exclusive consumptions there exists a continuum of different structural models that generate the same aggregate demand. More precisely, identifiability obtains only up to two functions, in a manner that is precisely described in the paper.

- A simple exclusivity assumption is in general sufficient to guarantee full, non-parametric identifiability of the welfare-relevant structure. Specifically, we define the *collective indirect utility* of each member as the utility level that member ultimately reaches for given prices, household income and possibly distribution factors, taking into account the allocation of resources prevailing within the household. We show the following result: if, for each agent of the group, there exists a commodity which is exclusively consumed by that agent, then, in general, the *collective indirect utility* of each member can be recovered (up to some increasing transform), irrespective of the total number of commodities. Our general conclusion, hence, is that *one exclusive commodity per agent is sufficient to identify all welfare-relevant aspects of the collective model.* Moreover, when distribution factors are available, one assignable good (i.e., a private good for which individual consumptions are observed) only is sufficient for identifiability.

Section 2 describes the model. The formal structure of the identifiability problem is analyzed in Section 3. Sections 4, 5 and 6 consider the case of two-person groups. Section 4 characterizes the limits to identification in a general context. Identification under exclusivity or generalized separability assumptions is discussed in Section 5, and applied to specific economic frameworks in Section 6. Section 7 briefly discusses the extension to the general case of S-person groups. Empirical issues are briefly discussed in Conclusion.

2 The model

2.1 Preferences

We consider a S person group. There exist N commodities, n of which are privately consumed within the group while the remaining $K = N - n$ commodities are public. Purchases[5] are denoted by the vector $x \in \mathbb{R}^N$. Here, $x = (\sum_s x_s, X)$, where $x_s \in \mathbb{R}^n$ denotes the vector of private consumption by agent s and $X \in \mathbb{R}^K$ is the household's public consumption[6]. The corresponding prices are $(p, P) \in \mathbb{R}^N = \mathbb{R}^n \times \mathbb{R}^K$, and household income is y, giving the budget constraint:

$$p' (x_1 + ... + x_S) + P'X = y$$

Each member has her/his own preferences over the goods consumed in the group. In the most general case, each member's preferences can depend on other members' private and public consumptions; this allows for altruism, but also for externalities or any other preference interaction. The utility

[5]Formally purchases could include leisure; then the price vector includes the wages - or virtual wages for non-participants.

[6]Throughout the paper, x_s^i denotes the private consumption of commodity i by agent s, and x_s is the vector of private consumption for agent s.

of member s is then of the form $U^s(x_1, ...x_S, X)$. We shall say that the function U^s is *normal* if it is strictly increasing in (x_s, X), twice continuously differentiable in $(x_1, ...x_S, X)$, and the matrix of second derivatives is negative definite.

We shall see that identification does not obtain in the general setting of normal utilities. Therefore, throughout most of the paper we use a slightly less general framework. Specifically, we concentrate on egoistic preferences, defined as follows:

Definition 1 *The preferences of agent s are* egoistic *if they can be represented by a utility function of the form $U^s(x_s, X)$.*

In words, preferences are egoistic if each agent only cares about his private consumption and the household's vector of public goods. Most of our results can be extended to allow for preferences of the 'caring' type (i.e., agent s maximizes an index of the form $W^s(U^1, ..., U^S)$; however, we will not discuss the identifiability of the W^s.[7] Finally, we shall denote by $z \in \mathbb{R}^d$ the vector of distribution factors.

[7]Each allocation that is efficient with respect to the W^s must also be efficient with respect to the U^s. The converse is not true (e.g., an allocation which is too unequal may fail to be efficient for the W^s), a property that has sometimes been used to achieve identification (see Browning and Lechène 2003).

2.2 The decision process.

We now consider the mechanism that the group uses to decide on what to buy. Note, first, that if the functions $U^1, ..., U^S$ represent the same preferences then we are in a 'unitary' model where the common utility is maximized under the budget constraint. The same conclusion obtains if one of the partners can act as a dictator and impose her (or his) preferences as the group's maximand. Clearly, these are very particular cases. In general, the 'process' that takes place within the group is more complex.

Following the 'collective' approach, we shall throughout the paper postulate efficiency, as expressed in the following axiom :

Axiom 2 (Efficiency) *The outcome of the group decision process is Pareto efficient; that is, for any prices* (p, P)*, income* y *and distribution factors* z*, the consumption* $(x_1, ...x_S, X)$ *chosen by the group is such that no other vector* $\left(\bar{x}_1, ...\bar{x}_S, \bar{X}\right)$ *in the budget set could make all members better off, one of them strictly so.*

The axiom can be restated as follows: there exists S scalar functions $\mu^s(p, P, y, z) \geq 0,\ 1 \leq s \leq S$, the Pareto weights, normalized by $\sum_s \mu^s = 1$.

such that $(x_1, ...x_S, X)$ solves:

$$\begin{cases} \max_{x_1,...x_S,X} \sum \mu^s (p, P, y, z) U^s(x_1, ...x_S, X) \\ p' (x_1 + ... + x_S) + P'X = y \end{cases} \tag{Pr}$$

For any given utility functions U^1,...,U^S and any price-income bundle, the budget constraint defines a Pareto frontier for the group. From the Efficiency Axiom, the final outcome will be located on this frontier. It is well-known that, for every (p, P, y, z), any point on the Pareto frontier can be obtained as a solution to problem (Pr): the vector $\mu (p, P, y, z)$, which belongs to the $(S - 1)$-dimensional simplex, summarizes the decision process because it determines the final location of the demand vector on this frontier. The map μ describes the distribution of power. If one of the weights, μ^s, is equal to one for every (p, P, y, z), then the group behaves as though s is the effective dictator. For intermediate values, the group behaves as though each person s has some decision power, and the person's weight μ^s can be seen as an indicator of this power[8].

[8]This interpretation must be used with care, since the Pareto coefficient μ^s obviously depend on the particular cardinalization adopted for individual preferences; in particular, $\mu^s > \mu^t$ does *not* necessarily mean that 's has more power than t'. However, the *variations of* μ^s are significant, in the sense that for any given cardinalization, a policy change that increases μ^s while leaving μ^s constant unambiguously ameliorates the position of s relative to t.

It is important to note that the weights μ^s will in general depend on prices p, P, income y and distribution factors z, since these variables may in principle influence the distribution of 'power' within the group, hence the location of the final choice over the Pareto frontier. Three additional remarks can be made:

- while prices enter both Pareto weights and the budget constraint, distribution factors matter only (if at all) through their impact on μ.
- we assume throughout the paper the absence of monetary illusion. In particular, the μ^s are zero-homogeneous in (p, P, y)
- Following Browning and Chiappori (1998), we add some structure by assuming that the $\mu^s(p, P, y, z)$ are continuously differentiable for $s = 1, ..., S$

From now on, we set $\pi = (p, P) \in \mathbb{R}^N$.

2.3 Characterization of aggregate demand.

In a companion paper, Chiappori and Ekeland (2005), we derive necessary and sufficient conditions for a function $\xi(\pi, y, z)$ to be the aggregate demand of an efficient S-member group. For the sake of completeness, we briefly remind these conditions. Let us first omit the distribution factors. Then:

- if a function $\xi(\pi)$ is the aggregate demand of an efficient S-member group, then its Slutsky matrix can be decomposed as:

$$S(\pi) = \Sigma(\pi) + R(\pi) \tag{1}$$

  where the matrix Σ is symmetric, negative definite, and the matrix R is of rank at most $S-1$. Equivalently, there exists a subspace $\mathcal{R}$ of dimension at least $N-(S-1)$ such that the restriction of $S(\pi)$ to $\mathcal{R}$ is symmetric and negative definite

- Conversely, if a 'smooth enough' map $\xi(\pi)$ satisfies Walras' law $\pi'\xi = y$ and condition (1) in some neighborhood of $\bar{\pi}$, and if the Jacobian of ξ at $\bar{\pi}$ has maximum rank, then $\xi(\pi)$ can locally be obtained as the aggregate demand of an efficient S-member group; we say that ξ is *S-admissible.*

Relation (1) was initially derived by Browning and Chiappori (1998); it is known as the SNR($S-1$) condition. If a smooth map ξ satisfies Walras' law and SNR($S-1$), then one can (locally) recover S utility functions of the general form $U^s(x_1, ...x_S, X)$ and S Pareto weights $\mu^s(\pi, y) \geq 0$ such that $\xi(\pi, y)$ is the collective demand associated with problem (Pr).

A natural question is whether more knowledge about intragroup consumption will generate stronger restrictions. Assume, for instance, that commodities are known to be privately consumed, so that the utility functions are of the form $U^s(x_s)$, or, alternatively, that consumption is exclusively public, so that the preferences are $U^s(X)$. Does the integration result still hold when utilities are constrained to belong to these specific classes? Interestingly enough, the answer is positive. In fact, it is impossible to distinguish the two cases by looking at the aggregate demand only. In the paper mentioned above, we prove that whenever a function ξ is locally $S-$admissible, then it can be (locally) obtained as a Pareto-efficient aggregate demand for a group in which all consumptions are public, and it can also be (locally) obtained as a Pareto-efficient aggregate demand for a group in which all consumptions are private.

Finally, the same paper provides necessary condition on the effect of distribution factors.

2.4 Identifiability: the general problem

Following the discussion above, we now raise the question of identifiability:

Question A (Identifiability): *Take an arbitrary demand function* $\xi(\pi, y, z)$ *satisfying the SNR*$(S-1)$ *condition. Is there a* unique *family of preference relations on* $\mathbb{R}^N$*, represented by utility functions* $U^s(x_1, ..., x_S, X)$ *(unique up to an increasing transformation) and, for each cardinalization of preferences, a* unique *family of differentiable Pareto weights* $\mu^s(\pi, y, z)$, $1 \leq s \leq S$, *with* $\sum \mu^s = 1$,*,such that* $\xi(\pi, y, z)$ *is the aggregate demand associated with problem (Pr) ?*

Question I refers to what could be called a *non-parametric* definition of identifiability, because uniqueness is required within the general set of well-behaved functions, rather than within the set of functions sharing a specific parametric form in which only a finite number of parameters can be varied.

It should be clear that in the most general version of the model we consider, identifiability cannot obtain. A demand function that satisfies SNR$(S-1)$ is compatible with (at least) two different structural models: one where all commodities are privately consumed, and one in which all consumption is public. Quite obviously, these models have very different welfare implications, although they generate the same aggregate demand.

