

Public Goods, Transferable Utility and Divorce Laws*

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Abstract

We reconsider the well known Becker-Coase (*BC*) argument, according to which changes in divorce laws should not affect divorce rates, in the context of households which consume public goods in addition to private goods. For this result to hold, utility must be transferable both within marriage and upon divorce, and the marginal rate of substitution between public and private consumption needs to be invariant in marital status. We develop a model in which couples consume public goods and show that if divorce alters the way some goods are consumed (either because some goods that are public in marriage become private in divorce or because divorce affects the marginal rate of substitution between public and private goods), then the Becker-Coase theorem holds only under strict quasi-linearity. We conclude that, in general, divorce laws will influence the divorce rate, although the impact of a change in divorce laws can go in either direction.

1 Introduction

A highly pertinent question in the economics of the family is whether changes in divorce laws would impact divorce rates. In the words of Becker (1993, p. 331), “A husband and wife would both consent to a divorce if, and only if, they both expected to be better off divorced. Although divorce might seem more difficult when mutual

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consent is required than when either alone can divorce at will, the frequency and incidence of divorce should be similar with these and other rules if couples contemplating divorce can easily bargain with each other. This assertion is a special case of the Coase theorem (1960) and is a natural extension of the argument [...] that persons marry each other if, and only if, they both expect to be better off compared to their best alternatives.”¹ Thus, according to the Becker-Coase theorem, the move from mutual consent to unilateral divorce laws ought to have no impact on the rates of marriage dissolution, although it would affect the division of family resources within marriage and in the aftermath of divorce.² Given, this result one can go further and show that divorce laws have no effect on the joint expected utility from marriage and therefore the entry into marriage, implying that any laws that determine divisions of property rights conditioned on marital status can be undone by individual recontracting and therefore have no effect on either marriage or divorce. However, empirical studies on this topic which exploit the cross-state variation in the adoption of the no-fault, unilateral divorce laws in the United States show that unilateral divorce laws *did* raise divorce rates in the short run, although longer term effects are harder to establish (see, for example, Peters, 1986, Friedberg, 1998, Stevenson and Wolfers, 2006 and Wolfers, 2006). There is also some evidence that the change in divorce laws have reduced marriage rates (see Rasul 2005, and Matouschek and Rasul, 2006).

The goal of this paper is to reassess the theoretical validity of the Becker-Coase argument. We do so by assuming the same basic environment as in Becker (1993). In particular, we assume away inefficiencies generated by frictions or asymmetric information between the spouses. In such an environment there are three specific requirements for the Becker-Coase theorem: (a) transferable utility between the spouses *within marriage* (*TUM*); (b) transferable utility between the spouses *upon divorce* (*TUD*); and (c) invariance of the “exchange rate” of the utilities of the two partners to changes in marital status (*IER*). Together, these requirements imply that the decision to divorce is determined by the aggregate ‘real income’ of the two partners (including non-monetary benefits). If the couple’s real income is higher upon divorce, the partners will choose to split. But if it is lower in divorce, they will stay married. This holds true irrespective of the individual incomes (utilities) of the partners in the

¹The generalized Becker-Coase theorem as it applies to couples and intra-household allocations was originally formulated in Becker (1981).

²By definition, *mutual consent* divorce laws require both spouses to agree to a divorce whereas under *unilateral* divorce legislation divorce occurs when one spouse files for it.

two situations and the type of the divorce legislation in effect.

Recent analysis such as Zelder (1993), Clark (1999) and Fella et al. (2004) demonstrate that, in the absence of transferable utility, divorce rates can—and typically will be—influenced by changes in the divorce legislation. The question, however, is whether *TUM*, *TUD* and *IER*, taken together, are reasonable. To address this issue, we consider the implications of these conditions for household behavior. We apply a collective approach which presumes that partners to marriage reach an efficient outcome and consider a situation in which couples may consume both private and public goods. The presence of public goods generates economic marital gains. In addition, marriage generates non-pecuniary benefits which are revealed with some lag and may be different for the two spouses. A poor realization of these benefits can trigger divorce, although the economic surplus generated by marriage can mitigate the likelihood of this outcome. Having an explicit household model with endogenous divorce allows us to go beyond the implications of the three requirements listed above for divorce and explore their implications for other observable features of household behavior, such as household demand functions.

We begin by identifying spousal preferences that enable transferability of utility between spouses within marriage and characterizing their implications for household demand. A standard sufficient condition for *TUM* is quasi-linearity of spousal utilities, which implies that all goods, but one, have zero income elasticity. However, in an important contribution, Bergstrom and Cornes (1981, 1983) show that transferable utility within marriage does *not* require quasi-linearity, because the marginal utility of private consumption of the partners can depend on their common consumption of the public good; in particular, all commodities can have positive income elasticity. They provide a simple characterization of the preferences of the two spouses that maintain *TUM*. Basically, spouses' preferences are such that one good (at least) can be used to transfer utility between the partners. For both partners, this good generates a positive and identical level of marginal utility that depends only on the goods that are publicly consumed. We refer to such preferences as '*generalized quasi-linear (GQL)*'. We note, however, that from the perspective of the collective approach, the assumption of *GQL* is quite restrictive because it implies that couples behave in a 'unitary' way. When *GQL* holds, couples act as if they maximize a single utility function and household demand satisfies income pooling as well as the Slutsky conditions. Accordingly, holding total family income fixed, transferring income from

the husband to the wife should have no impact on household consumption except on those goods that are used to transfer utility. This prediction is often rejected by empirical evidence.

We argue that even if *GQL* holds, structural changes in the domains of public and private consumption following divorce will cause violations of *TUD* or *IER* (unless quasi-linearity of spousal utility representations is imposed). There are two basic reasons for this. First, some commodities that are public in marriage become private in divorce (e.g. housing). In this case, *GQL* preferences imply that the Pareto frontier upon divorce is convex (rather than linear). Second, even for those goods that remain public after divorce (e.g., children), the marginal rate of substitution between public and private consumption is likely to change (for instance, due to the distance of non-custodial parents to their offspring), implying different slopes for the linear frontiers in marriage and divorce. Due to either of these two reasons, the utility frontiers facing a couple upon marriage and divorce, respectively, can intersect. Consequently, the divorce outcome will depend on the initial sharing rule, the realization of match-specific qualities and the distribution of property rights as they are defined by the prevailing divorce legislation.

In sum, we find that the violations of Becker-Coase may be more complex than usually assumed. If divorce settlements are characterized by an uneven allocation of income and wealth, say because one spouse is much richer than the other and divorce laws allows the wealthier spouse to keep most of his/her wealth, then there are realizations of the shocks to match quality such that the marriage will dissolve under unilateral divorce laws but will remain intact under mutual consent. In contrast, when divorce laws produce more even divorce settlements in terms of income and wealth, there are realizations of the shocks to match quality such that the marriage will remain intact under unilateral divorce laws but *not* under mutual consent—a possibility already identified by Clark (1999). In general, couples who experience different match-quality shocks can react differently to changes in divorce legislation and, without further information on the prevailing divorce settlement laws, it is impossible to predict whether or not a switch from mutual consent divorce laws to no-fault divorce would lead to higher or lower aggregate divorce rates.

2 The Basic Ingredients

We consider a static model in which individuals live for two periods. Each person has preferences over commodities and marital status. Commodities are classified into two types; private goods, denoted by x and public goods denoted X that can be consumed jointly when two individuals are married. Preferences are represented by a pair of state dependent utility functions, $v(x, X)$ if a person is single and $u(x, X) + \theta$, if a person is married. The additive component θ represents non-economic benefits from marriage, such as love and companionship, that are assumed to be separable from the preferences over goods and are match specific. We do not consider altruism between spouses for now; the extension to an altruistic context is discussed in the last section. We view children as one of the public goods and in this manner allow for the possibility that both parents, whether married or divorced, care about the welfare of their child(ren).

We assume that the quality of the match is not observable at the date of marriage but that it is fully revealed after one period. At the date of marriage, the partners agree upon the consumption levels of all goods, public and private. We assume that this allocation is Pareto efficient and not contingent on future realizations of the quality of the match.³ We do not restrict the mechanism that determines the outcome within marriage in any particular way (such as Nash Bargaining) but recognize that it can depend on the spouses' incomes and on marriage market conditions. At the end of the period, the individual marital match qualities of the two partners are drawn from some given distribution and each spouse obtains a utility payoff derived from the predetermined levels of consumption, as well as the realized values of θ_h for the husband and θ_w for the wife. On that basis, and given the laws governing divorce, the spouses decide either to dissolve their marriage or remain married. If the partners split, their incomes can be modified by some redistribution determined by law but the partners can renegotiate around this stipulation and, if *both* partners agree, the courts will “rubber-stamp” the agreement. If they choose to continue the marriage, the partners can also renegotiate the intra-household allocation defined by the initial sharing rule. In this regard, couples bargain in the “shadow of the law” (see Mnookin and Kornhauser, 1979).

