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GOD DOES NOT PLAY DICE: REVISITING EINSTEIN'S REJECTION OF PROBABILITY IN QUANTUM MECHANICS

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Abstract

Einstein's struggle with the use of probability in quantum mechanics is revisited. It is argued that Einstein was a statistical physicist who understood probability well, but the use of probability in quantum theory represented a radical departure which troubled Einstein. The theory denied the existence of physical reality until an observation was made, and probability replaced that reality. Einstein later put forward the powerful EPR thought experiment to show problems with quantum theory, but subsequent actual experiments have all supported quantum theory, instead of his local arguments.

Keywords: Probability; double-slit experiment; wave function; EPR

2010 Mathematics Subject Classification: Primary 81-03
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1. Introduction

The 1920s should have been a period of great self-satisfaction and happiness for Albert Einstein. Starting in the *annus mirabilis* of 1905, Einstein published five groundbreaking papers which included his papers on the photoelectric effect, Brownian motion, and the special theory of relativity. In 1916, Einstein published another fundamental paper dealing with the general theory of relativity. Relativity revolutionized our understanding of the world in the same way as Newton's theory of universal gravitation or Darwin's theory of evolution had done. The special theory of relativity established the speed of light as being absolute and the greatest achievable bound, and showed mass and energy to be equivalent through the formula $E = mc^2$ (where E denotes energy, m denotes mass, and c denotes the speed of light). However, the special theory dealt with inertial frames of reference. The general theory of relativity was Einstein's crowning achievement, extending the theory to noninertial frames of reference. A unified theory of gravity was presented by showing the latter to be a geometric property of space and time. To top up Einstein's list of achievements, 1921 (which was the same year Einstein first visited the United States) saw him being awarded the Nobel Prize for his work on the photoelectric effect.

However, the 1920s also saw a revolution in physics, namely the rise of quantum mechanics. Classical mechanics dealt with macroscopic objects moving at speeds much less than that of light, and relativistic mechanics considered bodies moving at speeds close to that of light. The new quantum mechanics focused on atomic and subatomic particles whose energy, momentum, and other properties consisted of discrete (quantum) values. The quantum revolution officially started in 1900 with the pioneering work of Max Planck in the context of blackbody radiation (see Planck (1900)). The trigger that brought attention to the field was the observation that

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heat (and later light) waves also exhibited particle behavior. Einstein himself had been an early contributor to quantum theory though his work on the photoelectric effect in 1905.

But Einstein later became deeply troubled with the way probability was used in the formulation of quantum theory and even more so by the implications of such a formulation. Thus, in a letter dated 4 December 1926 to his physicist friend Max Born (see Born *et al.* (1971, p. 91)), Einstein made his feelings known in no uncertain terms.

> Quantum mechanics is certainly imposing. But an inner voice tells me that it is not yet the real thing. The theory says a lot, but does not really bring us any closer to the secret of the 'old one' I, at any rate, am convinced that He is not playing at dice.

A similar sentiment as in this quote was expressed by Einstein on several later occasions. Einstein did not embrace any organized religion and was not a religious person in the true sense of the word, although he often described himself as religious. His spirituality was more directed to a 'cosmic religion'. Einstein expressed it in his own words as follows (see Einstein (1949)):

> The most beautiful and deepest experience a man can have is the sense of the mysterious. It is the underlying principle of religion as well as all serious endeavour in art and science. He who never had this experience seems to me, if not dead, then at least blind. To sense that behind anything that can be experienced there is a something that our mind cannot grasp and whose beauty and sublimity reaches us only indirectly and as a feeble reflection, this is religiousness. In this sense I am religious. To me it suffices to wonder at these secrets and to attempt humbly to grasp with my mind a mere image of the lofty structure of all that there is.

To the statistician, the above quote from Born *et al.* (1971) seems an outright rejection of the role of probability in quantum mechanics, and, more importantly, perhaps also in nature. (Quantum theory and the natural world are inextricably linked since the theory purports to describe the workings of nature at its most fundamental level, namely the quantum level.) If that was really the case, it would have been an unfortunate statement given that, even in 1926, scientists knew the pivotal role of randomness and chance in the workings of myriads of natural phenomena. The fundamental contributions of probability to astronomy, evolutionary biology, mortality, sociology, and thermodynamics could not be overlooked, even in 1926. But, as we shall see in the following paragraphs, there was something unique in the way probability was used in the theoretical formulation of quantum theory, something *radically different* from its use in all of the above-mentioned fields of scientific study. It was this radical difference that disquieted Einstein and put him increasingly at odds with most other physicists, who had rallied around the mainstream ideas of the two stalwarts of quantum mechanics, namely Niels Bohr and Weiner Heisenberg.