This suggests that more specific assumptions are needed. In what follows, we assume that each commodity is either known to be privately consumed or known to be publicly consumed. Also, preferences are egoistic in the sense defined above. While these assumptions are natural, we shall actually see that they are not sufficient. The nature of the indeterminacy is deeper than suggested by the previous remark. Even with egoistic preferences - and, as a matter of fact, even when consumptions are assumed to be either all public or alternatively all private - it is still the case that a continuum of different structural models generate the same group demand function. In other words, identifying restrictions are needed, that go beyond egoistic preferences.

In the remainder of the paper, we analyze the exact nature of such restrictions. We first investigate the mathematical structure underlying the model. We then prove a general result regarding uniqueness in this mathematical framework. Finally, we derive the specific results of interest.

3 The mathematical structure of the identifiability problem

3.1 The duality between private and public consumption

3.1.1 Basic intuition

With egoistic preferences, program (Pr) above becomes:

$$\begin{cases} \max_{x_1,\ldots x_S,X} \sum \mu^s\left(p,P,y,z\right)U^s(x_s,X) \\ p'\left(x_1+\ldots+x_S\right)+P'X=y \end{cases} \tag{Pr'}$$

and let $x_1\left(p,P,y,z\right),\ldots,x_S\left(p,P,y,z\right),X\left(p,P,y,z\right)$ denote its solution. The household demand function is then $\left(x\left(p,P,y,z\right),X\left(p,P,y,z\right)\right)$ where $x=\sum_s x_s$. In what follows, we repeatedly use the duality between private and public consumption, a standard tool in public economics.

In the neighborhood of a point (p,P,y,z) such that D_PX is of full rank, we can consider the following change in variables:

$$\begin{cases} \psi : \mathbb{R}^{n+K+1+d} \rightarrow \mathbb{R}^{n+K+1+d} \\ \psi\left(p, P, y, z\right) = \left(p, X\left(p, P, y, z\right), y, z\right) \end{cases}$$

The economic motivation for such a change in variables is clear. A basic insight underlying the duality between private and public goods is that, broadly speaking, quantities play for public goods the role of prices for private goods and conversely. Intuitively: in the case of private goods, all agents face the same price but consume different quantities, which add up to the group's demand; with public goods, agents consume the same quantity, but face different (Lindahl) prices, which add up to the market price if the allocation is efficient. This suggest that whenever the direct demand function $x\left(p\right)$ is a relevant concept for private consumption, then the inverse demand function $P\left(X\right)$ should be used for public goods. The change of variable ψ allows to implement this intuition.

In particular, instead of considering the demand function (x, X) as a function of (p, P, y, z), we shall often consider (x, P) as a function of (p, X, y, z) (then the public prices P are implicitly determined by the condition that demand for public goods must be equal to X while private prices are equal to p,income is y and distribution factors are z). While these two viewpoints are clearly equivalent (one can switch form the first to the second and back using the change ψ), the computations are much easier (and more natural) in the second case. Finally, for the sake of clarity, we omit distribution factors for the moment, and consider functions of (p, X, y) only.

3.1.2 Conditional sharing rules

We now introduce the notion of a *conditional sharing rule*. It stems from the following result:

Lemma 3 *For given (p, P), let $\left(\bar{x}_1, ..., \bar{x}_S, \bar{X}\right)$ denote a solution to (Pr'). Define*

$$\rho^s\left(p, X, y\right) = p'\bar{x}_s\left(p, X, y\right)$$

for $s = 1, ..., S$. Then $\bar{x}_s$ solves:

$$\begin{array}{c} \max_{x_s} U^s(x_s, X) \\ p'x_s \leq \rho^s \end{array} \tag{Pr$_s$}$$

Proof. *Assume not, then there exists some x'_s such that $p'x'_s \leq \rho^s$ and $U^s(x'_s, X) > U^s(x_s, X)$. But then the allocation $\left(\bar{x}_1, ..., x'_s, ..., \bar{x}_S, \bar{X}\right)$ is fea-*

sible and Pareto dominates $\left(\bar{x}_1, ..., \bar{x}_S, \bar{X}\right)$*, a contradiction.* ■

In words, an efficient allocation can always be seen as stemming from a two stage decision process[9]. At stage 1, members decide on the public purchases X, and on the allocation of the remaining income $y - P'X$ between the members; member s receives ρ^s. At stage 2, agents each chose their vector of private consumption, subject to their own budget constraint and taking the level of public consumption as given. The *conditional sharing rule* is the vector $(\rho_1, ..., \rho_S)$; it generalizes the notion of sharing rule developed in collective models with private goods only (see for instance Chiappori (1992)) because it is defined conditionally on the level of public consumption previously chosen. Of course, if all commodities are private ($K = 0$) then the conditional sharing rule boils down to the standard notion. In all cases, it satisfies the budget constraint

$$\sum_s \rho^s = y - P'X. \tag{2}$$

[9]Needless to say, we are not assuming that the actual decision process is in two stages. The result simply states that any efficient group behaves *as if* it was following a process of this type.

As above, the conditional sharing rule can be expressed either as a function of (p, P, y, z), as above, or, using the change in variable ψ, as a function of (p, X, y, z). In the first case, ρ is one-homogeneous in (p, P, y) ;in the second case, ρ is one-homogeneous in (p, y)[10].

We define the *conditional indirect utility* of member s as the value of program (Pr_s); hence

$$V^s(p, X, \rho) = \max\{U^s(x_s, X) \text{ st } p' x_s = \rho\} \tag{3}$$

which can be interpreted as the utility reached by member s when consuming X and being allocated an amount ρ for her private expenditures.

3.1.3 Some duality results

The envelope theorem applied to (3) gives:

$$D_p V^s = -\lambda^s x_s$$

$$D_X V^s = D_X U^s$$

$$D_\rho V^s = \lambda^s$$

[10]With a slight notational abuse, we use the same notation ρ in both cases. This convention avoids tedious distinctions, in a context in which confusions are easy to avoid.

where λ^s denotes the Lagrange multiplier of the budget constraint (or equivalently the marginal utility of private income for member s) in (3), and where

$$D_pV = \begin{pmatrix} \frac{\partial V}{\partial p_1} \\ \vdots \\ \frac{\partial V}{\partial p_n} \end{pmatrix}, \; D_XV = \begin{pmatrix} \frac{\partial V}{\partial X_1} \\ \vdots \\ \frac{\partial V}{\partial X_K} \end{pmatrix}, \; D_\rho V = \frac{\partial V}{\partial \rho}$$

Hence we have a direct extension of Roy's identity:

$$\frac{D_pV^s}{D_\rho V^s} = -x_s$$

The first stage decision process can then be modelled as:

$$\max_{X,\rho^1,...,\rho^S} \textstyle\sum_s \mu^s (p, P, y) V^s (p, X, \rho^s)$$
$$\textstyle\sum_s \rho^s + P'X = y$$

First order conditions give:

$$\mu^s D_\rho V^s (p, X, \rho^s) = \gamma (p, P, y), \quad s = 1, ..., S$$
$$\sum_s \mu^s D_X V^s (p, X, \rho^s) = \gamma (p, P, y) P$$

where γ denotes the Lagrange multiplier of the constraint. Setting $\gamma^s = \mu^s/\gamma$, we have that

$$D_\rho V^s(p, X, \rho^s) = \frac{1}{\gamma^s(p, P, y)}, \qquad s = 1, ..., S$$

which expresses the fact that individual marginal utilities of private incomes, $D_\rho V^s(p, X, \rho^s)$, are inversely proportional to Pareto weights. Finally, we have the following conditions:

$$\sum_s \gamma^s(p, P, y) D_p V^s = -\sum_s x_s = -x \tag{4}$$

$$\sum_s \gamma^s(p, P, y) D_X V^s = P$$

which determine $x(p, P, y)$ and $X(p, P, y)$. Using the change in variable ψ, we see that (x, P), as a function of (p, X), can be decomposed as a linear combination of the partial derivatives of the V^s. The nice symmetry of these equations illustrates the duality between private and public consumptions.

3.2 Collective indirect utility

Following Chiappori (2006), we introduce the following, key definition:

Definition 4 *Given a conditional sharing rule* $\rho^s(p, X, y)$*, the* collective indirect utility *of agent* s *is defined by:*

$$W^s(p, X, y) = V^s(p, X, \rho^s(p, X, y))$$

In words, W^s denotes the utility level reached by agent s, at prices p and with total income y, in an efficient allocation such that the household demand for public goods is X, taking into account the conditional sharing rule $\rho^s(p, X, y)$. Note that W^s depends not only on the preferences of agent s (through the conditional indirect utility V^s) but also on the decision process (through the conditional sharing rule ρ^s). Hence W^s summarizes the impact on s of the interactions taking place within the group. As such, it is the main concept required for welfare analysis: *knowing the* W^s *allows to assess the impact of any reform (i.e. any change in prices and incomes) on the welfare of each group member.*

Most of what follows is devoted to the identification of the collective indirect utility of each member.

3.3 Identifying collective indirect utilities : the mathematical structure

Condition (4) can readily be translated in terms of collective indirect utilities. Since:

$$\frac{D_p W^s}{D_\rho V^s} = \gamma^s D_p W^s = \gamma^s D_p V^s + D_p \rho^s = -x_s + D_p \rho^s$$
$$\frac{D_X W^s}{D_\rho V^s} = \gamma^s D_X W^s = \gamma^s D_X V^s + D_X \rho^s$$
$$\frac{D_y W^s}{D_\rho V^s} = \gamma^s D_y W^s = D_y \rho^s$$

and using the budget constraint (2), we conclude that:

$$\sum_s \gamma^s D_p W^s = -x - (D_p P) X$$
$$\sum_s \gamma^s D_X W^s = -(D_X P) X$$
$$\sum_s \gamma^s D_y W^s = 1 - (D_y P)' X$$

Let $A(p, X, y) = P(p, X, y)' X$ denote household total expenditures on public goods. The above equations become:

$$\sum_s \gamma^s D_p W^s = -x - D_p A$$
$$\sum_s \gamma^s D_X W^s = P - D_X A \tag{5}$$
$$\sum_s \gamma^s D_y W^s = 1 - D_y A.$$

The identifiability question described above can thus be rephrased in mathematical terms as follows:

Question B: *Take an arbitrary,* Sadmissible *demand* $(x(p,P,y), X(p,P,y))$, *and apply the change of variable* ψ *in the neighborhood of a regular point to obtain a function* $(x(p,X,y), P(p,X,y))$*. Let* $A(p,X,y)$ *be the total household expenditure on public goods. Is there a* unique *family of differentiable functions* $W^s(p,X,y)$ *on* $\mathbb{R}^N$*, each defined up to some increasing transformation, such that the vector*

$$\begin{pmatrix} -x - D_p A \\ P - D_X A \\ 1 - D_y A \end{pmatrix}$$

can be expressed as a linear combination of the gradients of the $W^s, 1 \leq s \leq$

S?

