Phys C2601, Physics III: Classical and Quantum Waves Homework Assignment 1: Solutions

September 28, 2009

1

(a) **5 points**

$$\begin{aligned}\Psi(t) &= A\cos(\omega_0 t - \delta) \\ &= A(\cos\omega_0 t\cos\delta + \sin\omega_0 t\sin\delta)\end{aligned} \Rightarrow \boxed{B_1 = A\cos\delta, B_2 = A\sin\delta}$$

(b) **5 points**

$$\begin{aligned}\Psi(t) &= C\exp(i\omega_0 t) + C^*\exp(-i\omega_0 t) \\ &= C(\cos\omega_0 t + i\sin\omega_0 t) + C^*(\cos\omega_0 t - i\sin\omega_0 t) \\ &= 2\mathrm{Re}(C)\cos\omega_0 t - i2\mathrm{Im}(C)\sin\omega_0 t\end{aligned}$$

$$\Rightarrow \boxed{C = \frac{A\cos\delta}{2} - \frac{A\sin\delta}{2}i}$$

$$\begin{aligned}\Psi(t) &= \mathrm{Re}(D\cos\omega_0 t + iD\sin\omega_0 t) \\ &= \mathrm{Re}(D)\cos\omega_0 t - \mathrm{Im}(D)\sin\omega_0 t\end{aligned}$$

$$\Rightarrow \boxed{D = A\cos\delta - iA\sin\delta}$$

2

(a) **3 points**

The potential energy could be written as below:

$$V(x) = \frac{V_0}{a^2}(x-a)^2 - V_0$$

$$m\frac{d^2x}{dt^2} = -\frac{dV}{dx} = -\frac{2V_0}{a^2}(x-a)$$

thus the center of the oscillation is $\boxed{x_0\text{=a}}$.

(b) **3 points**

For a simple harmonic oscillator, $\omega = \sqrt{\frac{k}{m}}$, since

$$F = m\frac{d^2x}{dt^2} = -\frac{dV}{dx} = -\frac{2V_0}{a^2}(x-a) \Rightarrow k = \frac{2V_0}{a^2}$$

thus $\boxed{\omega = \sqrt{\frac{2V_0}{ma^2}}}$.

(c) **4 points**

For a simple harmonic oscillator, try solution $x = A\cos\omega t + a$, with $\omega = \sqrt{\frac{2V_0}{ma^2}}$

$$E_{tot} = \frac{1}{2}mA^2\omega^2\sin^2\omega t + \frac{V_0}{a^2}A^2\cos^2\omega t - V_0 = \frac{V_0A^2}{a^2} - V_0 \Rightarrow \boxed{A = \sqrt{\frac{E_{tot}+V_0}{V_0}a^2}}$$

3

(a) **3 points**

$$\left.\begin{aligned} E_{tot} = \frac{m}{2}\nu^2 = \frac{m}{2}[(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2] + mgy \\ x = L\sin\theta, y = L - L\cos\theta \\ \frac{dx}{dt} = \frac{dy}{dt} = 0, \text{at } \theta_{max} \end{aligned}\right\} \Longrightarrow \boxed{\theta_{max} = \arccos(1 - \frac{\nu^2}{2gL})}$$

(b) **3 points(c) 4 points**

$$\left.\begin{aligned} \frac{m}{2}\nu^2 = \frac{mL^2}{2}(\frac{d\theta}{dt})^2 + mgL(1-\cos\theta) \\ 1-\cos\theta \approx \frac{\theta^2}{2}, \text{considering } \theta_{max} \ll 1 \\ \theta = A\sin\omega t, \text{since } \theta = 0 \text{ when t=0} \end{aligned}\right\} \Longrightarrow \theta = A\sin\omega t, \omega = \sqrt{\frac{g}{L}}, A = \frac{\nu}{L\omega}, T = 2\pi/\omega$$

$$\Rightarrow \boxed{\theta = \frac{\nu}{gL}\sin\sqrt{\frac{g}{L}}t, t = T/2 = \pi\sqrt{\frac{L}{g}}}$$

4

(a) **3 points**

x is the displacement from the equilibrium position.

$$E_{tot} = \frac{m}{2}(\frac{dx}{dt})^2 + \frac{1}{2}k_1x^2 + \frac{1}{2}k_2x^2 \Rightarrow \boxed{\omega = \sqrt{\frac{k_1+k_2}{m}}}$$