Could Einstein have misapprehended the nature of probability and its application in quantum mechanics? We shall argue that this is definitely not the case. After reading this paper, we hope the reader will realize that there is more to Einstein's statement about 'God not playing dice' that meets the eye. Rather than discard Einstein as maybe a man who was much more versed in physics than probability, and therefore more prone to misunderstanding the latter, we hope the reader will be more sympathetic to Einstein's concerns than perhaps many of his fellow physicists have been.

2. Free use of probability and statistics in early physics papers

Lest the reader thought that perhaps Einstein's understanding of probability was not as sharp as his mastery of physics, let us quickly dispel this misconception: Einstein was in every sense of the word on top of his game with respect to probability. Max Born backed this claim categorically (see Born (1949, p. 163)):

> He [Einstein] has seen more clearly than anyone before him the statistical background of the laws of physics.

Not only did Einstein understand probability well, he also used it freely in his papers. As a first example, let us consider Einstein's derivation of the classical diffusion equation in the second paper of the 'miraculous' 1905 (see Einstein (1905a), (2005b)). Einstein considered particles suspended in a liquid in a state of thermodynamic equilibrium. He first carefully stated the assumption of statistical independence (see Einstein (2005b, p. 94)):

> We now introduce a time interval τ, which is very small compared with observable time intervals but still large enough that the motions performed by a particle during two consecutive time intervals τ can be considered as mutually independent events.

Let the total number of suspended particles be n, and let the x-coordinate of a given particle increase by Δ in the time interval τ. Then Einstein wrote the probability density of Δ as

$$\phi(\Delta) = \frac{1}{n}\frac{\mathrm{d}n}{\mathrm{d}\Delta},$$

where $\int_{-\infty}^{+\infty} \phi(\Delta)\,\mathrm{d}\Delta = 1$ and $\phi(-\Delta) = \phi(\Delta)$. Suppose now that the number of particles per unit volume at a distance x along the x-axis and at time t is $f(x,t)$. Then, by considering the number of particles at time $t+\tau$ between the two planes perpendicular to the x-axis with abscissas x and $x + \mathrm{d}x$, Einstein wrote

$$f(x, t+\tau)\,\mathrm{d}x = \mathrm{d}x \int_{-\infty}^{+\infty} f(x+\Delta, t)\phi(\Delta)\,\mathrm{d}\Delta.$$

He next expanded each of $f(x, t+\tau)$ and $f(x+\Delta, t)$ as Taylor series, obtaining

$$f + \frac{\partial f}{\partial t}\tau = f\int_{-\infty}^{+\infty} \phi(\Delta)\,\mathrm{d}\Delta + \frac{\partial f}{\partial x}\int_{-\infty}^{+\infty} \Delta\phi(\Delta)\,\mathrm{d}\Delta + \frac{\partial^2 f}{\partial x^2}\int_{-\infty}^{+\infty} \frac{\Delta^2}{2}\phi(\Delta)\,\mathrm{d}\Delta + \cdots .$$

The final step was to use the facts that $\int_{-\infty}^{+\infty} \phi(\Delta)\,\mathrm{d}\Delta = 1$, $\int_{-\infty}^{+\infty} \Delta\phi(\Delta)\,\mathrm{d}\Delta = 0$, and to set $(1/\tau)\int_{-\infty}^{+\infty} (\Delta^2/2)\phi(\Delta)\,\mathrm{d}\Delta = D$, resulting in the one-dimensional diffusion equation

$$\frac{\partial f}{\partial t} = D\frac{\partial^2 f}{\partial x^2}$$

(see Einstein (2005b, p. 96)).

In the above, we see that probability was central to Einstein's work. But what was his interpretation of probability? Let us first hear it in his own words (see Einstein (1905b), (2005a, p. 187)):

> In calculating entropy by molecular-theoretical methods, the word 'probability' is often used in a sense differing from the way the word is defined in probability theory. In particular, 'cases of equal probability' are often hypothetically stipulated when the theoretical models employed are definite enough to permit a deduction rather than a stipulation. I will show in a separate paper that, in dealing with thermal processes, it suffices completely to use the so-called statistical probability, and hope thereby to eliminate a logical difficulty that still obstructs the application of Boltzmann's principle.

