

Estimating Consumption Economies of Scale, Adult Equivalence Scales, and Household Bargaining Power

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Abstract

How much income would a woman living alone require to attain the same standard of living that she would have if she were married? What percentage of a married couple's expenditures are controlled by the husband? How much money does a couple save on consumption goods by living together versus living apart? We propose and estimate a collective model of household behavior that permits identification and estimation of concepts such as these. We model households in terms of the utility functions of its members, a bargaining or social welfare function, and a consumption technology function. We demonstrate generic nonparametric identification of the model, and hence of a version of adult equivalence scales that we call "indifference scales," as well as consumption economies of scale, the household's resource sharing rule or members' bargaining power, and other related concepts.

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1 Introduction

On average, how much income would a woman living alone require to attain the same standard of living that she would have if she were married? What percentage of a married couple's expenditures benefit the husband? How much money does a couple save on consumption goods by living together versus living apart? The goal of this paper is to propose a collective model of household behavior aimed at answering questions such as these.

A large literature exists on specification and estimation of ordinary demand systems, collective and household bargaining models, and on identification (and lack thereof) of equivalence scales, bargaining power measures, resource shares, consumption economies scale, and other related household welfare measures. Some surveys of this literature include Deaton and Muellbauer (1980), Blundell (1988), Browning (1992), Pollak and Wales (1992), Blundell, Preston, and Walker (1994), Blackorby and Donaldson (1994), Bourguignon and Chiappori (1994), Lewbel (1997), Jorgenson (1997), Slesnick (1998), and Vermeulen (2000).

We propose a new model of household consumption behavior that has three components, which are separate utility functions over goods for each household member, a consumption technology function that characterizes the jointness or publicness of goods and hence the economies of scale and scope in consumption, and a sharing rule that defines the relative allocation of household resources among the household members. The basic structure of this model is that households purchase a bundle (an n vector of quantities) of goods z . By economies of scale and scope in consumption, that is, by sharing, this z for the household is equivalent to purchasing bundle of privately consumed goods x where each element of x is typically greater than or equal to the corresponding element of z . The vector of quantities x is then divided up among the household members, and each member i derives utility from consuming their bundle of goods x^i , so the sum of these x^i vectors across household members i is x . The conversion of z to x , which is embodied by the consumption technology function, is essentially an application of the models of Barten (1964) and Gorman (1976) that characterize how demands differ across households of different sizes. The allocation of shares of x to different household members, characterized by a sharing rule, is essentially the collective household model of Bourguignon and Chiappori (1994) and Vermeulen (2000). Our model combines the features of both approaches,

which is what allows us to identify consumption economies of scale, adult equivalence scales, and household bargaining power.

We provide a dual representation of our collective model that facilitates empirical application, and show that the model is generically nonparametrically identified. We then show how the model overcomes traditional problems regarding nonidentification of equivalence scales, and can be used to address the questions listed above. Our results only require ordinal representations of preferences, and do not depend on any utility cardinalization or interpersonal comparability assumptions. We apply the model to Canadian consumption data on couples and singles.

2 Equivalence Scales and Indifference Scales

Equivalence scales seek to answer the question, “how much money does a household need to spend to be as well off as a single person living alone?” Equivalence scales have many practical applications. They are commonly used for generating poverty lines for households of various compositions given a poverty line for single males. Income inequality measures have been applied to equivalence scaled income rather than observed income to adjust for household composition (see, e.g., Jorgenson 1997). Calculation of appropriate levels of alimony or life insurance also entail comparisons of costs of living for couples versus those of singles.

An equivalence scale is traditionally defined as the expenditures of the household divided by the expenditures of a single person that enjoys the same “standard of living” as the household. Just as a true cost of living price index measures the ratio of costs of attaining the same utility level under different price regimes, equivalence scales are supposed to measure the ratio of costs of attaining the same utility level under different household compositions. Unfortunately, unlike true cost of living indexes, equivalence scales defined in this way can never be identified from revealed preference data (that is, from the observed expenditures of households under different price and income regimes). The reason is that defining a household to have the same utility level as a single individual requires that the utility functions of the household and of the single individual be comparable. We cannot avoid this problem by defining the household and the single to be equally well off when they attain the same indifference curve, analogous to the construction of true cost of living indices, because the household and the single have different preferences and hence do not possess the same indifference curves. Pollak and Wales (1979, 1992) describe these identification problems in detail, while Blundell and Lewbel (1991) prove that only changes in traditional equivalence scales, but not their levels, can be identified by revealed preference. See Lewbel (1997) for a survey of equivalence scale identification issues.

We argue that the source of these identification problems is that the standard equivalence scale question is badly posed, for two reasons. First, by definition any comparison between the preferences of two distinct decision units entails interpersonal utility comparisons. Second, and perhaps more fundamental, the notion of a household utility is flawed. Individuals have utility, not households. What is relevant is not the 'preferences' of a given household, but rather the preferences of the individuals that compose it.

We propose therefore that meaningful comparisons must be undertaken at the individual level, and that the appropriate question to ask is, "how much income would an individual living alone need to attain the same indifference curve over goods that the individual attains as a member of the household?" This latter question avoids issues of interpersonal comparability and does not require us to compare the utility levels of different indifference curves. This question also does not depend on the utility level that is assigned to an indifference curve, i.e., it is unaffected by the fact that a person's utility associated with a particular indifference curve over goods might change as a result of living with a partner. The question only depends on ordinal preferences, and hence is at least in principle answerable from revealed preference data. Consequently, in sharp contrast with the existing equivalence scale literature, our framework does not assume the existence of a unique household utility function, nor does it require comparability of utility between individuals and collectives (such as the household). Instead, following the basic ideas of the collective approach to household behavior, we assume that each individual is characterized by his/her own ordinal utility function, so the only comparisons we make is between the same person's welfare (defined by indifference curves) in different living arrangements.

Define a *equivalent income (or expenditure)* to be the income or total expenditure level y^{i*} required by an individual household member i purchasing goods privately, to be as well off materially as he or she is while living with others in a household that has joint income y . In our model, this means that when the household spends y to buy a bundle z , household member i consumes a bundle x^i (determined by the consumption technology and sharing rule). Then y^{i*} is the least expenditure required to buy a bundle of goods that lies on member i 's same indifference curve as the bundle x^i . Then, instead of a traditional adult equivalence scale, we define individual i 's "*indifference scale*" to be $S^i = y^{i*}/y$.

If member i were given the fraction S^i of the household's total expenditures, then by buying goods on the open market individual i could get herself to the same indifference curve (defined in terms of her own utility function) that she attained as a member of the household, taking into account whatever economies of scale in consumption she enjoyed by sharing and joint consumption within the household. Indifference scales depend only on the indifference curves of household members, the resources of the household, and on the degree to which consumption is shared within the household, and so can be identified without any utility cardinalization or interpersonal comparability assumptions.

To see the usefulness of indifference scales, consider the question of determining an appropriate level of life insurance for a spouse. If the couple spends y dollars per year then for the wife to maintain the same standard of living after the husband dies, she will need an insurance policy that pays enough to permit spending $S^f y$ dollars per year. Note that this amount of money only compensates the wife enough to reach the same indifference curve over goods that she attained while she was a household member. It does not compensate for any loss of utility due to grief, or for any change in her preferences that might result from the death of her husband.

Similarly, in cases of wrongful death, juries are instructed to assess damages both to compensate for the loss in “standard of living,” (i.e., S^i) and, separately for “pain and suffering,” which would presumably be noneconomic effects (see Lewbel 2003). Another example is poverty lines. If poverty lines for individuals have been established, then the poverty line for a couple could be defined as the expenditures required for *each member* of the couple to attain his or her own poverty line indifference curve.

Traditional equivalence scales do not properly answer these questions, because they attempt to relate the utility of an individual to that of a household, instead of relating the utility of the same individual in two different settings, e.g. living with a husband versus without. This is similar to the distinction Pollak and Wales (1992) make between what they call a welfare comparison versus a situation comparison.

3 The Model

This section describes the proposed household model. Let superscripts refer to household members and subscripts refer to goods. Let $U^i(x^i)$ be the direct utility function for a consumer i , consuming the vector of goods $x^i = (x_1^i, \dots, x_n^i)$. We consider households consisting of two members, which we will for convenience refer to as the husband ($i = m$) and the wife ($i = f$). For many applications, it may be useful to interpret one of these utility functions as a joint utility function for all but one member of the household, e.g., U^f could be the joint utility function of a wife and her children.

3.1 Household Members

ASSUMPTION A1: Each household member i has a monotonically increasing, continuously twice differentiable and strictly quasi-concave utility function $U^i(x^i)$ over a bundle of n goods x^i .

If member i were to face an n vector of prices p with income level y^i and this utility function, he or she would solve the optimization program

$$\max_{x^i} U^i(x^i) \text{ subject to } p'x^i = y^i. \quad (1)$$

Let $x^i = h^i(p/y^i)$ denote the solution to this individual optimization program, so the vector valued function h^i is the set of Marshallian demand functions corresponding to $U^i(x^i)$. Define V^i by

$$V^i(p/y^i) = U^i h^i(p/y^i) \quad (2)$$

so V^i is the indirect utility function corresponding to U^i . The functional form of individual demand functions h^i could be obtained from a functional specification of V^i using Roy's identity.

3.2 The Household Decision Process

Now consider a household consisting of a couple living together, and facing the budget constraint $p'z \leq y$. Following the standard collective approach, our key assumption regarding decision making within the household is that outcomes are Pareto efficient. A standard result of welfare theory (see, e.g., Bourguignon and Chiappori 1994) is that, given ordinality, one can without loss of generality write Pareto efficient decisions as a constrained maximization of the weighted sum $\mu U^f(x^f) + U^m(x^m)$. Here the constraints are the technology constraints that define feasible values of individual consumption vectors x^f and x^m given z , and the budget constraint that defines feasible values of the vector of purchases z . It is important to note that the Pareto weight μ may in general depend on prices, total expenditures, and on a vector s of distribution factors, the latter being defined as variables with no direct impact on preferences, technology or budget constraint, but that may influence the decision process.

We assume the household does not suffer from money illusion, and so write the Pareto weight function as $\mu(p/y, s)$. Possible examples of distribution factors s include individual wages (as in Browning et al., 1994) or non labor income (Thomas 1990), sex ratio on the relevant marriage market and divorce legislation (Chiappori, Fortin and Lacroix 2002), generosity of single parent benefits (Rubalcava and Thomas 2000), spouses' wealth at marriage (Thomas, Contreras and Frankenberg 1997), and the targeting of specific benefits to particular members (Duflo 2000). See also Chiappori and Ekeland (2005) for a general discussion.

ASSUMPTION A2: Given budget and technology constraints and the absence of money illusion, the household makes Pareto efficient decisions, that is, it's choice of x^f and x^m

maximizes the weighted sum $\tilde{U} = \mu(p/y, s)U^f(x^f) + U^m(x^m)$. The Pareto weight function $\mu(p/y, s)$ is positive and twice continuously differentiable in p/y .

All of the functions we define may depend on other variables that we have suppressed for notational simplicity. For example, in our empirical application the functions U^f and U^m also depend on demographic characteristics. We will also often similarly suppress the vector s .

In Assumption A2, \tilde{U} can be interpreted as a social welfare function for the household, though one in which the relative effects of individual member's utility functions may vary with prices and distribution factors. Alternatively, \tilde{U} may stem from some specific bargaining model (Nash bargaining for instance), in which distribution factors affect individual threat points. This model also allows for effects such as a wife's utility depending on both her own attained utility level over goods (the value of U^f) and on her spouse's happiness, U^m .

From a bargaining perspective, the Pareto weight $\mu(p/y, s)$ can be seen as a measure of f 's influence in the decision process. The larger the value of μ is, the greater is the weight that member f 's preferences receive in the resulting household program, and the greater will be the resulting private equivalent quantities x^f versus x^m . One difficulty with using μ as a measure of the weight given to (or the bargaining power of) member f is that the magnitude of μ will depend on the arbitrary cardinalizations of the functions U^f and U^m . Later we will propose an alternative bargaining power measure, the sharing rule, that does not depend upon any cardinalizations.