(b) **4 points**

x_1, x_2, x are the displacements from the equilibrium position of k_1, k_2 and m.

$$\left.\begin{aligned} E_{tot} = \frac{m}{2}(\frac{dx}{dt})^2 + \frac{1}{2}k_1x_1^2 + \frac{1}{2}k_2x_2^2 \\ x_1 + x_2 = x \\ k_1x_1 = k_2x_2 \end{aligned}\right\} \Longrightarrow \boxed{\omega = \sqrt{\frac{k_1k_2}{m(k_1+k_2)}}}$$

(c) **3 points**

x is the displacement from the equilibrium position.

$$E_{tot} = \frac{m}{2}(\frac{dx}{dt})^2 + \frac{1}{2}k_1x^2 + \frac{1}{2}k_2x^2 \Rightarrow \boxed{\omega = \sqrt{\frac{k_1+k_2}{m}}}$$

5

(a) **3 points**

$$\left.\begin{aligned} \omega_0 = 2\pi f \\ f = 256Hz \\ \frac{1}{2} = e^{-\gamma t_0}, t_0\text{=1sec} \\ Q = \frac{\omega_0}{\gamma} \end{aligned}\right\} \Longrightarrow \boxed{Q_{(a)} = \frac{512\pi}{ln2}}$$

(b) **3 points**

$\boxed{Q_{(b)} = 2Q_{(a)}}$

(c) **4 points**

$$\left.\begin{aligned} Q &= \frac{\omega_0}{\gamma} \\ \omega_0 &= \sqrt{km} \\ \frac{1}{e} = e^{-\gamma t_1}, t_1&=4 \text{ sec} \Rightarrow \gamma = 0.25 \text{ Hz} \end{aligned}\right\} \Longrightarrow \boxed{\text{Q=12}}$$

$$\left.\begin{aligned} m\frac{d^2x}{dt^2} &= -kx - b\frac{dx}{dt} \\ x &= Ae^{i(\omega t+\alpha)} \end{aligned}\right\} \Rightarrow x = Ae^{-\frac{b}{2m}t}\cos(\omega' t + \alpha), \omega' = \frac{\sqrt{4mk - b^2}}{2m}$$

$$\Rightarrow \boxed{b = m\gamma = 0.025 kg/s}$$

6

(a) **2 points**

This problem could be considered as a damping oscillator with a solution Assuming the motion could be described by $x = A\sin 2\pi\nu t$ during any one cycle, the acceleration in this cycle is then $a = \frac{d^2x}{dt^2} = -A(2\pi\nu)^2 sin2\pi\nu t$, thus the energy loss per cycle is:

$$\boxed{\Delta E = \int_0^{1/\nu} \frac{Ke^2a^2}{c^3}dt = \frac{8K\pi^4\nu^3e^2A^2}{c^3}}$$

(b) **3 points**

$$E_{(t)} = E_0 \exp(-\gamma t) \Rightarrow dE/dt = -E_0\gamma \exp(-\gamma t) \Rightarrow \frac{\Delta E}{E_0 \exp(-\gamma t)} \approx \gamma \Delta t$$

$$Q = \frac{\omega_0}{\gamma} = \frac{2\pi E_0 \exp(-\gamma t)}{\Delta E} = -\frac{2\pi\frac{1}{2}m(2\pi\nu)^2A^2}{\frac{8K\pi^4\nu^3e^2A^2}{c^3}} = \boxed{\frac{mc^3}{2\pi\nu Ke^2}}$$

(c) **3 points**

$$\frac{E_0}{2} = E_0 \exp(-\frac{\omega_0 t}{Q}) \Rightarrow t = \frac{Qln2}{\omega_0} \Rightarrow \boxed{N = \frac{t}{2\pi/\omega_0} = \frac{Qln2}{2\pi}}$$

(d) **2 points**

Visible light frequency 790∼400 THz. $\nu \approx 5\times10^{14} Hz, m = 9.1\times10^{-31} kg, K = 6 \times 10^9 Nm^2/C^2, e = 1.6 \times 10^{-19} C, c = 3 \times 10^8 m/s$

$$\Rightarrow \boxed{Q = 5 \times 10^7} \boxed{t = 1.1 \times 10^{-8} s}$$