An Assignment Model with Divorce and Remarriage*

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Abstract

We develop a two-sided matching model with positive sorting. Match quality for each couple is revealed ex post and those with poor draws divorce. We show that full commitment and Nash bargaining mechanisms both yield the same non-contingent intra-household allocations. We then apply our framework to an analysis of changes in divorce settlement laws. Changes in laws governing property division in divorce can have temporary effects in their desired direction, but whether or not their longer term impact can be undone in the markets for marriage depends on commitment and the timing of marriage vis-a-vis the legislative policy change. In particular, we find that, if existing couples were able to commit to their allocations prior to marriage, legislative changes that favor a spouse yield their intended benefits only among the poor-quality matches. But if existing couples were not able to make pre-marital allocative commitments, then changes in property division rules in divorce impact all couples and generate marital gains for all of the intended beneficiaries. For individuals not yet married, in contrast, changes in property division rules generate offsetting intra-household transfers and lower intra-marital allocations for all future spouses who are the intended beneficiaries.

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1 Introduction

The spousal matching and assignment model has been an important workhorse of the literature on the economics of the family since the seminal contributions of Shapley and Shubik (1972) and Becker (1981). This model is applicable for the analysis of marriage patterns in a given isolated "marriage market" under static conditions. In particular, it can predict who marries whom and the division of marital gains between husbands and wives.[1]

In this paper, we propose a spousal matching model with divorce and remarriage that can be applied to modern marriage markets which are characterized by high turnover, whereby many individuals divorce and remarry (Browning et al., in progress, ch.1). Turnover is generated by introducing match-specific random shocks that are observed after marriage has taken place. In contrast to search models (e.g., Mortensen, 1988) that impose frictions in the form of random meetings that are spaced over time, we maintain the competitive spirit of the assignment model by putting no restrictions on meetings. However, some non-competitive elements appear in our model too due to ex-post rents that arise when match quality is revealed.

The main ingredients of our model are as follows: There is a continuum of men and women who live for two periods. Each agent is characterized by a single attribute, income (or human capital), with continuous distributions of incomes on both sides of the marriage market, so that each agent has a close substitute. The economic gains from marriage arise from joint consumption of a public good and a non-monetary common factor that is match specific. This match quality for each couple is revealed ex post and those with poor matches may divorce. For analytical purposes, we proceed in two steps: In a benchmark scenario, we analyze a model in which remarriage is ruled out following divorce. Then, in an extension, we examine a more comprehensive model in which divorcees can rematch with others in the same cohort (who could either be never married or divorcees themselves). We assume that a unilateral divorce law is in effect and that divorce laws determine the spousal distribution of incomes upon divorce. Therefore, income distributions in the remarriage market and the first-marriage market may differ. We assume throughout that utility is transferable between spouses and lending or borrowing is not an option.[2] Because of public

[1]In recent years, there have also been various efforts to unify these models with stages of the life cycle prior to marriage. Embedding a spousal assignment framework into a model of pre-marital investment is relevant to the extent that the educational attainment and labor supplies of men and women respond to not only the returns in the labor market, but also incentives in the markets for marriage. See, Iyigun and Walsh (2007) and Chiappori, Iyigun and Weiss (2007).

[2]Allowing saving raises issues of which married partner is responsible to family debts in the case of divorce which would complicate the analysis substantially.

goods, the incomes of the two spouses complement each other, which can be shown to generate positive assortative matching according to spousal incomes, even though divorce is endogenous. Finally, to avoid potential knife-edge outcomes, we assume that there are more women than men in the marriage market. Because there are more women than men, there will always have to be some unmarried women in the first-period marriage market.

Extending the assignment model in such fashion enables us to distinguish between the expected lifetime utility of an agent and the inter-temporal pattern of per period utilities. In a first marriage, before the match specific quality is realized, each individual has a very close substitute and competition fully determines the share that each person receives in marriage, based on the anticipated outcomes following divorce. These shares are determined by two basic principles: (a) Due to competition from single women with slightly lower income, but who are otherwise identical, married women in the lowest income classes cannot get any surplus in marriage (relative to being single), and (b) married spouses receive exactly their marginal contributions to marital surplus, or else they can be replaced by husbands or wives with slightly lower surpluses. In the second period, after the quality of match is revealed, marriage continues if and only if the surplus of the marriage, including its non-monetary benefit exceeds the sum of the outside options of the spouses, which depend on whether or not remarriage is possible. However, if the marriage is successful and the partners wish to continue it, competition alone can no longer determine the shares within marriage. We consider here two possibilities, a Nash bargaining model in which people systematically renegotiate at the beginning of the second period, taking their second period outside opportunities as threat points, and an alternative framework which allows early commitment over second-period allocations. In both cases, however, people cannot commit not to divorce, which imposes some second-period individual rationality constraints. We show that both settings generate similar outcomes. Specifically, the 'Nash-bargained' allocation is compatible with the equilibrium conditions of the 'commitment' case, and is actually the only equilibrium outcome under commitment in which intra-household resource allocations are not contingent on the realization of match quality.

We then utilize our framework to analyze how changes in divorce settlement laws might affect individuals' marriage patterns, and particularly the intra-marital allocation of resources. In particular, we consider a reform that increases the wives' share of household wealth after divorce. In our Beckerian framework, such a change in post-divorce property rights cannot affect divorce probabilities, but it can influence the sharing of resources within household, both before and after divorce — even for couples who do not divorce eventually. Our main conclusion is that the short- and long-term consequences of the reform are different and, in general, potentially offsetting. For couples already married, the reform can only improve the wives' welfare at the husbands' expense; while the exact scope of the reform depends on assumptions regarding commitment, either some or all women will strictly gain from the reform and no woman can lose (and no man can gain) from it. Regarding couples who marry after the reform, however, the logic is quite different, because the new divorce settlement is taken into account at the matching stage, resulting in a different inter-temporal allocation of resources and welfare between spouses. We show that a change in divorce settlements aimed at favoring women typically generates offsetting intra-household transfers, eventually resulting in *lower* intra-marital allocations for all married women. Finally, we show that the decision to get married or remain single could be affected by divorce settlement laws too. For instance, if men are forced to pay a large transfer to their ex-wives, some men may be better off by postponing marriage, in order to avoid the risk of paying a high divorce settlement in case of divorce.

The ability of our simple model to generate explicit predictions regarding the impact of divorce laws on intra-family allocations is important for empirical work using collective models of the household. Such models can estimate a couple-specific "sharing rule" based on observed work and consumption patterns of married couples. Researchers have specified sharing rules that relate the shares of husbands and wives in the marital surplus to "distribution factors" such as sex ratios and divorce laws (see Chiappori et al., 2002), welfare benefits (Rubacalva and Thomas, $$$), and the legalization of abortion (Oreffice, 2007). This paper provides a theoretical underpinning for such rules, showing how an equilibrium analysis of the marriage market can theoretically restrict the form of the sharing rule and predict the impact of specific policy changes, thus opening the possibility of direct empirical tests.

2 The Model

2.1 Preferences

The economy is made up of individuals who live two periods. Individuals are characterized by their income, y for men and z for women. In each period, individuals derive utility from consumption of a private good, q and a public good Q. Married people also derive satisfaction from the quality of their match, θ. The husbands' and wives' individual utilities take the form

$$U_i = u_i(q_i, Q) + \theta, \qquad i = h, w, \tag{1}$$

where q_i is private consumption, Q is consumption of the public good and θ is the quality of the match. We assume that preferences of married individuals are of the generalized quasi-linear (GQL) form (see Bergstrom, 1989).

$$u_i(q_i, Q) = A(Q)\, q_i + B_i^m(Q) + \theta, \tag{2}$$

where $A(Q)$ and $B_i^m(Q)$, $i = h, w$, are increasing concave functions such that $A(0) = 1$ and $B_i^m(0) = 0$. When single, preferences take the form:

$$u_i^s(q_i, Q) = q_i + B_i^s(Q), \tag{3}$$

where again the $B_i^m(Q)$, $i = h, w$, are increasing concave functions such that $B_i^s(0) = 0$.