3.3 The Consumption Technology Function

In our model the household purchases some bundle z , but individual consumptions of the household members add up to some other bundle x , with the difference due to sharing and jointness of consumption. In the technological relation $z = F(x)$, we can interpret z as the inputs and x as the outputs in the household's consumption technology, described by the production function F^{-1} , though what is being 'produced' is additional consumption. This framework is similar to a Becker (1965) type household production model, except that instead of using the purchased vector of market goods z to produce commodities that contribute to utility, the household essentially produces the equivalent of a greater quantity of market goods x via sharing. The unobserved n vectors x^f , x^m , and $x = x^f + x^m$ are private good equivalent vectors, that is, they are respectively the quantities of transformed goods that are consumed by the female, the male, and in total.

ASSUMPTION A3: Given a purchased bundle of goods z , the feasible set of private good equivalent bundles x^f and x^m are given by the consumption technology function

$$z = F(x^f + x^m).$$

It will often be convenient to work with a linear consumption technology, which is mathematically identical to Gorman's (1976) linear technology (a special case of which is Barten (1964) scaling), except that we apply it in the context of a collective model.

ASSUMPTION A4: The consumption technology function is linear, so $F(x) = Ax + a$, where A is a nonsingular n by n matrix and a is a n vector.

Most of our theoretical results and all of our empirical work use a linear consumption technology, but in an appendix we will show how our main identification and duality results can be generically extended to arbitrary smooth consumption technologies.

Consider some examples. Let good j be food. Suppose that if an individual or a couple buy a quantity of food z_j , the then total amount of food that the individual or couple can actually consume (that is, get utility from) is $z_j - a_j$, where a_j is waste in food preparation, spoilage, etc.. If the individuals lived apart, each would waste an amount a_j , so the total amount wasted would be $2a_j$, while living together results in only a waste of a_j . In this simple example the economies of scale to food consumption from living together are a reduction in waste from $2a_j$ to a_j , implying that $x_j = z_j + a_j$, so the consumption technology function for food takes the simple form $z_j = F_j(x) = x_j - a_j$.

Economies of scale also arise directly from sharing. For example, let good j be automobile use, measured by distance traveled (or some consumed good that is proportional to distance, perhaps gasoline). If x_j^f and x_j^m are the distances traveled by car by each household member, then the total distance the car travels is $z_j = (x_j^f + x_j^m)/(1 + r)$, where r is the fraction of distance that the couple ride together. This yields a consumption technology function for automobile use of $z_j = F_j(x) = x_j/(1 + r)$. This example is similar to the usual motivation for Barten (1964) scales, but it is operationally more complicated, because Barten scales fail to distinguish the separate utility functions, and hence the separate consumptions, of the household members.

More complicated consumption technologies can arise in a variety of ways. The fraction of time r that the couple share the car could depend on the total usage, resulting in F_j being a nonlinear function of x_j . There could also be economies (or diseconomies) of scope as well as scale in the consumption technology, e.g., the shared travel time percentage r could be related to expenditures on vacations, resulting in $F_j(x)$ being a function of other elements of x in addition to x_j .

The model also allows for possible diseconomies of scale, e.g., diagonal elements of A could be larger than one or elements of a could be negative.

In the collective model literature sharing and jointness of consumption is usually modeled by assuming some goods are purely private and others purely public (see, e.g., Ver-

meulen 2000), with additional generality obtained by defining some goods to be the sum of public and private components, e.g., solo car travel and joint car travel could be modeled as two separate goods, one private and one public. Recalling that $x_j = x_j^f + x_j^m$, in our model a purely private good has $z_j = x_j = x_j^f + x_j^m$, while a purely public good would have $z_j = x_j/2$, with the additional constraint that $x_j^f = x_j^m$. Our model includes both $z_j = x_j$ and $z_j = x_j/2$ as special cases, and so includes purely private goods and can at least approximate purely public goods, but we do not impose the constraint $x_j^f = x_j^m$. Such constraints can be imposed in our model, but doing so would interfere with the simple duality results we derive that make our model empirically tractable. We later show how our model can be extended to include purely public goods and other, more general consumption technologies.

Goods that are often cited as purely public, such as heating, may actually be consumed in different quantities by different household members, as our model permits. For example a spouse that stays at home consumes more of the household's heat than one that goes to a job in the daytime. Finally, we note that even a linear technology function F includes many types of joint and shared consumption that cannot be captured by models that only consider purely private and purely public goods (such as economies of scope to consumption), and that our model produces household demand functions that nest as special cases the Barten (1964) and Gorman (1976) models, which are widely used in the empirical demand modeling literature.

3.4 The Household's program

ASSUMPTION A5: The household faces the budget constraint $p'z \leq y$.

Here z is the vector of quantities of the n goods the household purchases, p is market prices and y is the household's total expenditures. Assumptions A2, A3, and A5 together say that the household's consumption behavior is determined by the optimization program

$$\max_{x^f, x^m, z} \mu(p/y)U^f(x^f) + U^m(x^m) \text{ subject to } x = x^f + x^m, z = F(x), p'z = y \quad (3)$$

To save notation we have suppressed distribution factors s that affect the Pareto weight μ and any demographic or other attributes that may affect μ or the utility functions U^f and U^m or the consumption technology F . The solution of the program (3) yields the household's demand functions, which we denote as $z = h(p/y)$, and private good equivalent demand functions $x^i = x^i(p/y)$.

One way to interpret the program (3) is to consider two extreme cases. If all goods were private and there were no shared or joint consumption, then $F(x) = x$, so z would equal

$x^f + x^m$ and the program (3) would reduce to $\max_{x^f, z} \mu(p/y)U^f(x^f) + U^m(z - x^f)$ such that $p'z = y$, which is the standard specification of a collective model for purely private goods (see, e.g., Bourguignon and Chiappori 1994 or Vermeulen 2000). At the other extreme, imagine a household that has a consumption technology function F , but assume that the household's utility function for transformed goods just equaled $U^m(x)$, as might happen if the male were a dictator that forced the other household members to consume goods in the same proportion that he does. In that case, the model would reduce to $\max_z U^m F^{-1}(z)$ such that $p'z = y$, which for linear F (Assumption A4) is equivalent to Gorman's (1976) general linear technology model, a special case of which is Barten (1964) scales (corresponding to A diagonal and a zero). Our general model, program (3), combines the consumption technology logic of the Gorman or Barten framework with the collective model of a household as either a bargaining or a social welfare maximizing group.

3.5 Duality

To prove identification results and to facilitate empirical application of our model, we derive a dual representation of the household's program. To do so, consider the household as an open economy. From the second welfare theorem, any Pareto efficient allocation can be implemented as an equilibrium of this economy, possibly after lump sum transfers between members. We summarize these transfers by the *sharing rule* $\eta(p/y)$, which is defined as the fraction of a suitably constructed measure of household resources consumed by member f . The household's behavior is equivalent to allocating the fraction $\eta(p/y)$ of resources to member f , and the fraction $1 - \eta(p/y)$ to member m . Each member i then maximizes their own utility function U^i given a Lindahl (1919) type shadow price vector π and their own shadow income η^i to calculate their desired private good equivalent consumption vectors x^f and x^m . This concept of a sharing rule is borrowed directly from the collective model literature (see, e.g., Browning and Chiappori 1998 and Vermeulen 2000 for a survey), though our model is richer than these earlier households models because of the inclusion of the consumption technology function. Formally:

PROPOSITION 1: Let Assumptions A1, A2, A3, and A5 hold. There exists a Lindahl shadow price vector $\pi(p/y)$ and a scalar valued sharing rule $\eta(p/y)$, with $0 \leq \eta(p/y) \leq 1$, such that

$$\begin{aligned}
x^f(p/y) &= h^f\left(\frac{\pi(p/y)}{\eta(p/y)}\right) \\
x^m(p/y) &= h^m\left(\frac{\pi(p/y)}{1-\eta(p/y)}\right) \\
z &= h(p/y) = Fx^f(p/y) + x^m(p/y)
\end{aligned} \tag{4}$$

This and other propositions are proved in the Appendix. Here $\pi(p/y)$ denotes a vector of equilibrium (shadow) prices within the household economy. One feature of this result that will be used later for identification and empirical tractability is that these shadow prices are the same for both household members. This follows from assuming consumption technologies of the form $z = F(x^f + x^m)$. Our conceptual framework can be immediately extended to arbitrary technologies $z = F^*(x^f, x^m)$, but we would then lose this feature of members facing the same shadow prices.

Without loss of generality, shadow prices are scaled so to make total shadow income of the household be $\pi'(x^f + x^m) = 1$. The sharing rule η is then given by $\eta = \pi'x^f$, which is the fraction of total shadow income consumed by member f . It follows that $\eta(p/y)$ is a direct measure of the weight given to member f in the outcome of the household decision process. The following proposition shows the relationship between the sharing rule η and the Pareto weight μ .

PROPOSITION 2: Let Assumptions A1, A2, A3, and A5 hold, so by Proposition 1, the functions $\eta(p/y)$ and $\pi(p/y)$ exist. Then

$$\mu(p/y) = - \left[\frac{\partial V^m\left(\frac{\pi(p/y)}{1-\eta(p/y)}\right)}{\partial \eta} \right] / \left[\frac{\partial V^f\left(\frac{\pi(p/y)}{\eta(p/y)}\right)}{\partial \eta} \right] \tag{5}$$

Given utility functions U^f and U^m and a technology F , Proposition 1 shows there exists a unique sharing rule function $\eta(p/y)$ corresponding to any Pareto weight function $\mu(p/y)$ and Proposition 2 shows the converse. Our model does not require identification or estimation of μ (since we use η as our measure of resource allocation or bargaining power), however, if one wanted to use Proposition 2 empirically to recover μ , one would need variation in η that keeps π fixed. For that purpose, it suffices that $\eta(p/y)$ depend on some parameter that can be varied holding shadow prices $\pi(p/y)$ fixed. Distribution

factors s are obvious candidates. In general, shadow prices will depend in a complicated way on the consumption technology function, on both members' demand functions and on distribution factors. In particular, changing the members' respective bargaining powers will generally affect shadow prices because it modifies the structure of household demand and shadow prices are not independent of this structure. There is, however, an interesting exception to this rule, which is that $\pi(p/y)$ has a simple tractable functional form that only depends on the consumption technology when that technology is linear (Assumption A4), as follows.

PROPOSITION 3: Let Assumptions A1, A2, A3, A4, and A5 hold. Then Propositions 1 and 2 hold with

$$\pi(p/y) = \frac{A'p}{y - a'p}$$

$$z = h(p/y) = Ah^f \left(\frac{A'p}{y - a'p} \frac{1}{\eta(p/y)} \right) + Ah^m \left(\frac{A'p}{y - a'p} \frac{1}{1 - \eta(p/y)} \right) + a. \quad (6)$$

4 Identification

Given functional forms for the consumption technology function, the sharing rule, and member demand functions, Proposition 1 shows how the demand functions for households would be constructed. In the case of linear consumption technologies, these household demands have the simple, explicit form given in Proposition 3. Specifically, given linear technologies a simple parametric modeling strategy is to posit functional forms for indirect utility functions of members, and a functional form for the sharing rule η . Roy's identity then gives functional forms for the household member demand functions h^f and h^m , and the resulting functional form for the demand functions of couples is given by equation (6).

While this result is convenient for empirical work, it would be useful to know if all the features of our model are nonparametrically identified, since it would be undesirable if our estimates of economies of scale, bargaining power, indifference scales, etc., were based in part on untestable parametric assumptions. The question is, given the observable demand functions h^m , h^f , and h , can we identify (and later, estimate) the consumption technology function $F(x)$, the shadow prices $\pi(p/y)$, the sharing rule $\eta(p/y)$, and the private good equivalent demand functions $x^f(p/y)$ and $x^m(p/y)$? We show below that these functions are 'generically' nonparametrically identified, meaning that identification will only fail if the utility and technology functions are too simple (for example, a linear F is not identified if demands are of the linear expenditure system form). This identification in turn implies nonparametric identification of objects of interest such as bargaining power, economies of scale, and our indifference scales.

Two concepts that are relevant for identification are private goods and assignable goods. Define a good j to be *private*, or privately consumed, if $z_j = x_j$, so private goods have no economies or diseconomies of scale in consumption. Define a good j to be *assignable* if x_j^f and x_j^m are observed, so assignable goods are goods where we know how much is consumed separately by the husband and by the wife. For example, if we observe how much time each household member uses the family car then auto use is assignable (and not necessarily private, since some of that usage is shared when they ride together). Clothing is private if we observe total purchases of clothing and there are no economies or diseconomies of scale or scope in clothing use. Clothing is both private and assignable if it is private and if we separately observe husbands' clothing use and wives' clothing use. Having some goods be private and/or assignable helps identification, by reducing the information required to go from purchases z_j to private good equivalent consumptions x_j^f and x_j^m .