In what follows, we do not consider the possibility of remarriage and whenever

³Contingent contracts would raise complex implementation issues, insofar as the realization of match qualities are not verifiable by a third party.

a marriage dissolves, the partners remain single. This assumption is made for expositional simplicity and could easily be relaxed. However, the ex-spouses remain connected in the sense that they transfer resources to each other, if forced by law.⁴

3 A benchmark case without public goods

To reiterate Becker's results, consider a model with only one private good, x , and strictly quasi linear preferences given by

$$\begin{aligned} u(x_m) &= x_m + \theta_m, \\ v(x_m) &= x_m, \quad m = w, h, \end{aligned} \tag{1}$$

where h denotes the husband and w denotes the wife. This specification satisfies *TUM*, *TUD* and *IER*, which implies that the utility frontiers upon continued marriage and divorce are parallel straight lines. Then, for *any* allocation of the property rights upon marriage and divorce and irrespective of whether consent is required, divorce occurs if $\theta_h + \theta_w < 0$ and marriage continues if $\theta_h + \theta_w > 0$. To understand this result, consider Figure 1 that represents the case in which $\theta_h + \theta_w < 0$. The points M and D on the marriage and divorce frontiers, respectively, represent the prescribed allocations of a given family income, y , upon divorce and upon continued marriage. The point D is determined by the existing divorce law and the point M is a (random) outcome of the predetermined choice of consumption goods and the specific realizations of θ_h and θ_w . Under unilateral divorce, there must be one partner for whom utility upon divorce is higher than in marriage and, because the point D is outside the utility frontier in marriage, he\she cannot be bribed by his\her spouse to stay in the marriage. Under mutual consent, for any M , there is a feasible revision of D , such that *both* partners can be made strictly better off upon divorce. The partners can agree on such a new contract and usually courts are willing to enforce divorce contracts to which both partners agree. Hence, under both unilateral divorce and mutual consent, divorce is the predicted outcome. By an analogous argument marriage will continue if $\theta_h + \theta_w > 0$. Then, for any D there is a revision of M such that no one wants to leave under unilateral divorce, while under mutual consent, for

⁴If the couple had children, it is possible that transfers in the aftermath of divorce will occur voluntarily. We ignore here such voluntary transfers, assuming that they are completely crowded out by the transfers determined by law.

any M , the party who wants to leave cannot compensate his ex-spouse upon divorce, without becoming worse off.

Moving backward in time, we see that the expected utilities of the partners, conditioned on marriage in the first period, are

$$U_m = x_m + p(x_m + E\{\theta_m \mid \theta_h + \theta_w \geq 0\}) + (1 - p)u_m^D, \quad (2)$$

where $p = \text{prob}(\theta_h + \theta_w \geq 0)$ and u_m^D is the utility outcome upon divorce which depends on the division of income between the two spouses upon divorce that the law prescribes and on whether mutual consent is required. Adding the utilities of the two partners, we see that the Pareto frontier upon marriage is linear and given by

$$U_h + U_w = 2y + pE\{\theta_h \mid \theta_h + \theta_w \geq 0\} + pE\{\theta_w \mid \theta_h + \theta_w \geq 0\} \quad (3)$$

Due to the assumed transferable utility, marriage occurs whenever the RHS of (3) exceeds $2y$ which is what the partners obtain together as singles. Under plausible assumptions on the distributions of θ_h and θ_w , the expected non monetary gains from marriage are positive. Hence, if all individuals are identical and there are no frictions all matched couples will choose to marry. The important conclusion here is that the joint gains from marriage are *independent* of the divorce laws.⁵

4 A simple example with public goods

Consider now a case with 2 commodities; one that is purely private (the price of which is normalized to 1) and another that can be publicly consumed if two individuals marry. Assume that spouses' respective preferences regarding the private and the public good can be represented by the utility function:

$$v_m(x_m, X) = X + x_m X,$$

when single and by

$$u_m(x_m, X) = X + x_m X + \theta_m, \quad m = w, h, \quad (4)$$

⁵Note further that if law specifies a particular division upon divorce, the partners are still free to choose any division of life time utilities upon this expected utilities frontier. In particular, a partner with high entitlement upon divorce may be "punished" upon marriage by lower level of consumption. .

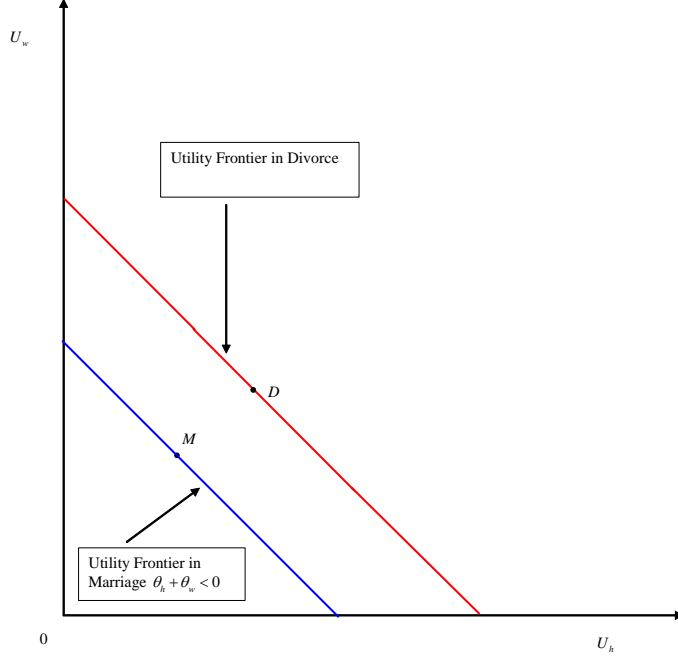


Figure 1: Pareto frontiers in marriage and divorce, no public goods

if they are married. This specification imposes some complementarity between public and private goods and, in particular, the marginal utility from private good is given by X for *both* spouses. By construction, it then follows that it is possible to transfer utility within marriage on a one-to-one basis between the spouses, using the private consumption good. Initially, we assume that preferences over commodities are the same for the two spouses and independent of marital status so that $u_m(x_m, X) = v_m(x_m, X)$ for $m = w, h$. Alternative scenarios will be discussed shortly.

4.1 Consumption and utility if marriage continues

For the preferences described here, the efficient level of X in an interior solution with $x_h > 0$ and $x_w > 0$ depends only on its price P and on the *aggregate* family income y . It is given by

$$X = \min \left(\frac{y}{P}, \frac{y+2}{2P} \right), \quad (5)$$

where y is the pooled income of the two spouses (see Bergstrom and Cornes, 1983). We shall assume hereafter that $y > 2$. Then household demand takes the *LES* form:

$$X = \frac{y+2}{2P}, \quad x = x_h + x_w = \frac{y-2}{2}. \quad (6)$$

Note that the consumption of the public good and the aggregate consumption of the private goods both rise with family income. Furthermore, any pair of private consumption levels that satisfies

$$x_h > 0, \quad x_w > 0, \quad x_h + x_w = \frac{y-2}{2} \quad (7)$$

is efficient.⁶

The implied utility levels of the two partners, conditioned on the efficient and interior level of the public good, are

$$U_m^M = (1 + x_m) \frac{2+y}{2P} + \theta_m, \quad m = w, h. \quad (8)$$

Hence, the Pareto frontier is given by

$$U_h^M + U_w^M = \frac{1}{4} \frac{(2+y)^2}{P} + \theta_h + \theta_w, \quad (9)$$

which defines a straight line with slope -1 . This indicates transferable utility. That is, by shifting the amount of private consumption goods between the two partners, it is possible to transfer utility at a fixed ‘rate of exchange’ that can be normalized to unity.

The significance of this feature is that the new Pareto set generated by *any* change in circumstances, such as a rise in family income or shocks to marital match quality of either spouse, either includes or is included in the initial Pareto set. In this respect, there is no conflict between the partners in their evaluations of the overall situation that they face—they always prefer the context in which the Pareto set is largest.⁷ Note, however, that this is true only on the segment of the Pareto frontier which corresponds to $0 < x_h + x_w < (y-2)/2$. There are also other regions of the Pareto frontier in which the non-negativity constraints on consumption bind and the Pareto

⁶To determine the division of private consumption goods between the two spouses, one must go beyond the principle of efficiency and specify a ‘sharing rule’ that selects (implements) a particular point on the Pareto frontier as a function of the individual incomes of the spouses and other ‘distribution factors’. See Browning et al. (2006, ch. 3).

⁷This idea of potential compensation plays an important role in the early literature on welfare economics, especially the evaluation of national income (see Samuelson, 1950).

frontier is no longer linear. One such region covers $x_h = 0$ and $x_w = y - PX$. Then, $U_h^M = X + \theta_h$ and $U_w^M = X(1 + y - PX) + \theta_w$, which represents a concave segment of the Pareto frontier given by

$$U_w^M = (U_h^M - \theta_h) [1 + y - P(U_h^M - \theta_h)] + \theta_w . \quad (10)$$

Another concave segment exists when $x_w = 0$, and $x_h = y - X$. Then,

$$U_w^M = \frac{1}{2P} \left[1 + y + \sqrt{(1 + y)^2 - 4P(U_h^M - \theta_h)} \right] + \theta_w . \quad (11)$$

When either of these ‘corner’ solutions arises, utility is no longer transferable at a fixed rate of exchange between the spouses even in marriage. But the higher is family income, the wider is the range in which utility is transferable.

4.2 Consumption and Utility in Divorce

Consider, now, the case in which agents decide to divorce. The utilities actually achieved depend on two features. One is the division of income following divorce, as determined either by law or a private contract between the parties. Without making explicit which of those mechanisms determine the post-divorce allocations, we simply assume that the husband gets βy and the wife gets $(1 - \beta)y$ for some parameter $1 \geq \beta \geq 0$.