If a man with income y is matched with a woman with income z, they can pool their incomes. Given GQL preferences, utility is *transferable* between spouses and there is a unique efficient amount of the public good that depends only on the total income of the partners. The Pareto frontier is linear and given by

$$u_h + u_w = \max_Q A(Q)(y + z - Q) + B_h(Q) + B_w(Q) + 2\theta \;\equiv\; \eta(t) + 2\theta, \tag{4}$$

where $t \equiv y + z$ is the *total* family income, $\eta(t)$ is *increasing and convex* in t, while u_h and u_w are the attainable utility levels that can be implemented by the allocations of the private good q between the two spouses, given the efficient level of Q and family income, t. [3]

If a man (woman) with income of y (z) remains single he (she) consumes all his (her) income. Since his utility is quasi linear, the marginal utility of income, beyond what is needed to purchase the optimal level of commodity Q, is constant. Thus, utility is transferable both between married couples and divorced ex-spouses.[4]

Due to the consumption of a public good, the two individual traits, y and z are *complements* within the household. This complementarity generates positive economic gains from marriage in the sense that the material output $\eta(t)$ the partners

[3] Using the envelope theorem,

$$\eta'(t) = A(Q) \geq 1.$$

The optimal level of Q is given by the unique solution to

$$tA'(Q) - A(Q) - B_h'(Q) + B_w'(Q) = 0,$$

which implies that Q is an increasing in family income. We thus obtain that $\eta(t)$ is increasing convex function with $\eta(0) = 0$ and $\eta(t) > t$ for $t > 0$.

[4] See Chiappori, Iyigun and Weiss (2007) for a detailed investigation of the transferability in the presence of public goods.

generate together exceeds the sum of the outputs that the partners can obtain separately. Specifically, the marital surplus $\eta(t) - t$ rises with the total income of the partners, t, and equality holds only when both partners have no income.

For any couple, match quality θ is drawn from a fixed distribution Φ with a mean $\bar{\theta} \geq 0$. Upon marriage, both spouses expect to derive the same non-monetary utility from marriage, $\bar{\theta}$. At the end of the first period, the match quality is revealed and the realized value of θ may be either a above or below it's expected value, $\bar{\theta}$. A realized value of θ that is below $\bar{\theta}$ constitutes a negative surprise that may trigger divorce.[5]

2.2 Endowments

There exists a continuum of men and a continuum of women. The measure of men is normalized to unity and the measure of women is denoted by r, where $r > 1$. Each man has an amount y at the beginning of each period. Individual incomes y are distributed over the support $[a, A]$, $0 < a < A$, according to some distribution F. Similarly, each woman has an income z at the beginning of each period, and the z's are distributed over the support $[b, B]$, $0 < b < B$ according to the distribution G.

Following divorce, there can be an incomes transfer between the ex-spouses. Thus, if a man with income y marries a woman with income z, her income following divorce is $z\prime = \beta(y + z)$ and his income is $y\prime = (1 - \beta)(y + z)$. Note that the net income of a divorced person is generally different from what his/her income would have been had he/she not married. Therefore, marriage in the first period is associated with a potential cost (benefit) that depends on the identity of the prospective spouse. Consequently, the distribution of incomes among divorcees can differ from the distribution of income in the whole population.

Incomes in our model can be interpreted as either labor or property income.[6] Redistribution corresponds to a legal approach where the property incomes or the earnings of the spouses are treated as a common resource and each spouse has some claim on the income of the other. The special case in which all incomes are considered private, implying no redistribution, is represented by a β that is couple-specific, namely $\beta \equiv \frac{z}{y+z}$.

Finally, an interesting illustration of our results obtains under the assumption that the income distribution of women can be obtained from that of men by a *linear*

[5]Hence, marriage match quality, θ, is couple specific and does not vary by spouse. If our model were extended so that marriage match quality were individual specific, then our main conclusions would remain intact although such an extension could introduce other interesting aspects of marital matching, spousal allocations and divorce not covered below.

[6]For simplicity, we do not allow savings or human capital investments during marriage so that both the property and human capital are constant.

change in variables such that

$$G(z) = F(\lambda z + \delta), \tag{5}$$

for some fixed λ and δ. This is the case, for instance, when both distributions are lognormal for $y \geq a$ and $z \geq b$ with mean and variance (μ_m, σ_m) for men and (μ_f, σ_f) for women, if we assume that $\sigma_m = \sigma_f$. Then, $\lambda = \exp(\mu_m - \mu_f)$ and $\delta = a - \lambda b$. We shall refer to this property as a linear shift or LS. If $\lambda > 1$ and $\delta \geq 0$, the distribution of men dominates that of women in a first degree sense; in particular, under assortative matching each man is wealthier than his spouse.

3 The Basic Framework

We begin our analysis assuming no "second hand" marriage market so that, following divorce, a person remains single for the rest of his\her life. In the first period, all men and women wish to marry because the expected economic and non monetary gains from marriage are positive. However, because $r > 1$, some woman will remain single.

3.1 Divorce

At the end of the first period, the true value of match quality is revealed and each partner of a couple (y, z) can decide whether or not to stay in the marriage, based on the continuation of the realized θ. If the marriage dissolves, each partner receives the income stipulated by law, namely $(1 - \beta)(y + z)$ for the husband and $\beta(y + z)$ for the wife. Because utility is transferable within marriage, the 'Becker-Coase theorem' applies and divorce occurs whenever the total surplus generated outside the relationship is larger than what can be achieved within it. That is, if the total income of the partners, $t = y + z$, is such that

$$\eta(t) + 2\theta < t \ , \tag{6}$$

or, equivalently,

$$\eta(t) + 2\theta < t \Leftrightarrow \theta < \hat{\theta}(t) = -\frac{1}{2}[\eta(t) - t] \ . \tag{7}$$

On this basis, the ex-ante probability of divorce for a couple with endowments of y and z is

$$\alpha(t) \ \equiv \ \Phi[\hat{\theta}(t)] \ . \tag{8}$$

Note that the threshold $\hat{\theta}(t)$ rises with the income of the couple, t, and consequently the probability of divorce $\alpha(t)$ declines. Because of the complementarity of individual incomes in the household production process, the economic loss generated by divorce is higher for wealthier couples.

The expected marital output generated over the two periods by Mr. y and Mrs. z is

$$S(t) = \eta(t) + [1 - \alpha(t)] \left\{ \eta(t) + 2E\left[\theta \mid \theta \geq \hat{\theta}(t) \right] \right\} + \alpha(t)\, t \ .$$

Note, first, that $S(t) > 2t$. Hence, all individuals prefer to get married rather than stay single. Secondly, $S(t)$ is increasing in t, hence in each partner's income. In particular, whenever women strictly outnumber men (so that $r > 1$), women belonging to the bottom part of the female income distribution remain single. Finally, individuals will sort positively into marriage because utility is transferable and $S(t)$ is convex (so that the traits of the two partners are *complements*) even after the risk of divorce is taken into account. This last result follows from the fact that the gains from marriage depend only on total family income (see Appendix).