In the above, Einstein stated that the classical (Laplacian) definition of probability, which regarded probability as the ratio of the number of favorable cases to the total number of equally likely cases, was inadequate for his purposes. This was because, in the classical definition, the total number of cases was finite. In thermodynamic applications, where in the limit the number of molecules approached infinity, a different kind of probability was required, namely statistical probability, which was based on the limiting-frequency concept. This was made even more explicit by Einstein a few years later (see Einstein (1909, p. 187)). If, out of a total time T, a system spends a time t in some state, then the probability of that state is

$$\lim_{T\to\infty} \frac{t}{T}.$$

Thus, Einstein's use of probability was based on its statistical or frequency-based definition, and was a limit of time-average.

Let us now examine an instance of Einstein's use of statistical probabilities in the fifth paper (see Einstein (1905b), (2005a)). Einstein first started with Wien's law for the intensity distribution ρ of the light spectrum at frequency ν of a black body at absolute temperature T,

$$\rho = \alpha\nu^3 \exp\left(-\frac{\beta\nu}{T}\right)$$

(see Einstein (2005a, p. 186)), where α and β are constants. From the above, he obtained the entropy S of a volume V of radiation as

$$S - S_0 = \frac{E}{\beta\nu} \ln\left(\frac{V}{V_0}\right). \tag{1}$$

Here S_0 is the known entropy of a volume V_0 of radiation and E is the total energy in V. The last formula is now compared to Boltzmann's fundamental entropy equation,

$$S - S_0 = \frac{R}{N} \ln W, \tag{2}$$

where R is the universal gas constant, N is Avogadro's number, and W is the probability of the current state. As originally derived by Boltzmann, the above formula was meant to obtain S as a function of W, which was calculated based on combinatorial arguments. However, by comparing (1) and (2), Einstein used the latter equation in reverse fashion to obtain the *statistical probability* W,

$$W = \left(\frac{V}{V_0}\right)^{(N/R)(E/\beta\nu)}$$

(see Einstein (2005a, p. 191)). By the multiplication law of probabilities for independent events, we see from the above that, within the conditions of validity of Wien's law, the radiation behaved as if it consisted of $n = NE/R\beta\nu$ independent quanta. Since the total energy is E, each quantum would have energy $R\beta\nu/N$.

3. The quantum conundrum

The end of the previous section shows Einstein's thorough acquaintance with the meaning and application of probability, yet the use of the latter in quantum mechanics deeply troubled him. What was so unique in the way probability was used in the formulation of quantum mechanics that Einstein would have none of it?

An important insight into the gist of the problem involves the double-slit experiment (see Figure 1) which Feynman described (see Feynman (2011, pp. 1–2)) "…has in it the heart of quantum mechanics…[and] in reality, it contains the *only* mystery". A beam of atoms of the same wavelength is shot from a source on the left. The atoms pass through the double-slit barrier and land on the screen on the right. There a record is made as to where each atom lands. It is found that the atoms form a pattern on the screen, only landing in certain regions (see Figure 2(a)). The interference pattern formed shows the wave-like behavior of the atoms. This is an instance of the wave-particle duality first put forward by Louis de Broglie in 1923

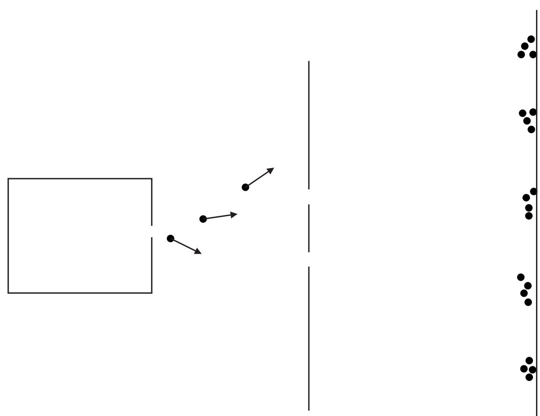

FIGURE 1: The double-slit experiment.

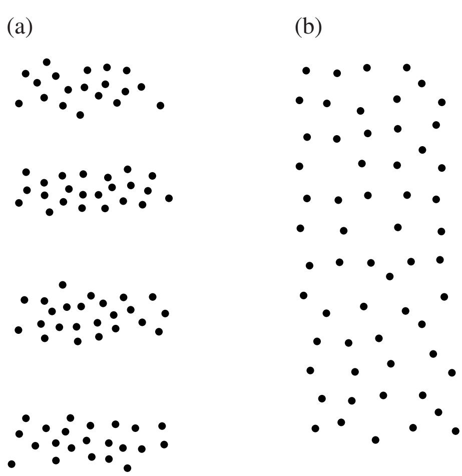

FIGURE 2: Results of the double-slit experiment when (a) both slits are open and (b) a single slit is open.

and 1924, which stated that not only waves can also display particle-like behavior, but particles can also display wave-like behavior (see de Broglie (1924)).