The other determinant of intra-household welfare is the technology of consumption that prevails after divorce. Specifically, while the private good certainly remains private, the impact of divorce on consumption of the other (previously public) commodity is less clear. One possibility is that good X is *privately* consumed after divorce. This would be the case, for instance, of housing assuming that the spouses quit cohabiting after divorce. Alternatively, the consumption of good X may remain public. For instance, we may think of ‘child quality’ as such a good, since parents generally care about their children’s wellbeing regardless of the state of their marriage. However, divorce may still influence the allocations for this ‘consumption’. A plausible reason for this is that loss of custody or shared custody arrangements reduce the interaction time between at least one of the parents and the child(ren). In essence, this ‘distance’ effect reduces the marginal benefit of child-quality investments.

To capture these ideas in our simple framework, we consider two alternative settings: In one, the public good becomes purely private upon divorce. In the other,

the good remains public but the consumption utility of one of the ex-spouses is ‘discounted’ in the sense that the person’s marginal willingness to pay for this public consumption is smaller than it was in marriage.

4.2.1 Public Good Become Private

We start with the private consumption case. The ex-husband solves

$$\max_{x_h, X_h} X_h + (\beta y - P X_h) X_h, \quad (12)$$

which yields the consumption levels and the utility

$$X_h = \frac{1 + \beta y}{2P}, \quad x_h = \beta y - P X_h = \frac{\beta y - 1}{2}, \quad (13)$$

$$U_h^D = u_h^D = \frac{(1 + \beta y)^2}{4P}.$$

Similarly, the ex-wife solves

$$\max_{x_w, X_w} X_w + ((1 - \beta) y - P X_w) X_w, \quad (14)$$

which gives

$$X_w = \frac{(1 - \beta) y + 1}{2P}, \quad x_w = (1 - \beta) y - P X_w = \frac{(1 - \beta) y - 1}{2}, \quad (15)$$

$$U_w^D = u_w^D = \frac{(1 + (1 - \beta) y)^2}{4P}.$$

Using transfers, that is, by changing β , it is possible to move along the Pareto frontier in the aftermath of divorce. Hence, the Pareto frontier in divorce is given by

$$U_w^D = \frac{(2 + y - 2\sqrt{P U_h^D})^2}{4P}, \quad (16)$$

which is decreasing and convex with a slope given by

$$\frac{dU_w^D}{dU_h^D} = - \frac{1 + (1 - \beta) y}{1 + \beta y}. \quad (17)$$

In Figure 2, we compare the Pareto frontiers in marriage and divorce, assuming that $p = 1$. The location of the marriage frontier depends on the realization of the θ 's.

More precisely, while the location of the linear segment depends on the sum $\theta_h + \theta_w$, those of the ‘corner’ portions depend on the specific values of each marital match-quality shock. In the special case in which $\theta_h = \theta_w = 0$, the utility frontier in marriage is completely outside the utility frontier in divorce, because of the economic gains resulting from the joint consumption of public goods when partners live together. Variation in θ_h and θ_w shift the utility frontier in marriage upwards and downwards. However, when $\theta_h + \theta_w < 0$ the two utility frontiers can intersect, as indicated by the broken blue line. Having assumed that consumption is determined before the values of θ_h and θ_w are realized, different realizations are associated with different utility outcomes for the two partners in marriage, as indicated by the points M and M' . In contrast, the utility outcomes upon divorce, represented by the point D are independent of θ_h and θ_w .

It is important to note that in comparing the two Pareto frontiers in the case of marriage and divorce, we maintain the *same* cardinal representation of individual preferences. That is, if there exists a representation such that utility is transferable in marriage but it is not fully transferable in divorce, then the two Pareto frontiers can intersect. For this example, there will be two intersections provided that

$$0 > \theta_h + \theta_w > -\frac{(2+y)^2}{8P}. \quad (18)$$

If monotone transformations are applied to individual preferences in marriage and divorce, the shapes of the two Pareto frontiers will change but the pattern of intersections will remain the same.⁸

4.2.2 ‘Distance’ in Public Consumption

Assume, alternatively, that the marital public goods remain public upon divorce (e.g., ex-spouses still care about the welfare of their offspring after divorce). However, the ‘distance’ created by divorce between the non-custodial parent and the child(ren) affects preferences in terms of the relative importance of public and private goods and the evaluation of the marital state compared to being single.

Suppose, for instance, that the mother’s preferences are unaffected by divorce because she maintains custody over the child(ren), but the father’s utility upon divorce becomes

$$v_h(x_h, X) = \gamma \left(X + \frac{1}{\delta} x_h X \right), \quad (19)$$

with $\gamma < \delta < 1$.

In words, divorce has two effects. First, the public good is discounted by some factor γ , compared to its value in marriage. Second, the marginal willingness to pay for the public good, which is equal to $(1 + x_h) / X$ within marriage, drops to $(\delta + x_h) / X$ after divorce. Both of these changes reflect the reduction in the interaction between the father and the child(ren) when the mother is the custodial parent.

⁸At an intersection, we have

$$U_h^M = u_h^M + \theta_h = u_h^D = U_h^D,$$

$$U_w^M = u_w^M + \theta_w = u_w^D = U_w^D.$$

Thus, any monotone transformation $H_m(U)$ will maintain the equality (as well as an inequality).

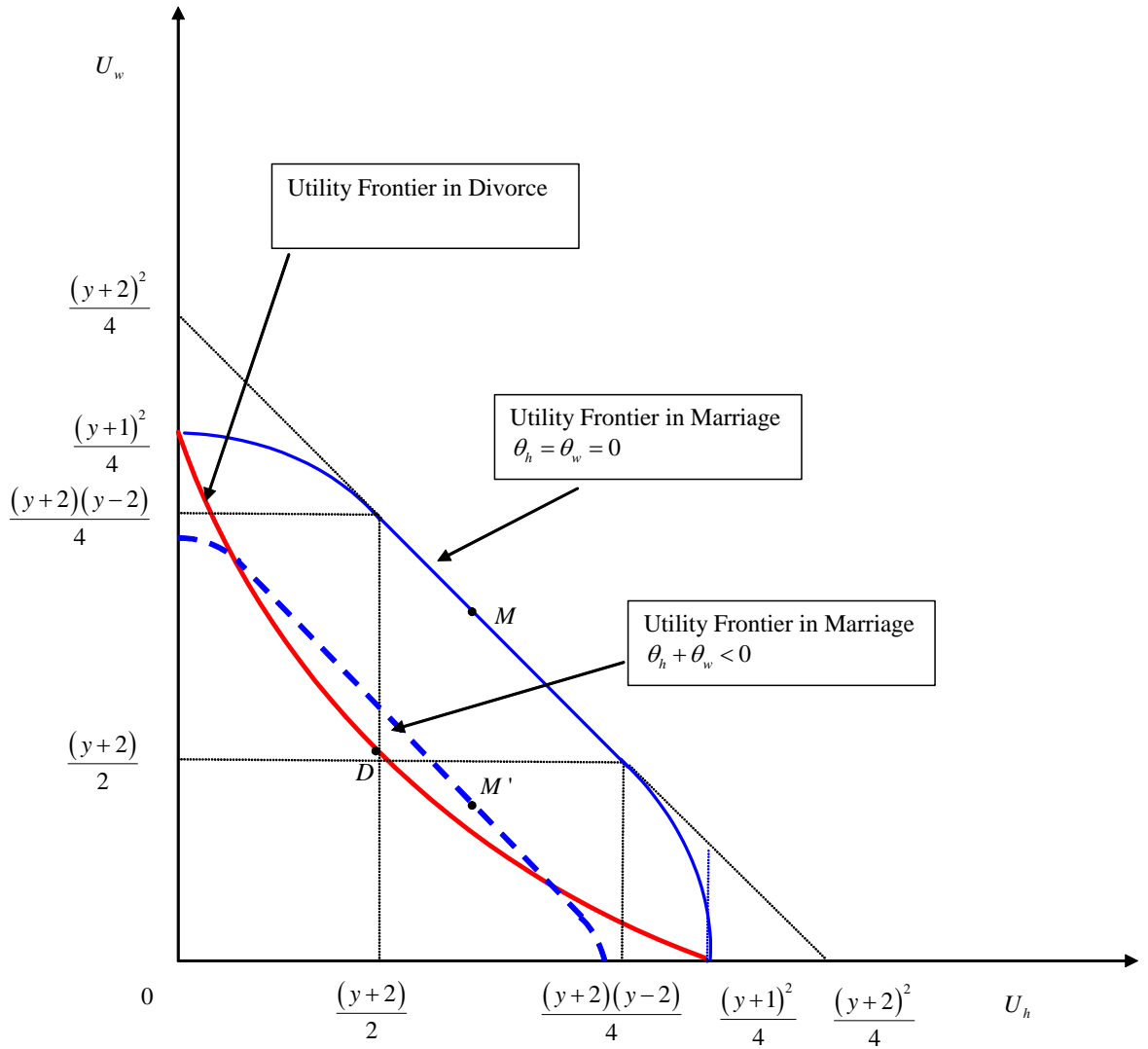


Figure 2: Pareto frontiers in marriage and divorce, with public goods

Following divorce, the Pareto frontier remains linear with a constant (income-independent) slope. In particular, one can easily check that, for any interior solution, we have

$$\frac{\delta}{\gamma}U_h^D + U_w^D = \max_X [(\delta + 1)X + (y - PX)X] . \quad (20)$$