4.1 Generic Identification

ASSUMPTION A6: The household demand function $h(\cdot)$ and the household member demand functions $h^f(\cdot)$ and $h^m(\cdot)$ are identified.

Identification of the household demand function is straightforward, since we would generally estimate $z = h(p/y)$ using ordinary demand data on observed prices p , total expenditures y , and corresponding bundles z purchased by couples. Identification of member demand functions h^f and h^m is not immediate, but can be obtained in a few different ways. If individual's indifference curves over goods do not change when they marry or otherwise form households, then h^f and h^m could be estimated (and hence identified) using ordinary demand data on observed prices, total expenditures, and quantities purchased by individual men and women living alone, that is, from observing the consumption demands of singles. More generally, h^f and h^m can be identified if preferences do change upon marriage, as long as those changes can be identified, since in that case we can use singles data, couples data, and knowledge of the changes (e.g., estimated parameters that change values after marriage) to recover h^f and h^m .

Identification could alternatively be obtained if we can directly observe the consumption of goods by each individual within the household under various price and expenditure regimes, since that would provide direct observation of the functions $x^m(p/y)$ and $x^f(p/y)$ along with $h(p/y)$, which could be used to recover h^f and h^m . In other words, h^f and h^m will be identified if goods are assignable.

In practice, identification and associated estimation of h^f and h^m may be obtained by some mix of all of the above, e.g., h^f and h^m may be identified from a combination of

estimated demand functions of singles and of couples, parameterization of changes in preferences over goods resulting from marriage, and make use of assignability of some goods, such as separate observation of men's and women's clothing purchases in the household. We will later illustrate these identification methods in the context of an empirical application.

Older demand models such as Barten (1964), Gorman (1976), and Lewbel (1985), and newer specifications such as shape invariance (see, e.g., Pendakur (1999)) are models of how demands vary across households of different sizes, and so exploit data from both singles and couples to jointly identify parameters that are common to both, as well as identifying parameters that characterize the differences between the two. The idea of using singles data to identify some couple's parameters has also been used in some labor supply models, including Barmby and Smith (2001) and Bargain et. al. (2004). An example of using smoothness assumptions to identify features of individual household member consumptions is Chesher (1998).

Ordinary revealed preference theory shows that the utility functions of members U^m and U^f , or equivalently the indirect utility functions V^m and V^f , are identified up to arbitrary monotonic transformation given the member demand functions h^f and h^m . Our goal now is to use the additional information of observed household demand functions h to identify the other features of the household's program.

PROPOSITION 4: Let Assumptions A1, A2, A3, A4, A5, and A6 hold. If the number of goods in the system is $n \geq 3$, then the functions $\pi(p/y)$, $\eta(p/y)$, $x^m(p/y)$ and $x^f(p/y)$, and the technology parameters represented by A and a , are all generically identified.

The Pareto weight $\mu(p/y)$ depends upon the unobservable cardinalization of member utilities, but if those cardinalizations are known and distribution factors are present, then $\mu(p/y)$ may also be identified using equation (5), given Propositions 2 and 3.

The identification in Proposition 4 is only generic, in the sense that it only shows that we have (many) more equations than unknowns. However these equations (equation (24) in the appendix for $t = 1, \dots, T$) could fail to be linearly independent for particular functional forms (roughly analogous to the order versus the rank condition for identification in traditional linear simultaneous systems). For example, it can be readily verified that identification fails when the individual demands h^f and h^m have the Linear Expenditure System functional form. However, such problems disappear for sufficiently complicated functional forms for members demands, since nonlinearities in functional form tend to eliminate linear dependencies across equations. This is demonstrated in the next subsection with the Almost Ideal (AIDS) and Quadratic Almost Ideal (QUAIDS) functional forms. The latter will be used for our empirical estimation.

In the appendix we show informally how Proposition 4 could be extended; generic nonparametric identification should hold not just for linear consumption technologies $z = Ax + a$, but also for arbitrary technologies, that is, with $z = F(x)$ for general monotonic vector valued functions F . In other words, our general methodology does not depend on functional form assumptions to obtain identification.

4.2 The case of QUAIDS individual demands

For our empirical application, we use the following convenient method of constructing functional forms for estimation. First, choose ordinary indirect utility functions for members m and f , and let h^m and h^f be the corresponding ordinary Marshallian demand functions. Assume a linear consumption technology F . Next choose a functional form for the sharing rule η , which could simply be a constant, or a function of measures of bargaining power such as relative wages of the household members or other distribution parameters. Proposition 3 then provides the resulting functional form for the household demand function $h(p/y)$, and ensures that a corresponding household program exists that rationalizes the choice of functions h^m , h^f , and η . To satisfy Assumption A6 we assume initially that members have the same demand functions h^m and h^f as single men and single women living alone, respectively (we later consider some very simple parameterizations of preference change resulting from marriage).

In our empirical application we assume singles have preferences given by the Integrable QUAIDS demand system of Banks, Blundell and Lewbel (1997). For $i = f$ or m , let $w^i = \omega^i(p/y^i)$ denote the n -vector of member i 's budget shares w_k^i ($k = 1, \dots, n$) when living as a single, facing prices p and having total expenditure level y^i . The QUAIDS demand system we estimate takes the vector form

$$\omega^i(p/y^i) = \alpha^i + \Gamma^i \ln p + \beta^i [\ln(y^i) - c^i(p)] + \frac{\lambda^i}{b^i(p)} [\ln(y^i) - c^i(p)]^2 \quad (7)$$

where $c^i(p)$ and $b^i(p)$ are price indices defined as

$$c^i(p) = \delta^i + (\ln p)' \alpha^i + \frac{1}{2} (\ln p)' \Gamma^i \ln p \quad (8)$$

$$\ln b^i(p) = (\ln p)' \beta^i. \quad (9)$$

Here α^i , β^i and λ^i are n -vectors of parameters, Γ^i is an $n \times n$ matrix of parameters and δ^i is a scalar parameter which we take to equal zero, based on the insensitivity reported in Banks, Blundell, and Lewbel (1997). Adding up implies that $e' \alpha^i = 1$ and $e' \beta^i = e' \lambda^i = \Gamma^i e = 0$ where e is an n -vector of ones. Homogeneity implies that $\Gamma^{i'} e = 0$ and Slutsky

symmetry is equivalent to Γ^i being symmetric. The above restrictions yield the integrable QUAIDS demand system, which has the indirect utility function:

$$V^i \left(\frac{p}{y^i} \right) = \left[\left(\frac{\ln(y^i) - c^i(p)}{b^i(p)} \right)^{-1} + \lambda^{i'} \ln(p) \right]^{-1} \quad (10)$$

for $i = f$ and $i = m$. The singles demand functions $\omega^i(p/y^i)$ in equation (7) are obtained by applying Roy's identity to equation (10). Deaton and Muellbauer's (1980) Almost Ideal Demand System (AIDS) is the special case of the integrable QUAIDS in which $\lambda^i = 0$. Each element of $h_k^i(p/y^i)$ of $h^i(p/y^i)$ for $i = m$ and $i = f$, that is, member i 's quantity demand function for good j is given by $h_k^i(p/y^i) = \omega_k^i(p/y^i)y^i/p_k$.

PROPOSITION 5: Let Assumptions A1, A2, A3, A4, A5, and A6 hold. Assume members have demand functions given by the integrable AIDS or QUAIDS. Assume $\beta^f \neq \beta^m$ and each element of β^f , β^m , and the diagonal of A is nonzero. Then the functions $F(x)$, $\eta(p/y)$, $x^m(p/y)$ and $x^f(p/y)$ are identified.

Proposition 5 confirms that the QUAIDS model for member's preferences, with a linear household technology, is sufficiently nonlinear to permit identification of all the components of the couple's model. The assumption regarding nonzero and unequal elements in Proposition 5 can be relaxed. See the proof in the Appendix for details.

That proof shows that not only are the relevant household model functions are identified, but that they are massively overidentified. In fact, given member demands, most of the parameters comprising these functions can be identified just from observing the household's demand function for one good. This suggests that there is considerable scope for maintaining identification even if one adds many additional parameters to characterize changes in tastes that result from marriage.

5 Applications

Here we summarize some potential uses for our model of household consumption behavior. These uses are in addition to the standard applications of demand models, such as evaluation of price and income elasticities.

5.1 Resource Allocation Across Members and Bargaining Power

The sharing rule $\eta(p/y)$ provides a direct measure of the allocation of household resources among the household members, and hence may also be interpreted as a measure of relative

bargaining power after taking altruism into account. If all goods were private, with no economies of scale or scope in consumption (i.e., if $x = z$) then η would exactly equal the share of total expenditures y that were used to purchase the bundle x^f consumed by member f . Browning and Chiappori (1998), using household data alone, show that the relative bargaining power measures can only be identified up to an arbitrary location. By contrast, Propositions 4 and 5 show that in our model, by combining demand functions of households and the demand functions of individual members, the sharing rule η is completely identified.

We argue that the Pareto weight μ is a less tractable measure of bargaining power than η , because μ depends on the unobservable cardinalizations of the utility functions of individual household members, while η is recoverable just from observable demand functions. However, if these cardinalizations are known, then $\mu(p/y)$ can be calculated and used with our model.

5.2 Economies of Scale in Consumption

Previous attempts to measure economies of scale in consumption have required very restrictive assumptions regarding the preferences of household members (see, e.g., Nelson 1988). In contrast, given estimates of the private good equivalents vector $x = x^f + x^m$ from our model, we may calculate $y/p'x$, which is a measure of the overall economies of scale from living together, since y is what the household spends to buy z and $p'x$ is the cost of buying the private good equivalents of z . This economy of scale measure takes the form

$$\frac{y}{p'x} = \frac{y}{p' (h^f \pi / \eta + h^m \pi / (1 - \eta))}.$$

This measure does *not* directly provide an estimate of adult equivalence scales or indifference scales. The reason is that, in general, the shadow prices used within the household are not proportional to market prices (this is exactly Barten's intuition). It follows that individuals living alone would *not* buy the bundles x^f and x^m , which were optimal given shadow prices. They would in general reach a higher level of utility by re-optimizing using market instead of shadow prices.

We may also calculate the corresponding economies of scale in consumption for each good k separately as z_k/x_k . This can also be interpreted as a measure of the publicness or privateness of each good. For a purely private good with no jointness of consumption, z_k/x_k equals one, while goods that are mostly shared will have z_k/x_k close to one half. When the consumption technology has the Barten form, z_k/x_k equals the Barten scale for good k .

5.3 Indifference Scales

Given each household member i 's private good equivalents x^i , we define member i 's collective based equivalent income, y^{i*} , as the minimum expenditures required to buy a vector of goods that is on the same member i indifference curve as x^i . The ratio $S^i = y^{i*}/y$ is then what we call member i 's *indifference scale*, so S^i is the fraction of the household's income that member i would need to buy a bundle of privately consumed goods at market prices that put her on the same indifference curve over goods that she attained as a member of the household.

Like the above described economies of scale and bargaining power measures, these indifference scales do not depend on any utility cardinalizations or assumptions of interpersonally comparable utility.

Recall that $V^f(p/y^f)$ is the indirect utility function of member f , and the private good equivalent vector consumed by member f in the household is $x^f(p/y) = h^f[\pi(p/y)/\eta(p/y)]$, where h^f is the Marshallian demand function obtained from $V^f(p/y^f)$ by Roy's identity. The indifference scale $S^f(p/y, \eta)$ is defined as the solution to the equation

$$V^f\left(\frac{p/y}{S^f(p/y, \eta)}\right) = V^f\left(\frac{\pi(p/y)}{\eta}\right) \quad (11)$$

This definition of $S^f(p/y, \eta)$ only depends upon an ordinal representation of utility, i.e., it does not depend on the chosen cardinalization for member f 's utility function. Specifically, and in marked contrast to traditional adult equivalence scales, replacing V^f with any monotonic transformation of V^f in equation (11) leaves S^f unchanged. The expression for member m defining S^m , is the same as equation (11), replacing f with m and replacing η with $1 - \eta$.