Decomposing a given function as a linear combination of gradients is an old problem in mathematics, to which much work has been devoted in the first half of the XXth century, particularly by Elie Cartan, who developed exterior differential calculus for that purpose (among others). We shall use these tools in what follows.

3.3.1 Particular case 1: public goods only

Before addressing question B, we may briefly consider two special cases. Assume, first, that all goods are public. Then W^s only depends on X:

$$W^s(X) = U^s(X)$$

In particular, identifying the W^s is equivalent to identifying the U^s. Since $A = P'X = y$, equations (5) become:

$$\sum_s \gamma^s D_X U^s = P(X) \tag{6}$$

The existence (or characterization) problem is whether one can find functions $U^s(X)$ and $\gamma^s(X)$, $1 \leq s \leq S$ this equation. It has been addressed in

Chiappori and Ekeland (2006). Here we are interested in the uniqueness (or identification) question: are the $U^s(X)$ unique, up to an increasing transformation ?

3.3.2 Particular case 2: private goods only

The opposite polar case obtains when all commodities are private. This is a case that has been repeatedly studied in the literature, starting with Chiappori (1988, 1992). Then $A = 0$, and W^s is a function of (p, y) only. Moreover, W^s, as V^s, is zero-homogeneous, so we can normalize y to be one. Equations (5) become:

$$\sum_s \gamma^s(p)\, D_p W^s = -x(p) \tag{7}$$

Remember that W^s is not identical to the standard indirect utility function V^s; the difference, indeed, is that W^s captures both the preferences of agent s (through V^s) and the decision process (which, in that case, is fully summarized by the sharing rule ρ). In particular, identifying W^s is not equivalent to identifying V^s (hence U^s).

For any given functions W^s, there exist a continuum of choices for the V^s

and ρ^s, since the latter must only satisfy (using homogeneity of V^s):

$$W^s(p) = V^s(p, \rho^s(p)) = V^s\left(\frac{p}{\rho^s(p)}\right) \tag{8}$$

It is easy to prove that, taking W^s as given, for *any* ρ that is not one homogeneous, one can locally construct a V^s such that (8) is satisfied. We conclude that, in contrast with the public good case, the knowledge of the collective indirect utilities is not sufficient under private consumption to identify preferences and the decision process (as summarized by the sharing rule). This indeterminacy is a direct generalization of a previous result derived by Chiappori (1992) in a three commodity, labor supply setting.

The crucial remark, however, is that this indeterminacy is *welfare irrelevant.* If $\left(\bar{V}^1, ..., \bar{V}^S; \bar{\rho}^1, ..., \bar{\rho}^S\right)$ is a particular solution, the various, alternative solutions $\left(V^1, ..., V^S; \rho^1, ..., \rho^S\right)$ have a very simple interpretation in welfare terms. Namely, the V^s are such that the *(ordinal) utility of each member s, when facing the sharing rule ρ, is always the same as under the $\bar{V}$ and* $\left(\bar{\rho}^1, ..., \bar{\rho}^S\right)$. It follows, in particular, that any reform that is found to increase the welfare of member s for $\bar{V}^s$ and $\bar{\rho}^s$ will also increase her welfare for V^s and ρ^s. Again, identifying the collective indirect utilities W^s is sufficient for welfare analysis. This remark is general, and applies to any context, whatever the number of private and public goods.

Finally, in the pure private good case, not only is the indeterminacy welfare irrelevant, but it is also behavior irrelevant; i.e., two households with the same collective indirect utilities W^s and the same weights γ^s will exhibit the same *market* behavior, irrespective of the particular sharing rule (although intrahousehold allocation will obviously differ). Hence estimation of collective indirect utilities and weights allows to predict household behavior and derive comparative statics conclusions.

Both equations (6) and (7) have the same mathematical structure: a given function has to be equal to a linear combination of gradients. Necessary conditions for this to happen have been recalled in Section 2. In what follows, we first consider the identifiability problem from a mathematical perspective, then derive the corresponding economic conclusions. For expositional convenience, Sections 4, 5 and 6 exclusively consider the case of two-person groups ($S = 2$). The general case is discussed in Section 7.

4 Identifiability in the general setting: a negative result

We now state the central mathematical result that underlies most of the following conclusions. It will be applied in various settings. We denote by $\pi = (\pi_1, ..., \pi_N)$ the independent variables, and by $\xi(\pi) = \left(\xi^1(\pi), ..., \xi^N(\pi)\right)$ a given vector field. All functions are assumed to be smooth (at least C^2).

Lemma 5 *Suppose four functions* $\left(\bar{W}^1(\pi), \bar{W}^2(\pi), \bar{\lambda}_1(\pi), \bar{\lambda}_2(\pi)\right)$ *satisfy :*

$$\lambda_1 DW^1 + \lambda_2 DW^2(\pi) = \xi \tag{9}$$

in some neighborhood Ω_1 *of* $\bar{\pi}$*, and that* $D_\pi \bar{W}^1, D_\pi \bar{W}^2, D_\pi\left(\frac{\bar{\lambda}_1}{\bar{\lambda}_2}\right)$ *are linearly independent at* $\bar{\pi}$*. Then for any other family* $(W^1, W^2, \lambda_1, \lambda_2)$ *satisfying (9) in some neighborhood* Ω_2 *of* $\bar{\pi}$*, there exists a neighborhood* $\Omega_3 \subset \Omega_1 \cap \Omega_2$ *and two functions* F *and* G*, defined on some neighborhood of* $\left(\bar{W}^1(\bar{\pi}), \bar{W}^2(\bar{\pi}), \frac{\bar{\lambda}_1}{\bar{\lambda}_2}(\bar{\pi})\right)$ *in* $\mathbb{R}^3$*, such that, for all* $\pi \in \Omega_3$*:*

$$W^1(\pi) = F\left[\bar{W}^1(\pi), \bar{W}^2(\pi), \frac{\bar{\lambda}_1}{\bar{\lambda}_2}(\pi)\right] \tag{10}$$

$$W^2(\pi) = G\left[\bar{W}^1(\pi), \bar{W}^2(\pi), \frac{\bar{\lambda}_1}{\bar{\lambda}_2}(\pi)\right] \tag{11}$$

The functions $F(t_1, t_2, t_3)$ and $G(t_1, t_2, t_3)$ must moreover satisfy the partial differential equation:

$$\frac{\partial G}{\partial t_1}\frac{\partial F}{\partial t_3} - \frac{\partial G}{\partial t_3}\frac{\partial F}{\partial t_1} = t_3\left(\frac{\partial G}{\partial t_2}\frac{\partial F}{\partial t_3} - \frac{\partial G}{\partial t_3}\frac{\partial F}{\partial t_2}\right). \tag{12}$$

Finally, λ_1 and λ_2 are completely determined by the choice of F and G.

Proof. *See Appendix.*

In practice, one can pick one arbitrary function F of three variables. Then (12) becomes a linear partial differential equation for the unknown function G, which is therefore fully determined by its values on any hypersurface not tangent to the vector field $(\frac{\partial F}{\partial t_3}, -t_3\frac{\partial F}{\partial t_3}, -\frac{\partial F}{\partial t_1} + t_3\frac{\partial F}{\partial t_2})$. Hence the solution is defined up to the arbitrary choice of one function of three variables and one function of two variables. ■

Lemma 5 provides two conclusions. One is that whenever a given function can be decomposed as a linear combination of two gradients, this can be done in a continuum of different ways. Secondly, one can exactly characterize the set of such solutions. Specifically, the decomposition is identified up to a pair of functions of three variables; moreover, these functions are linked by a partial differential equation.

An immediate implication is the following:

Corollary 6 *In the collective model, the collective indirect utilities of the members are* not *uniquely determined by the knowledge of household demand.*

To get a better intuition of this result, it is useful to consider the particular case in which all goods are public. Remember that in that case the collective indirect utility of a member is simply the person's direct utility. Now, let $\left(\bar{U}^1, \bar{U}^2\right)$ be a particular solution; the inverse demand $P\left(X\right)$ can be written as a linear combination of the gradients of $\left(\bar{U}^1, \bar{U}^2\right)$ - say:

$$\bar{\lambda}^1 D_X \bar{U}^1 + \bar{\lambda}^2 D_X \bar{U}^2 = P\left(X\right)$$

Define

$$U^1\left(X\right) = F\left(\bar{U}^1\left(X\right), \bar{U}^2\left(X\right)\right)$$
$$U^2\left(X\right) = G\left(\bar{U}^1\left(X\right), \bar{U}^2\left(X\right)\right)$$

where F and G are arbitrary, increasing functions. Clearly, a linear combination of the gradients of U^1 and U^2 must be a linear combination of the gradients of $\bar{U}^1$ and $\bar{U}^2$. The economic intuition is that any allocation X

which is Pareto efficient for (U^1, U^2) must also be Pareto efficient for $\left(\bar{U}^1, \bar{U}^2\right)$ (otherwise it would be possible to increase both $\bar{U}^1$ and $\bar{U}^2$, but this would increase U^1 and U^2 as well, a contradiction). This gives a first intuition why the Pareto efficiency assumption is not sufficient to distinguish between the two solutions: if a demand function can be collectively rationalized by a couple with utilities $\left(\bar{U}^1, \bar{U}^2\right)$ then it can also be collectively rationalized by a couple with utilities $\left(F\left(\bar{U}_1, \bar{U}_2\right), G\left(\bar{U}_1, \bar{U}_2\right)\right)$.

Interestingly enough, this is not the only degree of indeterminacy. To get a different example, set $\bar{\theta}(X) = \frac{\bar{\lambda}^1(X)}{\bar{\lambda}^2(X)}$, and define:

$$U^1(X) = \bar{U}^1(X) + \bar{U}^2(X) + \bar{\theta}(X)$$

$$U^2(X) = -\bar{U}^1(X) + \log\left(1 - \bar{\theta}(X)\right)$$

Here, $F(t_1, t_2, t_3) = t_1 + t_2 + t_3$, and $G(t_1, t_2, t_3) = -t_1 + \ln(1 - t_3)$ is a particular solution of equation (12). It is easy to check that:

$$\bar{\lambda}^2 D_X U^1 + \left(\bar{\lambda}^2 - \bar{\lambda}^1\right) D_X U^2 = \bar{\lambda}^1 D_X \bar{U}^1 + \bar{\lambda}^2 D_X \bar{U}^2 = P(X)$$

and we conclude that the inverse demand function $P(X)$ can also be collectively rationalized by a couple with utilities (U^1, U^2).