The slope is no longer equal to -1 . Instead, it is now $-\gamma/\delta > -1$, reflecting the father's 'discount' factor in divorce.⁹

Figure 3 compares the divorce and marriage frontier in this case. Again, in the special case in which $\theta_h = \theta_w = 0$, the utility frontier in marriage is completely outside the utility frontier in divorce and the divorce frontier upon divorce is steeper because of the husband suffers a larger utility loss upon separation. Variation in θ_h and θ_w shift the utility frontier in marriage upwards and downwards. However, when $\theta_h + \theta_w < 0$ the two utility frontiers can intersect, as indicated by the broken blue line. Under unilateral divorce, the couple will divorce if and only if the division prescribed upon divorce D falls outside the utility frontier in marriage. Under mutual consent, divorce will occur if and only if the (random) division M implied by choice of consumption prior to the realization of θ_h and θ_w falls inside the utility frontier upon divorce. The points M and D in Figure 3 illustrates a case in which divorce occurs under mutual consent but marriage continues under divorce at will. It is important to note, however that under mutual consent the points M and D are not independent, because commitment made at marriage affect the range of renegotiated outcomes if marriage continues and, therefore, the probability of divorce. Taking D as given and looking forward, a couple may choose an initial allocation of the private consumption goods such that, *on the average*, M is close to D , thereby making divorce less likely. The partner who is hurt by this shift in the first period, may be compensated in term of life time utility by higher gains from joint consumption if marriage continues. Nevertheless, with sufficient variation in θ_h and θ_w , there will be ex-post realizations such that M and D are as drawn in Figure 3, implying that for such couples divorce laws matters.

⁹By the same token, the optimal level of public consumption is still identical for all efficient allocations, but it is smaller than before divorce; namely,

$$X^D = \frac{y + \delta + 1}{2P} < \frac{y + 2}{2P} .$$

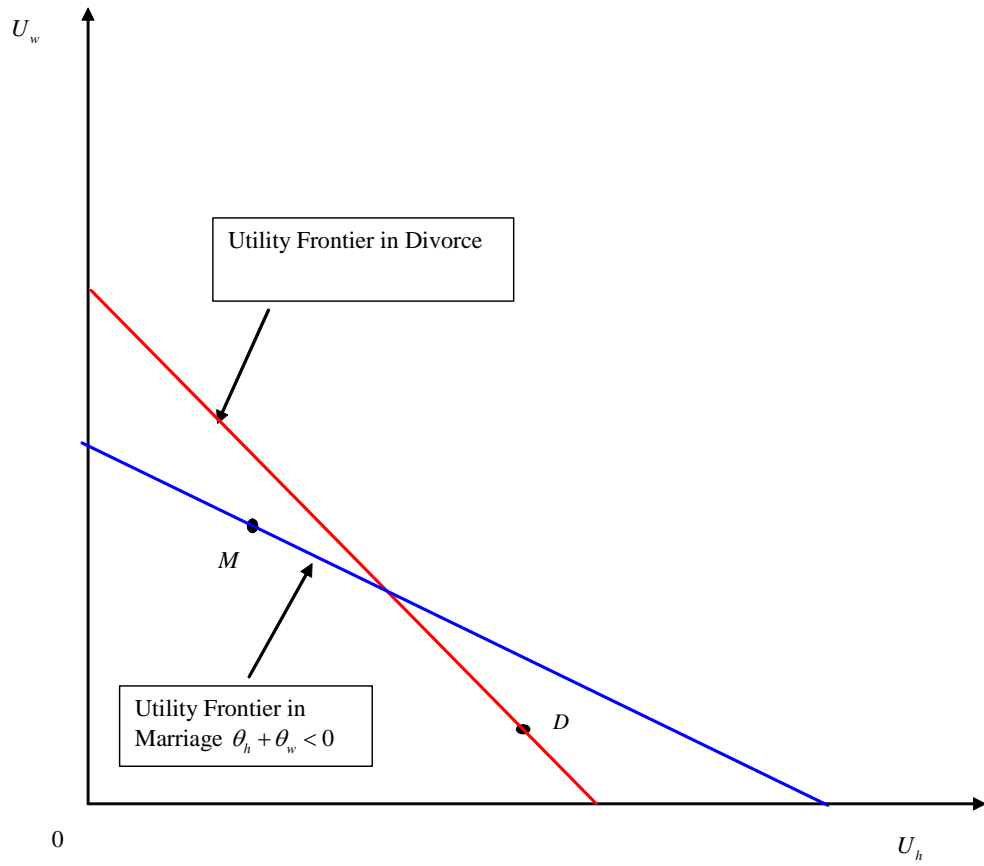


Figure 3: Pareto frontiers in marriage and divorce with "distance" in public goods

4.3 Becker-Coase Theorem Revisited

Our key result is that in both cases described above, regarding the nature of marital public goods and how they might be altered in divorce, the strong version of the Becker-Coase theorem does not hold. Indeed, for some realization of the match-quality shocks, the divorce decision depends on the legal framework. In particular, we provide below two examples in which different divorce laws lead to opposite outcomes. The first one is somewhat standard, in the sense that the couple in question would split under unilateral divorce, but would remain married if mutual consent is required. The second example is more counter-intuitive, because the same couple would divorce if mutual consent is required but they would remain married under unilateral divorce. The reason for this is that unilateral divorce would trigger a renegotiation of the intra-household allocations and enable the couple to find efficient reallocations they can agree upon within marriage.

We present both examples utilizing the ‘public-good-becomes-private’ framework we worked out above. Moreover, we ignore the possibility of corner solutions for the Pareto frontier when married. None of these choices is important, as these examples can easily be transposed into a different context. The key condition is that the realizations of match quality generate a Pareto frontier under marriage that intersects the utility frontier under divorce.¹⁰ In addition, we require that the match-quality shocks apply at the individual level and for them not be identical for the husband and the wife, so that any distribution of the marital gains upon marriage is possible.

Counter-example 1 (Mutual Consent Reduces Divorce) Consider the case depicted in Figure 2 and suppose that

1. Divorce legislation is rather favorable to men (i.e. β is large; for instance, property is mostly private and the husband is much wealthier than his wife);
2. The realization of the shocks is such that marriage is more valuable to the wife, $\theta_w > 0 > \theta_h$.

Figure 4 depicts the corresponding points on the Pareto frontiers as M if married and as D if divorced.¹¹ One can readily see from the diagram that the couple will

¹⁰In the case under consideration, this condition is met whenever equation (15) is satisfied.

¹¹For simplicity, we ignore here the non linear segments of the utility frontiers in marriage.

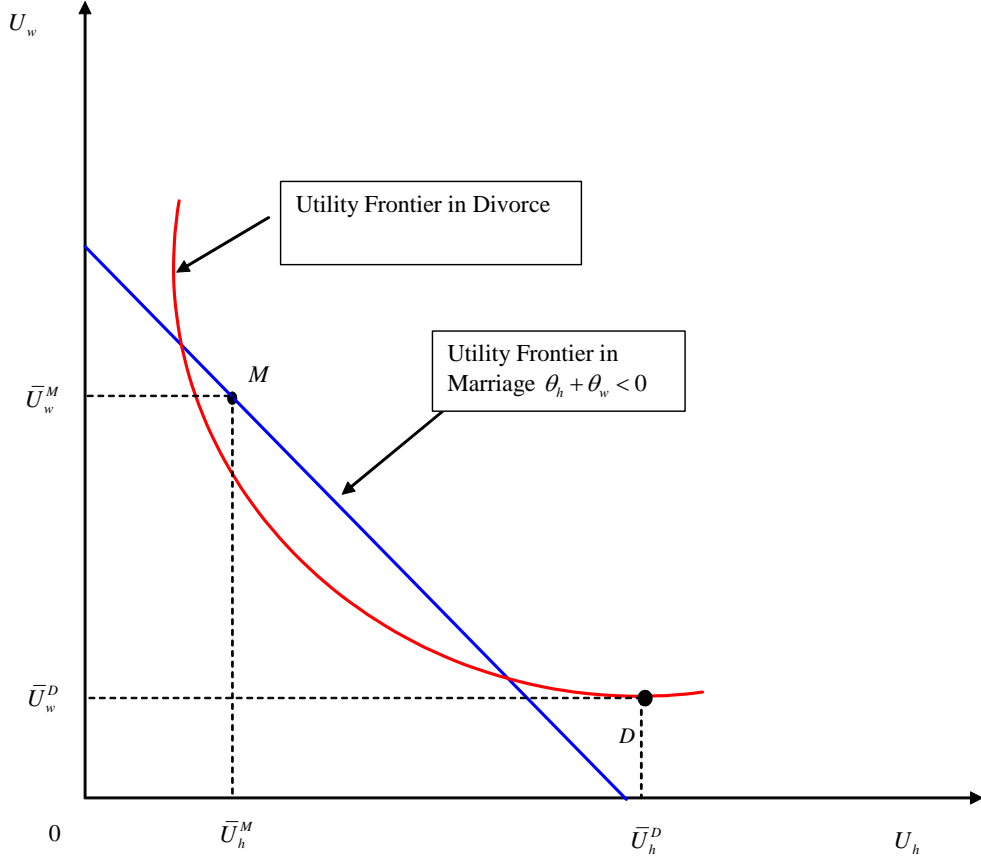


Figure 4: Mutual consent reduces divorce

remain married mutual consent is required for divorce. Because of the veto power conferred to either spouse, the wife can guarantee a utility level that is at least equal to what she gets when she is married, namely \bar{U}_w^M and because no point on the divorce frontier can offer the wife a similar level of welfare without making the husband worse off. Assume, on the contrary, that the husband can unilaterally decide to divorce. Then, he can reach the utility level \bar{U}_h^D . This is more than what he can achieve when married and the wife is unable to bribe him to stay. Therefore, the couple will split.¹²

The general intuition is clear: whenever the two Pareto frontiers intersect, one can find two utility levels—one for the couple when they are married and another for

¹²Note that this case is similar to Figure 2 in Clark (1999). Our model can thus be seen as providing an explicit foundation for Clark's discussion.

when they are divorced—such that the wife cannot, under unilateral divorce, ‘bribe’ the husband into staying married and the husband cannot, under mutual consent, pay off his wife to accept a separation.