3.2 Matching

Who Marries Whom? Given the results of transferable utility and the complementarity of individual incomes in generating marital surplus, a stable assignment must be characterized by *positive assortative matching*. That is, if a man with an endowment y is married to a woman with an endowment z, then the mass of men with endowments above y must exactly equal the mass of women with endowments above z. This implies the following marriage market clearing condition:

$$1 - F(y) = [r1 - G(z)]. \tag{10}$$

As a result, we have the following, spousal matching functions:

$$y = F^{-1}[1 - r(1 - G(z))] \equiv \phi(z) \tag{11}$$

and

$$z = G^{-1}\left[1 - \frac{1}{r}(1 - F(y))\right] \equiv \psi(y)\,. \tag{12}$$

For $r > 1$, all men are married and women with incomes below $z_0 = G^{-1}(1 - 1/r)$ remain single. Women with incomes exceeding z_0 are then assigned to men according to $\psi(y)$ which indicates positive assortative matching.

Positive assortative matching has immediate implications for the analysis of divorce. Because divorce is less likely when a couple has higher total income and individuals sort into marriage based on income, individuals with higher income are less likely to divorce.[7]

Finally, if $r = 1$ and the linear shift property (5) holds, then spousal matching is simply described by

$$\phi(z) = \lambda z + \delta \qquad \text{and} \qquad \psi(y) = \frac{y - \delta}{\lambda} \ . \tag{13}$$

3.3 Stability Conditions

In a transferable-utility world, while the size of the surplus generated by each marriage is given, its distribution between spouses is endogenously determined by the equilibrium (or stability) conditions. The allocations which support a stable assignment must be such that the implied expected lifetime utilities of the partners satisfy

$$U_h(y) + U_w(z) \ \geq S(t) \ ; \qquad \forall \ y, \ z \ , \tag{14}$$

where $U_h(y)$ and $U_w(z)$ respectively represent the expected lifetime utilities of the husband and the wife over the two periods. For any stable marriage, equation (14) is satisfied as an equality, whereas for a pair that is not married, (14) would be satisfied as an inequality. In particular, we have

$$\begin{aligned} U_h(y) &= \max_z \left[S(t) - U_w(z)\right] \ , \\ U_w(z) &= \max_y \left[S(t) - U_h(y)\right] \ . \end{aligned} \tag{15}$$

While the stability conditions above constrain the total (two-period) expected utilities U_h and U_w, they have no implication for the distribution of utility intertemporally over the two periods.

3.4 Determination of Expected Lifetime Utilities

Condition (15) leads to an explicit characterization of the intra-household allocations. The envelope theorem applied to these conditions yields the differential equations :

$$U_h'(y) \ = S'[y + \psi(y)], \tag{16}$$

and

[7] Such a result is consistent with empirical findings on marriage and divorce patterns by schooling. Individuals sort positively into marriage based on schooling and individuals with more schooling are less likely to divorce. See Browning, Chiappori, Weiss (in progress, ch. 1).

$$U_w{}'(z) \;\; = S'[\phi(z) + z] \; . \tag{17}$$

To derive the expected spousal allocations over the two periods and along the assortative marital order, we integrate the expressions in (16) and (17). Hence, surplus share of a *married man* with income y is

$$U_h(y) \;\; = \;\; k^h + \int_a^y U_h'(x)\, dx \; , \tag{18}$$

and the surplus share for a *married woman* with income z is

$$U_w(z) = k^w + \int_b^z U_w'(x)\, dx \; , \tag{19}$$

for some constants k^h and k^w which we determine below.

Pinning down the Constants The constants k^h and k^w are pinned down by two conditions. First, for all married couples, the total output is known as expressed by equations (18) and (19). Hence,

$$k^h \; + \; k^w \;\; = S[y + \psi(y)] - \int_a^y U_h'(x)\, dx - \int_b^{\psi(y)} U_w'(x)\, dx \; , \tag{20}$$

where the left-hand side, by construction, does not depend on y. Secondly, it must be the case that 'the last married person is just indifferent between marriage and singlehood'. In the case with more women than men, $r > 1$, we have

$$U_w(z_0) \;\; = 2z_0 \qquad \Leftrightarrow \qquad k^w = 2z_0 - \int_b^{z_0} U_w'(x)\, dx \; , \tag{21}$$

with $z_0 \equiv \Phi(1 - r)$. Hence,

$$k^h = S[\phi(z_0) + z_0] - k^w$$

$$U_w(z) = 2z_0 + \int_{z_0}^z U_w'(x)\, dx \tag{22}$$

$$U_h(y) \;\; = S[y + \psi(y)] - U_w[\psi(y)] = S[y + \psi(y)] - \left(2z_0 + \int_{z_0}^{\psi(y)} U_w'(x)\, dx\right) .$$

The Linear Spousal-Income Shift Case In the linear shift case (5), these expressions can be further simplified. Indeed, $\psi(y) = \frac{y-\delta}{\lambda}$ and equation (16) becomes:

$$U_h'(y) = S'\left(y + \frac{y-\delta}{\lambda}\right) = S'(t) \tag{23}$$

where $t = y + \frac{y-\delta}{\lambda} = \frac{\lambda+1}{\lambda}y - \frac{\delta}{\lambda}$ is the household's total income. It follows that, using the change in variables $u = \frac{\lambda+1}{\lambda}x - \frac{\delta}{\lambda}$:

$$\begin{aligned} U_h(y) &= k^h + \int_a^y S'\left(x + \frac{x-\delta}{\lambda}\right)dx = k^h + \frac{\lambda}{\lambda+1}\int_{t_m}^t S'(u)\,du \\ &= k^h + \frac{\lambda}{\lambda+1}[S(t) - S(t_m)] \end{aligned} \tag{24}$$

where $t_m = a + b$ is the income of the poorest married couple. Similarly,

$$U_w(z) = k^w + \frac{1}{\lambda+1}[S(t) - S(t_m)] \tag{25}$$

and since utilities add up to $S(t)$, finally:

$$\begin{aligned} U_h(y) &= \frac{\lambda}{\lambda+1}S(t) + k, \\ U_w(z) &= \frac{1}{\lambda+1}S(t) - k. \end{aligned} \tag{26}$$

In words, each spouse's utility consists of a fixed share of the surplus plus or minus some constant. Note that the share reflects the local characteristics of the two distributions; specifically, it only depends on the linear shift parameter λ. The constant, on the other hand, is indeterminate since the numbers of males and females are exactly equal in that case. It is however bounded by the constraint that no married person would be better off as a single:

$$\begin{aligned} U_h(y) &= \frac{\lambda}{\lambda+1}S\left(\frac{\lambda+1}{\lambda}y - \frac{\delta}{\lambda}\right) + k \geq 2y \\ U_w(z) &= \frac{1}{\lambda+1}S((\lambda+1)z + \delta) - k \geq 2z \end{aligned} \tag{27}$$

for all y, z. Since $S'(t) \geq 2$ for all t, it is sufficient to check these inequalities for $y = a$ and $z = b$. Therefore,

$$\frac{\lambda}{\lambda+1}S(t_m) - 2a \leq k \leq \frac{1}{\lambda+1}S(t_m) - 2b. \tag{28}$$

4 Intra-temporal Spousal Utilities

4.1 The Commitment Issue

It is important to stress that the previous results are derived without any assumption on the level of commitment attainable by the spouses. The insight is that the conditions on the marriage market determine the allocation of lifetime utilities between spouses: because of competition, a wife would not agree to marry a husband who would provide less than the equilibrium utility (since many perfect substitutes exist), and conversely for the husband. Also, note that in our Becker-Coase context, divorce takes place if and only if it is efficient, in the sense that the average quality of the match, θ, is so poor that it offsets the economic gains resulting from the intra-household sharing of public consumption. Therefore, whether a marriage will end up in divorce or not does not depend on the type of commitment available to the spouses; in a sense, spouses are always unanimous regarding their divorce decision.