5 Identifiability under exclusivity

The previous section concludes that identifiability does not obtain in the general version of the collective model. The next step is to find specific, additional assumptions that guarantee identifiability. We now show that a simple exclusivity condition is sufficient in general.

5.1 The main result

Again, suppose four functions $\left(\bar{W}^1(\pi), \bar{W}^2(\pi), \bar{\lambda}_1(\pi), \bar{\lambda}_2(\pi)\right)$ satisfy equation (9) in some neighborhood Ω_1 of $\bar{\pi}$. We shall say that this solution is *generic* if $D_\pi \bar{W}^1, D_\pi \bar{W}^2$ and $D_\pi \bar{\theta}$ are linearly independent at $\bar{\pi}$, and

$$\frac{\partial \bar{W}^2}{\partial \pi_1}(\pi) \neq 0 \text{ and } \bar{D}_{a,b,c,d}(\bar{\pi}) \neq 0 \text{ for some } a, b, c, d \in \{1, ..., N\} \tag{13}$$

where:

$$\bar{\theta}(\pi) = \frac{\bar{\lambda}^1(\pi)}{\bar{\lambda}^2(\pi)}, \quad \bar{\Phi}(\pi) = \frac{\partial \bar{\theta}}{\partial \pi_1}(\pi)\left[\frac{\partial \bar{W}^2}{\partial \pi_1}(\pi)\right]^{-1} \tag{14}$$

$$\bar{D}_{a,b,c,d} = \begin{vmatrix} \frac{\partial \bar{\Phi}}{\partial \pi_a} & \frac{\partial \bar{W}^1}{\partial \pi_a} & \frac{\partial \bar{W}^2}{\partial \pi_a} & \frac{\partial \bar{\theta}}{\partial \pi_a} \\ \frac{\partial \bar{\Phi}}{\partial \pi_b} & \frac{\partial \bar{W}^1}{\partial \pi_b} & \frac{\partial \bar{W}^2}{\partial \pi_b} & \frac{\partial \bar{\theta}}{\partial \pi_b} \\ \frac{\partial \bar{\Phi}}{\partial \pi_c} & \frac{\partial \bar{W}^1}{\partial \pi_c} & \frac{\partial \bar{W}^2}{\partial \pi_c} & \frac{\partial \bar{\theta}}{\partial \pi_c} \\ \frac{\partial \bar{\Phi}}{\partial \pi_d} & \frac{\partial \bar{W}^1}{\partial \pi_d} & \frac{\partial \bar{W}^2}{\partial \pi_d} & \frac{\partial \bar{\theta}}{\partial \pi_d} \end{vmatrix} \tag{15}$$

Note that if equations (13) hold at $\bar{\pi}$, they will hold in a neihgborhood of $\bar{\pi}$ as well. Note also that in a practical situation, there is no reason to expect that the functions $\bar{W}^1$, $\bar{W}^2$ and $\bar{\theta} = \bar{\lambda}_1/\bar{\lambda}_2$ would satisfy these equations, and if they do, a slight perturbation of the model would ensure that they do not.

We shall say that $\bar{W}^1$is *exclusive* in Ω if:

$$\frac{\partial \bar{W}^1}{\partial \pi_1}(\pi) = 0 \quad \forall \pi \in \Omega \tag{16}$$

We now show that exclusivity is sufficient to guarantee identifiability for generic models. Note, first, that the result is obvious if the number of commodities is exactly 2 - since individual consumptions are perfectly observed in that case. Also, the case $N = 3$ has been already solved by

Chiappori (1992) for private goods and Blundell et al. (2006) for public goods. We are thus left with the general case $N \geq 4$. The main result is the following:

Proposition 7 *Let $N \geq 4$, assume that equation (9) has a generic solution $\left(\bar{W}^1, \bar{W}^2, \bar{\lambda}_1, \bar{\lambda}_2\right)$ in some neighborhood Ω_1 of $\bar{\pi}$, where $\bar{W}^1$ is exclusive. Let $(W^1, W^2, \lambda_1, \lambda_2)$ be another solution on Ω_1, with W^1 exclusive Then there exist a function F and a neighborhood $\Omega_2 \subset \Omega_1$ of $\bar{\pi}$ such that $W_1 = F\left(\bar{W}_1\right)$ on Ω_2, i.e. W_1 is ordinally identified.*

Proof. *From Lemma 5, we know that there is some neighborhood $\Omega_2 \subset \Omega_1$ and some function F such that:*

$$W^1 = F\left(\bar{W}^1, \bar{W}^2, \bar{\theta}\right) \quad \text{on } \Omega_2$$

Without loss of generality, it can be assumed that equations (13) hold for all $\pi \in \Omega_2$. From the 1-exclusivity condition for W^1 and $\bar{W}^1$, we derive, for all $\pi \in \Omega_2$:

$$0 = \frac{\partial W^1}{\partial \pi_1}(\pi) = F_2\left(\bar{W}^1, \bar{W}^2, \bar{\theta}\right)\frac{\partial \bar{W}^2}{\partial \pi_1} + F_3\left(\bar{W}^1, \bar{W}^2, \bar{\theta}\right)\frac{\partial \bar{\theta}}{\partial \pi_1} \qquad (17)$$

Suppose $F_3\left(\bar{W}^1(\pi), \bar{W}^2(\pi), \bar{\theta}(\pi)\right) \neq 0$ for some $\pi \in \Omega_2$. Equation (17) *then implies that:*

$$\bar{\Phi}(\pi) = \frac{\partial \bar{\theta}}{\partial \pi_1}(\pi) \left[\frac{\partial \bar{W}^2}{\partial \pi_1}(\pi) \right]^{-1} = -\frac{F_2\left(\bar{W}^1, \bar{W}^2, \bar{\theta}\right)}{F_3\left(\bar{W}^1, \bar{W}^2, \bar{\theta}\right)}$$

in some neighborhood $\Omega_3 \subset \Omega_2$ of π. So $\bar{\Phi}$ is a function of $\left(\bar{W}^1, \bar{W}^2, \bar{\theta}\right)$, and its gradient must be a linear combination of the gradients of $\bar{W}^1, \bar{W}^2$ and $\bar{\theta}$. For $N \geq 4$, this translates into $\bar{D}_{a,b,c,d}(\bar{\pi}) = 0$ for all $a, b, c, d \in \{1, ..., N\}$. But this contradicts the fact that equation (13) holds for all π in Ω_2.

So we must have $F_3\left(\bar{W}^1(\pi), \bar{W}^2(\pi), \bar{\theta}(\pi)\right) = 0$ for all $\pi \in \Omega_2$. Then equation (17) becomes:

$$0 = F_2\left(\bar{W}^1(\pi), \bar{W}^2(\pi), \bar{\theta}(\pi)\right) \frac{\partial \bar{W}^2}{\partial \pi_1}(\pi)$$

which in turn implies $F_2\left(\bar{W}^1(\pi), \bar{W}^2(\pi), \bar{\theta}(\pi)\right) = 0$. We are left with $W^1(\pi) = F\left[\bar{W}^1(\pi)\right]$, as announced. ■

Proposition 7 states that, broadly speaking, a simple exclusivity condition is sufficient to obtain identifiability. Note that the result is specific to one of the functions: if W^1 does not depend on π_1, then W^1 is ordinally identifiable, irrespective of W^2 (which may not be).

5.2 Generalized separability

The previous result has an important extension, that generalizes the notion of separability. Let $\rho : \Omega_1 \to \mathbb{R}^N$ be given, where Ω_1 is a neighborhood of $\bar{\pi}$ in $\mathbb{R}^N$, and assume that

$$\frac{\partial \rho}{\partial \pi_N}(\pi) \neq 0 \quad \forall \pi \in \Omega_1 \tag{18}$$

Suppose four functions $\left(\bar{W}^1(\pi), \bar{W}^2(\pi), \bar{\lambda}_1(\pi), \bar{\lambda}_2(\pi)\right)$ satisfy equation (9) in some neighborhood $\Omega_2 \subset \Omega_1$ of $\bar{\pi}$. We shall say that this solution is *generic* if $\frac{\partial \bar{W}^1}{\partial \pi_N}$ does not vanish on Ω_1, $D_\pi \bar{W}^1(\bar{\pi})$, $D_\pi \bar{W}^2(\bar{\pi})$, $D_\pi \bar{\theta}(\bar{\pi})$ are linearly independent, and:

$$\frac{\partial \bar{W}^2}{\partial \pi_1}\left[\frac{\partial \bar{W}^2}{\partial \pi_N}\right]^{-1}, \frac{\partial \bar{\theta}}{\partial \pi_1}\left[\frac{\partial \bar{\theta}}{\partial \pi_N}\right]^{-1} \text{ and } \frac{\partial \rho}{\partial \pi_1}\left[\frac{\partial \rho}{\partial \pi_N}\right]^{-1} \tag{19}$$

are well defined and pairwise distinct on Ω_1 (with $\bar{\theta} = \bar{\lambda}_1/\bar{\lambda}_2$, as above).

We now introduce a generalized separability conditions, which states that all solutions depend on two particular variables, say π_1 and π_N, only through the same function ρ. This is a well-known consequence of separability in standard consumer theory (when a set of commodities is separable, their demand depends on the other prices only through an income effect), which explains the expression 'generalized separability'.