What is intriguing is that even when the atoms are shot one at a time, separated by some time between each shot, the same interference pattern builds over time. Therefore, it is not that different atoms are interfering with each other, rather *each atom somehow interacts with itself*.

Now, when the atoms are shot through a single slit (i.e. one of the slits is closed), the pattern on the screen changes (see Figure 2(b)). We obtain a uniform distribution, showing that the atoms now display particle behavior.

What comes out of the double-slit experiment is that, by keeping both slits open, *we can choose* to show the wave-like behavior of atoms and, by keeping a single slit open, *we can choose* to show the particle-like behavior of atoms. By choosing to keep both (or one) slits open, we have somehow forced the atoms to come out of the source on the left as waves (or particles), i.e. *we have created the past*. This turns the entire concept of causality on its head because the future is seen to cause the past!

As if what we have written in the previous paragraph was not weird enough, still a fundamental question remains: how does an atom somehow interact with itself when both slits are open to produce an interference pattern? Clearly *something* must be going through *both* slits since the results are different from when a single slit is open. Could it be that one part of the atom moves through one slit, and another part through the other slit? This hypothesis is immediately rejected by shining a light source near both slits. We then *see* that when an atom is shot, it actually passes through one (and only one) of the slits. But the pattern on the screen now looks uniform, like the one for a single slit in Figure 2(b)! So, it seems that our mere act of observing the atom causes a fundamental change in its behavior, as if the atom somehow *knew* we were observing it. This is known as the *measurement problem* in quantum physics.

In the next section we shall see that quantum mechanics offers a surprising answer involving probability to the questions and observations of the last paragraph. Not only this, quantum mechanics offers several other bewildering insights, again using probability. In the process, we hope the reader will appreciate Einstein's qualms about the role of chance in quantum mechanics.

4. Wave-like behavior (waviness): what is it really?

The interpretation of quantum physics given in this section follows the Copenhagen interpretation, which was put forward by Bohr and Heisenberg in 1925–1927. It represents the most commonly taught interpretation (see Rosenblum (2011, Chapter 10) for more details).

Consider a quantum particle of mass m which moves along the x-axis such that it has position $x(t)$ at time t. At the heart of quantum mechanics lies the wave function $\Psi(x,t)$ which provides a *complete description* of the quantum particle and which is obtained from the Schrödinger equation,

$$\mathrm{i}\overline{h}\frac{\partial \Psi}{\partial t} = -\frac{\overline{h}^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + V\Psi. \tag{3}$$

In the above, $\mathrm{i} = \sqrt{-1}$, $\overline{h}$ is Planck's constant ($\overline{h} = 1.054572 \times 10^{-34}$ Js), and V is the potential energy of the particle. Equation (3) was first obtained by Erwin Schrödinger in 1926 (see Schrödinger (1926)) and was directly motivated by de Broglie's wave-particle duality of 1923–1924. The Schrödinger equation can be regarded as the new universal law of motion and replaces Newton's second law of motion. (The Bohr correspondence principle was enunciated in 1924 and stated that the predictions of quantum mechanics are those of classical (Newtonian) mechanics in the limit of large quantum numbers (i.e. for large orbits and large energies).)

Schrödinger won the Nobel Prize in 1933 for his work but he initially made a mistake in the physical interpretation of $\Psi(x, t)$. As we just mentioned, Schrödinger put forward his equation right in the wake of de Broglie's wave-particle duality. It might therefore have seemed reasonable for Schrödinger to think that a particle's wave-like behavior (or waviness), which was represented by the wave function $\Psi(x, t)$, was a smeared out region in space representing the material of the particle. This could perhaps explain, he might have thought, how a single atom interacts with itself in the two-slit experiment with both slits open, to produce an interference pattern: the atom's waviness would split into two physical waves and each wave would pass thorough a slit. As tantalizing as Schrödinger's hypothesis was, it was wrong!

A little thought might convince us that Schrödinger's initial interpretation that $\Psi(x, t)$ represented the physical wave associated with a quantum particle could not be correct: from (3) we see that $\Psi(x, t)$ is a complex number, being a function of the imaginary number i. So $\Psi(x, t)$ could not possibly represent a physical wave.

The real nature of $\Psi(x, t)$ was revealed by Max Born in 1926 (see Born (1926)). Born correctly postulated that the Schrödinger equation (3) gave us *probabilities*. More precisely, $|\Psi(x, t)|^2$ *is the probability density of the position* x *at which the quantum particle will be found when observed at time* t, i.e.

$$\mathbb{P}\{\text{particle will be found between } a \text{ and } b \text{ at time } t\} = \int_a^b |\Psi(x, t)|^2 \, \mathrm{d}x.$$

Thus, it is perfectly legitimate to speak of the wake-like nature or waviness of a particle as long as we understand that the waviness has no physical reality, it is simply a probabilistic abstraction.