We now consider the allocation of lifetime utilities U_h and U_w between the two periods. At this point, commitment issues become crucial. While some degree of commitment is clearly achievable, there may be limits on the extent to which couples are able to commit — after all, couples could not (and would not) commit not to divorce. Two broad views emerge from the existing literature.[8] Some contributors argue that only short term commitment is attainable and that long-term decisions are generally open to renegotiation at a further stage. Others authors point out that a set of instruments (including prenuptial agreements) are available to sustain commitment. They, therefore, claim that divorce is the only limitation to commitment. Technically, marriage contracts should be seen as long-term efficient agreements under one constraint — namely, a person who wants to divorce can always choose to do so.

In our framework, the two alternative views about commitment described above each have a natural translation. Specifically, we can entertain two scenarios: In the first case ('commitment'), couples can commit to their spousal allocations in both periods conditional on the continuation of their marriage; the corresponding (contingent) allocations are ex-ante efficient under the sole constraint that divorce is unilateral. Therefore, the only constraint on intra-temporal allocation of resources is that second-period utility should exceed singles' utility, at least insofar as divorce is not an efficient outcome. Finally, should an unexpected event occur between the two periods (such as a reform of divorce laws, an example we consider below), this does

[8] See Lundberg and Pollak (1993) and especially Mazzocco ($$$$, $$$$) for a discussion of commitment issues within marriage.

not trigger a renegotiation of the initial agreement, unless the new individual rationality constraint is violated for one spouse. In the latter case, such a spouse receives an additional share of household resources, such that she becomes just indifferent between marriage and singlehood under the new law.[9]

In the alternative, polar case ('no commitment'), serious limits exist on the spouse's ability to commit. To capture this idea, we assume that couples can only commit on the immediate (i.e. first period) allocation of resources and that future allocations cannot be contracted upon, and will therefore be determined by a bargaining mechanism at the beginning of the second period. Of course, this feature is known ex ante by the agents, and influences the decisions regarding first-period allocations. Finally, if a reform occurs between the two periods, the new situation is taken into account during the second-period bargaining; i.e., bargaining always take place 'in the shadow of the law'.

4.2 The Commitment Case

Second-period Utilities We first consider couples who can commit to their spousal allocations in marriage; remember that no renegotiation can take place unless divorce is credible, and that if renegotiation does occur, it results in the minimal change needed for a marriage to continue, if that is optimal.

Let $u_h^2(y)$ and $u_w^2(z)$ denote the *monetary* components of utility derived from the intra-marital allocations respectively of husband (with endowment y) and wife (with endowment z) in the second period should they continue with their marriage. Hence, the husband's (wife's) total second-period utility is $u_h^2(y) + \theta_h$ (resp. $u_w^2(z) + \theta_w$) if the marriage continues. Feasibility constraints require that

$$u_h^2(y) + u_w^2(z) = \eta(t). \tag{29}$$

Under unilateral divorce, the wife may elect to leave her husband. If she chooses to do so, she would walk away with $\beta(y+z)$. Individual rationality implies that her utility when married should be at least as large as what she gets under divorce. The same must also hold for the husband. Therefore, it must thus be the case that

$$u_h^2(y) + \theta \geq (1-\beta)t \quad \text{and} \quad u_w^2(z) + \theta \geq \beta t, \tag{30}$$

which we shall hereafter refer as the *individual rationality constraints* (IR). Note that these conditions jointly imply that

$$u_h^2(y) + u_w^2(z) + 2\theta = \eta(t) + 2\theta \geq t\ ,$$

[9] Similar ideas are used in different contexts, in particular risk sharing agreements under limited commitment; see Ligon, Thomas and Worrall ($$$) and Kocherlakota ($$$).

or equivalently that $\theta \geq \hat{\theta}(t)$, so that divorce is not the efficient outcome.

Any allocation such that (30) is satisfied can be implemented as part of a feasible marital contract. A natural question is whether the monetary allocation (u_h^2, u_w^2) can be contingent upon the realization of θ. Contingent allocations raise specific problems. For instance, depending on the enforcement mechanism, they may require that the quality of the match be verifiable by a third party. Whether such verifiability is an acceptable assumption is not clear. It turns out, however, that verifiability is not needed in our context. Specifically:

Proposition 1 *With commitment and unilateral divorce, there exists exactly one allocation that is not θ-contingent and guarantees that all the constraints are satisfied for any realization of θ.*

Proof. *The key remark is that the time consistency constraints (30) must be binding when $\theta = \hat{\theta}(t)$ since, for that value, the couple is indifferent between marriage and divorce. Hence,*

$$u_h^2(y) = (1-\beta)t - \hat{\theta}(t) = \frac{1}{2}(\eta(t) + (1-2\beta)t), \tag{31}$$

$$u_w^2(z) = \beta t - \hat{\theta}(t) = \frac{1}{2}(\eta(t) - (1-2\beta)t). \tag{32}$$

Note that for any realization of θ, either $\theta < \hat{\theta}(t)$ and divorce takes place, or $\theta \geq \hat{\theta}(t)$ and utilities are equal to $(1-\beta)t + \theta - \hat{\theta}(t)$ and $\beta t + \theta - \hat{\theta}(t)$ for the husband and the wife respectively, so that the time consistency constraints are fulfilled for both spouses.

■

Interestingly, the second-period utilities in marriage exactly reflect the utilities if divorced, with the addition of the difference between the actual match quality θ and the threshold $\hat{\theta}$. In particular, any increase of, say, the wife's utility in divorce is exactly reflected in her second-period utility even if divorce does not take place.

First-period Utilities For each choice of k, we can recover the first-period allocations. The expected two-period utilities equal

$$U_h(y) = u_h^1(y) + (1-\alpha(t))\left\{u_h^2(y) + E\left[\theta \mid \theta \geq \hat{\theta}(t)\right]\right\} + \alpha(t)(1-\beta)t\ , \tag{33}$$

$$U_w(z) = u_w^1(y) + (1-\alpha(t))\left\{u_w^2(z) + E\left[\theta \mid \theta \geq \hat{\theta}(t)\right]\right\} + \alpha(t)\beta t\ . \tag{34}$$

where $\alpha(t) = \Pr\left(\theta < \hat{\theta}\right)$ is the divorce probability. These utilities must coincide with the equilibrium values derived above. Therefore, for $r > 1$,

$$u_w^1(z) = z_0 + \int_{z_0}^{z} S'[\phi(x) + x]\, dx$$

$$- (1 - \alpha(t)) \left\{ u_w^2(z) + E\left[\theta \mid \theta \geq \hat{\theta}(t)\right] \right\} - \alpha(t)\beta t \ ,$$

$$u_h^1(y) = S[y + \psi(y)] - z_0 - \int_{z_0}^{\psi(y)} S'[\phi(x) + x]\, dx \tag{35}$$

$$+ (1 - \alpha(t)) \left\{ u_h^2(y) + E\left[\theta \mid \theta \geq \hat{\theta}(t)\right] \right\} - \alpha(t)(1 - \beta)t \ .$$

4.3 No Commitment

For couples who won't divorce but also cannot make pre-marital allocative commitments, renegotiation systematically takes place at the beginning of the second period. We assume that such couples reach a Nash-bargaining solution with the utility of the husband and the wife in case of divorce as the relevant threat points. Hence, the allocations which we denote $(v_h^2(y), v_w^2(z))$, satisfy

$$v_h^2(y) + v_w^2(z) = \eta(t) \tag{36}$$

and solve

$$\max_{v_h^2(y),\ v_w^2(z)} \left[v_h^2(y) + \theta - (1 - \beta)t\right] \left[v_w^2(z) + \theta - \beta t\right] \ . \tag{37}$$

The solution is given by the following statement:

Proposition 2 *In the no-commitment case, the Nash-bargained, second-period utilities are:*

$$v_h^2(y) = \frac{1}{2}[\eta(t) + (1 - 2\beta)t],$$

$$v_w^2(z) = \frac{1}{2}[\eta(t) + (2\beta - 1)t] \ . \tag{38}$$

Proof. *The program maximizes the product of two terms, the sum of which is constant and equal to $\eta(t) + 2\theta - t$. Therefore*

$$v_h^2(y) + \theta - (1 - \beta)t = \frac{\eta(t) + 2\theta - t}{2} = v_w^2(z) + \theta - \beta t,$$

hence, the conclusion. ■

These values are exactly the same as in the non θ contingent allocation under commitment. In other words, the unique second-period allocation that is not θ-contingent and guarantees that the individual rationality constraints are satisfied for any realization of θ is also the Nash solution to a second-period bargaining.