Technically, let ρ be a given, smooth function. We shall say that $\bar{W}^1$ is *separable for* ρ if:

$$\frac{\partial \rho}{\partial \pi_N}(\pi)\frac{\partial \bar{W}^1}{\partial \pi_1}(\pi) = \frac{\partial \bar{W}^1}{\partial \pi_N}(\pi)\frac{\partial \rho}{\partial \pi_1}(\pi) \tag{20}$$

Corollary 8 *Let $N \geq 4$. Assume that $\left(\bar{W}^1, \bar{W}^2, \bar{\lambda}_1, \bar{\lambda}_2\right)$ is a generic solution of equation (9)on Ω_2 and that $\bar{W}^1$ is separable for ρ. Let $(W^1, W^2, \lambda_1, \lambda_2)$ be another solution on Ω_2, with W^1 separable for the same ρ. Then there exist a function F and a neighborhood $\Omega_3 \subset \Omega_2$ of $\bar{\pi}$ such that $W_1 = F\left(\bar{W}_1\right)$ on Ω_3, i.e. W_1 is ordinally identified.*

Proof. *We claim that W^1 can be written under the form:*

$$W^1(\pi) = K^1\left[\pi_2, ..., \pi_{N-1}, \rho(\pi)\right] \tag{21}$$

Indeed, the map:

$$(\pi_1, \pi_2, ..., \pi_N) \rightarrow (\pi_1, \pi_2, ..., \pi_{N-1}, \rho(\pi)) \tag{22}$$

is invertible in a neighborhood Ω_3 *of* $\bar{\pi}$ *by condition (18). So* W^1 *can be written as:*

$$W^1(\pi) = K^1[\pi_1, \pi_2, ..., \pi_{N-1}, \rho(\pi)]$$

Differentiating, we get:

$$\begin{aligned}
\frac{\partial W^1}{\partial \pi_1} &= \frac{\partial K^1}{\partial \pi_1} + \frac{\partial K^1}{\partial \rho}\frac{\partial \rho}{\partial \pi_1} \\
\frac{\partial W^1}{\partial \pi_N} &= \frac{\partial K^1}{\partial \rho}\frac{\partial \rho}{\partial \pi_N} \\
\frac{\partial W^1}{\partial \pi_1}\left[\frac{\partial W^1}{\partial \pi_N}\right]^{-1} &= \frac{\partial K^1}{\partial \pi_1}\left[\frac{\partial K^1}{\partial \rho}\frac{\partial \rho}{\partial \pi_N}\right]^{-1} + \frac{\partial \rho}{\partial \pi_1}\left[\frac{\partial \rho}{\partial \pi_N}\right]^{-1}
\end{aligned}$$

Condition (20) then implies that $\partial K^1/\partial \pi_1 = 0$ *for all* π*, and (21) is proved. Taking* $(\pi_1, \pi_2, ..., \pi_{N-1}, \rho)$ *as local coordinates, we find that* $W^1 = K^1(\pi_2, ..., \pi_{N-1}, \rho)$ *does not depend on the first coordinate, and proposition 7 applies.* ■

Corollary 8 shows that generalized separability is but a particular form of exclusivity restriction, so that the general identifiability result applies in that case as well. As it turns out, while Proposition 7 directly relates to public goods, Corollary 8 is easier to use in a private good context.

Finally, note that if either the exclusivity or the generalized separability property apply to both members, then both W^1 and W^2 are ordinally identifiable. Then for any cardinalization of W^1 and W^2, the coefficients λ^i are identifiable as well.

5.3 The meaning of genericity.

Before discussing the economic interpretation of the previous results, a remark is in order. In Proposition 7 and Corollary 8, identifiability is only 'generic'. Technically, what Proposition 7 shows is that identifiability obtains unless the structural functions $\bar{W}^1, \bar{W}^2, \bar{\theta}$ satisfy a set of partial differential equations that can be explicitly derived. It is thus important to understand the exact scope of these partial differential equations, and particularly to discuss examples in which they are satisfied everywhere.

An obvious case in which the equations are always fulfilled obtains when $\bar{\theta}$ is constant: $\bar{\Phi}$ is then identically null, and so are the determinants $\bar{D}_{a,b,c,d}$ in condition (15). This result is by no means surprising. Indeed, if $\bar{\theta}$ is a constant (say k), then $\bar{\lambda}^2(\pi) = k\bar{\lambda}^1(\pi)$ for all π, and (9) becomes:

$$
\begin{aligned}
\xi(\pi) &= \lambda^1(\pi)\left[D_\pi W^1(\pi) + kD_\pi W^2(\pi)\right] \\
&= \lambda^1(\pi)\, D_\pi\left[W^1(\pi) + kW^2(\pi)\right]
\end{aligned}
$$

In that case, the function ξ is in fact proportional to a single gradient; economically, the group behaves as a single consumer with an indirect utility equal to $W^1(\pi) + kW^2(\pi)$. The non identifiability conclusion is indeed expected in that case: clearly, there exists a continuum of pairs $(W^1(\pi), W^2(\pi))$ that add up to the same weighted sum $W^1(\pi) + kW^2(\pi)$. Note that exclusivity does not help here, unless it applies to all variables. If there exists one variable, say N, that enter both utilities, then if $(W^1(\pi), W^2(\pi))$ is a solution, so is $(W^1(\pi) + kf(\pi_N), W^2(\pi) - f(\pi_N))$ for any arbitrary function f.

We conclude that when a group behaves as a single consumer, then individual preferences are not identifiable. Ironically, a large fraction of the literature devoted to household behavior tends to assume a unitary setting, in which the group is described as a unique decision maker. Our conclusions show that this approach, while analytically convenient, entails a huge cost, since it precludes the (non parametric) identification of individual consumption and welfare. In a general sense, *non unitary models are indispensable for addressing issues related to intrahousehold allocation.*[11]

[11]More generally, Proposition 7 does not apply if $\partial\bar{\theta}(\pi)/\partial\pi_1 = 0$; again, Φ is then the null function. It is thus important to check that this condition is satisfied. For empirical applications, however, we shall see that it is likely to hold unless $\bar{\theta}$ is constant.

Finally, except for these particular cases, the identifiability result is quite robust. It is easy to check, for instance, that whenever a particular solution $\left(\bar{W}^1, \bar{W}^2, \bar{\theta}\right)$ is non-generic, so so that the equations (**??**) and (13) are satisfied, the property is not robust to slight perturbations of either preferences or weights. Consider, for instance, linear versions of all functions:

$$\bar{\lambda}^1\left(\pi\right) = \sum_i \lambda_i^1 \pi_i, \bar{\lambda}^2\left(\pi\right) = \sum_i \lambda_i^2 \pi_i$$

so that:

$$\bar{\theta}\left(\pi\right) = \frac{\sum_i \lambda_i^1 \pi_i}{\sum_i \lambda_i^2 \pi_i}, \quad \bar{W}^1\left(\pi\right) = \sum_i w_i^1 \pi_i, \bar{W}^2\left(\pi\right) = \sum_i w_i^2 \pi_i$$

Assume that the conditions (16) are satisfied:

$$w_1^1 = 0, w_1^2 \neq 0, \sum_i \left(\lambda_1^1 \lambda_i^2 - \lambda_1^2 \lambda_i^1\right) \pi_i \neq 0$$

(note that the last inequality is satisfied whenever one at least of the $\left(\lambda_1^1 \lambda_i^2 - \lambda_1^2 \lambda_i^1\right)$ is non zero - i.e., whenever λ^1 and λ^2 are not proportional).

Then

$$\bar{\Phi}\left(\pi\right) = \frac{\partial \bar{\theta} / \partial \pi_1}{\partial \bar{W}^2 / \partial \pi_1} = \frac{\sum_i \left(\lambda_1^1 \lambda_i^2 - \lambda_1^2 \lambda_i^1\right) \pi_i}{w_1^2 \left(\sum_i \lambda_i^2 \pi_i\right)^2}$$

therefore

$$\bar{D}_{a,b,c,d} = \begin{vmatrix} \frac{\left(\lambda_1^1\lambda_a^2-\lambda_1^2\lambda_a^1\right)\left(\sum_i \lambda_i^2\pi_i\right)-2\lambda_a^2\left(\sum_i\left(\lambda_1^1\lambda_i^2-\lambda_1^2\lambda_i^1\right)\pi_i\right)}{w_1^2\left(\sum_i \lambda_i^2\pi_i\right)^3} & w_a^1 & w_a^2 & \frac{\sum_i\left(\lambda_a^1\lambda_i^2-\lambda_a^2\lambda_i^1\right)\pi_i}{\left(\sum_i \lambda_i^2\pi_i\right)^2} \\ \frac{\left(\lambda_1^1\lambda_b^2-\lambda_1^2\lambda_b^1\right)\left(\sum_i \lambda_i^2\pi_i\right)-2\lambda_b^2\left(\sum_i\left(\lambda_1^1\lambda_i^2-\lambda_1^2\lambda_i^1\right)\pi_i\right)}{w_1^2\left(\sum_i \lambda_i^2\pi_i\right)^3} & w_b^1 & w_b^2 & \frac{\sum_i\left(\lambda_b^1\lambda_i^2-\lambda_b^2\lambda_i^1\right)\pi_i}{\left(\sum_i \lambda_i^2\pi_i\right)^2} \\ \frac{\left(\lambda_1^1\lambda_c^2-\lambda_1^2\lambda_c^1\right)\left(\sum_i \lambda_i^2\pi_i\right)-2\lambda_c^2\left(\sum_i\left(\lambda_1^1\lambda_i^2-\lambda_1^2\lambda_i^1\right)\pi_i\right)}{w_1^2\left(\sum_i \lambda_i^2\pi_i\right)^3} & w_c^1 & w_c^2 & \frac{\sum_i\left(\lambda_c^1\lambda_i^2-\lambda_c^2\lambda_i^1\right)\pi_i}{\left(\sum_i \lambda_i^2\pi_i\right)^2} \\ \frac{\left(\lambda_1^1\lambda_d^2-\lambda_1^2\lambda_d^1\right)\left(\sum_i \lambda_i^2\pi_i\right)-2\lambda_d^2\left(\sum_i\left(\lambda_1^1\lambda_i^2-\lambda_1^2\lambda_i^1\right)\pi_i\right)}{w_1^2\left(\sum_i \lambda_i^2\pi_i\right)^3} & w_d^1 & w_d^2 & \frac{\sum_i\left(\lambda_d^1\lambda_i^2-\lambda_d^2\lambda_i^1\right)\pi_i}{\left(\sum_i \lambda_i^2\pi_i\right)^2} \end{vmatrix} \tag{23}$$

The determinant $\bar{D}_{a,b,c,d}(\bar{\pi})$ is a polynomial of degree 21 in (λ, w). The set of values (λ, w) for which it vanishes clearly has measure zero, so identifiability holds for almost all values of the true parameters.

6 Application: identifiability in the collective model ($S = 2$)

We may now specialize our result to specific economic contexts.