A word of caution is necessary here, and this brings us closer to understanding Einstein's reluctance towards the quantum theory. $|\Psi(x, t)|^2$ is *not* the probability density of the position of the particle, and it *does not* give us the relative probability of where the particle could be. It *is* the probability density of the position at which the particle will be found *once an observation is made*. (An observation is here to be understood as an interaction between a microscopic and a macroscopic object.) Indeed, quantum theory denies the physical reality of a particle before an observation is made: the particle is not at any particular point in space! It is only when an observation is made that the physical reality of the particle is created. Bohr put this succinctly as follows. 'Isolated material particles are abstractions, their properties being definable and observable only through their interaction with other systems' (Bohr (1934)).

Furthermore, given that the particle is completely described by the wave function $\Psi(x, t)$, we see that probability is not merely a tool of understanding reality in quantum mechanics, as it is in several other scientific fields. In quantum mechanics, *probability is the only reality before an observation is made.*

How can quantum mechanics explain the results of the double-slit experiment in the previous section? Two questions need to be answered.

(a) When both slits are open how does an atom interact with itself to produce an interference pattern?

(b) When both slits are open and a light source is put close to both slits, how is it that just observing each atom causes the pattern to be uniform on the screen, like the one when a single slit is open?

Regarding (a), quantum theory postulates that the waviness associated with each atom is split into two states with each state going though one slit. When split, the waviness is said to be in a *superposition state*. The two states then interact with each other to produce the inference pattern on the screen. Concerning (b), the quantum theory reasoning is that, as soon as an observation is made, the superposition collapses to the spot where the atom is actually observed. The atom thus shows only particle behavior and no interference is obtained. *Observation thus collapses the waviness (or wave function) to a specific position.*

5. Positivism versus objective realism

Even if the reader feels bemused by the quantum theory postulates of the previous section, he/she should at least take some comfort of being in good company, for Einstein too felt the same. Earlier in his career Einstein embraced Ernst Mach's positivist philosophy, according to which meaningful science should be based only on events or objects that can be experienced and measured. Science is thus about the *description* of empirical facts not the *explanation* in terms of abstract and intangible entities. To the statistician, Einstein's viewpoint at this stage finds a parallel in Karl Pearson's embrace of Mach's positivism when, for instance, Pearson stated (see Pearson (1892, p. 136)):

> Science for the past is a description, for the future a belief; it is not, and has never been, an explanation, if by this word is meant that science shows the necessity of any sequence of perceptions.

Einstein's positivist influence was clearly seen in his development of special relativity. He focused specifically on what could be measured and observed. Motion and time were not absolute, but only relative to the observer. The same positivism permeated his 1916 general theory of relativity. For instance, when using a particular verificationist argument, Einstein stated that such an argument '…takes away from space and time the last remnants of physical objectivity' (see Einstein (1952, p. 117)). However, in the face of the radical probabilistic abstraction that quantum mechanics embodied, Einstein's philosophy turned to one of objective realism. Around 1920, Einstein started to believe that there was an absolute reality in the universe, and it was the function of the scientist to understand it:

> We do not only wish to know how nature is (and how her processes develop) but also wish, if possible to arrive at the perhaps utopian and pretentious-seeming goal to know why nature *is as it is and not otherwise*. In this domain lies the highest satisfaction of the scientist.

(See Pais (1994, pp. 131–132).) Bohr, one of the staunchest defenders of quantum theory, adopted a diametrically opposite viewpoint (see McEvoy (2001, p. 291)).

> It is wrong to think that the task of physics is to find out how nature *is*. Physics concerns what we can say *about* nature.

Recall that one of the major tenets of quantum mechanics was the phenomenon of observer-created reality. On the other hand, Einstein believed that an objective world should exist independent of the human observer (see Einstein (1934, p. 60)):

> The belief in an external world independent of the perceiving subject is the basis of all natural science. Since, however, sense perception only gives information of this external world or of 'physical reality' indirectly, we can only grasp the latter by speculative means. It follows from this that our notions of physical reality can never be final. We must always

> be ready to change these notions – that is to say, the axiomatic structure of physics – in order to do justice to perceived facts in the most logically perfect way.