Counter-example 2 (Mutual Consent Raises Divorce) We now reverse the two assumptions introduced above. In particular, consider the following scenario:

1. Divorce legislation is not too favorable to men (say, it splits household wealth equally between the spouses, $\beta = 1/2$);
2. The realization of the shocks favors the husband, $\theta_h > 0 > \theta_w$.

This situation is represented by the Pareto frontiers in Figure 5, where M is the division when the couple remains married and D is the division if the couple divorces. This case is similar to Figure 3 in Clark (1999) and the argument goes as follows: When mutual consent divorce laws apply, the husband will use his veto power to make sure that his utility is at least as much as what he gets when he stays married, namely \bar{U}_h^M . Therefore, the final outcome must be located to the northeast of point M . Actually, there exist achievable outcomes that Pareto improve upon the current situation. However, they are all located on the divorce frontier. From a straightforward efficiency argument, the partners will renegotiate the divorce contract, allowing them to reach upon divorce a point such as D' , which is preferred by both of them to remaining married.

Now consider the case of unilateral divorce. The wife may, by divorcing, achieve the utility level \bar{U}_w^D that exceeds her payoff within marriage \bar{U}_w^M . Hence, the initial allocation within marriage, M , is no longer implementable and will be renegotiated. The outcome of the renegotiation must provide the wife with a payoff within marriage that exceeds \bar{U}_w^D and there is a continuum of points on the Pareto frontier when married that are located to the northeast of point D and Pareto dominate the outcome in the case of divorce. Therefore, the couple will remain married and the private consumption goods will be redistributed in favor of the wife to yield an allocation such as M' .

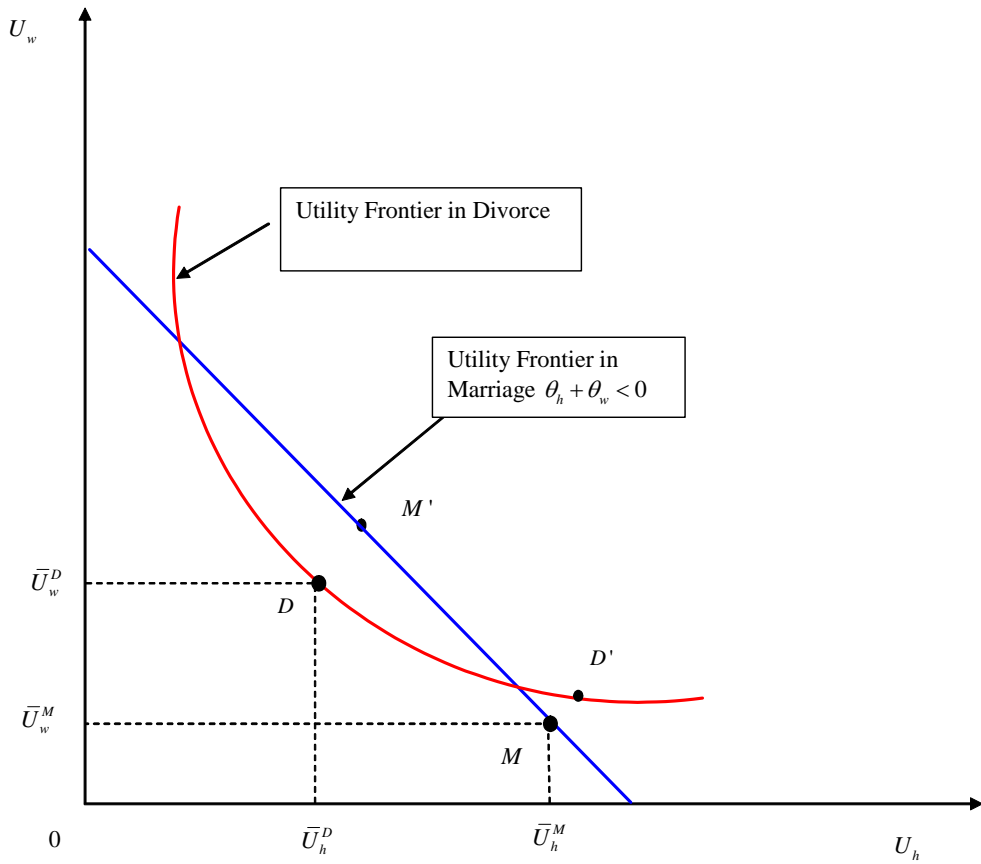


Figure 5: Mutual consent raises divorce

In the first example, the outcomes within marriage and upon divorce that were agreed upon ex-ante (at the time of marriage) are efficient ex-post (after the shocks to the match quality have been realized) and, therefore, are executed. In the second example, these agreements are ex-post inefficient and, therefore, are renegotiated. However, in both cases, *the decision ultimately taken is efficient* (regardless of whether it is divorce or not) in the conventional sense that no alternative could make both spouses better off. Therefore, a weaker version of the Becker-Coase theorem still holds; outcomes are efficient irrespective of the distribution of property rights. However, it is not generally true that efficiency arguments are sufficient to select a specific decision. On the contrary, there may exist a *continuum* of different Pareto efficient allocations which correspond to different decisions and outcomes. Efficiency alone cannot determine one particular divorce decision, except in the very peculiar case in which utility remains fully transferable after divorce (i.e., *TUD* holds, which requires the X commodity to remain public) and the slope of the Pareto frontier is unchanged (i.e., *IER* holds so that there are no ‘distance’ effects).

In general, whenever one spouse could lose in divorce while the other one could gain, there may exist efficient outcomes with and without divorce. And whether or not the marriage continues depends on the initial sharing rule, the realization of match-specific qualities and the distribution of property rights as defined by the prevailing divorce legislation.

5 A General Model

We now study the robustness of the insights we sketched out above. We consider a general model of household behavior with an arbitrary number of private and public consumption goods. We provide necessary and sufficient conditions for the Becker-Coase conclusion to hold and then argue that they are unduly restrictive. Specifically, we show that the requirements for transferable utility within marriage imply that utility *cannot* be transferable in the aftermath of divorce, unless the preferences of the two partners can be represented by quasi-linear utility functions. As is well known, quasi-linear preferences imply that the income elasticity of demand for all goods but one be zero—an assumption that can hardly be maintained as a basis for the empirical analysis of family behavior. In practice, while the Bergstrom conditions for transferable utility within marriage provide a possibly realistic framework for

the economic analysis of married couples, they also imply that utility *cannot* be transferable upon divorce. This suggests that, in general, divorce laws would impact divorce rates.

5.1 Goods and Preferences

There are n private and N public commodities.¹³ Let $x_m = (x_m^1, \dots, x_m^n)$, where $m = h, w$, denote member m 's private consumption and $p = (p^1 = 1, \dots, p^n)$ the corresponding price vector where p^1 is normalized to 1. Similarly, $X = (X^1, \dots, X^N)$ denotes the household's public consumption purchased at price $P = (P^1, \dots, P^N)$. Finally, let y_m denote member m 's income, and let $y = y_h + y_w$ be the household's total (pooled) income.

We assume that utility is transferable between married spouses; clearly, the Becker-Coase intuition cannot apply if this condition is not satisfied. Technically, the transferable utility (*TUM*) property is satisfied if, for each agent m , there exists a cardinal representation u_m of m 's utility such that, for each (p, P) , the Pareto set defined by the budget constraint is the hyperplane $\sum_m u_m = 1$.

Necessary and sufficient conditions for transferability have been known for some time. Specifically, define the notion of *generalized quasi-linearity* as follows:

Definition 1 *The utility functions u_m is generalized quasi linear (GQL) if there exist increasing functions F_m, A_m and b_m such that:*

$$u_m(x_m, X) = F_m [A_m(x_m^2, \dots, x_m^n, X) + x_m^1 b_m(X)], \quad m = h, w. \quad (21)$$

Obviously, whenever we are interested in the properties of individual or household demands generated by such utilities, we may assume that both F_h and F_w are the identity transform ($F_h = F_w = Id$). Note that the requirement that A_h, A_w and b be *each* increasing is needed for consistency. Indeed, if b could be decreasing, then u_m would also be decreasing for sufficiently large values of x_m^1 . And similarly, if A_i could be decreasing in its arguments, then u_m would also be decreasing for sufficiently small values of x_m^1 .

In a seminal contribution, Bergstrom (1989) has shown that transferable utility (*TUM*) requires three properties for utility functions:

¹³The goods are public within the household only; they are privately purchased on the market. One may think of housing or expenditures on children as typical examples.

