

Economics of the Internet (W4490)

Problem Set 2-Due Mon. Feb. 11th

Answer all questions.

1. Give a formal definition of (strictly) dominated strategy and Nash equilibrium.
2. Consider the game of Battle of the Sexes you saw in class. How would you modify the payoffs to (f,o) and (o,f) to reflect the following: the husband is unhappiest when he is at the opera by himself, he is a little happier if he is at the football game by himself, he is happier still if he is with his wife at the opera, and he is the happiest if they are both at the football game? (Likewise, the wife is unhappiest when she is at the football game by herself, she is a little happier if she is alone at the opera, happier still if she is with her husband at the football game, and the happiest if they are both at the opera). Check for Nash equilibria in this modified game.
3. *Colonel Blotto's game*. In this war game, Colonel Blotto has two infantry units that he can send to any pair of four locations (1,2-1,3-1,4-2,3-2,4-3,4). While Colonel Tlobbo has one unit that he can send to anyone of the four locations. A unit wins a location if it arrives uncontested, and a unit fights to a standstill if an enemy unit also comes to the same location. A win counts as one unit of utility; a standstill yields zero utility. The strategic form of the game is the following:

<i>Tlobbo\Blotto</i>	<i>1,2</i>	<i>1,3</i>	<i>1,4</i>	<i>2,3</i>	<i>2,4</i>	<i>3,4</i>
<i>1</i>	0,1	0,1	0,1	1,2	1,2	1,2
<i>2</i>	0,1	1,2	1,2	0,1	0,1	1,2
<i>3</i>	1,2	0,1	1,2	0,1	1,2	0,1
<i>4</i>	1,2	1,2	0,1	1,2	0,1	0,1

Check for dominated (both weakly and strictly) strategies and Nash equilibria.

4. *Hawk-Dove game*. Consider the following game between two spiders:

Spider 1/Spider 2	Concede	Fight
Concede	5,5	0,10
Fight	10,0	X,X

The spiders are fighting for a web whose utility is 10. If one spider fights and the other concedes, the first has the web. If neither fights they have a 50-50 chance to get the web. If they both fight they have again a 50-50 chance to get the web but there is also a possibility that they will be physically harmed. If the physical costs are higher than the expected value of the web, then X will be less than 0, otherwise it will be greater than 0.

- Find the Nash equilibria with both X less and greater than 0.
- Now suppose that the payoff of winning the web is greater than 10. Clearly explain how the payoff to (f,f) might change as well.
- Show that what determines whether or not the spiders fight is the difference between the payoffs to fighting and conceding when the other spider chooses to fight.
- Argue that both spiders conceding can never be a Nash equilibrium of this game.

5. *Bertrand price competition*. Suppose we have two firms (duopoly) that set prices in a market whose demand is given by: $Q=6-P$, where P is the lower of the two prices. If there is a lower priced firm, then it meets all of the demand. If the two firms post the same price P, then they each get half of the market, that is, they each get $(6-P)/2$. Suppose that prices can only be quoted in dollar units (0, 1, 2, 3, 4, 5, 6 dollars) and that costs of production are zero.

- Show that posting a price of zero dollars and posting a price of six dollars are both dominated strategies. What about the strategy of posting a price of four and five dollars?
- Suppose for a moment that this market had only one firm. Show that the price at which this monopoly firm maximizes profits is \$3.