As discussed earlier, these indifference scales $S^i(p/y, \eta)$ could be used for poverty, life insurance, and wrongful death, calculations. For example, in the case of a wrongful death or insurance calculation, a woman (or a mother and her children, if V^f is defined as the joint utility function of a mother and her children) would need income $S^i(p/y, \eta(p/y))y$ to attain the same standard of living without her husband that she (or they) attained while in the household with the husband present and total expenditures y . This would take into account the share of household resources $\eta(p/y)$ that she consumed, and would be exactly sufficient to compensate for the loss of economies of scale and scope from shared consumption, but would not compensate her for the husband's consumption, or for grieving, loss of companionship, or other components of utility that are assumed to be separable from consumption of goods. It would also not compensate for any change in preferences over goods that might occur as a result of the death.

Another interesting indifference scale to construct is $S^f(p/y, \eta)/S^m(p/y, 1 - \eta)$, the ratio of how much income a woman needs when living alone to the income a man needs

when living alone to make each as well off as they would be in a household. Even if men and women had identical preferences, this ratio need not equal one, because if the sharing rule η is bigger than one half, then the wife receives more than half of the household's resources, and hence would need more income when living alone to attain the same standard of living. To separate bargaining effects from other considerations, one might instead calculate $S^i(p/y, 1/2)$, which is the indifference scale assuming equal sharing of resources. For example, the ratio $S^f(p/y, 1/2)/S^m(p/y, 1/2)$ might better match the intuition of an equivalence scale comparing women to men.

In other applications, one might want to consider the roles of equivalent incomes and the sharing rule jointly. For example, given poverty lines for singles, one might define the corresponding poverty line for the couple as the minimum y such that, by choosing η optimally, each member i of the couple would have an equivalent income $S^i y$ equal to his or her poverty line as a single.

This discussion illustrates an important feature of our model, which is the ability to separately evaluate and measure the roles of individual household member preferences, of intrahousehold allocation and control of resources, and of economies of scale from sharing and joint consumption, on indifference scales and other welfare calculations.

6 Additional Results

6.1 Barten and Gorman Scales

Gorman's (1976) general linear technology model assumes that household demands are given by

$$z = Ah \left(\frac{A'p}{y - a'p} \right) + a \quad (12)$$

Barten (1964) scaling (also known as demographic scaling) is the special case of Gorman's model in which $a = 0$ and A is a diagonal matrix; see also Muellbauer (1977). Demographic translation is Gorman's linear technology with A equal to the identity matrix and a non zero, and what Pollak and Wales (1992) call the Gorman and reverse Gorman forms have both A diagonal and a nonzero. These are all standard models for incorporating demographic variation (such as the difference between couples and individuals) into demand systems.

The motivation for these models is identical to the motivation for our linear technology F , but they fail to account for the structure of the household's program. Even if the household members have identical preferences ($h^f = h^m = h$) and identical private equivalent incomes ($\eta = 1/2$), comparison of equations (6) and (12) shows that household demand

functions will still not actually be given by the Gorman or Barten model. In fact, comparison of these models shows that household demands will take the form of Gorman's linear technology, or some special case of Gorman such as Barten, only if demands are linear in prices (i.e., the linear expenditure system), or if η is zero or one, corresponding to one household member consuming all the goods (or more realistically, one member using their own preferences to dictate the entire household's purchases). Gorman's (1976) famous comment, "If I have a wife and child, a penny bun costs threepence," rationalizes the Barten scale model but implicitly assumes a dictator imposing his taste for buns on his family members.

Gorman (1976) allows A to be rectangular rather than just square, as in Becker (1965) type models where the outputs are composites that generate utility. Our theoretical framework can be immediately extended to include this case, but then it becomes considerably more difficult to obtain identification because we would not observe singles consuming these composites when they arise due to economies of scale and scope in consumption.

6.2 Other Consumption Technologies

We first show how our framework could be extended to allow for the existence of pure public goods within the household. For notational simplicity, assume for now that there is only one purely public and one purely private good, denoted by X and x respectively (with market prices P and p respectively), and that the household technology is separable across goods. Pareto efficiency implies that the vector (x^f, x^m, X) solves the program (with obvious notations):

$$\begin{aligned} & \max_{x^f, x^m, z, Z} \mu(p/y, P/y)U^f(x^f, X) + U^m(x^m, X) \\ \text{subject to } z &= f(x^f + x^m), \quad Z = F(X), \quad pz + PZ = y \end{aligned}$$

An equivalent formulation is

$$\begin{aligned} & \max_{x^f, x^m, z, Z} \mu(p/y, P/y)U^f(x^f, X^f) + U^m(x^m, X^m) \\ \text{subject to } z &= f(x^f + x^m), \quad Z = F(\max(X^f, X^m)), \quad pz + PZ = y \end{aligned} \tag{13}$$

Here, X^i can be interpreted as the quantity of the public good desired by member i . In principle, we allow X^f and X^m to differ, though the solution to this program will impose $X^f = X^m$. Equation (13) is formally equivalent to equation (3), except that F is now a function of the pair of individual consumptions, not of their sum only. In this model the demands of member f depend on the vector $(\pi/\eta, \pi^i/\eta)$, and similarly for member m . Each member i 's (Lindahl) decentralizing price π^i for the public good has $\pi^i > 0$.

If the technologies for the public and private goods are both linear, so $f(x) = Ax + a$ and $F(X) = BX + b$, then the shadow prices satisfy

$$\pi = \frac{Ap}{y - ap - bP}, \quad \pi^f + \pi^m = \frac{BP}{y - ap - bP}$$

so the sum of individual prices for the public good plays the same role as the price of a private commodity. A similar analysis could be used to incorporate externalities within the household, whether positive or negative. Note, however, that with negative externalities individual prices π^i for the externality could be negative.

Returning to the general case of n goods, a more general program that encompasses both models (13) and (3) as special cases is

$$\max_{x^f, x^m, z} \mu(p/y)U^f(x^f) + U^m(x^m) \text{ subject to } z = F(x^f, x^m), \quad p'z = y \quad (14)$$

which allows for completely general consumption technology functions $F(x^f, x^m)$. Like the other programs, the general household model (14) can be decentralized, but now the complete vector of Lindahl shadow prices will be different for members m and f , being functions of the separate derivatives of F (assuming these derivatives exist) with respect to x^m and x^f . In contrast, our program (3) assumption of an additive technology $z = F(x^f + x^m)$ makes the derivatives of F with respect to x^m and x^f equal, resulting in shadow prices $\pi(p/y)$ that are the same for m and f . The problem with nonadditive technology models is that our nonparametric identification result does not hold for them. In particular, it may be the case that several different technology functions $F(x^f, x^m)$ could generate the same observable demand functions. Identification would then need to rely either on detailed functional form assumptions or on the availability of additional information, such as assignability of goods.

Suppose that our model, which assumes an additive consumption technology $z = F(x^f + x^m)$, were applied to data derived from a more general technology $z = F(x^f, x^m)$. How would this misspecification bias the results? Our additive technology implies that both individuals face the same shadow prices π . This will provide a good approximation to a more general technology if agents have similar marginal valuations for the goods, and hence similar shadow prices.

In the particular example of a purely public good, our additive model will provide a good approximation to actual behavior if the household members have a similar willingness to pay for the public good. For example, the shadow price of purely public good should be one half the market price, leading in the Barten technology $\pi_k = A_k p_k / y$ to a coefficient of $A_k = 1/2$. If additional economies of scale are present, the coefficient may become smaller than one half; conversely, diseconomies of scale (or partly private use) would generate Barten coefficients between .5 and 1.

6.3 Preference Changes

Our model assumes that the demand functions of household members, h^f and h^m , are identified. The simplest way to satisfy this assumption is to assume that individual's preferences for goods do not change when they marry. This does not mean that individuals consume the same bundles as singles in couples (because the model of a couple permits economies of scale and scope in consumption) nor does it rule out individuals getting utility from marriage, rather, all it implies is that the indifference curves of single men or women living alone are the same as the indifference curves associated with the utility functions U^m and U^f in the household model. If this assumption holds, then the functions h^m and h^f can be estimated directly by observing the consumption behavior of single men and single women.

Instead of assuming tastes do not change, we could also attain identification by estimating the demand functions for singles, parameterize how those preferences change as a result of marriage, and use couple's data to estimate these preference change parameters along with the other features of the household models, that is, the consumption technology and sharing rule. For example, suppose that the utility function of individual i living alone is ordinaly represented by $U^i(L^{-1}z^i)$ for some matrix L . We could write this more generally as utility given by $T(U^i(L^{-1}z^i), L)$, where T is monotonic in its first element. The utility function of individual i in the household is ordinaly represented by $U^i(z^i)$, so the matrix L embodies the changes in taste for goods that result from marriage. Then the demand functions for a single i living alone will be

$$z^i = \tilde{h}^i(p/y^i) = L^{-1}h^i(L^{-1}p/y^i)$$

and therefore, given a linear consumption technology, the demand functions of the couple, expressed in terms of the demand functions \tilde{h}^i for the members, will be

$$z = h(p/y) = AL\tilde{h}^f\left(\frac{L'A'p}{y - a'p} \frac{1}{\eta(p/y)}\right) + AL\tilde{h}^m\left(\frac{L'A'p}{y - a'p} \frac{1}{1 - \eta(p/y)}\right) + a \quad (15)$$

In particular, if A and L are diagonal and $a = 0$, so both the technology and preference change have a Barten form, then for each good k , A_k is a technological economies of scale parameter that should lie between $1/2$ and 1 (assuming no diseconomies of scale), while L_k captures common taste changes over the private good equivalent $x_k = (z_k/A_k)$. If $L_k = 1$ then tastes for good k do not change with marriage. If $L_k > 1$ then married individuals like good k less than when they are single.

In this example, taste changes and economies of scale affect couple's demands in exactly the same way. If we had estimated the model assuming no taste change, we would

obtain Barten technology estimates of $L_k A_k$ instead of just A_k , and so would end up attributing taste changes to economies of scale. Additional information would then be required to separately identify L and A . For example, any good k that is known to be purely private would have $A_k = 1$, so an estimated Barten scale other than one for that good would equal L_k . Similarly, the estimated Barten scale for any good k that is known to have $L_k = 1$ (no taste change) would equal A_k . Any good k that is assignable (meaning that x_k is directly observed) would allow direct estimation of h_k^i , which could then be used in the household's model to estimate some or all of the parameters of A and compared to the singles demand functions \tilde{h}_k^i to estimate some or all of the taste change parameters L . All prices appear in the demand functions for each good, so it may be possible to identify many of the technology and taste change parameters from observations of \tilde{h}_k^i and h_k^i for a small number of goods k that are private or assignable or do not undergo a change in taste upon marriage.

In our empirical application we estimate a Barten technology and apply the model with two different interpretations of the result. One is to assume no taste change, interpreting the estimated parameters as equalling each A_k . We also apply the model assuming the estimated Barten parameters equal $L_k A_k$, where $L_k = 1$ if A_k is less than one, otherwise we assume $A_k = 1$ and the estimated parameter corresponds to taste change L_k . We also experiment with exploiting assignability of clothing.

In our indifference scale and other welfare calculations, we do not attempt to compensate for taste changes, since that, like traditional adult equivalence scales, would require untestable cardinalization assumptions, e.g., equivalence scales based on equating $U^i(L^{-1}x^i)$ to $U^i(x^i)$ would differ from equivalence scales based on equating the observationally equivalent representations of preferences $T(U^i(L^{-1}z^i), L)$ to $T(U^i(z^i), I)$ for monotonic transformations T . This is precisely the problem we avoid with our indifference scale calculations, i.e., this problem does not arise for technology parameters A instead of L , because preferences do not depend upon A . As a result, using our indifference scales, compensation is purely for resources one received as a household member, and the loss of economies of scale and scope in consumption. We believe this more closely matches the usual notion of maintaining a standard of living than any attempt to also compensate for changes in tastes.

Note that separating L from A will not be important if the dollar effect of a change in tastes for goods is small, implying that L_k is close to one for most goods. For example, joint consumption of heating is likely to have a much larger effect on measured cost savings of living together than on any change in tastes for heat. Separating L from A is largely irrelevant for some relevant calculations such as demand elasticities, or the interpretation of the sharing rule η as a measure of bargaining power.

In our empirical application we also considered other models of taste change, such

as allowing some of the intercept parameters α^i to differ for singles living alone versus those in couples, but attempts to estimate such models were abandoned when the resulting models, which are identified through functional form, failed to converge numerically.