1. individual utilities are of the *GQL* form;
2. the $b_m(X)$ functions are identical across agents: $b_h(X) = b_w(X) = b(X)$;
3. the allocation of resources between members is such that each member has a positive consumption of commodity 1, $x_h^1, x_w^1 > 0$.

We maintain these assumptions throughout the paper. In addition, it is natural to assume the following:¹⁴

Assumption N: *Preferences are such that all public goods are normal, in the sense that for any efficient household, the demand for each public good weakly increases with income.*

A consequence of this setting is that, for the particular cardinal representation of preferences corresponding to $F_h = F_w = Id$, the marginal utility of private commodity 1 that serves to transfer utility increases with the consumption of the public goods, hence with income. The imposition of this restriction is fairly benign, however, because our results—and, in particular, the examples of violations of the Becker-Coase theorem—are *ordinal*. Thus, they remain valid for any choice of F_h and F_w .

Finally, except for the increase in the number of goods, we maintain all other assumptions on preferences and timing of events of the simple example presented in Section 2.

5.2 Consumption and Utility in Marriage

Following the standard collective approach (see Chiappori, 1988, 1992), we assume that household decisions are Pareto efficient. As demonstrated elsewhere (for example, Chiappori and Ekeland, 2006), the decision can be analyzed as a two-stage process. At stage one, the spouses jointly decide the vector of public consumptions, X , and how to split the remaining amount between them. If ρ_i denotes the amount received by member i (the ‘conditional sharing rule’ in Chiappori and Ekeland’s terms),

¹⁴An interesting implication of this framework is in terms of spousal matching. Assume that agents with preferences of this type are respectively endowed with individual incomes and that marriage is the outcome of a standard matching process. Assuming public goods are normal, all stable equilibria exhibit *positive assortative matching* on income if and only if $b(X)$ is increasing in X .

we have

$$\rho_h + \rho_w + \sum_i P^i X^i = y_h + y_w = y . \quad (22)$$

At the second stage, individuals choose their private consumption vector by maximizing their utility subject to the budget constraint and taking the public consumption vector X as given. They solve

$$v_m(x_m, X) = \max_{x_m} A_m(x_m^2, \dots, x_m^n, X) + x_m^1 b(X) , \quad (23)$$

under the constraint

$$\sum_i p_i x_m^i = \rho_m \quad (24)$$

for $m = h, w$.

Now the key remark is that, *conditional on X* , the utility v_m is quasi-linear in x_m . It follows that, if ρ_m is ‘large enough’ (which we assume to be the case throughout the paper), then the optimal consumption of commodities 2, ..., n does not depend on ρ_m :

$$x_m^{-1} = (x_m^2, \dots, x_m^n) = \xi_m^{-1}(p, X) \quad \text{and} \quad x_m^1 = \rho_m - \sum_{i \geq 2} p_i \xi_m^i(p, X) . \quad (25)$$

The utility of member m is thus

$$v_m(x_m, X) = A_m(\xi_m^{-1}(p, X), X) + \left(\rho_m - \sum_{i \geq 2} p_i \xi_m^i(p, X) \right) b(X) , \quad (26)$$

and we may define a reduced-form utility u_m as

$$u_m(\rho_m, X, p) = a_m(p, X) + \rho_m b(X) , \quad (27)$$

where

$$a_m(p, X) = A_m(\xi_m^{-1}(p, X), X) - \sum_{i \geq 2} p_i \xi_m^i(p, X) b(X) . \quad (28)$$

Regarding the first-stage, Pareto efficiency implies that the household solves

$$\max_{\rho_h, \rho_w, X} a_h(p, X) + \rho_h b(X) \quad (29)$$

under the constraints

$$a_w(p, X) + \rho_w b(X) \geq \bar{u}_w , \quad (30)$$

$$\rho_h + \rho_w + \sum_i P^i X^i = y .$$

This program satisfies the transferable utility properties. Therefore, the optimal vector X does not depend on the parameter \bar{u}_w , and it solves

$$\max_X \sum_i a_i(p, X) + \left(y - \sum_i P^i X^i \right) b(X) . \quad (31)$$

Let $\bar{X}(p, P, y)$ denote the solution and $\eta(p, P, y)$ the value of the maximum. Since we do not consider price variations in what follows, we simply write $\bar{X}(y)$ and $\eta(y)$.

A particular Pareto efficient allocation is defined by the choice of the private good consumptions ρ_m . Individual utilities are thus

$$V_m(p, P, y, \rho_m) = a_m(p, \bar{X}(p, P, y)) + \rho_m b(\bar{X}(p, P, y)) + \theta_m , \quad (32)$$

where the shares ρ_m satisfy

$$\rho_h + \rho_w + \sum_i P^i \bar{X}^i(p, P, y) = y . \quad (33)$$

The Pareto frontier is the set of couples (U_M^h, U_M^w) satisfying the condition

$$U_M^h + U_M^w = \eta(y, p, P) = \max_X \sum_i a_i(p, X) + \left(y - \sum_i P^i X^i \right) b(X) . \quad (34)$$

Defining the function A by

$$A(x^2, \dots, x^n, X) = \max_{(x^1, x_h^2, \dots, x_h^n, x_w^2, \dots, x_w^n)} \left\{ \begin{array}{l} A_h(x_h^2, \dots, x_h^n, X) + A_w(x_w^2, \dots, x_w^n, X) \\ \text{s.t.} \quad x_h^i + x_w^i = x^i, i = 2, \dots, n \end{array} \right\} , \quad (35)$$

we see that household demand can be derived from the maximization, subject to the family budget constraint, of a single utility function U^H which is also of the generalized quasi-linear form:

$$U^H(x^1, x^2, \dots, x^n, X) = A(x^2, \dots, x^n, X) + x^1 b(X) \quad (36)$$

Conversely, if a household demand stems from the maximization of a unique utility of the form (36), then it can always be derived as the Pareto efficient aggregate demand of a household satisfying the *TUM* property. We now study the properties of such demand functions.

5.3 Testable implications of the *TUM* Assumption

Since the *TUM* assumption plays a key role in the theoretical analysis, it is natural to investigate its *empirical* implications (if any). We thus address the following questions:

- What testable restrictions does the form under consideration imply for *observable* behavior - i.e. demand functions?
- If such restrictions exist, are they ‘likely’ to be empirically fulfilled? In other words, how easy is it to empirically falsify the *TUM* property?

Unitary restrictions We know that the *TUM* assumption is satisfied if and only if household demand can be derived from the maximization of a single utility function, namely U^H . Therefore, the resulting household demand must satisfy the standard, ‘unitary’ restrictions reflecting this fact. Specifically:

Proposition 2 *If the TUM assumption is satisfied, household demand must satisfy*

- *Income pooling: it only depends on total income $y = y_h + y_w$,*
- *Slutsky symmetry and negativity: if $\xi = (x, X)$ and $\pi = (p, P)$ then*

$$\frac{\partial \xi^i}{\partial \pi^j} + \xi^j \frac{\partial \xi^i}{\partial y} = \frac{\partial \xi^j}{\partial \pi^i} + \xi^i \frac{\partial \xi^j}{\partial y}$$

In practice, a large literature has been devoted to testing these restrictions. Most recent works concentrate on the income pooling property (see, for instance, Thomas (1990), Schultz (1990), Bourguignon *et al* (1993), Browning *et al* (1994), Lundberg, Pollak and Wales (1997), Thomas *et al.* (1997) and Duflo (2003) to mention just a few). All these papers reject the income pooling assumption. The few contributions testing Slutsky symmetry also reject the property for couples, although, quite interestingly, they fail to reject it for singles (Browning and Chiappori 1998, Kapan 2006). These findings suggest that the unitary representation of household behavior, a necessary consequence of the *TUM* assumption, is far from being an innocuous property, and that it may fail to be supported by the data.

Generalized quasi-linearity Additional restrictions come from the specific form of the household utility. Indeed, the *TUM* property requires aggregate demand be generated by a generalized quasi-linear (*GQL*) utility.