6.1 Identifiability with purely public consumptions.

We first consider the benchmark case where all commodities are publicly consumed; then $n = 0$ and $N = K$, and the problem can be expressed as (see section 3.3.1):

$$\sum_{s=1,2} \gamma^s D_X U^s = P(X, y)$$

where $\gamma^s = \mu^s/\gamma$. In that case, the exclusivity assumption has a natural translation, namely that commodity 1 (resp. 2) is exclusively consumed by member 1 (resp. 2). We conclude from Proposition 7 that U^1 and U^2 are ordinally identifiable. Moreover, for any cardinalization of these utilities, the coefficients γ^s are exactly identifiable. Since they are proportional to the Pareto weights and that the latter add up to one, we conclude that the Pareto weights are identifiable as well. In summary:

Corollary 9 *In the collective model with two agents and public consumption only, if member 1 does not consume at least one good, then generically the utility of member 1 is exactly (ordinally) identifiable from household demand. If each member is the exclusive consumer of at least one good, then generically individual preferences are exactly (ordinally) identifiable from household demand; and for any cardinalization of individual utilities, the Pareto weights are exactly identifiable.*

We may briefly discuss the conditions needed for identifiability. One is that $\frac{\partial W^2(\pi)}{\partial \pi_1} \neq 0$, i.e. in this case $\frac{\partial U^2(\pi)}{\partial X^1} \neq 0$. Clearly, if commodity one is valued neither by member 1 nor by member 2, household demand for this good is zero and cannot be used for identification.

More demanding is the requirement that $\frac{\partial \theta}{\partial X^1} \neq 0$. In our context, θ is the ratio of individual Pareto weights. In general, Pareto weights are functions of prices P and income y. In Proposition 7, however, θ is expressed as a function of (X, y) to exploit the private/public goods duality. The link between the two is straightforward: if μ denotes the ratio of Pareto weights, we have $\theta(X, y) = \mu[P(X, y), y]$, and hence:

$$\frac{\partial \theta}{\partial X^1} = \sum_{k=1}^{K} \frac{\partial \mu}{\partial P_k} \frac{\partial P_k}{\partial X^1}$$

This partial $\frac{\partial \theta}{\partial X^1}$ cannot be zero unless the vector $\left(\frac{\partial P_1}{\partial X^1}, ..., \frac{\partial P_K}{\partial X^1}\right)$ is orthogonal to the gradient of μ in P. This will surely occur if μ is constant, since the gradient is null. This is the case, discussed above, in which the household behaves as a single decision maker and maximizes the unitary utility $U^1 + \mu U^2$. If μ is not constant, however, then $\frac{\partial \theta}{\partial X^1} = 0$ leads to the equation:

$$\sum_{k=1}^{K} \frac{\partial \mu}{\partial P_k} [P(X, y), y] \frac{\partial P_k}{\partial X^1}(X, y) = 0$$

Taking (P, y) as independent variables instead of (X, y), we get an equation of the form:

$$\sum_{k=1}^{K} \frac{\partial \mu}{\partial P_k}(P, y)\, \varphi_k(P, y) = 0 \tag{24}$$

where the $\varphi_k, 1 \leq k \leq K$ depend on μ, U^1 and U^2. It can be shown that, generically[12] in (μ, U^1, U^2), equation (24) defines a submanifold Σ of codimension 1 (and hence a set of measure 0) in $\mathbb{R}^K \times \mathbb{R}$. In other words, if the actual μ, U^1 and U^2 are generic (in particular, if μ is not constant), there is a set Σ of measure 0 such that, whenever $(P, y) \notin \Sigma$, Pareto weights and preferences can be uniquely recovered from observing the collective demand function $X(P, y)$ near $(\bar{P}, \bar{y})$.

6.2 Application: collective models of labor supply with public consumptions

An immediate application is to the collective model of household labor supply, initially introduced by Chiappori (1988, 1992). The idea is to consider

[12] Here, genericity is taken in the sense of Thom; it means that the set $\mathcal{N}$ of (μ, U^1, U^2) where the property does not hold is small in an appropriate sense. To be precise, it is, in an appropriate function space, a countable union of closed sets with empty interiors. As a consequence, it has itself an empty interior, so that if $(\bar{\mu}, \bar{U}^1, \bar{U}^2)$ happens to belong to $\mathcal{N}$, there are neighbours (μ, U^1, U^2), which are as close as one wishes to $(\bar{\mu}, \bar{U}^1, \bar{U}^2)$ and which do not belong to $\mathcal{N}$.

the household as a two-person group making Pareto efficient decisions on consumption and labor supply; let L^s denote the leisure of member s, and w_s the corresponding wage. Various versions of the model can be considered. In each of them Corollary 9 applies, leading to full identifiability of the model.

1. **Leisure as an exclusive good**

   In the first model, each member's leisure is exclusive and there is no household production. Labor and non labor incomes are used to purchase commodities $X^1, ..., X^K$ that are publicly consumed within the household; utilities are thus of the form $U^s\left(L^s, X^1, ..., X^K\right)$.

2. **Leisures are public, one exclusive good per member**

   In the second model, leisure of one member is also consumed by the other member; again, there is no household production. The identifying assumption is that there exists two commodities (say, 1 and 2) such that commodity i is exclusively consumed by member s.[13] One can think, for instance, of clothing as the exclusive commodity (as in Browning et al 1994), but many other examples can be considered. Utilities are then of the form $U^s\left(L^1, L^2, X^s, X^3, ..., X^K\right)$. Again, Proposition

[13] This framework is close to (but less general than) that of Fong and Zhang (2000)

9 applies: from the observation of the two labor supplies and the K consumptions as functions of prices, wages and non labor income, it is possible to uniquely recover preferences and Pareto weights. This is a strong result indeed, since it states that one can, from the sole observation of household labor supply and consumption, identify the partials $\partial U^i / \partial L^j$, $i \neq j$, that is, deduce to what extent individual leisures are publicly consumed.

3. **Leisure as exclusive goods with household production**

As a third example, assume that individual time can be devoted to three different uses: leisure, market work and household production. The domestic good Y is produced from domestic labor under some constant return to scale technology, say $Y = f(t^1, t^2)$,[14] and publicly consumed within the household. Preferences are of the form $U^s\left(L^s, Y, X^1, ..., X^K\right)$ and one can define

$$\tilde{U}^s\left(L^s, t^1, t^2, X^1, ..., X^K\right) = U^s\left(L^s, f\left(t^1, t^2\right), X^1, ..., X^K\right) \tag{25}$$

Here, Y is not observable in general, but t^1 and t^2 are observed, which

[14] Other inputs can be introduced at no cost, provided they are observable.

typically requires data over time use (obviously, there is little chance to identify household production if neither the output nor the input are observable).

From Proposition 9, the $\tilde{U}^s$ are identified. Then the production technology can be recovered up to a scaling factor, using the assumption of constant return to scale, from the relation:

$$\frac{\partial \tilde{U}^1/\partial t^1}{\partial \tilde{U}^1/\partial t^2} = \frac{\partial \tilde{U}^2/\partial t^1}{\partial \tilde{U}^2/\partial t^2} = \frac{\partial f/\partial t^1}{\partial f/\partial t^2}$$

which in addition generates an overidentifying restriction. Finally, (25) allows to recover the U^s; again, the separability property in (25) generates additional, testable restrictions.

4. **Leisures are public, one exclusive good per member and household production**

   Finally, one can combine models 2 and 3 by assuming that leisure is a public good, but the demand for two other exclusive goods can be observed. Again, identifiability generically obtains in this context.

6.3 Identifiability in the general case

We may now address the general case with private and public consumptions, that is, we want to solve the system of equations (5) for a given $A(p, X, y)$. A first remark is that a commodity exclusively consumed by a member can arbitrarily be considered as private or public. In what follows, we adopt the convention that such a good is always considered as private.

In the presence of private consumption, the problem is more complex, because the exclusivity assumption of the type introduced in Proposition 7 does not have a direct economic translation. Indeed, remember that:

$$W^s(p, X, y) = V^s(p, X, \rho^s(p, X, y))$$

hence:

$$\frac{\partial W^1}{\partial p_1} = \frac{\partial V^1}{\partial p_1} + \frac{\partial V^1}{\partial \rho^1}\frac{\partial \rho^1}{\partial p_1}$$

We see, in particular, that even when consumer 1 does not consume commodity 2, so that $\frac{\partial V^1}{\partial p_1} = \frac{\partial U^1}{\partial X^1} = 0$, we still have that $\frac{\partial W^1}{\partial p_1} = \frac{\partial V^1}{\partial \rho}\frac{\partial \rho^1}{\partial p_1} \neq 0$ in general. Intuitively, even when a public good is not consumed by an agent, the corresponding expenditures may still impact the agent's share of resources, therefore the agent's welfare, through an income effect.

However, Corollary 8 is now the relevant tool. Take a point $\left(\bar{p}, \bar{X}, \bar{y}\right)$ and assume that:

$$\frac{\partial \rho^1}{\partial y}\left(\bar{p}, \bar{X}, \bar{y}\right) \neq 0. \tag{26}$$

From now on, all results are understood to be local, that is, to hold in some neighborhood of $\left(\bar{p}, \bar{X}, \bar{y}\right)$.

Assume, now, that commodity 1 is not consumed by member 1, and commodity 2 is not consumed by member 2:

$$\frac{\partial V^i}{\partial p_i} = \frac{\partial U^i}{\partial X^i} = 0, \ i = 1, 2$$

It follows, first, that for any solution W:

$$\frac{\partial W^1\left(p, X, y\right) / \partial p_1}{\partial W^1\left(p, X, y\right) / \partial y} = \frac{\partial \rho^1\left(p, X, y\right) / \partial p_1}{\partial \rho^1\left(p, X, y\right) / \partial y}$$

Let $(U^1, U^2, \rho^1, \rho^2 = y - A - \rho^1)$ and $\left(\bar{U}^1, \bar{U}^2, \bar{\rho}^1, \bar{\rho}^2 = y - A - \bar{\rho}^1\right)$ be the utilities and conditional sharing rules corresponding to two different solutions of equations (5). We have:

$$\frac{\partial W^1 / \partial p_1}{\partial W^1 / \partial y} = \frac{\partial \rho^1 / \partial p_1}{\partial \rho^1 / \partial y} \text{ and } \frac{\partial \bar{W}^1 / \partial p_1}{\partial \bar{W}^1 / \partial y} = \frac{\partial \bar{\rho}^1 / \partial p_1}{\partial \bar{\rho}^1 / \partial y} \tag{27}$$

Consider the household demand x_1 for commodity 2. We have:

$$x_2(p, X, y) = \xi_2^1\left(p_2, ..., p_n, X, \rho^1(p, X, y)\right)$$
$$= \bar{\xi}_2^1\left(p_2, ..., p_n, X, \bar{\rho}^1(p, X, y)\right)$$

where ξ_i^1 (resp. $\bar{\xi}_i^1$) is the conditional Marshallian demand corresponding to U^1 (resp. $\bar{U}^1$). Therefore:

$$\frac{\partial\rho^1/\partial p_1}{\partial\rho^1/\partial y} = \frac{\partial x_2/\partial p_1}{\partial x_i/\partial y} = \frac{\partial\bar{\rho}^1/\partial p_1}{\partial\bar{\rho}^1/\partial y}. \tag{28}$$

Comparing (27) with (28), we get:

$$\frac{\partial\bar{W}/\partial p_1}{\partial\bar{W}^1/\partial y} = \frac{\partial\rho^1/\partial p_1}{\partial\rho^1/\partial y}$$

which is condition (20). Applying corollary 8, we get:

Corollary 10 *In the general, collective model with two agents, under assumption (26), if each member does not consume at least one good, then generically the indirect collective utility of each member is exactly (ordinally) identifiable from household demand. For any cardinalization of indirect collective utilities, the Pareto weights are exactly identifiable.*

Note, however, that the conditions (19) have to be satisfied. Although they hold true in a 'generic' sense, checking that they are fulfilled in a specific context may be tedious. In such cases, using distribution factors may facilitate identifiability (see below).

