Week 7: Product Differentiation II – Vertical Differentiation

(This week's lecture is based on Tirole, Chap. 7.5)

1. Overview

Recall that with horizontal differentiation, some easing of price competition, as niche markets develop. Similar phenomenon with vertical differentiation except now products can be ranked on "quality".

2. Vertical Differentiation I – Model

All consumers agree on the ranking of two products; let the ranks be denoted s_1 and s_2 (and suppose $s_2 > s_1$). Consumers can differ on the intensity of the rank difference; consumer I derives utility $s_1.x_I$ from good 1 and $s_2.x_I$ from good 2. If the two prices are p_1 and p_2 respectively, then consumer I derives net utility of $s_1.x_I - p_1$ from good 1 and $s_2.x_I - p_2$ from good 2. Write $s_2 - s_1$ as s – the difference in quality and assume that $s\, x_2$ - $2s\, x_1$.

Suppose that consumers are uniformly distributed in their intensity between $x = x_1$ and $x = x_2$, where x_1 and x_2 are two numbers, with $x_1 < x_2$. Assume that $s\, x_2$ -> $2s\, x_1$. For purposes of illustration we will often consider the case $x_1 = 1$ and $x_2 = 4$. In that latter case, between intensities 1 and 2.5 there are exactly 50% of the consumers, between intensities 1 and 3.4 there are exactly 80% of the consumers, etc. Suppose also that each consumer wants to buy one unit of the good and will do so from one of the two firms.

Hence consumer I prefers good 1 if sx_I is smaller than $p_2 - p_1$. Clearly high x_I makes it more likely that the consumer will buy from firm 2. Let the consumer who is indifferent between the two firms be denoted X (where X is in between x_1 and x_2 and is defined by $sX = p_2 - p_1$

Suppose finally that each firm has a constant AC of production equal to c.

(How is this model different from the Hotelling model? What happens if both prices are equal?)

3. Vertical Differentiation II – Price Competition

By the assumptions above, firm 1 sells $X - x_1$ units (and 2 sells x_2-X units). . For purposes of illustration when we consider the case $x_1 = 1$ and $x_2 = 4$, firm 1 will service the quantity X – 1 and 2 will sell 4 – X units. Hence, firm 1's profits are 1/s times

$$(p_1 - c)(p_2 - p_1 - s\, x_1)$$

These profits are maximized when marginal profits are zero, i.e., at

$$2\, p_1 = c + p_2 - s\, x_1$$

Similarly, firm 2's profits are 1/s times

$$(p_2 - c)(s\, x_2 - p_2 + p_1)$$

This maximum profit price is called firm 1's best response (to p_2) price. A similar exercise yields firm 2's best response price to be given by $2\, p_2 = c + s\, x_2 + p_1$

Recall that a Nash equilibrium is a pair of prices such that each firm is playing a best response. Hence, it is a solution to the above best response price equations. It follows that the equilibrium prices are

$$3p_2 = 3c + 2s\, x_2 - s\, x_1$$
$$3p_1 = 3c + s\, x_2 - 2s\, x_1$$

(Work out the best response pice functions as well as the Nash equilibrium in the illustrative case.)

Note that prices are above cost. That follows from the differentiation in products. (What happens when the goods are homogeneous in this model? What about completely differentiated?)

Show that equilibrium profits increase in the amount of the differentiation s.

4. Vertical Differentiation III – Choosing Quality

Since profits of both firms are increasing in the amount of differentiation, if the two firms chose quality levels they would want the quality difference to be as high as possible. Of course firm with the higher quality would make higher pofits and hence they would both want to be the higher quality firm.