We are thus looking for conditions *on the resulting demand function* that fully characterize *GQL* utilities. Chiappori (2007) provides a comprehensive answer to that question. Specifically, he proves the following result: Take some specific demand (x, X) , expressed as a function of $(p^2, \dots, p^n, P^1, \dots, P^N, y)$, and consider some open set \mathcal{O}' on which its Jacobian determinant does not vanish. Then, one can invert it, thus defining the inverse demand function

$$\begin{aligned} p^i &= \pi^i(x, X), \quad i = 2, \dots, n \\ P^k &= \Pi^k(x, X), \quad k = 1, \dots, N \\ y &= \theta(x, X) \end{aligned} \tag{37}$$

On this basis, we can state the following:

Proposition 3 *Assume that (x, X) , as functions of (p, P, y) , stems from the maximization of some generalized quasi-linear utility U^H under the budget constraint. Consider some open set \mathcal{O} on which its Jacobian determinant does not vanish, and such that $x^1(p, P, y) > 0$ over \mathcal{O} . Then:*

1. *Each $x^i, i = 2, \dots, n$, can be written as a function of (p, X) alone. In particular, the vector $D_{P,y}x^i = \left(\frac{\partial x^i}{\partial P^1}, \dots, \frac{\partial x^i}{\partial P^N}, \frac{\partial x^i}{\partial y} \right)'$ can be written as a linear combination of $D_{P,y}X^j, j = 1, \dots, N$.*
2. *The inverse demand function (π, Π, θ) satisfies the following conditions:*

- *For any $1 \leq k \leq N$, $\Pi^k(x, X)$ has the affine (in x^1) form:*

$$\Pi^k(x, X) = \alpha_k(x^{-1}, X) + x^1\beta_k(X) \tag{38}$$

for some functions α_k and β_k (where $x^{-1} = (x^2, \dots, x^n)$). In particular,

$$\frac{\partial^2 \Pi_k}{\partial x^1 \partial x^i} = 0, \quad i = 1, \dots, n \tag{39}$$

- *For any $1 \leq k \leq N, 1 \leq j \leq N$,*

$$\frac{\partial \beta_k}{\partial X^j} = \frac{\partial \beta_j}{\partial X^k} \tag{40}$$

- If $b(X)$ is such that $\beta_k(X) = \frac{\partial \log b(X)}{\partial X^k}$, $k = 1, \dots, N$, the matrix M of general term

$$M_{k,j} = \frac{\partial [\alpha_k(x^{-1}, X) b(X)]}{\partial X^j} \quad (41)$$

is symmetric:

$$\frac{\partial [\alpha_k(x^{-1}, X) b(X)]}{\partial X^j} = \frac{\partial [\alpha_j(x^{-1}, X) b(X)]}{\partial X^k} \quad (42)$$

Proof. See Chiappori (2007). Note that condition (40) guarantees the existence and the uniqueness (up to a multiplicative constant) of the function $b(X)$ used in condition (42). Indeed, from (40), the vector $(\beta_1, \dots, \beta_N)'$ is the gradient of some function $B(X)$ and we may then define $b(X) = \exp(\beta(X))$. ■

Intuitively, property 1 generalizes the standard characteristic of quasi-linear functions. Indeed, in the absence of public goods, it implies that $\partial x^i / \partial y = 0$ for all $i \geq 2$, a property characteristic of strict quasi-linearity. Clearly, the condition above is much less restrictive than quasi-linearity; it simply requires that there be a link between the income effects of private and public goods. On the other hand, the presence of public goods, while it relaxes the quasi-linearity requirement, generates additional conditions, which happen to be easily expressed in terms of the inverse demand function. These constitute the second set of restrictions.¹⁵

Finally, Chiappori (2007) shows that these conditions are also sufficient, at least locally. If a demand function satisfies Slutsky and the conditions of Proposition 3, then it is possible to construct a household utility of the *GQL* family from which demand can be derived.

We can thus conclude that the *TUM* assumption generates strong testable predictions on behavior. These predictions are of two types. First, the *TUM* assumption leads to a *unitary* representation of household behavior. A second set of conditions characterizes the specific form the household utility must take in a *TUM* context. While the latter are indeed restrictive, it is important to note that they can be empirically tested only to the extent that *variations in the prices of public goods can be*

¹⁵Household's demand prices for public goods equal the *sum* of the willingness to pay of the household members for each public goods (see Samuelson, 1954). Under *GQL*, this sum equals

$$\sum_m \frac{\frac{\partial A_m(x_m^2, \dots, x_m^n, X)}{\partial X^k} + x_m^1 \frac{\partial b(X)}{\partial X^k}}{b(X)},$$

which is affine in x^1 .

observed. If not, as is often the case with cross-sectional data, then the restrictions stemming from the generalized quasi-linear form of household utility cannot be empirically rejected: any household behavior that is compatible with (unitary) utility maximization can be derived from a *TUM* framework. In other words, in the absence of public-good price variations, the testable implications of the *TUM* framework boil down to those of the unitary model—namely, the existence of a ‘representative’ household utility.

5.4 Consumption and Utility in Divorce

The previous results fully characterize the conditions under which utility is transferable between the married spouses. However, a crucial remark suggested by the example above is that even if the transferable utility (*TUM*) property is satisfied when agents are married, it need not hold after divorce (*TUD*). Even if it does, the slope of the Pareto frontier need not—and, in general, will not—be the same as within marriage. We now formally investigate the robustness of this claim.

We first make a regularity assumption:

Assumption R: *The function $b(X)$ is not constant and the function $A_m(x^2, \dots, x^n, X)$ is not itself of the generalized quasi-linear form (36).*

In words, demand for public goods increases with income and household utility is neither quasi-linear (which it would be if $b(X)$ was constant) nor ‘twice generalized quasi-linear’ (if the function $A_m(x^2, \dots, x^n, X)$ was itself of the generalized quasi-linear form, U^H would be of the form (36) for two different commodities, x^1 and x^i for $i \geq 2$).

The first assumption is uncontroversial. Clearly, quasi-linear utilities exhibit demand properties (zero income elasticity for all goods but one) that are grossly counterfactual. As for the second, the assumption is in fact very mild. Indeed, consider a utility of the *GQL* form (36), and assume that, furthermore, A is *GQL*, say in x^2 :

$$A(x^2, \dots, x^n, X) = \alpha(x^3, \dots, x^n, X) + x^2\beta(X) \quad (43)$$

Then

$$U^H(x^1, x^2, \dots, x^n, X) = \alpha(x^3, \dots, x^n, X) + x^2\beta(X) + x^1b(X) . \quad (44)$$

When this function is maximized under a budget constraint, generically prices are such that $\beta(X) / p^2 \neq b(X) / p^1$. Assume, for instance, that we are considering a price-income bundle such that $\beta(X) / p^2 < b(X) / p^1$. Then $x^2 = 0$ in a neighborhood of that price-income bundle, and we may simply ignore (locally) commodity 2; therefore A is not *GQL* in the relevant neighborhood. In practice, a function is almost never ‘twice generalized quasi-linear’: A form like (44) is in fact *GQL* ‘only once’ almost everywhere, although the specific good with respect to which generalized quasi-linearity obtains may vary with prices and income.

Our main result is then the following:

Proposition 4 *Assume that Assumptions N and R are satisfied. Then:*

- *If at least one marital public commodity j such that $\partial b(X) / \partial X^j \neq 0$ becomes private, TUD is violated. If all public goods become private, the Pareto frontier after divorce is actually strictly convex.*
- *If all commodities j such that $\partial b(X) / \partial X^j \neq 0$ remain public, but utilities functions are altered so that, after divorce,*

$$v_h(x_h, X) = A_h^D(x_h^2, \dots, x_h^n, X) + x_h^1 b_h^D(X)$$

and

$$v_w(x_w, X) = A_w^D(x_w^2, \dots, x_w^n, X) + x_w^1 b_w^D(X) \tag{45}$$

with $b_h^D(X) \neq b_w^D(X)$, TUD is violated unless $b_w^D(X) = \Omega b_h^D(X)$ for some scalar Ω . Even if TUD is not violated, the slope of the Pareto frontier after divorce is not equal to -1 unless $\Omega = 1$.

Proof. *We first show the second statement. Consider the post-divorce equivalent of program (29); with obvious notations, post-divorce Pareto efficient allocations solve:*

$$\max_{\rho_h, \rho_w, X} a_h^D(p, X) + \rho_h b_h^D(X) \tag{46}$$

under the constraints

$$a_w^D(p, X) + \rho_w b_w^D(X) \geq \bar{u}_w , \tag{47}$$

$$\rho_h + \rho_w + \sum_i P^i X^i = y .$$

where ρ_m is now the amount member m can spend on private consumption after divorce. Let μ and λ be the Lagrange multiplier of the first and the second constraint respectively. Equivalently, μ is the Pareto weight of the wife when the husband's weight is normalized to 1. In the (v_h, v_w) plane, therefore, the slope of the Pareto frontier at the particular solution of this program is $-\mu$. But first-order conditions give

$$b_h^D(X) = \lambda \quad \text{and} \quad \mu b_w^D(X) = \lambda, \quad \text{therefore} \quad \mu = \frac{b_h^D(X)}{b_w^D(X)}. \quad (48)$$

Assume the ratio $b_h^D(X) / b_w^D(X)$ is not constant. By the Bergstrom-Cornes result mentioned above, X is not constant across the various Pareto efficient outcomes. But then slope of the Pareto frontier is not constant either, and TUD is violated. Assume, now, that $b_h^D(X) / b_w^D(X)$ is constant and equal to some scalar Ω . Then the slope of the Pareto frontier is constant and equal to $-\Omega$, hence the conclusion.