Links with the existing literature The statements derived above generalize several results existing in the literature.

1. In an earlier work, Chiappori (1992) analyzed a collective model of labor supply in three goods framework (two labor supplies and an Hicksian composite good). In this model, all commodities are privately consumed, and each member's labor supply is exclusive. Chiappori shows that:

    - efficiency is equivalent to the existence of a sharing rule
    - the sharing rule is identifiable from labor supply up to an additive constant; for any choice of the constant, individual preferences are exactly identified.
    - finally, the additive constant is welfare-irrelevant.

    Our paper generalizes these conclusions to a general framework, including an arbitrary number of private and public consumptions. In particular:

- the sharing rule of Chiappori's model (which entails private goods only) can be extended to a conditional sharing rule in the general context.
- identifiability up to an additive constant is only true in a three goods context; in the general case, the indeterminacy of direct utilities and the sharing rule is deeper. In fact, one can show that under exclusivity, ρ is determined up to an additive function of public consumptions and non exclusive prices; that is, if ρ is a solution, any other solution $\bar{\rho}$ is of the form

$$\bar{\rho}(p, X, y) = \rho(p, X, y) + H(p_3, ..., p_n, X)$$

for some mapping H.[15]

- however, the indeterminacy is welfare-irrelevant, which generalizes

[15] Exclusivity implies that

$$\frac{\partial \xi_2^1 / \partial p_1}{\partial \xi_2^1 / \partial y} = \frac{\partial \rho / \partial p_1}{\partial \rho / \partial y}$$

and

$$\frac{\partial \xi_1^2 / \partial p_2}{\partial \xi_1^2 / \partial y} = \frac{\partial (A - \rho) / \partial p_2}{\partial (A - \rho) / \partial y} \text{ hence } \frac{\partial \rho}{\partial p_2} \equiv L \frac{\partial \rho}{\partial y} + M$$

Chiappori's conclusion.

2. In a recent paper, Blundell, Chiappori and Meghir (from now on BCM) analyze a similar model, with the difference that the non exclusive good is public (and can be interpreted as children expenditures or welfare). They prove that their model is generically identifiable.[16] The identifiability result in BCM stems directly from Corollary 9 (the model is

---

where

$$L = \frac{\partial \xi_1^2 / \partial p_2}{\partial \xi_1^2 / \partial y}, M = \frac{\partial A}{\partial p_2} - \frac{\partial \xi_1^2 / \partial p_2}{\partial \xi_1^2 / \partial y} \frac{\partial A}{\partial y}$$

The first condition implies that for any $\bar{\rho}$,

$$\bar{\rho}(p, X, y) = H(\rho(p, X, y), p_2, p_3, ..., p_n, X)$$

hence

$$\frac{\partial \bar{\rho}}{\partial p_2} = \frac{\partial H}{\partial \rho} \frac{\partial \rho}{\partial p_2} + \frac{\partial H}{\partial p_2} \quad \text{and} \quad L \frac{\partial \bar{\rho}}{\partial y} + M = L \frac{\partial H}{\partial \rho} \frac{\partial \rho}{\partial y} + M$$

and the second becomes

$$\frac{\partial H}{\partial \rho} \left(L \frac{\partial \rho}{\partial y} + M \right) + \frac{\partial H}{\partial p_2} = L \frac{\partial H}{\partial \rho} \frac{\partial \rho}{\partial y} + M$$

or

$$\frac{\partial H}{\partial p_2} = \left(1 - \frac{\partial H}{\partial \rho} \right) M$$

If $\frac{\partial H}{\partial \rho} \neq 1$ then

$$M = \frac{\partial H / \partial p_2}{1 - \partial H / \partial \rho}$$

and the function M can be written as a function of $(\rho, p_2, p_3, ..., p_n, X)$. Generically, this cannot be the case; therefore $\frac{\partial H}{\partial \rho} = 1$, $\frac{\partial H}{\partial p_2} = 0$, hence the conclusion.

[16] BCM provide a detailed analysis of the comparative statics of such a model and of its welfare implications. They show, in particular, that increasing the Pareto weight of a particular member increases the household demand for a public good if and only if the person's marginal willingness to pay for the public good is more income sensitive than that of her spouse.

a particular case of the 'public goods only' version of our setting, in the case $K = 3$). Again, the present paper shows that identifiability obtains in the BCM context with an arbitrary number of public goods, and actually an arbitrary number of private and public consumptions.

6.4 Identifiability using distribution factors

The results derived so far do not use distribution factors. We now show how such factors can help the identification process by alleviating the necessary conditions required. Let z denote a distribution factor which is behavior relevant, in the sense that $\partial\theta/\partial z \neq 0$ where $\theta = \lambda_1/\lambda_2$ as usual. Also, we maintain the regularity assumption (26). From:

$$W^1(p, X, y, z) = V^1\left(p, X, \rho^1(p, X, y, z)\right)$$

we get that:

$$\frac{\partial W^1/\partial z}{\partial W^1/\partial y} = \frac{\partial \rho^1/\partial z}{\partial \rho^1/\partial y} \tag{29}$$

and corollary 8 applies if we can show that the right-hand side does not depend on the particular solution considered. This can be done, as before, when each member is the exclusive consumer of (at least) one commodity.

But this requirement can be relaxed. We only need either one exclusive good (instead of two) or an *assignable* commodity. A good is assignable when it is consumed by both members, and the consumption of each member is independently observed.

Assume that good 1 is either assignable or exclusively consumed by member 1, so that $x_1^1(p, X, y, z)$ is observed. Now, for any two solutions, we have that

$$x_1^1(p, X, y, z) = \xi_1^1\left(p, X, \rho^1(p, X, y, z)\right) = \bar{\xi}_1^1\left(p, X, \bar{\rho}^1(p, X, y, z)\right)$$

where ξ_1^1 and $\bar{\xi}_1^1$ denote the conditional demand of member 1 in each solution. Therefore:

$$\frac{\partial \rho^1/\partial z}{\partial \rho^1/\partial y} = \frac{\partial x_1^1/\partial z}{\partial x_1^1/\partial y} = \frac{\partial \bar{\rho}^1/\partial z}{\partial \bar{\rho}^1/\partial y}$$

and the right-hand side of (29) does not depend on the particular solution considered, so that identifiability obtains by corollary 8.

The genericity condition (19) may also be easier to check in that case. Consider, in particular, the case of private goods only. Then

$$W^2(p, y, z) = V^2\left(p, \rho^2(p, y, z)\right) = V^2\left(p, y - \rho^1(p, y, z)\right)$$

hence:

$$\frac{\partial W^2/\partial z}{\partial W^2/\partial y} = \frac{\partial \rho^2/\partial z}{\partial \rho^2/\partial y} = -\frac{\partial \rho^1/\partial z}{1 - \partial \rho^1/\partial y}$$

which cannot equal $\frac{\partial \rho^1/\partial z}{\partial \rho^1/\partial y}$ unless $\partial \rho^1/\partial z = 0$, i.e. unless the distribution factor is behavior irrelevant.

7 The general case ($S \geq 2$)

Finally, how do these results generalize to groups of arbitrary sizes? We only indicate here the main results. First, Lemma 5 extends as follows. Consider the equation

$$\xi(\pi) = \sum_{s=1}^{S} \lambda^s(\pi) D_\pi W^s$$

where $\pi \in \mathbb{R}^N$ and ξ maps $\mathbb{R}^N$ into itself. Let $\left(\bar{W}^1, ..., \bar{W}^S, \bar{\lambda}^1, ..., \bar{\lambda}^S\right)$ be a particular solution. Under a regularity condition, for any other solution $\left(W^1, ..., W^S, \lambda^1, ..., \lambda^S\right)$, there exists S functions $F^1, ..., F^S$ such that:

$$W^1(\pi) = F^1\left[\bar{W}^1, ..., \bar{W}^S, \frac{\bar{\lambda}^1}{\bar{\lambda}^S}, ..., \frac{\bar{\lambda}^{S-1}}{\bar{\lambda}^S}\right]$$

$$\vdots$$

$$W^S(\pi) = F^S\left[\bar{W}^1, ..., \bar{W}^S, \frac{\bar{\lambda}^1}{\bar{\lambda}^S}, ..., \frac{\bar{\lambda}^{S-1}}{\bar{\lambda}^S}\right]$$

where the F^s satisfy moreover a set of partial differential equations. Secondly, identifiability under exclusivity still holds in the following sense. Assume that there exists some variable, say π_1, such that $\frac{\partial W^1}{\partial \pi_1} = 0$. Then generically W^1 is identifiable. The generalized separability property extends in a similar manner. However, both results require at least $2S$ commodities to apply.

Regarding economic implications, we conclude that while the general model (without exclusivity) is clearly not identifiable, one exclusive good per agent is sufficient to obtain generic identification in the general case, with the same caveats as above. The techniques are similar to (although more tedious than) those used above.