5 Some Extensions

5.1 Different Match-Quality Valuations

We now consider various extensions of our model. A first variant relates to the quality of the match. So far, we have assumed that the quality of the match was perceived in similar terms by the husband and the wife. What if they disagree and individual utilities take the form

$$U_i = u_i(q_i, Q) + \theta_i, \qquad i = h, w, \tag{39}$$

where the pair (θ_h, θ_w) is jointly distributed over some support in $\mathbb{R}^2$ which needs not be the diagonal? If we denote $\theta = (\theta_h + \theta_w)/2$ the average valuation, the total surplus generated by marriage is $\eta(t) + \theta_h + \theta_w = \eta(t) + 2\theta$. In particular, the analysis of divorce probabilities and lifetime utilities remains unchanged, since in our Becker-Coase framework they are driven by the total marital surplus only. The main difference relates to the inter-temporal distribution of welfare. Specifically, second-period utilities in the no commitment case are given by:

$$\begin{aligned} v_h^2(y) &= \frac{1}{2}\left[\eta(t) + (1-2\beta)t\right] + (\theta - \theta_h), \\ v_w^2(z) &= \frac{1}{2}\left[\eta(t) + (2\beta-1)t\right] + (\theta - \theta_w). \end{aligned} \tag{40}$$

These allocations are contingent on the realization of (θ_h, θ_w), reflecting the fact that, should the marriage continue, a spouse whose evaluation is poor must be compensated by an adequate monetary transfer. Again, there exists in the commitment case a continuum of feasible inter-temporal allocations of welfare, among which one coincides with the no commitment allocation.

5.2 Limits to Transferability

The previous analysis entirely relies on the assumption that utility is freely transferable between spouses. However, transferability has its limits. Specifically, in a common property regime, consider a couple (y, z) such that $y > (1-\beta)(y+z)$ —

i.e., what the husband gets after divorce is less than what he would have had as a single. It is possible that the maximum surplus that he can get within marriage does not compensate the expected loss in case of divorce (due to the existing divorce law). Then, he is better off remaining single despite the fact that the total surplus generated by marriage is still positive.[10] This occurs if

$$d_H(t) = \eta(t) + [1-\alpha(t)]\{\eta(t) + E\left[\theta \mid \theta \geq \hat{\theta}(t)\right]\} \\ + \alpha(t)(1-\beta)t - 2y \quad \leq \; 0 \; . \tag{41}$$

Clearly, an increase in the share of the wife upon divorce reduces the husband's inclination to marry, because

$$\frac{\partial d_H}{\partial \beta} = -\alpha(t) \;\; t < 0 \; . \tag{42}$$

However, which marriages will be discouraged is not clear. The surplus generated by marriage increases with income, while the divorce probability decreases with it. Both factors go in the direction of making marriages between wealthy spouses harder to discourage. But the husband's loss from entering marriage, $y-(1-\beta)t = \beta y - (1-\beta)\psi(y)$, may increase or decrease with income, depending on the wife that is assigned to him, given the respective distributions of income. If

$$\beta < (1-\beta)\psi'(y) = (1-\beta)\left[\frac{1}{r}\frac{f(y)}{g\psi(y)}\right] , \tag{43}$$

then the loss is smaller for wealthy couples, and we conclude that a reform of divorce laws that favors the wives will mostly discourage marriage between poorer spouses. However, for very low incomes (i.e. t close to zero), we have

$$d_H = \frac{1}{2}\{E\left[\theta \mid \theta \geq \hat{\theta}(t)\right]\} > 0 \; , \tag{44}$$

and marriages between very poor individuals cannot be discouraged by divorce laws.

6 An Application: Reforming Divorce Laws

Various reforms of divorce laws have been discussed and implemented in the past. Much of the analysis of these reforms focused on the impact of the switch from mutual

[10] Matouschek and Rasul (2008) identify a similar finding whereby changes in the cost of divorce affect not only the incentive of those existing couples to stay married, but also the flow of those individuals that are selected into the marriage market.

consent to unilateral divorce (see our companion paper, Chiappori, Iyigun and Weiss, 2007). In this paper, we focus on the impact of the changes in the division of the property rights to family assets, including human capital. In several countries, the law has shifted towards viewing all family assets as common to some extent, implying that each partner has some property rights on the income of his\her spouse if the marriage dissolves.

Clearly, a shift from private to common property favors the less wealthy spouse. To keep the presentation simple, we assume that the husband is the wealthier spouse. Then, we consider a change in divorce laws such that the share of aggregate household resources rewarded to the wives is increased form β to $\hat{\beta} > \beta$. Note, however, that β may be couple-specific (as it would in a private property regime). Moreover, the analysis could apply to couples for whom the wife has higher income by simply switching the genders.

In our transferable utility framework, the Becker-Coase theorem applies and such a change *does not affect divorce probabilities.* In particular, the threshold $\hat{\theta}(t)$ only depends on the surplus generated by marriage, not on its post-divorce division between (ex-)spouses; a couple splits if and only if its realized θ lies below the threshold, irrespective of the β in place. But, under unilateral divorce laws, changes in β typically result in a redistribution of the surplus between spouses during marriage. Whether a wife would benefit from the new property division rules would depend on her income, her marriage match quality, and the level of commitment achieved between the spouses.

Our main claim is that *it is crucial to distinguish between existing couples, who are already married when the change becomes effective, and those who are not yet married*. For the former, unexpected legislative changes may trigger a renegotiation within the household and they may alter the original contract implemented. For the latter, the new legislation would be taken into account at the matching stage and reflected in the expected allocations entering marriage. We now consider these two cases successively.

6.1 Existing Marriages

Consider a married couple with endowments y and z for the husband and wife, respectively, whose match quality θ strictly exceeds the threshold $\hat{\theta}(t)$. Since the intra-household spousal allocations, as determined in the marriage market, were individually rational, it must be the case that neither spouse has an incentive to get divorced with the original β in place.