6.4 Externalities

The analysis of positive externalities within a household is similar to that of jointly consumed goods, while negative externalities affect shadow prices in the opposite direction. Suppose, e.g., that a commodity consumed by one member has a negative impact on the other person. Since the second person's marginal willingness to pay is negative, the first person's individual shadow price must exceed the household price. In the absence of economies of scale or scope for this commodity, the estimated shadow price would be larger than the market price, leading to a Barten coefficient greater than one. More generally, if a commodity (say, tobacco) is consumed by both members and each individual's consumption has a negative impact on the other person's welfare, then each individual price may exceed the household price (because at the optimum each member will have a positive marginal willingness to pay to *get rid of* the other person's consumption). Overall, in the absence of preference changes, an estimated Barten coefficient larger than one would be suggestive of the presence of negative consumption externalities for the commodity under consideration.

The differences between externalities versus changes in preferences would be hard if not impossible to distinguish empirically. For example, if we found that married couples consume proportionally more restaurant meals than either single men or single women, this could be due either to externalities (each member's marginal utility of restaurant dining might be enhanced by the presence of their spouse) or by a simple change in preferences (members liking restaurant dining better when married). The two explanations have different implications, e.g., under the externality story only dinners taken with one's spouse are increased in value, in contrast to the preference change explanation. In practice, available data would hardly allow the estimation of such subtle distinctions.

7 Empirical Application

7.1 Data

We use Canadian FAMEX data from 1974, 1978, 1982, 1984, 1986, 1990 and 1992 on annual expenditures, incomes, labour supply and demographics for individual ('economic') households. Prices are taken from Statistics Canada. Composite commodity prices are constructed as the weighted geometric mean of the component prices with budget shares

averaged across the strata (couples, single males and single females) for weights. The (non-durable) goods we model are: food at home, restaurant expenditures, clothing, vices (alcohol and tobacco), transport, services and recreation.

We sample single females, single males and couples with no one else present in the household, with women and men aged less than or equal to 42 and 45, respectively. All agents are in full year full time employment with non-negative net and gross incomes and all couples own a car (to avoid modeling the demand patterns of the 5% of couples in the original sample that did not own cars). Table 1 presents summary statistics for our data.

7.2 Singles Budget Shares

Our model starts with a utility derived functional form for the budget shares of singles. For this we use the QUAIDS model described in equations (7), (8), (9), and (10), with $i = m$ for single men and f for single women. We also estimate a QUAIDS for couples to provide an initial comparison with singles and with our collective household model.

We allow the α and β parameters in the QUAIDS to depend on demographics. Specifically we take:

$$\alpha_k^i = \alpha_{k0}^i + \sum_{m=1}^{M_\alpha} \alpha_{km}^i d_m \quad (16)$$

$$\beta_k^i = \beta_{k0}^i + \sum_{m=1}^{M_\beta} \beta_{km}^i d_m \quad (17)$$

where $M_\beta = 2$ for singles (dummies for owning a car and being a home owner) and $M_\beta = 1$ for couples (a home owner dummy). For the intercept demographics we have $M_\alpha = 18$ for couples and $M_\alpha = 13$ for singles. These demographics include four regional dummies; a house ownership dummy and a dummy for living in a city. For singles we also have a car ownership dummy. As well we include individual specific demographics: age and age squared, a dummy for having more than high school education, dummies for being French speaking or neither English nor French speaking and an occupation dummy for being in a white collar job. Although these demographics are highly correlated within couples we need to include both sets so that we can allow for the dependence of couples' budget shares on both sets of individual characteristics. We have 24 and 28 parameters per good/equation for singles and couples respectively.

As a preliminary analysis we estimate separate QUAIDS models for single men, single women, and couples. We estimate by GMM and take account of the endogeneity of total expenditure by using as instruments for singles: all of the demographics and log relative prices (the included variables) plus log real net income (defined as the log of nominal net

	Single	Single	Couples	
	females	males	Husband	Wife
Sample size	1393	1574	1610	
Net income*	26, 137	30, 890	56, 578	
Total expenditure*	11, 855	13, 645	23, 175	
Wife's share of gross income	-	-	0.43	
bs(food at home)	0.18	0.16	0.18	
bs(restaurant)	0.11	0.15	0.11	
bs(clothing)	0.16	0.09	0.14	
bs(vices)	0.07	0.12	0.08	
bs(transport)	0.21	0.25	0.25	
bs(services)	0.17	0.10	0.12	
bs(recreation)	0.11	0.13	0.11	
car	0.64	0.78	1	
home owner	0.13	0.23	0.58	
city dweller	0.85	0.81	0.83	
age	29.9	31.1	30.2	28.2
higher education	0.20	0.25	0.21	0.19
Francophone	0.19	0.17	0.21	0.18
Allophone	0.08	0.08	0.08	0.07
White collar	0.43	0.40	0.39	0.37
Notes. * mean for 1992 (when prices are unity).				

Table 1: Descriptive statistics

income divided by a Stone price index computed for our seven non-durables goods) and its square, the product of real net income with the car and home ownership dummies and absolute log prices. For instruments for couples we use the instruments used for singles (without, of course, the car dummy variables), with individual specific values for husbands and wives, where appropriate. We also include logs of the individual gross incomes and the ratio of the wife's gross income to total gross income (we discuss these variables below).

The singles models are used for first stage GMM estimates in our joint model, since by themselves they are consistent but inefficient in the context of our joint model. The QUAIDS model for couples, which we only estimate for comparison, is a unitary model, while our actual proposed model is a collective model, essentially a mixture of the two single's QUAIDS models.

To illustrate the differences in demands of single men, single women, and couples, Figure 1 present fitted demand (Engel curve) plots for our seven goods, calculated for agents at the mean level of demographic variables, who live in Ontario in 1992 (the year and region for which prices are set to unity), with total expenditures ranging from the first to the ninth decile. We shift the plots for couples to the left in these figures to make them comparable to the singles plots. We find that food at home is a necessity for all and that transport and services are necessities for singles. Restaurants and recreation are luxuries for all and vices are a luxury for singles. Clothing is a necessity for low income single men but a luxury for high income single men (the QUAIDS quadratic term is significant). Vices are a necessity and transport is a luxury for low income couples. In both Table 1 and in these figures, the demands of couples more closely resemble the demands of single women than those of single men. The question now is whether our model of a couple as a combination of two singles can capture these effects.

7.3 The Joint Model

For our empirical application, we assume singles demands have the integrable QUAIDS functional form described in equations (7), (8), (9), (10), (16) and (17). The joint model therefore includes one set of α^i , β^i , Γ^i and λ^i parameters (including demographic components) for men, $i = m$, and another complete set of these parameters for women, $i = f$. Let $w_k^f = \omega_k^f(p/y^f)$ and $w_k^m = \omega_k^m(p/y^m)$ denote the QUAIDS budget share of consumption of good k for single women and single men, respectively.

For couples we assume a Barten type technology function defined as

$$z_k = A_k x_k \tag{18}$$

for each good k , and so is equivalent to the general linear technology $z = Ax + a$ when the matrix A is diagonal and overheads a are zero. The shadow prices for this technology

are

$$\pi_k = \frac{A_k p_k}{y} \quad (19)$$

where the couple faces prices p and has total expenditure level y . This technology function is particularly convenient for budget share models. Let $w_k = \omega_k(p/y)$ be the budget share of good k for the household, so $\omega_k(p/y) = p_k h_k(p/y)/y$. With the technology function (18) and corresponding shadow prices (19), equation (6) yields the following simple expression for the couple's budget shares

$$\omega_k(p/y) = \eta \omega_k^f \left(\frac{\pi}{\eta} \right) + (1 - \eta) \omega_k^m \left(\frac{\pi}{1 - \eta} \right) \quad (20)$$

Equation (20) shows that, with a Barten type technology, the budget shares of the couple equal a weighted average of the budget shares of its members, with weights given by the income sharing rule η and $1 - \eta$. Equation (20) shows that η represents both the fraction of resources controlled by the wife and the extent to which the household's demands resemble her demands, evaluated at shadow prices.

The parameters of the couples model consist of all the QUAIDS parameters of both the single's models ω^f and ω^m , the Barten scales A_k which we model as constants, and the parameters of the sharing rule η . Our model for η is

$$\eta = \frac{\exp(s'\delta)}{1 + \exp(s'\delta)} \quad (21)$$

where s is a household specific vector of distribution factors and δ is a vector of parameters. This logistic form bounds the sharing rule between zero and one. Based in part on the bargaining models of Browning, Bourguignon, Chiappori and Lechene (1994) and Browning and Chiappori (1998), the distribution factors that comprise s are a constant, the wife's share in total gross income, the difference in age between husband and wife, a home-ownership dummy and log deflated (by a Stone price index) household total expenditure.

The joint system consists of the vectors of budget shares for singles $\omega^f(p/y^f)$ and $\omega^m(p/y^m)$ and the vector of budget shares for couples $\omega(p/y)$. For efficiency these are all estimated simultaneously, since all the parameters in the singles models also appear in the couples model.

We estimate the joint system by GMM. We assume that the error terms are uncorrelated across households but are correlated across goods within households. Let ω_h be the $(n - 1)$ vector of budget shares for the first $n - 1$ goods consumed by household h . Denote the vector of all parameter values by θ and let $\widehat{\omega}_h(\theta)$ be the predicted budget shares for

household h . The error vector for household h is thus given by $u_h(\theta) = \omega_h - \widehat{\omega}_h(\theta)$. Let the numbers of couples, single females and single males be given by H_c , H_f and H_m respectively. Denote the $(1 \times g_c)$ vector of instruments for couples by z_h^c and similarly for singles (where $g_c = 32$ and $g_f = g_m = 25$). The vector of moment conditions for couples is given by the $((n-1)g_c \times 1)$ vector:

$$v^c(\theta) = \sum_{h=1}^{H_c} u_h' (I_{n-1} \otimes z_h^c)$$

and similarly for singles. Define the weighting matrix

$$W_c = \left(\sum_{h=1}^{H_c} (I_{n-1} \otimes z_h^c)' \tilde{u}^c \tilde{u}^{c'} (I_{n-1} \otimes z_h^c) \right)^{-1}$$

and similarly for singles. The residuals \tilde{u}^c are taken from a first stage GMM with an identity weighting matrix. For the singles these first stage residuals are from the QUAIDS estimates reported earlier. For couples, these first stage residuals are from estimates of the couples budget share system (20), estimated using just couples data. Without singles data, the parameters from the couples model by itself are not all identified, but all that is required for the first stage are unrestricted residual estimates. The GMM criterion is then

$$\min_{\theta} \{v^c(\theta)' W_c v^c(\theta) + v^f(\theta)' W_f v^f(\theta) + v^m(\theta)' W_m v^m(\theta)\}$$

where θ is the full parameter vector. In our system we have at least 258 preference parameters ($6 * 24 - 15 = 129$ symmetry constrained QUAIDS parameters for each of men and women, plus the sharing rule and Barten scale parameters). We have 492 instruments (for each good there are 25 instruments for each single strata and 32 for couples), giving a maximum degrees of freedom of 234 for the simplest model.

7.4 Parameter Estimates and Analysis

For large, highly non-linear systems such as ours, we caution that conventional limiting distribution measures of standard errors can often be quite misleading; see e.g., Gregory and Veall (1985). For our preferred specification we found ten distinct local minima, each of which required a few days of iterations to converge, so our ability to experiment with alternative specifications was limited.

Table 2 present estimates of our model and of two simpler specifications. As a benchmark, model 1 has no consumption technology function and equal sharing ($\eta = 0.5$ and

	Model		
	1	2	3
Sharing rule	0.5	0.68	0.70*
Barten scale, food	1	0.93	0.82
Barten scale, rest	1	0.78	0.79
Barten scale, cloth	1	1.70	1.88
Barten scale, vices	1	3.19	3.00
Barten scale, trans	1	0.55	0.51
Barten scale, serv	1	0.59	0.59
Barten scale, recr	1	1.31	1.38
df	234	226	222
χ^2	432.0	236.1	224.1
P-value (%)	0	30.8	44.8
* at mean of sharing of factors			

Table 2: Results

$A_k = 1$ for all k), and is decisively rejected. Model 2 specifies the sharing rule and Barten parameters (η, A_1, \dots, A_7) as constants, and improves the fit dramatically; the χ^2 (8) statistic for these parameters versus model 1 is 195.9, and this model gives a satisfactory value for the test of the over-identifying restrictions. Model 3, our preferred model, replaces the constant sharing rule in model 2 with the model of equation (21) which includes the four household specific distribution factors described earlier. The χ^2 (4) statistic for the inclusion of these demographic factors is 12.0, so they are jointly significant. We shall take this variant as our preferred specification. Attempts to add fixed costs a to the consumption technology and other extensions were not successful.