Perhaps more important is the conclusion in the presence of *one* distribution factor (at least). Indeed, the argument previously described for two agents directly extends to $S \geq 2$. For any two solutions W^s and $\bar{W}^s$, we have that

$$\frac{\partial W^s/\partial z}{\partial W^s/\partial y} = \frac{\partial x_s^1/\partial z}{\partial x_s^1/\partial y} = \frac{\partial \bar{W}^s/\partial z}{\partial \bar{W}^s/\partial y}$$

Provided that the genericity conditions are satisfied (in particular, the $\frac{\partial \rho^s/\partial z}{\partial \rho^s/\partial y}$ must be pairwise different), we then conclude that the W^s are (ordinally) exactly identified, without restriction on the number of distribution factors.

8 Conclusion

The main goal of the paper is to assess under which conditions the aggregate behavior of a group provides enough information to recover the underlying structure (i.e., preferences and the decision process) even when nothing is known (or observed) about the intra household decision making mechanism beyond efficiency. We reach two main conclusions. First, the general version of the model is not identifiable: a continuum of different models generate the same household demand function. Secondly, the existence of an exclusive good for each member is sufficient to generically guarantee full identifiability of the collective indirect utility of each member, i.e. of the welfare-relevant concept that summarize preferences and the decision processes.

We adopt throughout the paper a 'non parametric' standpoint, in the sense that our results do not rely on specific functional form assumptions. Obviously, the introduction of a particular functional form is likely to considerably facilitate identifiability; that is, it may well be the case that, for models that are not identifiable in the non parametric sense, all parameters of a given functional form can be exactly identifiable, even when the form is quite flexible. In the end, the results above show that not much is needed to formulate normative judgements that take into account the complex nature of collective decision processes; one exclusive commodity per agent is 'generically' sufficient.

Constant Pareto weights Our identification is only 'generic', in the sense that one can always construct examples in which it does not hold. A polar case in which identifiability does not obtain is when Pareto weights are constant; then we are in a 'unitary' context, since the group behaves as a single consumer. We conclude that non-unitary models are necessary for identifiability.

It is fair to argue, however, that in basically any 'reasonable', non unitary model that generates Pareto efficient allocations, Pareto weights will not be constant. Take, for instance, a simple bargaining framework, as used (among many others) by Mc Elroy (1990) or Chiappori and Donni (2005). A key role in determining the outcome on the Pareto frontier is played by agents' 'threat points'. Various versions have been proposed for these: utilities in case of divorce, non cooperative outcomes, 'separate sphere' allocations, etc. In all cases, however, the threat point is an indirect utility, representing what the welfare an agent would achieve in some particular context; as such, they are price dependent.

The exclusivity assumption These ideas have actually been empirically implemented in a number of existing contributions. Most of the time, the exclusive good is taken to be leisure, as in Fortin and Lacroix (1997), Blundell et al. (2000), Dauphin and Fortin (2001), Vermeulen (2001), Chiappori, Fortin and Lacroix (2002), Dauphin (2003), Mazzocco (2003a,b) and Donni (2004). Exclusivity of leisure is certainly a strong assumption, for two reasons. First, household members may spend time on household production; note, however, that household production can readily be taken into account in this context, as discussed in Subsection 7.2 and in Apps and Rees (1996, 1997), Chiappori (1988b) and Rapoport et al. (2003). Secondly, leisure may be partly public, in the sense that the husband may derive direct utility from his wife's leisure and conversely. Therefore, if leisure is 'mainly' private (in the sense that the externalities just mentioned are of second order), the approach adopted in the literature is valid. If, on the contrary, public effects are paramount and drive most couples' behavior, the strategy under consideration may be problematic. In the end, whether leisure may be considered as exclusive is an empirical issue. It is therefore interesting to note that (i) almost none of the empirical tests of the collective model with exclusive labor supply reject the specific predictions generated by the exclusivity assumption, and (ii) in particular, papers aimed at comparing the respective empirical performances of the unitary and the collective approach with exclusive household labor supply invariably favor the latter (Fortin and Lacroix 1997, Vermeulen 2001).

Anyhow, the results derived in the present paper show that the exclusivity assumption can be relaxed in two directions. First, in the presence of distribution factors, one leisure only needs to be exclusive. This finding should lead to new empirical approaches. Secondly, even if both individual labor supplies are public, the model remains identifiable provided that two alternative, exclusive commodities (one only with a distribution factor) can be found. See Fong and Zhang (2001) for a careful analysis along these lines.

An alternative strategy considers specific consumption goods as exclusive. For instance, several papers, following an early contribution by Browning et al. (1984), use clothing expenditures. While assignability of clothing is not too problematic (except maybe for same sex couples), exclusivity may be trickier in our context, since it requires different prices for the various exclusive goods. In practice, while different prices for male and female clothing may be available, they tend to be highly correlated. Here, the result, demonstrated in the present paper, that assignability is sufficient for identifiability in the presence of a distribution factor provides a strong justification to these approaches.

Identification Finally, what about actual identification of an identifiable model? Specifically, what type of stochastic structure should be used, and to what data can it be applied?

Data set are readily available (and have been used) for models of household labor supply (see the abundant literature referred to above). The case of consumption goods is somewhat different. A standard problem of demand analysis is that while income effects are usually easy to estimate, the measurement of price effects require price variations, which may be hard to obtain. Two strategies have been used in the literature. One relies on variations through time and regions (see for instance Browning and Chiappori 1998 and Browning, Chiappori and Lewbel 2005). Alternatively, one may use natural experiments to generate exogenous price variations. For instance, Kapan (2006) uses Turkish data collected in a period of high inflation; similar households then face widely different relative prices depending on the month during which they are surveyed. Both works reject the unitary model when applied to couples, but not for singles; moreover, none rejects the collective model for couples.

Regarding the stochastic structure, on the other hand, existing works use mostly the standard approach of consumer theory, in which stochastic error terms (reflecting measurement errors, but also unobserved heterogeneity, etc.) enter demand equations additively. Identification is in general easy to achieve in this context. Some models, however consider more sophisticated stochastic structure, and discuss the related identification issues; see Blundell et al (2000) in the case of labor supply.

All in all, it is fair to conclude that much still has to be done in terms of empirical applications of the collective model. The main goal of the present paper is to provide a theoretical background for such works. One thing seems however clear: whenever issues related to intrahousehold allocation of commodities, welfare or decision power are at stake, non unitary models are needed. The collective approach provides a consistent and promising theoretical framework for that purpose.

APPENDIX

Proof of Lemma 5

The proof is in three steps:

1. Consider the 1-form $\omega = \sum \xi_i d\pi_i$. For information about differential forms, we refer to Chiappori and Ekeland (1999, 2004) and to the references therein. Equation (9) means that $\omega = \bar{\lambda}_1 d\bar{W}^1 + \bar{\lambda}_2 d\bar{W}^2$. Differentiating, and then taking wedge products, we get:

$$d\omega = d\bar{\lambda}_1 \wedge d\bar{W}^1 + d\bar{\lambda}_2 \wedge d\bar{W}^2 \tag{30}$$

$$\omega \wedge d\omega = \left(\bar{\lambda}_1 d\bar{\lambda}_2 - \bar{\lambda}_2 d\bar{\lambda}^1\right) \wedge d\bar{W}^1 \wedge d\bar{W}^2 \tag{31}$$

$$= -\left(\bar{\lambda}_2\right)^2 d\left(\frac{\bar{\lambda}_1}{\bar{\lambda}_2}\right) \wedge d\bar{W}^1 \wedge d\bar{W}^2 \tag{32}$$

$$d\omega \wedge d\omega = 2\ d\bar{\lambda}_1 \wedge d\bar{W}^1 \wedge d\bar{\lambda}_2 \wedge d\bar{W}^2 \tag{33}$$

Now, following Ekeland and Nirenberg (2002), we introduce two sets of differential forms:

$$\mathcal{E}_1 = \{\alpha \mid \alpha \wedge d\omega \wedge d\omega = 0\}$$

$$\mathcal{E}_2 = \{\alpha \in \mathcal{E}_1 \mid \alpha \wedge \omega \wedge d\omega = 0\}$$

Both $\mathcal{E}_1$ and $\mathcal{E}_2$ are linear subspaces, and differential ideals. By the Frobenius theorem (see Bryant et al. (1991), ch. 2. Theorem 1.1), $d\left(\frac{\bar{\lambda}_1}{\bar{\lambda}_2}\right)$, $d\bar{W}^1, d\bar{W}^{\ 2}$ form a linear basis of $\mathcal{E}_2$.

If $(W^1, W^2, \lambda_1, \lambda_2)$ is another solution of (9), then $\omega = \lambda_1 dW^1 + \lambda_2 dW^2$, so that formulas (30), (31), (33) hold with $(W^1, W^2, \lambda_1, \lambda_2)$ replacing $(\bar{W}^1, \bar{W}^2, \bar{\lambda}_1, \bar{\lambda}_2)$. It follows that $d\left(\frac{\lambda_1}{\lambda_2}\right)$, $dW^1, dW^{\ 2}$belong to $\mathcal{E}_2$, and by the Frobenius theorem they are linear combinations of $d\left(\frac{\bar{\lambda}_1}{\bar{\lambda}_2}\right)$, $d\bar{W}^1, d\bar{W}^{\ 2}$. It follows that there are functions F, G, H such that (10) and (11) hold, together with:

$$\frac{\lambda_1}{\lambda_2}(\pi) = H\left[\bar{W}^1(\pi), \bar{W}^2(\pi), \frac{\bar{\lambda}_1}{\bar{\lambda}_2}(\pi)\right]$$

2. Set $\bar{\theta} = \frac{\bar{\lambda}^1}{\bar{\lambda}^2}$, $F_i = \frac{\partial F}{\partial t_i}$ and $G_i = \frac{\partial G}{\partial t_i}$. From (10), we have:

$$\omega = \lambda_1 dW^1 + \lambda_2 dW^2 = (\lambda_1 F_1 + \lambda_2 G_1)\, d\bar{W}^1 + (\lambda_1 F_2 + \lambda_2 G_2)\, d\bar{W}^2 + (\lambda_1 F_3 + \lambda_2 G_3)\, d\bar{\theta}$$

Comparing with $\omega = \bar{\lambda}_1 d\bar{W}^1 + \bar{\lambda}_2 d\bar{W}^2$, we get:

$$\lambda^1 F_1 + \lambda^2 G_1 = \bar{\lambda}$$

$$\lambda^1 F_2 + \lambda^2 G_2 = \bar{\mu}$$

$$\lambda^1 F_3 + \lambda^2 G_3 = 0$$

which gives (12).

3. Once W^1 and W^2 are determined, λ^1 and λ^2 follow from $\omega = \lambda_1 dW^1 + \lambda_2 dW^2$.

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