6.1.1 Commitment

Assume, first, that the spouses feel committed by the contract they initially chose, although they do not feel obligated to remain married. If θ is large enough, the wife's individual rationality requirements given by equations (30) above are satisfied for both β *and* $\hat{\beta}$. This occurs if

$$\theta \geq \hat{\beta} t - u_w^2(z), \tag{45}$$

where $u_w^2(z)$ denotes the continuation utility of the wife under the current agreement. Then, due to the commitment assumption, the change in divorce laws has no impact on intra-household allocations. If, on the contrary, θ is such that

$$\hat{\beta} t - u_w^2(z) > \theta \geq \beta t - u_w^2(z) \ , \tag{46}$$

then the initial agreement is no longer enforceable, since it would violate the wife's individual rationality. Hence, her second-period allocation must be adjusted upwards to $\hat{u}_w^2(z) = \hat{\beta} t - \theta$, which requires an additional transfer equal to

$$T = \left(\hat{\beta} - \beta\right) t - \theta - \frac{\eta(t) - t}{2} \geq 0, \tag{47}$$

From a comparative statics perspective, the probability of a renegotiation taking place depends on the distribution of θ. In the benchmark case where θ is more or less uniform over a 'large enough' support, the probability is proportional to $\left(\hat{\beta} - \beta\right) t$. When both β and $\hat{\beta}$ are identical across couples, the reform affects a larger proportion of higher-income couples. Regarding the size of the transfer, one can readily check that if β and $\hat{\beta}$ are identical across couples, the transfer T given by (47) is concave in total wealth t. It increases in t for small t; however, if the surplus function $\eta(t)$ is convex enough, it decreases in t when t is large enough. Then, the magnitude of the transfer is non-monotonic in income; it is smaller for the poorest and the highest-income couples, and maximal for intermediate income levels. In the special case of a move from private to common property, then $\beta = z/(y+z)$. The reform, not surprisingly, is more likely to affect those couples for whom the initial distribution of incomes was biased in favor of the husband. The transfer can be written as

$$T = \hat{\beta} t - z - \theta - \frac{\eta(t) - t}{2}.$$

It is still concave in t; moreover, for any given t, it decreases in z, implying that it is larger fore initially unequal couples.

We conclude that the reform will affect intra-household allocations of some - but not all - couples. For couples with a low realized match quality, the second period marital allocation of the wife may no longer be sustainable in marriage. As a result, there will be more recontracting in favor of women among such couples. And since first-period spousal allocations would have already been sunk for all of the existing marriages at the time of the legislative change, a more generous settlement rule for the wives would imply higher allocations for them in the second period *and* over their lifetimes.

6.1.2 No Commitment

In the absence of commitment, renegotiation takes place between all spouses. The reform directly impacts the respective threat points. Therefore, it will affect all couples.

Assuming, as above, a Nash bargaining solution with utilities in case of divorce as threat points, we see that the wife's gain from the reform is

$$\hat{v}_w^2(z) - v_w^2(z) = \left(\hat{\beta} - \beta\right) t \ , \tag{48}$$

while the husband loses the same amount.

We conclude that when a reform of divorce laws is favorable to women and there is no commitment to ex-ante spousal allocations between spouses, all wives will benefit and all husbands will lose. This exemplifies the case of 'bargaining in the shadow of the law'.

6.2 Future Marriages

Now consider a couple who is not yet married at the time of the legislative change. The expected lifetime allocations of such a couple, as given by equations (33) and (34), can be decomposed into three parts: the first-period utility, the second-period utility if marriage is continued, and the second-period utility in case of divorce. Unlike existing marriages, however, this effect is fully anticipated by the agents in the matching phase, and reflected in the equilibrium allocations. This fact has two consequences. First, the reform influences intra-household allocation in *both* periods. This is because the allocation of *lifetime* utility, which involves first- and second-period welfare, is decided during the matching process, taking into account the new law. A second and more subtle implication is that the impact of the reform on future marriage is the same whether or not agents are able to commit to specific intra-household allocations ex ante. Indeed, we have seen in Section 4 that the (non-θ-contingent) allocation decided ex ante is the same in both contexts.

Using (31) and (32), we can compute the impact of a change in post-divorce allocations on individual utilities. If β is identical across couples before and after the reform (one may think of a redefinition of ‘equitable distribution’ in a sense more favorable to women), the variations in individual utilities are given by:

$$\begin{aligned} \Delta u_h^1 &= \left(\hat{\beta}-\beta\right)t, & \Delta u_h^2 = -\left(\hat{\beta}-\beta\right)t, \\ \Delta u_w^1 &= -\left(\hat{\beta}-\beta\right)t, & \Delta u_w^2 = \left(\hat{\beta}-\beta\right)t\ . \end{aligned} \tag{49}$$

while if the switch is from private to common property, then,

$$\begin{aligned} \Delta u_h^1 &= \hat{\beta}t-z, & \Delta u_h^2 = -\left(\hat{\beta}t-z\right), \\ \Delta u_w^1 &= -\left(\hat{\beta}t-z\right), & \Delta u_w^2 = \hat{\beta}t-z\ . \end{aligned} \tag{50}$$

In both cases, a divorce law that mandates more generous divorce settlements for women increases their utility in the second period whether or not the couple divorces. However, the reform also *lowers* their first-period allocations by the same amount. Implicit in the above argument is what we have already established in (18) and (19): in marriages not yet formed, a legislative change has no effect on the expected *lifetime* allocations of each spouse, $U_h(y)$ and $U_w(z)$. But given that equilibrium spousal allocations need to be individually rational, more favorable divorce rules may lead to a more rapidly rising allocation path for the wives-to-be in order to ensure that their marital commitments are time consistent. In particular, all wives' expected intra-marital allocations *conditional on remaining married* are reduced and the reduction exactly offsets their gain in case of divorce.

We conclude with the following general proposition

Proposition 3 *A change in the rules governing property rights over the distribution of family assets has* no *impact on welfare as measured by expected lifetime utilities at the time of marriage. To the extent that the policy raises the utility of women following divorce, it must* reduce *their total utility while married.*

This neutrality result is reminiscent of the Ricardian equivalence result (see Barro 1974) in that an attempt by the government to redistribute income among agents is completely undone by a redistribution over time *within* family units. Our result is also quite different in that it relies on market forces rather than altruism to endogenize redistribution between spouses.

7 The Remarriage Market

Finally, we relax the assumption that divorcees must remain single during the second period. Instead, we introduce at the beginning of the second period a remarriage market in which all singles — whether never married or recent divorcees — can find a new spouse.

The analysis of a matching model with remarriage is difficult in general, in particular because the possibility of remarriage has a complex impact on the initial marital choice. We first consider a particular case, characterized by two additional assumptions under which the model can be completely solved. Then, we briefly discuss the general framework.

7.1 A Particular Case

In this subsection, we maintain the following two assumptions:

A1 The average match quality $\bar{\theta}$ is 'large' so that all agents are willing to marry in the first period.

A2 Define, as above, $z_0 = G^{-1}(1 - 1/r)$. Then $\beta(z_0 + a) > z_0$.

A characteristic feature of remarriage markets is that they may generate 'strategic postponement', whereby some agents decide not to marry during the first period, in order to improve their marital prospects in the second. A large enough expected first-period surplus eliminates such strategies, which is the motivation of Assumption A1. Regarding A2, note that if matching is assortative, the last married woman has income z_0 and marries a man with the lowest income a; then A2 requires that she is wealthier after divorce than before. A consequence is that on the remarriage market, she will be in a better position than any never married woman, which helps pinning down the remarriage matching.

We shall see that under Assumptions A1 and A2, one can fully characterize the stable matches. A technical difficulty is that, while the game must be solved backward (starting with the remarriage matching game), the analysis of remarriage depends on post-divorce income distributions, which themselves reflect the characteristics of the initial match (especially, whether or not it was assortative). It is important to note, however, that since utility remains transferable, the stable matching profile maximizes total surplus; therefore it is generically unique. Our strategy is, thus, to (i) assume that lifetime surplus is increasing and supermodular, so that initial matching is positively assortative; (ii) solve the game backward under this assumption; and (iii) show that the resulting total surplus is indeed supermodular; indeed, if a candidate equilibrium satisfies all the stability conditions, it must be the unique equilibrium.

For expositional simplicity, we start with the case $r = 1$ — i.e., there are equal measures of men and women so that all agents marry initially. Couples who draw a very poor match quality may choose to divorce; in that case, they enter the remarriage market with their post-divorce allocations, respectively equal to $y^D = (1-\beta)(y+z)$ for men and $z^D = \beta(y+z)$ for women. Then, the static matching game is played once. Note, however, that the income distribution is not the same as the initial one, because of the transfers between genders induced by divorce settlements.