Our preferred model imposes strong restrictions on the differences between budget shares of singles and couples. In figure 2 we present the Engel curve fits for our model based on the same reference groups as in figure 1. Our model has far fewer parameters than the separate unconstrained QUAIDS models in figure 1 (411 versus 270), so the fact that our model generates quite similar estimates is reassuring. The main differences between the two are the couples Engel curves for vices (though the fits for this good were rather imprecise), transport, and perhaps food. Non-nested hypothesis tests we performed comparing the two models were inconclusive, which is not surprising given the size and complexity of the models.

Household characteristics	Wife's share η
Benchmark	0.65
First quartile total expenditure	0.60
Third quartile total expenditure	0.68
Renters	0.77
Wife's income share = 0.25	0.65
Wife's income share = 0.75	0.66
Husband ten years older	0.65
Wife ten years older	0.65

Table 3: Sharing rule implications

7.4.1 The sharing rule

Table 3 gives the values of the sharing rule η for different sets of characteristics. The benchmark household has a home-owning husband and wife with the same gross income and age and with *median* total expenditure for Ontario in 1992. Table 3 presents the estimates of the level and variations in the sharing rule.

At our benchmark value the wife's share is 0.65. If this was purely for private goods then this would be implausibly high but it also reflects the effects of joint or public consumption. Essentially, this number says that household demand functions look more like women's demand functions than men's demand functions.

From rows 2 and 3 we see that the wife's share is strongly increasing in total expenditure so that in poorer households the wife's share is much lower. This impact of total expenditures on the sharing rule means that household's demands are not unitary, that is, they are not equal to demands that would arise from the maximization of a single, ordinary utility function.

Other rows of this table show that the home-ownership status variable has a very strong impact with renting households apparently having a much higher share for the wife. As we saw above, the pattern of home ownership between singles and couples is very different so this result is probably due at least in part to a selection effect rather than a genuine difference in intra-household allocations. Neither the wife's share of income nor their relative ages has much impact on the sharing rule. This is very different from the results reported in Browning, Bourguignon, Chiappori and Lechene (1994), which examines conventional sharing rules for private goods using similar data on couples. Note that the latter paper did not include the home ownership dummy in the sharing rule and included real household income instead of total expenditures. Nonetheless the qualitative differences are striking and suggest that allowing for household technology may have a major impact

on our conclusions regarding intrahousehold allocations.

7.4.2 Assignability of Clothing

We do not model men's and women's clothing as assignable, showing that our methodology works without any assignability. However, we do observe men's and women's clothing as separate expenditure categories in the data, so assuming clothing is purely private and that women never use men's clothing or vice versa, then clothing is assignable in our data.

One way to use privateness and assignability of clothing is to generate an (inefficient) alternative estimate of the sharing rule, as the value of η that equates relative clothing shares observed within couples to estimates of clothing shares within the couple based on clothing demand functions estimated from singles data. This simple estimator for η also assumes no taste change for clothing after marriage. At base prices this yields an estimate of η of 0.55, somewhat lower than our full model estimates, but still implying that the wife controls the majority of household resources.

We also reestimated our entire model treating men's and women's clothing as separate, assignable goods in the couple's model. We do not report these estimates to save space, since relative to Table 2, the results showed no gains in plausibility or precision resulting from imposing the additional assignability assumption.

7.4.3 Barten scales and Economies of scale

We now consider estimates of the private good equivalent consumption for married men and married women and the resulting economies of scale. Given estimates of the budget share systems for singles, $w_k^i = \omega_k^i(p/y^i)$, and for households (equation 20), the private good equivalent quantities for each household member are given by $x_k^f = \eta\omega_k^f(\pi/\eta)/\pi_k$ and $x_k^m = (1 - \eta)\omega_k^m(\pi/(1 - \eta))/\pi_k$. Define R , the relative economies of scale to consumption, as

$$R = \frac{p'(x^f + x^m)}{y} - 1 = \frac{p'(x^f + x^m - z)}{p'z}$$

In Table 4 we present estimates of R and of member's private good equivalent budget shares, given $\eta = .5$ (equal sharing), and for the median benchmark estimate $\eta = .65$. We also present results using estimated Barten scales and Barten scales capped at one. This latter case assumes each estimated Barten scale equals $L_k A_k$ instead of just A_k , as described in the section on preference changes. The results that impose $A_k \leq 1$ in Table 4 (models B and D) assume for example that the estimated scale of 3.0 for vices (see

Model	A		B		C		D	
Wife's share η	0.5		0.5		0.65		0.65	
$A_k \leq 1, \forall k$	No		Yes		No		Yes	
	f	m	f	m	f	m	f	m
Ebs, food	19.2	13.1	17.8	16.4	15.7	16.9	13.9	20.7
Ebs, rest	8.8	15.8	9.0	14.6	9.5	13.7	9.9	12.8
Ebs, cloth	9.4	8.7	13.1	6.8	12.5	10.8	16.3	7.1
Ebs, vices	5.8	13.9	5.6	13.3	6.5	12.1	6.3	11.8
Ebs, tran	25.7	27.3	23.9	26.3	23.6	28.0	22.0	27.6
Ebs, serv	17.2	8.1	14.9	11.6	17.5	8.4	15.2	12.1
Ebs, recr	13.8	13.1	15.7	11.0	14.7	10.1	16.4	7.8
Scale economy	27		41		27		39	
Ebs = budget share $\times 100$ of equivalent expenditures								
Scale economy = (equivalent/actual expenditures) minus one = $R \times 100$								

Table 4: Implications of estimates

Table 2) represents a change (a decrease) in tastes for vices rather than substantial diseconomies of scale in the consumption of vices, and hence that $L_k = 3$ and $A_k = 1$ instead of $A_k = 3$. This L_k could also embody negative externalities of vices within the household. As discussed above, the distinction between the two explanations is hardly identifiable empirically. Given data limitations, whether some individuals smoke less when married because their taste for smoking has decreased or because they take into account their spouse's discomfort, or both, may be more a matter of speculation than of objective analysis. Notably, we find this strong effect, with an estimated scale much larger than one, only in commodities for which both negative externality effects and taste changes are likely to take place.

In Table 4 the first set of estimates (labelled "Ebs, good") give the equivalent budget shares for different goods for each member at unit market prices these are defined as $100\omega_k^f(\pi/\eta)$ and $100\omega_k^m(\pi/(1-\eta))$ evaluated at shadow prices $\pi_k = A_k$ for each good k . Each column of this table therefore shows, in per cent terms, how each member f and m allocates their fraction of household resources to the various goods under different sharing rules and different assumptions regarding taste change.

The differences in budget shares between m and f in the $\eta = .5$ models A and B shows the considerable effects of differences in tastes of men and women within the household, e.g. wives purchase more than twice as much of the 'services equivalent' as do their husbands but less than one half of the 'vices equivalent'. Comparing model A to C and B to D shows the impact on allocations across goods of transferring some total resources

from husbands to wives. The difference between models A and B, like the differences between C and D, is that B and D rule out diseconomies of scale in clothing, vices, and recreation (see Table 2), attributing the Barten scales that are greater than one to taste change. This implies lower shadow prices for these goods, with consequent own price, cross price, and income effects on the resulting budget shares of each member, and a larger total estimate of household consumption economies of scale.

The overall scale economy measure R times 100 is given in the last row of Table 4. In models A and C the estimate is $R = .27$, which means that it would cost the couple 27% more to buy the (private equivalent) goods they consumed if there had been no shared or joint consumption. The models that rule out diseconomies of scale of course imply greater savings, with R equal 0.39 or 0.41, showing that singles buying bundles x^f and x^m would in total pay about 40% more than the couples expenditures y . Note that both models C and D are based on the same estimates, and so correspond to our benchmark estimates; they differ only in how the estimated Barten scales are interpreted.

7.4.4 Welfare comparisons and Indifference scales

The finding that two singles require 27 to 41 percent more total expenditure than a couple to buy the same equivalent consumption levels $x^f + x^m$ represents an upper bound on the total amount needed if they actually physically lived apart (while maintaining the same preferences). This is because differences between market prices and household shadow prices mean that singles living alone can more cheaply attain the same indifference curves that x^f and x^m respectively lie on by rebudgeting. That is what our indifference scales $S^i(p/y, \eta)$ measure.

In Table 5 we present indifference scales and also the (technological) scale economy parameter, for comparison purposes. For example, in model B in this table we have $S^f + S^m = 1.38$, that is, 38% more income in total rather than 41% given by R , the difference being the gains each can get by rebudgeting. In model A, which allows for diseconomies of scale, we get 14% more income required for singles in total compared to the same model's economies of scale measure of 27%, which implies considerable savings from singles' rebudgeting. However, when comparing models A and B recall that model B assumes taste differences between single and married individuals (the L_k parameters), and the indifference scales do not compensate for these taste changes.

Along with equal sharing, we choose the more moderate relative clothing based sharing rule estimate of 0.55 rather than 0.65 for this analysis, because at 0.65 we obtain $S^m < .5$ which implies that men are better off single even with less than half of household income. This is the result of two effects: we estimate that married women receive more private goods and they have more influence on the demand for shared goods. Both effects result in

Model	A	B	C	D
Wife's share η	0.5	0.5	0.55	0.55
$A_k \leq 1, \forall k$	No	Yes	No	Yes
Scale economy $R \times 100$	27	41	27	39
indiff. scale $S^f \times 100$	58	68	63	74
indiff. scale $S^m \times 100$	56	70	50	63

Table 5: Adult equivalence scales

generally higher estimated scales for women than for men. Note that individuals could still choose to marry when their indifference scale is less than one half, or when it is less than the fraction of income or other resources they bring to the marriage, because indifference scales are only based on indifference curves over goods. Utility from companionship can compensate for possible material losses.

Our ranges of indifference scales estimates are quite wide, with values between 58% and 74% for women and between 50% or less to 70% for men. Most of this range comes from differing interpretations of the estimated Barten scales. Future work on distinguishing between taste changes versus sharing and jointness of consumption, and generating better sharing rule estimates would go a long way towards improving the precision of these estimates. Both would greatly benefit from additional data, particularly direct observation of at least some features of the consumption patterns of individuals within households.

8 Conclusions

We model households in terms of the utility functions of its members, a bargaining or social welfare function, and a consumption technology function. By employing a collective model and a separate consumption technology, we combine data from singles and couples to identify a version of equivalence scales that we call indifference scales, as well as consumption economies of scale, individual members' control of resources within the household, and other related concepts, without assumptions regarding cardinalizations or interpersonal comparability of preferences. Unlike traditional adult equivalence scales that attempt to compare the utility of an individual to that of a household, our indifference scales directly compare the cost of living for an individual within a household to the cost of living of the same individual living alone.

We also provide duality results that facilitate the empirical application of our model. These include the use of indirect utility functions to describe individual household members' preferences, an income sharing rule to model the outcome of the household bargain-

ing or resource allocation process, and shadow prices that embody economies of scale and scope arising from joint consumption. We demonstrate generic nonparametric identification of our model.

We empirically apply our model to Canadian consumption data on single men, single women and married couples. Among other empirical results, we find that couples save the equivalent of about one third of their total expenditures through shared and joint consumption of goods, that wives on average control between a half and two thirds of household resources, and that singles need to spend between one half to three fourths as much as couples to attain the same standard of living by themselves as they attain as members of a two person household.

Given our framework, one useful area for further work would be the development of more detailed alternative identifying assumptions. Without preference changes or externalities within the household, our model is very much over identified, in fact, in our QUAIDS specification many of the parameters of the entire consumption technology could be separately identified from the couple's demands for a single good. This overidentification might be exploited by relaxing the model in ways that incorporate and identify possible externalities (among individuals within the household) or by allowing for more general models of individual's preference changes that result from marriage or household formation in general. Direct observation of some elements of the private good equivalent consumption vector x would also aid identification, by distinguishing changes in preferences from pure economies of scale or scope in consumption. Example of such data would be direct measures of the quantity of food wasted (discarded) by singles and couples, or measures of the fraction of time couples drive together versus alone. Exploiting assignability of some goods also helps theoretical identification, though we did not find empirical improvements from assuming assignability of clothing.

Alternative identifying conditions that minimize reliance on the consumption data of singles would also be valuable for extending the model to directly include the utility functions of children, which for now must be assumed to be joint with the utility of an adult household member. Another useful direction for future research would be the development of empirically tractable duality and identification results for more general, nonlinear consumption technology functions.