Let us now consider the alternative case in which some of the public goods - say, goods 1 to K - become private after divorce. The previous construct must be modified to account for the fact that the set of public commodities has been reduced. Specifically, we now define the post-divorce conditional sharing rule as:

$$\bar{\rho}_h = x_h^1 + \sum_{i \geq 2} p^i x_h^i + \sum_{j=1, \dots, K} P^j X_h^j, \quad \bar{\rho}_w = x_w^1 + \sum_{i \geq 2} p^i x_w^i + \sum_{j=1, \dots, K} P^j X_w^j \quad (49)$$

Also, let the husband's conditional utility, $\bar{v}_h(p, P^1, \dots, P^K, X^{K+1}, \dots, X^N, \bar{\rho}_h)$, be defined as the value of the program

$$\max_{x_h, X_h^1, \dots, X_h^K} A_h(x_h^{-1}, X_h^1, \dots, X_h^K, X^{K+1}, \dots, X^N) + x_h^1 b(X_h^1, \dots, X_h^K, X^{K+1}, \dots, X^N) \quad (50)$$

under the constraint

$$x_h^1 + \sum_{i \geq 2} p^i x_h^i + \sum_{j=1, \dots, K} P^j X_h^j \leq \bar{\rho}_h. \quad (51)$$

Note that, from the envelope theorem,

$$\frac{\partial \bar{v}_h}{\partial \bar{\rho}_h} = b(X_h^1, \dots, X_h^K, X^{K+1}, \dots, X^N). \quad (52)$$

Similarly, we define the wife's conditional utility $\bar{v}_w(p, P^1, \dots, P^K, X^{K+1}, \dots, X^N, \bar{\rho}_w)$, which is such that

$$\frac{\partial \bar{v}_w}{\partial \bar{\rho}_w} = b(X_w^1, \dots, X_w^K, X^{K+1}, \dots, X^N). \quad (53)$$

Now, the couple solves:

$$\max_{X^{K+1}, \dots, X^N, \bar{\rho}_h, \bar{\rho}_w} \bar{v}_h(p, P^1, \dots, P^K, X^{K+1}, \dots, X^N, \bar{\rho}_h) \quad (54)$$

under the constraints

$$\bar{v}_w(p, P^1, \dots, P^K, X^{K+1}, \dots, X^N, \bar{\rho}_w) \geq \bar{u}_w \quad (55)$$

$$\sum_{j \geq K+1} P^j X^j + \bar{\rho}_h + \bar{\rho}_w = y$$

With the same notations as above, the slope of the Pareto frontier at the particular solution of this program is:

$$-\mu = -\frac{\partial \bar{v}_h / \partial \bar{\rho}_h}{\partial \bar{v}_w / \partial \bar{\rho}_w} = -\frac{b(X_h^1, \dots, X_h^K, X^{K+1}, \dots, X^N)}{b(X_w^1, \dots, X_w^K, X^{K+1}, \dots, X^N)} \quad (56)$$

The end of the proof is as before: by Bergstrom-Cornes, the X_m^k are not constant across the various Pareto efficient outcomes, nor is slope of the Pareto frontier. Hence, TUD is violated. Finally, if $K = N$ (so that all goods become private), the slope becomes

$$-\mu = -\frac{b(X_h^1, \dots, X_h^N)}{b(X_w^1, \dots, X_w^N)} \quad (57)$$

Consider two Pareto efficient outcomes (x, X) and (\bar{x}, \bar{X}) , and assume the second is more favorable to the wife (hence the corresponding point is located to the southeast of the first on the Pareto frontier if her utility is on the horizontal axis). By normality, we get that

$$\bar{X}_w^j \geq X_w^j \text{ and } \bar{X}_h^j \leq X_h^j \text{ for all } j. \quad (58)$$

Therefore

$$\frac{b(\bar{X}_h^1, \dots, \bar{X}_h^N)}{b(\bar{X}_w^1, \dots, \bar{X}_w^N)} \leq \frac{b(X_h^1, \dots, X_h^N)}{b(X_w^1, \dots, X_w^N)} \quad (59)$$

and the slope is flatter, which describes a convex frontier. ■

This proposition generalizes the results in the previous section. They imply, indeed, that for some values of the shocks the Pareto frontiers in marriage and upon divorce may intersect, provided that (i) Assumptions N and R hold; (ii) the marital match qualities differ at the individual level; and (iii) either the sets of public goods and private commodities change depending on marital status or spousal utility from

marital public goods is altered in divorce. We conclude that *the situations depicted in Figure 4a and 4b are fully general; they obtain in the general setting considered in this section as well.*

5.5 Extensions

In our framework, a number of generalizations can be introduced at little or no cost. We list the main possible extensions below:

1. Altruism

Although our model assumes ‘egoistic’ preferences, in which individual care exclusively about their own (public and private) consumption, the extension to more altruistic preferences is straightforward. The key remark, here, is that, as argued for instance by Browning and Chiappori (2002), an allocation that is Pareto efficient for altruistic preferences of the form $W_m(u_w, u_h)$, $h = m, w$ is Pareto efficient for the egoistic preferences u_w, u_h ; therefore the collective approach, which only assumes efficiency, readily applies. Assume, furthermore, that the W are linear:

$$W_m(u_w, u_h) = u_m + k_m u_n, \quad n \neq m \quad (60)$$

Then:

$$\begin{aligned} \frac{1 - k_w}{1 - k_w k_h} W_h + \frac{1 - k_h}{1 - k_w k_h} W_w &= u_h + u_w \\ &= A_h(x_h^{-1}, X) + A_w(x_w^{-1}, X) + (x_h^1 + x_w^1) b(X) \end{aligned} \quad (61)$$

and the analysis goes through, the slope of the Pareto frontier now being now equal to $-(1 - k_h) / (1 - k_w)$.

Our results are therefore extended in the following way: if, after divorce, either (i) some public commodities become private, or (ii) the MRS between private and public consumptions are modified, or (iii) the ‘altruism coefficients k_h, k_w change in such a way that the ratio $(1 - k_h) / (1 - k_w)$ is modified, then the Becker-Coase theorem does not apply. In particular, (iii) is quite realistic: it is hard to believe that divorce will leave altruistic feelings unchanged within the

couple. We conclude that the introduction of altruism (if anything) strengthens our negative conclusions.

2. Externalities

Our model assumes away consumption externalities, whereby a person's private consumption directly affects the spouse's well being. One issue raised by externalities is the nature of the decision process; some authors have argued that non-cooperative decision processes should be considered (in which case the Becker-Coase theorem cannot apply). If, on the other hand, one sticks to a cooperative framework (thus assuming that the agents manage to internalize the interaction), then each member's private consumption of the externality-generating good can be considered as publicly consumed, and the analysis can readily be extended to that case.

3. Household production

Assume, now, that some of the commodities are produced within the household, according to some general technology described by

$$\phi(x_h^{-1}, x_w^{-1}, X) = 0 \quad (62)$$

where $x_m^{-1} = (x_m^2, \dots, x_m^n)$, $m = h, w$. Note that commodity 1 is not used in the production process. Define household utility by

$$U^H(x, X) = \max_{x_h^{-1}, x_w^{-1}} A_h(x_h^{-1}, X) + A_w(x_w^{-1}, X) + x^1 b(X) \quad (63)$$

under the constraint (62) .

By the envelope theorem, $\partial U^H / \partial x^1 = b(X)$, so that U^H is again *GQL* and the previous analysis applies. Note, however, that a change in the production technology upon divorce is not sufficient, in general, to generate a violation of Becker-Coase, at least as long as $b(X)$ remains unchanged.

4. Non-additive shocks

The exposition of our model is considerably simplified by the separability property of the ‘marriage quality’ shocks; indeed, the *MRS* between various commodities are not affected by their realization. Note, however, that non-separability can only make the three basic properties (*TUM*, *TUD* and *IER*) more difficult to fulfill (for instance, *TUM* should hold for *any* realization of the shocks). Hence, if marital shocks are non-additive, our negative result can only be strengthened.

5. Investment

The model presented here contained only consumption activities. Investment activities in schooling or on the job and match-specific investment, such as fertility choices, should be included in any realistic model of the household. The main added element is that the risk of divorce and the type of divorce settlement can affect the shape of the Pareto frontier as well as the sharing rule within marriage. Risk aversion and risk sharing may also become an issue. An analysis of these important issues require a dynamic setup that we have not considered here, but see Rasul (2006), Matuschek and Rasul (2006) and Wickelgren (2006) for their treatment.

6 Concluding Remarks

The Becker-Coase theorem, which posits that changes in divorce laws should have no impact on marriage dissolution rates relies on two main insights. One is that couples will be able, by bargaining in the shadow of the law, to reach Pareto efficient agreements; changes in divorce laws may therefore affect the distribution of the surplus between the spouses, but not the efficiency of the outcome. This intuition remains fully valid in our context. A second, and more fragile, ingredient is that *only one* marital status is compatible with efficiency - so that efficiency considerations de facto dictate the divorce decision.

Our main claim is that the *TU* assumption is unlikely to hold in *both* marriage and divorce. As a consequence, the utility frontiers facing a couple upon marriage and divorce can intersect. Then, the divorce outcome depends on the initial sharing rule, the realization of match-specific qualities and the distribution of property rights as they are defined by the prevailing divorce legislation. A switch from mutual consent

to unilateral may, as expected, increase divorce probability. This effect is more likely to be observed when divorce results in an unequal distribution of income or wealth welfare between spouses. We show, however, that the opposite effect is also possible. In some circumstances a couple may be *more* likely to divorce under mutual consent than under a unilateral rule. This counter intuitive situation may stem from the conjunction of a very unbalanced allocation of welfare when the couple is married and a relatively equal distribution of income and wealth after divorce. Then the spouse who becomes (relatively) unhappy in marriage would trigger a renegotiation that would raise his\her share within marriage, as compared to the share agreed upon at marriage when the quality of match was still unknown.

Using an explicit model of household consumption and divorce decisions allowed us to derive the testable restrictions implied by transferability in terms of household demand functions. Hence, the choice of a modeling strategy for the analysis of the household (and the marriage market in general) such as transferable utility vs. non-transferable utility can be based on evidence on household behavior rather than on considerations of tractability or faith alone. The available evidence suggests some caution in applying transferable utility, but it is hard to say whether or not we err much by maintaining it as a simplifying assumption for our analysis. In the end, the Becker-Coase theorem, although technically debatable, may remain an acceptable approximation.

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