The above should be seen as an attack on the very underpinning of quantum mechanics. To illustrate Einstein's realist viewpoint, suppose that an electron is observed to be at point P. The quantum theorist would state that, before the observation was made, the particle was not really anywhere. On the other hand, Einstein would contend that:

- since the particle was observed at P, *it must have been at P before the observation was made*,
- quantum mechanics therefore does not provide a complete theory since it is unable to tell us the particle was at P before the observation,
- in addition to $\Psi(x, t)$, some hidden variables are necessary to provide a complete description.

See the EPR experiment in Section 6, where this line of thought was further developed by Einstein.

Faced with the 'probabilitization' of quantum mechanics, Einstein adopted a more deterministic attitude towards nature: all phenomena were ultimately deterministic, where probability was only a measure of our ignorance of them. This is very reminiscent of Laplace's universal determinism (see Gorroochurn (2012, pp. 141–142), (2016, p. 78)). For Einstein, a physical reality denied and replaced by a probabilistic function made no sense (see Einstein (1987, p. 91)):

> I am working with my young people on an extremely interesting theory with which I hope to defeat modern proponents of probability-mysticism and their aversion to the notion of reality in the domain of physics.

Thus, Einstein was not opposed to the use of probability *per se* in physics, but he could not agree to a reality subsumed by probability. When he said 'He [God] is not playing at dice', Einstein was merely reacting to what he was seeing. To him, quantum mechanics implied that God gave us only probability, but no physical reality; the latter was created only by the observer. Einstein would have none of that.

6. Two powerful thought experiments

The later part of the 1920s saw a growth in the stature of quantum mechanics, led by the able hands of Bohr and Heisenberg. Convinced that quantum mechanics did not provide a complete picture of the world, Einstein devised several thought experiments mostly directed to his rival and friend Bohr, purporting to show defects in the theory.

6.1. Schrödinger's cat (1935)

Before we examine one of Einstein's most powerful thought experiments, we shall introduce Schrödinger's cat (see Schrödinger (1935) and Trimmer (1980)). This celebrated thought experiment was put forward by none other than the man who discovered the fundamental wave equation in (3), and yet grew increasingly uncomfortable by its implications.

A cat is placed inside a steel chamber which also contains a Geiger counter (see Figure 3). The latter contains a tiny amount of radioactive material such that it is equally likely for an atom to decay or not decay within the next hour. If a decay occurs, the counter triggers, causing

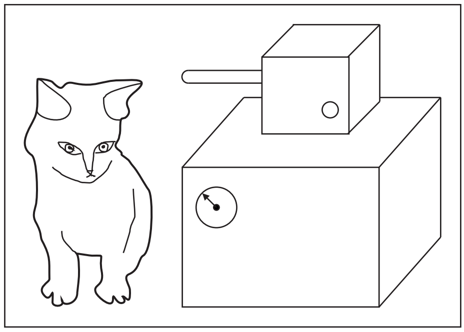

FIGURE 3: Schröndiger's cat experiment.

the release of cyanide in the chamber. If the entire system is left for one hour, then the cat will be alive if no atom has decayed, and will be dead otherwise. 'The Ψ-function of the entire system would express this by having in it the living and the dead cat (pardon the expression) mixed or smeared out in equal parts' (Trimmer (1980, p. 328)). That is to say, before we open the chamber to make the observation, the wave function of the cat has the following schematic form:

$$\Psi = \frac{1}{\sqrt{2}}(\Psi_{\text{alive}} + \Psi_{\text{dead}}).$$

Before the observation is made, it seems that the cat is neither alive nor dead, but a linear combination of the two! Quantum theory thus appears to lead to an absurd conclusion, but at least two counter-arguments can be made against such a charge. One such argument states that the actual observation occurred when the counter triggered, so there is no question of the cat actually being in a superposition state. A second argument points out that the wave function is really a probabilistic abstraction and in no way describes the physical state of the cat.

6.2. The EPR experiment (1935)

We now turn to Einstein's EPR thought experiment (so called because Einstein co-wrote the paper describing the experiment with his younger colleagues Podolsky and Rosen (see Einstein *et al.* 1935)), which is one of the most powerful attacks on the very foundations of quantum theory. The EPR experiment purported to show that the theory was not *complete*, in the sense that it failed to represent every element of the physical reality it deals with.