The generality of our estimated model in the number of goods and functional forms of its components resulted in many numerical and convergence difficulties. It would be useful to investigate simplifications that preserve the main features of the model. For example, the sharing rule can in principle be identified and estimated just from combining data on singles and couples for a single good.

An interesting extension of our model would be the introduction of time allocation as another good. In particular, our consumption technology function could directly embody

the extent to which couples achieve economies of scale in time use by combining chores (such as one member cooking meals for both), and economies of scope such as saving both time and gasoline by commuting together. More generally, the consumption technology function could be merged with home production functions, as in rural households that grow crops for personal consumption.

While our particular identifying and functional form assumptions should be open to debate and improvement, we believe that the general model we have provided is an appropriate framework for estimating and analyzing adult equivalence scales, consumption economies of scale, household members' bargaining power, and other concepts relating to household preferences, consumption, and demand behavior. It could be interesting to extend the model to other aspects of collective decision making, such as household formation and dissolution, fertility, the allocation of time, durables consumption and savings decisions.

9 Appendix

9.1 Proofs of Propositions

PROOF OF PROPOSITION 1: The Lagrangian for the household's program can be written as $\mu(p/y)U^f(x^f) + U^m(x^m) + \lambda [1 - (p/y)'F(x^f + x^m)]$ which gives first order conditions

$$\mu(p/y) \frac{\partial U^f(x^f)}{\partial x_k^f} = \frac{\lambda}{y} \sum_{j=1}^n p_j \frac{\partial F^j(x^f + x^m)}{\partial x_k^f}, \quad \frac{\partial U^m(x^m)}{\partial x_k^m} = \frac{\lambda}{y} \sum_{j=1}^n p_j \frac{\partial F^j(x^f + x^m)}{\partial x_k^m}$$

Define π to be the vector of elements $\pi_k, k = 1, \dots, n$ and define η by

$$\pi_k = \frac{\sum_{j=1}^n (p_j/y) \partial F^j(x)/\partial x_k}{\sum_{i=1}^n x_i \sum_{j=1}^n (p_j/y) \partial F^j(x)/\partial x_i} \quad (22)$$

$$\eta = \sum_{i=1}^n x_i^f \pi_i \quad (23)$$

evaluated at the optimizing values $x^f = x^f(p/y)$, $x^m = x^m(p/y)$, and $x = x^f(p/y) + x^m(p/y)$. By construction $1 - \eta = \sum_{i=1}^n x_i^m \pi_i$. The above first order conditions can then

be rewritten as

$$\begin{aligned}\mu(p/y) \frac{\partial U^f(x^f)}{\partial x_k^f} &= \kappa \pi_k \\ \frac{\partial U^m(x^m)}{\partial x_k^m} &= \kappa \pi_k\end{aligned}$$

where $\kappa = (\lambda/y) \sum_{i=1}^n x_i \sum_{j=1}^n p_j \partial F^j(x)/\partial x_i$. Then

$$\begin{aligned}\frac{\partial U^f(x^f)/\partial x_k^f}{\sum_{i=1}^n x_i^f \partial U^f(x^f)/\partial x_i^f} &= \frac{\pi_k}{\sum_{i=1}^n x_i^f \pi_i} = \frac{\pi_k}{\eta} \\ \frac{\partial U^m(x^m)/\partial x_k^m}{\sum_{i=1}^n x_i^m \partial U^m(x^m)/\partial x_i^m} &= \frac{\pi_k}{\sum_{i=1}^n x_i^m \pi_i} = \frac{\pi_k}{1-\eta}\end{aligned}$$

and these are identical to the first order conditions of the programs

$$\begin{aligned}\max_{x^f} U^f(x^f) \text{ subject to } \pi' x^f &= \eta \\ \max_{x^m} U^m(x^m) \text{ subject to } \pi' x^m &= 1 - \eta\end{aligned}$$

which define the functions h^f and h^m .

PROOF OF PROPOSITION 2: From the proof of Proposition 1 we have that a first order condition of the household's program is

$$\mu(p/y) \frac{\partial U^f(x^f)}{\partial x_k^f} = \kappa \pi_k$$

The first order condition of member f 's optimization $V^f(\pi/\eta) = \max\{U^f(x^f) \mid \pi' x^f \leq \eta\}$ gives $\partial U^f(x^f)/\partial x_k^f = \lambda^f \pi_k$ and using the envelope theorem we obtain

$$\mu(p/y) \frac{\partial V^f(\pi/\eta)}{\partial \eta} = \lambda_0.$$

Apply the same logic to member m gives

$$\frac{\partial V^m(\pi/(1-\eta))}{\partial(1-\eta)} = \lambda_0$$

The proposition follows from equating these two expression for λ_0 and solving for $\mu(p/y)$.

PROOF OF PROPOSITION 3: Let A_{jk} denote the j, k 'th element of A and a_j denote the j 'th element of a . Then Equation (22) with $F(x) = Ax + a$ is

$$\pi_k = \frac{\sum_{j=1}^n p_j A_{jk}}{\sum_{i=1}^n x_i \sum_{j=1}^n p_j A_{ji}} = \frac{\sum_{j=1}^n p_j A_{jk}}{\sum_{i=1}^n x_i \sum_{j=1}^n p_j A_{ji}}$$

so $\pi = A'p/(p'Ax)$ and $p'Ax = p'A(A^{-1}(z - a)) = p'(z - a) = y - p'a$. Given π , equation (6) follows from equation (4) with $z = A(x^f + x^m) + a$. Having $\pi = A'p/(y - a'p)$ means that π is not a function of U^f , U^m or μ , so any changes in those function that change the resulting optimal x^f for a given p/y can change $\eta = x^f \pi$ while holding π fixed.

PROOF OF PROPOSITION 4: Take a finite set of price vectors p^1, \dots, p^T where $T \geq n + 10$. These may be interpreted as prices in different time periods. For any p^t and any y , let $z^t = h(p^t/y)$ denote the corresponding household demands. From Proposition 3 we have

$$z^t = Ah^f \left(\frac{A'p^t}{y - a'p^t} \frac{1}{\eta(p^t/y)} \right) + Ah^m \left(\frac{A'p^t}{y - a'p^t} \frac{1}{1 - \eta(p^t/y)} \right) + a, \quad t = 1, \dots, T \quad (24)$$

This provides, for each t , $(n - 1)$ independent equations, hence a total of $(n - 1)T$ equations. The unknowns are the matrix A , the vector a and the scalars $\eta^t = \eta(p^t/y)$, $t = 1, \dots, T$; hence a total of $n^2 + n + T$ unknowns. If $n \geq 3$, then with $T \geq n + 10$, the number of equations exceeds the number of unknowns. This shows that, generically on the demand functions h^f and h^m and on the sharing rule η , the equations are linearly independent, precisely because of the non linearities introduced by these functions. Identification in this type of model is at best generic; see, e.g., Chiappori and Ekeland (2004). In our case, for instance, if η is constant and h^f and h^m are linear, the linear independence of the equations is no longer guaranteed. With linear independence, the system is generically (over) identified. Given identification of A and a , we then have identification of $F(x) = Ax + a$ and $\pi(p/y) = A'p/(y - a'p)$. The sharing rule $\eta(p/y)$ is then identified for all p/y from equation (6) and identification of x^m and x^f follows from equation (4).

PROOF OF PROPOSITION 5: The AIDS quantity demand for good j consumed by member i is

$$h_j^i(p/y^i) = \frac{y^i}{p_j} \left(\alpha_j^i + \Gamma_j^i \ln p + \beta_j^i [\ln(y^i) - c^i(p)] \right)$$

where

$$c^i(p) = (\ln p)' \alpha^i + \frac{1}{2} (\ln p)' \Gamma^i \ln p$$

$$\ln b^i(p) = (\ln p)' \beta^i$$

α^i and β^i are n -vectors of parameters, Γ^i is an $n \times n$ matrix of parameters, $e' \alpha^i = 1$, $e' \beta^i = \Gamma^i e = 0$ where e is an n -vector of ones, and Γ^i is symmetric.

Linear technology implies that the shadow price for good j is

$$\pi_j = \frac{A_{.j} p}{y - a' p} = \frac{\sum_k A_{kj} p_k}{y - a' p}$$

where $A_{.j}$ is the transpose of the j 'th column of A and A_{kj} is the k 'th element of $A_{.j}$.

With members having AID demand functions and linear consumption technology, the couple's quantity demand for good k is

$$h_k(p/y) = \sum_j A_{kj} \left[\frac{\eta}{\pi_j} \left(\alpha_j^f + \Gamma_j^f \ln \pi + \beta_j^f [\ln(\eta) - c^f(\pi)] \right) + \frac{1-\eta}{\pi_j} \left(\alpha_j^m + \Gamma_j^m \ln \pi + \beta_j^m [\ln(1-\eta) - c^m(\pi)] \right) \right] + a_k$$

$$h_k(p/y) = (y - a' p) \sum_j A_{kj} \left[\eta \left(\frac{1}{A_{.j} p} \right) \left(\alpha_j^f + \Gamma_j^f \ln \left(\frac{Ap}{y - a' p} \right) + \beta_j^f \left[\ln(\eta) - c^f \left(\frac{Ap}{y - a' p} \right) \right] \right) + (1-\eta) \left(\frac{1}{A_{.j} p} \right) \left(\alpha_j^m + \Gamma_j^m \ln \left(\frac{Ap}{y - a' p} \right) + \beta_j^m \left[\ln(1-\eta) - c^m \left(\frac{Ap}{y - a' p} \right) \right] \right) \right] + a_k$$

Given $h_k(p/y)$ for each good k , the constants a_k are identified as the intercept terms, and we may identify the summation by $h_k(p/y) - a_k / (y - a' p)$. Now use

$$c^i(\pi) = c^i \left(\frac{Ap}{y - a' p} \right) = (\ln Ap)' \alpha^i - (\ln y - a' p) + \frac{1}{2} (\ln(Ap))' \Gamma^i \ln(Ap)$$

to write this summation term as

$$\sum_j \frac{A_{jk}}{A_{.j} p} \left[\eta \left(\alpha_j^f + \Gamma_j^f \ln(Ap) + \beta_j^f [\ln(\eta) - ((\ln Ap)' \alpha^f - (\ln y - a' p) + \frac{1}{2} (\ln(Ap))' \Gamma^f \ln(Ap))] \right) + (1-\eta) \left(\alpha_j^m + \Gamma_j^m \ln(Ap) + \beta_j^m [\ln(1-\eta) - ((\ln Ap)' \alpha^m - (\ln y - a' p) + \frac{1}{2} (\ln(Ap))' \Gamma^m \ln(Ap))] \right) \right]$$

For each good k , the coefficient of $\ln y$ in this expression is

$$- \sum_j \frac{\eta \beta_j^f + (1-\eta) \beta_j^m}{\sum_\ell (A_{\ell j} / A_{kj}) p_\ell}$$

where the summation is now over all goods j for which A_{kj} is not equal to zero. Variation in y therefore provides identification of the above expression for each good k . Variation of p in the above expression for each k identifies

$$\frac{\eta\beta_j^f + (1 - \eta)\beta_j^m}{A_{\ell j}/A_{kj}}$$

for triplets of goods j, k, ℓ having $A_{kj} \neq 0$ and $A_{\ell j} \neq 0$. The above expression with $\ell = k$ identifies η , since β_j^m and β_j^f are identified from singles, and therefore $A_{\ell j}/A_{kj}$ is also identified. Define $\bar{A}_{\ell j} = A_{\ell j}/A_{jj}$ and $d_j = A_{jj}$. Identification of $A_{\ell j}/A_{kj}$ implies identification of $\bar{A}_{\ell j}$ for all j, ℓ . What remains is to identify each d_j .