Remarriage We start with the remarriage game. In the remarriage market, there are equal numbers of men and women, because each divorce increases the male and female supplies by one unit each. Moreover, for each gender, individual income rankings are not modified. Indeed, if a couple (y, z) was initially wealthier than $(\bar{y}, \bar{z})$, due to assortative matching, it must be the case that $y \geq \bar{y}$ and $z \geq \bar{z}$. If both divorce, the first husband, with an income $y^D = (1-\beta)(y+z)$, remains wealthier than the second, whose income is only $\bar{y}^D = (1-\beta)(\bar{y}+\bar{z})$ — and similarly for the wives. It follows that the number of men wealthier than y^D equals the number of women wealthier than z^D. Since matching is assortative in the remarriage market (due to the supermodularity of the one-period surplus η), we conclude that each divorced man marries a 'clone' of his former wife - i.e., a woman who just divorced an individual with the same initial income as his own.

In terms of income, if his initial income was y and hers was z, now his income is $(1-\beta)(y+z)$. Moreover, the incomes of both his *previous* and *new* wives are equal to $\beta(y+z)$ (and similarly for women). If $y^D = \phi^R(z^D)$ (or equivalently $z^D = \psi^R(y^D)$) denotes the new assignment, we have simply that

$$y^D = \phi^R(z^D) = \frac{1-\beta}{\beta} z^D \quad \text{or equivalently} \quad z^D = \psi^R(y^D) = \frac{\beta}{1-\beta} y^D, \tag{51}$$

implying a linear assignment profile. Note that, in general, β may vary across couples.

In the particular case where β is identical across couples, we see that *irrespective of the initial income distributions* (and the corresponding assignment profile), the male and female distributions of income in the remarriage market are deduced from each other by a simple, linear transform.

The new intra-household allocation of resources is again driven by the stability conditions in the remarriage market. Specifically, let $u_h^R(y^D)$ and $u_w^R(z^D)$ respectively denote the monetary components of the husbands' and wives' equilibrium utilities after remarriage (so that the true utilities are $u_h^R(y^D) + \theta^R$ and $u_w^R(z^D) + \theta^R$, where θ^R is the match quality of the new marriage). Stability requires that

$$u_h^R\left(y^D\right)=\max_{z^D}\left\{\eta\left(y^D+z^D\right)-u_w^R\left(z^D\right)\right\},$$

and

$$u_w^R\left(z^D\right)=\max_{y^D}\left\{\eta\left(y^D+z^D\right)-u_h^R\left(y^D\right)\right\}.$$

Again, the envelope theorem gives

$$\frac{du_h^R\left(y^D\right)}{dy^D}=\eta'\left(y^D+z^D\right)=\frac{du_w^R\left(z^D\right)}{dz^D}$$

Therefore,

$$\begin{aligned}
u_h^R\left(y^D\right) &= \int_{(1-\beta)(a+b)}^{y^D}\eta'\left(u+\psi^R(u)\right)du+K_h,\\
u_w^R\left(z^D\right) &= \int_{(1-\beta)(a+b)}^{z^D}\eta'\left(\phi^R(u)+u\right)du+K_w.
\end{aligned}$$

If β is identical across couples, we can use the linear transform property, and we finally get that

$$\begin{aligned}
u_h^R\left(y^D\right) &= (1-\beta)\int_{a+b}^{t}\eta'(u)\,du+K=(1-\beta)\,\eta(t)+K,\\
u_w^R\left(z^D\right) &= \beta\int_{a+b}^{t}\eta'(u)\,ds-K=\beta\eta(t)-K,
\end{aligned}$$

for some constant K. Here $t=y^D/(1-\beta)=z^D/\beta=y^D+z^D$ is the total income of the new couple $\left(y^D,z^D\right)$. In words, she gets a fraction β (and he gets a fraction $(1-\beta)$) of the surplus, plus some (positive or negative) constant K; note that since male and female are in equal number, the exact value of the constant K is indeterminate.

Divorce We now consider the divorce decision. Let $u_h^2(y)$ and $u_w^2(z)$ denote, as before, the monetary components of utility derived from the intra-marital allocations respectively of the husband (with endowment y) and the wife (with endowment z) in the second period should they continue with their marriage. Under unilateral divorce, individual rationality requires that spouses cannot remain married unless

$$u_h^2(y)+\theta\geq u_h^R\left(y^D\right)+\bar{\theta}\qquad\text{and}\qquad u_w^2(z)+\theta\geq u_w^R\left(z^D\right)+\bar{\theta}$$

This can be satisfied if

$$u_h^2(y) + u_w^2(z) + 2\theta = \eta(y+z) + 2\theta \geq u_h^R\left(y^D\right) + u_w^R\left(z^D\right) + 2\bar{\theta} = \eta(y+z) + 2\bar{\theta}$$

which boils down to $\theta \geq \bar{\theta}$: people divorce if and only if the quality of their current match is below the mean, thus exploiting the option to redraw provided by divorce. The divorce probability is therefore identical for all couples; if the distribution of θ is symmetric around its mean, the probability is now one half for all couples, a number that we keep as a benchmark in what follows. Note that, although second period utilities are no longer transferable between (former) spouses - neither $u_h^2(y)$ nor $u_w^2(z)$ are linear in general - the framework still satisfies the Becker-Coase property that divorce is independent of the laws governing settlements. This property, however, is due to the fact that each person remarries a clone of their former spouse; it would not hold in more general settings.

First-period Marriage We can now analyze the first-period matching game. Several remarks can be made:

- Despite the possibilities opened by the existence of a remarriage market, utility is still transferable in this game; this is because of the transferable structure of first-period utilities. In particular, we disregard in what follows the limits to transferability considered in the previous subsection.
- Moreover, the lifetime surplus generated by a first-period marriage still depends only on total income $t = y + z$. It is actually given by

$$\begin{aligned} S^R(t) &= \eta(t) + \frac{1}{2}\left[\eta(t) + 2E\left[\theta \mid \theta \geq \bar{\theta}\right]\right] + \frac{1}{2}\left(\eta(t) + 2\bar{\theta}\right) \\ &= 2\eta(t) + \bar{\theta} + E\left[\theta \mid \theta \geq \bar{\theta}\right]. \end{aligned} \tag{52}$$

  In particular, the surplus is supermodular (since η is convex), and the assortative matching conclusion is verified.

- The allocation of the lifetime surplus between spouses is now

$$U_h^R(y) = k^h + 2\int_a^y \eta'(x + \psi(x))\,dx \quad \text{and} \quad U_w^R(z) = k^w + 2\int_b^z \eta'(\phi(x) + x)\,dx \tag{53}$$

  where the constants k^h and k^w satisfy:

$$k^h + k^w = 2\eta(t) + \bar{\theta} + E\left[\theta \mid \theta \geq \bar{\theta}\right] - \int_a^y \eta'(x + \psi(x))\,dx - \int_b^z \eta'(\phi(x) + x)\,dx\ .$$

- Finally, the analysis of the inter-temporal allocation goes through as before. Taking, for instance, the non-commitment, Nash bargained solution, we find that the second-period allocation, which we denote $\left(v_h^{2R}(y), v_w^{2R}(z)\right)$, solves

$$\max_{v_h^2(y),\, v_w^2(z)} \left[v_h^2(y) + \theta - \left(\beta\eta(t) + K + \bar{\theta}\right)\right]\left[v_w^2(z) + \theta - \left((1-\beta)\eta(t) - K + \bar{\theta}\right)\right] . \tag{54}$$

which gives

$$v_h^2(y) = \beta\eta(t) + K \quad \text{and} \quad v_w^2(z) = (1-\beta)\eta(t) - K. \tag{55}$$

for some constant K. Again, this allocation satisfies the equilibrium conditions of the commitment case, and is not contingent on θ.