A simplified version of the EPR experiment by Bohm is as follows (see Figure 4). A neutral pi meson decays into an electron and a positron according to

$$\pi^0 \to e^- + e^+.$$

Assume that the electron and positron fly off in opposite directions. Since the pi meson has spin zero, by the conservation of angular momentum, the wave function of the electron and positron can be written schematically as

$$\Psi = \frac{1}{\sqrt{2}}\{\Psi_{\uparrow}^{(-)}\Psi_{\downarrow}^{(+)} - \Psi_{\downarrow}^{(-)}\Psi_{\uparrow}^{(+)}\},$$

where $\Psi^{(-)}$ is the wave function of the electron,$\Psi^{(+)}$ is the wave function of the positron, and the subscript indicates the spin orientation (up or down). Now, suppose that the electron

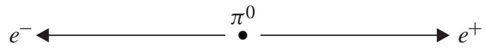

FIGURE 4: Bohm's version of the EPR experiment.

and positron are several light years away and we observe the spin of the electron. Say it is up (this event has probability $\frac{1}{2}$). Then we would know instantly that the spin of the positron must be down. There could be no causal influence from the electron to the positron since such an influence would have to travel at an infinite speed. *Thus, the reality of a down spin of the positron exists without the latter having been observed!* Since this reality is not accounted for by quantum theory, Einstein claimed that the theory was incomplete.

The EPR experiment showed that, though the electron and positron were light years apart, their initial interaction imposed a complete correlation between them. This phenomenon is known as *entanglement*. Entanglement in turn leads to *nonlocality*, i.e. instantaneous action at a distance. (Einstein had a more colorful word for this phenomenon: 'spooky-action-at-distance'.) Nonlocality thus formed the basis for Einstein's claim that quantum theory was incomplete.

Bohr's answer to Einstein's conundrum was long in coming, probably because he had not yet grasped the full implications of quantum theory. Months later, he published a long and somewhat obscure response to the EPR paper, with exactly the same title as the latter (see Bohr (1935)). The essence of Bohr's argument was that Einstein's condition for reality was too stringent. Einstein had insisted that 'If, without in any way disturbing a system, we can predict with certainty (i.e. with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity'. Bohr questioned the meaning of 'without in any way disturbing the system'. He maintained that, although there was no physical disturbance between the electron and positron in the EPR experiment, there was nevertheless a disturbance. The observation of the spin of the electron influences the spin of the positron, *for the latter is known only after observing the former*. Bohr concluded that, because Einstein's condition for reality was too severe, quantum theory was in fact as complete as it could be.

Although Einstein had tried to show a conundrum in the EPR experiment, from his local realist viewpoint there was really no puzzle if the fundamental assumptions of quantum theory could be done away with. The local realist would say that the electron and positron were perfectly correlated at the moment of their creation; since the spin of the electron was later observed to be up, *at the moment of creation it would have to have been up and that of the positron would have to have been down*. Therefore, the inference that the positron had a down spin once it was observed that the electron had a up spin was perfectly normal, since the positron had a down spin all along! The only problem is that quantum theory would reject such a seemingly reasonable explanation. According to quantum theory, before the observation, the spins of the electron and positron are in a superposition state: neither of the two particles has a definite state. It is only after the observation of an up spin of the electron that the wave function collapses to a down spin for the positron.

As reasonable as Einstein's local realist position was and as counterintuitive as the postulates of quantum theory were, it was the latter that was confirmed by a combination of theoretical and experimental work performed later. In 1964, John Bell published a paper where he developed a correlation function which could calculate the relationship between the measurements at two detectors when the settings of the latter were varied (Bell (1964)). He showed that, under local reality, the correlation must obey a certain inequality now called *Bell's inequality*, and that quantum theoretical predictions violated Bell's inequality. Experiments performed on photons by Alain Aspect in 1981 confirmed that indeed Bell's inequality was violated (see Aspect *et al.* (1981)). This meant that local reality (Einstein's position) was untenable, thus vindicating quantum theory. Many other subsequent tests were done, all of which provided further vindication.

7. Discussion

Einstein was not averse to the use of probability and statistics in physics. On the contrary, he was an able statistical physicist who put probability to good use in his papers. But quantum mechanics represented a radical departure from the way probability had been used before. Early applications included probability as an *aid* to understanding natural phenomena. But with quantum mechanics, probability took a fundamentally abstract form that completely subsumed the physical reality of the world. The theory denied the reality of a particle independent of its observation; instead it postulated the existence of a mathematical wave function, the square of the magnitude of which gave the probability density of the position at which the particle would be found if observed. Einstein's increasingly realist approach to science put him at odds with such an interpretation.

At the same time, Einstein was fully aware of the utility of quantum theory and of the fact that the theory had been very successful in its predictions. Thus, in his statement about 'God not playing dice' he acknowledged *a priori* that the theory was 'certainly imposing'. He was very careful to criticize only aspects of the theory and not to state that the whole theory was wrong. His famous EPR paper purported to show that the theory was *incomplete*, not that it was *entirely incorrect*. Throughout his later years, Einstein worked laboriously to come up with a unified theory that would subsume quantum theory. He died with his dream unfulfilled. So far, it appears that God will keep on playing dice.