The term $A_{kj}/A_{\cdot j}p = 1/\sum_{\ell}(\bar{A}_{\ell j}/\bar{A}_{kj})p_{\ell}$ is identified for each j , which implies based on the large summation term above that

$$\begin{aligned} & \eta \left(\alpha_j^f + \Gamma_j^f \ln(Ap) + \beta_j^f \left[\ln(\eta) - ((\ln Ap)'\alpha^f + \frac{1}{2}(\ln(Ap))'\Gamma^f \ln(Ap)) \right] \right) \\ & + (1 - \eta) \left(\alpha_j^m + \Gamma_j^m \ln(Ap) + \beta_j^m \left[\ln(1 - \eta) - ((\ln Ap)'\alpha^m + \frac{1}{2}(\ln(Ap))'\Gamma^m \ln(Ap)) \right] \right) \end{aligned}$$

is identified for each good j . The component of this expression that is quadratic in $\ln(Ap)$ is

$$\begin{aligned} & \ln(Ap)' \left(-\eta\beta_j^f \frac{1}{2}\Gamma^f - (1 - \eta)\beta_j^m \frac{1}{2}\Gamma^m \right) \ln(Ap) \\ & = \sum_k \sum_{\ell} \ln \left(\sum_j d_j \bar{A}_{kj} p_j \right) \ln \left(\sum_j d_j \bar{A}_{\ell j} p_j \right) \varphi_{k\ell} \end{aligned}$$

where $\varphi_{k\ell}$ and \bar{A}_{kj} are already identified for all j, k, ℓ . Variation in p in this expression then identifies d_j , which completes the identification.

Note that the technology and sharing rule are substantially overidentified. Given the demand functions of members, the couple's demands for each good k separately identify η , a (from the $y - a'p$ term) and $A_{\ell j}/A_{kj}$ for all j, k, ℓ having $A_{kj} \neq 0$. Therefore, observing couple's demands for all goods, not just one, greatly overidentifies these terms. In particular, if the k' th row of A has all nonzero elements, then (given the demands of singles) the technology and the sharing rule can be completely identified just by observing couple's demands for the k' th good. In addition to the overidentification resulting from observing the couple's demands for multiple goods, only the terms involving the intercept, the coefficient of $\ln y$, and the quadratic in $\ln(Ap)$ were used for identification, so all the other terms in the couple's demands, such as the linear in $\ln(Ap)$ term, provide additional overidentifying restrictions. These various overidentifying assumptions may be

used to relax the assumptions regarding unequal and nonzero parameters in the statement of Proposition 5.

Similarly, the above proof applies when single's have QUAIDS demands, since in that case the only difference in the model is the addition of more terms (in particular, a quadratic in $\ln y$) which provide more overidentifying restrictions.

9.2 Nonparametric Identification

We now show how the generic identification in Proposition 4 extends to the case where the consumption technology function F is an unknown smooth function. Let $\rho = p/y$ and let Ω_F be the space of increasing C^2 functions with range and domain equal to the positive orthant. Define a mapping \mathcal{T} by the following procedure. Given any element $\bar{F} \in \Omega_F$, treat \bar{F} as if it were the true technology function by defining the corresponding shadow price vector $\bar{\pi}(\rho) = D\bar{F}(x)' \cdot \rho / [x' D\bar{F}(x)' \cdot \rho]$ evaluated at $x = \bar{F}^{-1}h(\rho)$. Now let $\bar{x}(\rho, \eta) = h^m \bar{\pi}(\rho) / (1 - \eta) + h^f \bar{\pi}(\rho) / \eta$, and for every ρ in the positive orthant define $\bar{\eta}(\rho)$ by $\bar{\eta}(\rho) = \arg \min_{\eta^* \in (0,1)} \max |\bar{x}(\rho, \eta^*) - \bar{F}^{-1}h(\rho)|$. Finally, define the function \tilde{F} by $\tilde{F}[\bar{x}(\rho, \bar{\eta}(\rho))] = h(\rho)$. This procedure defines a mapping $\tilde{F} = \mathcal{T}(\bar{F})$.

The true technology function $F(x)$ is a fixed point of this mapping, and the true sharing rule $\eta(\rho)$ equals $\bar{\eta}(\rho)$ evaluated using $\bar{F} = F$. A sufficient condition to ensure the identification of η given identification of F is that there exist an element of the vector $\bar{x}(\rho, \eta)$ that is strictly monotonic in η (and a primitive sufficient condition for this is the existence of a good that is only consumed by one household member, who has a demand function for that good that is strictly monotonic in total expenditure).

\mathcal{T} might not be a contraction mapping, and so may not have a unique fixed point. However, for suitably regular demand functions (those for which the tangent application to \mathcal{T} is Fredholm) each fixed point will be locally unique as a consequence of Smale's (1973) generalized transversality theorem. This provides local generic identification. Ordinary regularity conditions such as monotonicity and domain constraints further restrict the set of feasible solution functions.

Global identification results if only one fixed point is present or if, among all fixed points, only one satisfies the required regularity conditions. This describes formally what we mean by 'generic' identification. A similar case of generic identification appears in Chiappori and Ekeland (1999). Informally, our construction of the mapping \mathcal{T} shows that a sufficient number of demand functions are identified to permit recovery of F and η , so these functions will be identified as long as the demand functions are not too simple. For example, as shown earlier, a linear technology is not identified if members' demand functions are also linear (specifically, the linear expenditure system), but are identified for nonlinear member demands like the Almost Ideal or QUAIDS.

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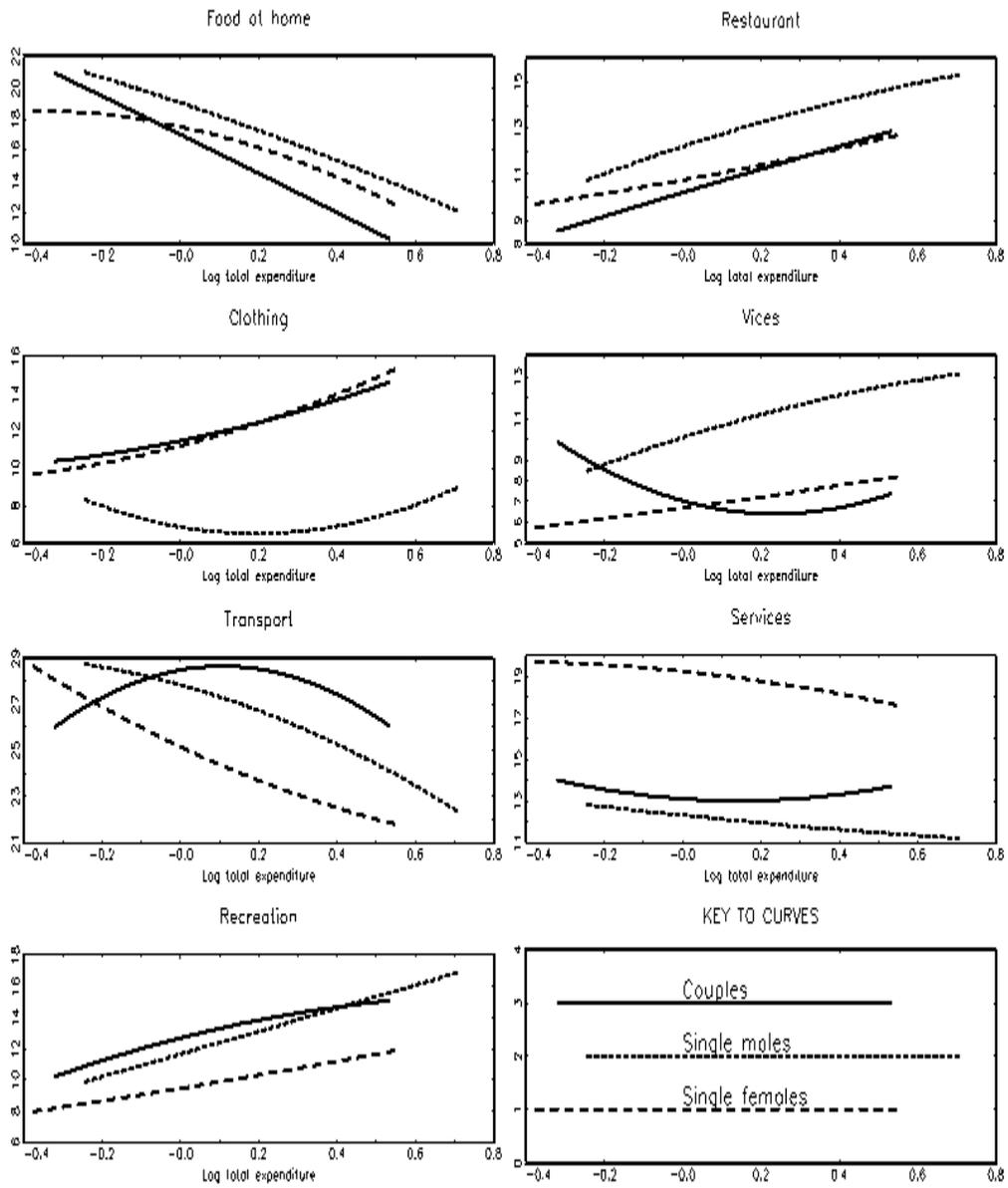


Figure 1: Q AIDS fits for singles and couples.

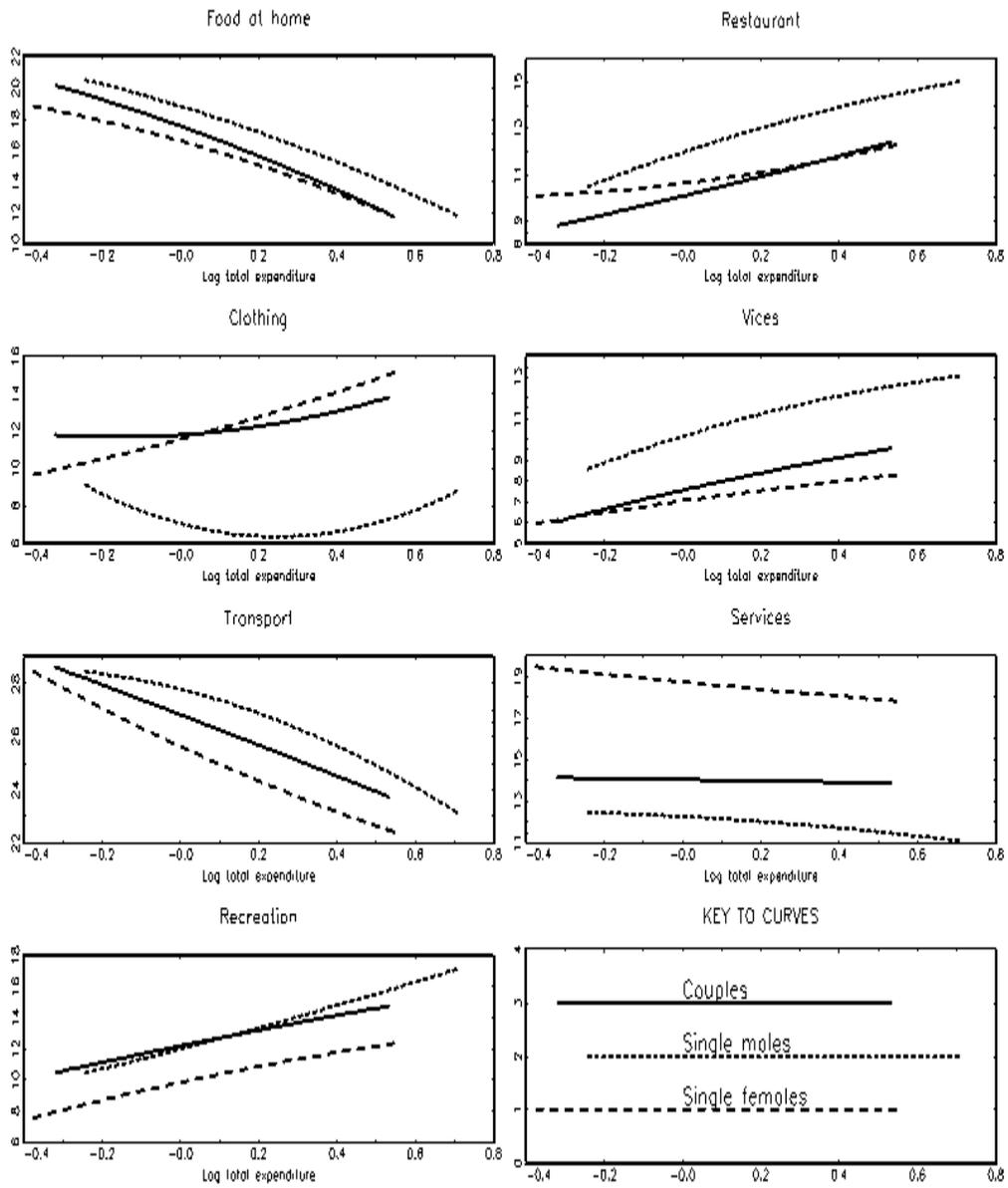


Figure 2: Model fits for singles and couples.