We now consider the general case $r \geqslant 1$. Then, some women are single at the end of the first period, and may enter the remarriage market during the second period. In this context, we show that the stable matching profile entails assortative matching, and that its main characteristics are actually the same as when $r = 1$. Indeed, if the first-period surplus is increasing and supermodular, single women are all at the bottom of the female income distribution. That is, there exists a threshold z_0 such that a woman is married if and only if her income is above z_0. At the beginning of the second period, all never married women have an income below z_0, whereas all divorcees have an income equal to $\beta(z_0 + a)$, which is larger than z_0 due to assumption $A2$. Therefore, the support of the second-period female income distribution consists of two disjoint intervals, $[b, z_0]$ and $[\beta(z_0 + a), \beta(A + B)]$. Since the second-period surplus η is supermodular and increasing, the second-period matching is positively assortative, and only women in the $[\beta(z_0 + a), \beta(A + B)]$ interval — all recent divorcees — remarry. In other words, the first-period marriage market splits the female population into two disjoint subsamples. Women who do not marry in the first period will remain single forever; the marriage market and the remarriage markets are de facto limited to women whose initial incomes were above z_0. By definition, such women are exactly as numerous as men. We conclude that the previous analysis, in which r was equal to 1, exactly applies in this case too. As seen before, the first-period surplus is indeed supermodular and increasing.

7.2 The General Case

If assumptions A1 or A2 fail to hold, the model becomes more complex. Indeed, divorce changes the relative rankings by income, both for men and for women. As a result, it is no longer the case that divorced men remarry a 'clone' of their ex-wives. A first consequence is that the Becker-Coase property (that divorce probability is independent of the legal system) does not hold. Changes is divorce settlements (our β) modify individual utilities in the remarriage market in a non-transferable way. Actually, the lifetime surplus generated by the initial marriage, while still transferable, also depends on β. Secondly, assortative matching is not guaranteed to hold; indeed, when the impact of the (couple-specific) divorce probability is taken into account, the lifetime surplus may fail to remain supermodular. Thirdly, agent's first period marital strategy becomes more complex, because some agents may choose to strategically postpone marriage. For instance, a never married man may have a much better ranking in the remarriage market than in the initial marriage market, especially if, in the latter market, most of his competitors are recent divorcees whose incomes are fairly depleted by the divorce settlements. While the stable matching profile is still generically unique (because of transferability), its determination must therefore rely on a fixed-point argument, whereby the agents' initial anticipations regarding the remarriage market turn out to be self-fulfilling.

Such an analysis is outside the scope of the present paper, but see, for instance, Kapan (2008) for a first investigation. One point must however be emphasized: the analysis proposed above, regarding the impact of a reform of divorce settlements, remains mostly valid. Insofar as the reforms change neither the initial income distributions nor the form taken by competition in the initial marriage market, any improvement in women's situation after divorce will, in the long run, be anticipated by agents in the initial matching phase, resulting in a compensation received by the husband in the initial phase of the marriage. The only qualification is that the changes are no longer fully neutral since, as noted above, the *size* of the aggregate surplus may vary. Note, however, that the latter effect is quite specific; for instance, the reform would typically increase (or decrease) the well being of *both* agents.

8 Conclusion

We have proposed a tractable assignment model with divorce and remarriage. Apart from making the assignment model more realistic, this extension has allowed us to address the impact of policy changes under different assumptions on commitments. We obtained a basic neutrality result which shows that any redistribution that the law imposes upon divorce can be undone within marriage, implying that, in the *long run,* changes in the laws governing divorce have no impact on the *lifetime utility* of the participants in the marriage market. In particular, if the policy improves the economic status of women upon divorce, then it must be the case that, while married, they receive a lower share of the monetary gains from marriage. Thus, an attempt to improve the status of women within marriage by transferring more resources to them in the case of divorce can be effective only in the *short run* for couples who were already married when the new policy is enacted. In this case, government intervention can have a different scope depending on the presence of prior commitments made at the time of marriage: only some of the pre-committed couples, but all of the non-commited ones will choose to renegotiate their ex-ante contracts.

It is important to compare the results of our paper with the results that would obtain in a search framework which include frictions. As shown by Mortensen (1988), such models can easily handle transitions across marital states, including divorce and remarriage, as well as learning about the quality of the match during marriage. However, in a search model, meetings occur randomly and are spaced over time. Therefore, when two agents meet and each of the matched partners can choose whether to marry or continue to search for an alternative mate, they are both aware of the cost of finding such an alternative. This creates a match-specific rent and some bargaining over the division of this rent takes place prior to marriage, with the continuation values of being single and maintaining the search serve as the natural threat points. In our model, there are no frictions and there are no ex-ante rents as, irrespective of traits, each agent has a close substitute in the marriage market. This absence of rents allows us to pin down the sharing of marital gains based only on competitive forces.

In principle, one can embed our model in a more realistic search model with frictions, and then our results will hold in the limit when meeting occur at high frequency and the discount factor is close to one (see Gale, 1986). However, several of our results would hold even without going to the limit. In particular, Garibaldi and Violante (2005) have shown that Lazear's result about the neutrality of severance payments also holds in a search economy. Likewise, we conjecture that the neutrality of divorce settlements can be extended to marriage markets with search frictions. The basic logic is that such government intervention has no direct impact on the value of remaining single. Hence, partners who bargain upon marriage will typically maintain the same distributions of the expected lifetime incomes as in the absence of government intervention; the latter only triggers a redistribution of consumption between periods.

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A Assortative matching

It is sufficient to show that the function

$$S(t) = \eta(t) + (1 - \alpha(t)) \left(h(t) + 2E\left[\theta \mid \theta \geq \hat{\theta}(y + z) \right] \right) + \alpha(t)(t) \tag{A.1}$$

where $t = y + z$, is convex. Recall that $\eta(t)$ is strictly convex. Then,

$$S(t) = \eta(t) + \int_{\hat{\theta}}^{\infty} (\eta(t) + 2\theta) f(\theta) d\theta + t \int_{-\infty}^{\hat{\theta}} f(\theta) d\theta \tag{A.2}$$

with

$$\hat{\theta}(t) = -\frac{1}{2}(\eta(t) - t) \tag{A.3}$$

Therefore,

$$\begin{aligned} S'(t) &= \eta'(t)\left(1 + \int_{\hat{\theta}}^{\infty} f(\theta)\, d\theta\right) + \int_{-\infty}^{\hat{\theta}} f(\theta) d\theta + f(\hat{\theta})[-\eta(t) - 2\hat{\theta} + t]\hat{\theta}'(t) \\ &= \eta'(t) + \eta'(t)\int_{\hat{\theta}}^{\infty} f(\theta)\, d\theta + \int_{-\infty}^{\hat{\theta}} f(\theta) d\theta > 2 \end{aligned} \tag{A.4}$$

since $\eta'(t) > 1$. Therefore:

$$\begin{aligned} S''(t) &= \eta''(t)\left(1 + \int_{\hat{\theta}}^{\infty} f(\theta)\, d\theta\right) + f(\hat{\theta})[-\eta'(t) + 1]\hat{\theta}'(t) \\ &= \eta''(t)\left(1 + \int_{\hat{\theta}}^{\infty} f(\theta)\, d\theta\right) + f(\hat{\theta})\frac{[\eta'(t) + 1]^2}{2} > 0 \end{aligned} \tag{A.5}$$

Hence, $S(t)$ is convex in t, implying that z and y are complements.