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References

Aspect, A., Grangier, P. and Roger, G. (1981). Experimental tests of realistic local theories via Bell's theorem. *Phys. Rev. Lett.* **47,** 460.

Bell, J. S. (1964). On the Einstein Podolsky Rosen paradox. *Physics* **1,** 195–200.

Bohr, N. (1934). *Atomic Theory and the Description of Nature*. Cambridge University Press.

Bohr, N. (1935). Can quantum-mechanical description of physical reality be considered complete? *Phys. Rev.* **48,** 696.

Born, M. (1926). Quantenmechanik der Stoßvorgänge. *Z. Physik* **38,** 803–827.

Born, M. (1949). Einstein's statistical theories. In *Albert Einstein: Philospher-Scientist*, ed. P. A. Schilpp, The Library of Living Philosophers, Evanston, IL, pp. 161–177.

Born, M. (1971). *The Born–Einstein Letters: Correspondence Between Albert Einstein and Max and Hedwig Born from 1916 to 1955*. Macmillan, London.

De Broglie, L. (1924). Recherches sur la théorie des quanta. Doctoral Thesis, Université de Paris.

Einstein, A. (1905a). Über die von der molekularkinetischen Theorie der Wärme geforderte Bewegung von in ruhenden Flüssigkeiten suspendierten Teilchen. *Ann. Physik* **17,** 549–560.

Einstein, A. (1905b). Über einen die Erzeugung und Verwandlung des Lichtes betreffenden heuristischen Gesichtspunkt. *Ann. Physik* **17,** 132–148.

Einstein, A. (1909). Zum gegenwärtigen Stand des Strahlungsproblems. *Physik. Z.* **10,** 185–193.

Einstein, A. (1934). Maxwell's influence on the evolution of the idea of physical reality. In *James Clerk Maxwell: A Commemoration Volume,* Cambridge University Press.

Einstein, A. (1949). *The World as I See It*. Philosophical Library, New York.

EINSTEIN, A. (1952). The foundation of the general theory of relativity (1916). In *The Principle of Relativity*, Dover, New York, pp. 111–173.

EINSTEIN, A. (1987). *Letters to Solovine*. Philosophical Library, New York.

EINSTEIN, A. (2005a). On a heuristic point of view concerning the production and transformation of light. In *Einstein's Miraculous Year: Five Papers That Changed the Face of Physics*, ed. J. Stachel, Princeton University Press, pp. 177–198.

EINSTEIN, A. (2005b). On the motion of small particles suspended in liquids at rest required by the molecular-kinetic theory of heat. In *Einstein's Miraculous Year: Five Papers That Changed the Face of Physics*, ed. J. Stachel, Princeton University Press, pp. 85–98.

EINSTEIN, A., PODOLSKY, B. AND ROSEN, N. (1935). Can a quantum-mechanical description of physical reality be considered complete? *Phys. Rev.* **47,** 777–780.

FEYNMAN, R. P. (2011). *The Feynman Lectures on Physics*, Vol. 3. Basic Books, New York.

GORROOCHURN, P. (2012). *Classic Problems of Probability*. John Wiley, Hoboken, NJ.

GORROOCHURN, P. (2016). *Classic Topics on the History of Modern Mathematical Statistics: From Laplace to More Recent Times*. John Wiley, Hoboken, NJ.

MCEVOY, P. (2001). *Niels Bohr: Reflections on Subject and Object*. Microanalytix, San Francisco, CA.

PAIS, A. (1994). *Einstein Lived Here*. Oxford University Press.

PEARSON, K. (1892). *The Grammar of Science*. Scott, London.

PLANCK, M. (1900). Zur Theorie des Gesetzes der Energieverteilung im Normalspectrum. *Verhandl. Deutsch. Phys. Ges.* **2,** 237–245.

ROSENBLUM, B. (2011). *Quantum Enigma: Physics Encounters Consciousness*, 2nd edn. Duckworth, London.

SCHRÖDINGER, E. (1926). An undulatory theory of the mechanics of atoms and molecules. *Phys. Rev.* **28,** 1049–1070.

SCHRÖDINGER, E. (1935). Die gegenwärtige Situation in der Quantenmechanik. *Naturwissenschaften* **23,** 807–812, 823–828, 844–849.

TRIMMER, J. D. (1980). The present situation in quantum mechanics: a translation of Schrödinger's 'Cat paradox' paper. *Proc. Amer. Phil. Soc.* **124,** 323–338.