Economics of the Internet (W4490)

Problem Set 2-Solutions

1. A strategy s'_i is *strictly dominated* for player i if there is at least another strategy s_i that does strictly better then s'_i against every strategy of the other players:

$$\pi_i(s_i, s_{-i}) > \pi_i(s'_i, s_{-i}) \quad for \quad all \quad s_{-i}$$

where $s_{-i} \equiv \{s_1, \ldots\ldots, s_{i-1}, s_{i+1}, \ldots\ldots, s_n\}$ is the strategy vector of the players other than i. If the inequality is weak the strategy is said to be *weakly dominated*.

A strategy s_i^* is a *best response* to a strategy vector s_{-i}^* of the other players if:

$$\pi_i(s_i^*, s_{-i}^*) \geq \pi_i(s_i, s_{-i}^*) \quad for \quad all \quad s_i \,.$$

The strategy vector $s^* = s_1^*, \ldots\ldots, s_n^*$ is a *Nash equilibrium* if:

$\pi_i(s_i^*, s_{-i}^*) \geq \pi_i(s_i, s_{-i}^*)$ for all s_i and all i.

Hence, in a Nash equilibrium:

i) Each player is playing a best response against a conjecture.

ii) The conjectures are correct.

2. *Battle of the Sexes*. A way to modify the game is the following:

Housband/Wife	Football	Opera
Football	3,1	0,0
Opera	0,0	1,3

Anyway, other modifications are possible. The Nash equilibria for this modified game are still (F, F) and (O, O).

1. *Colonel Blotto's game*. In this game there are clearly nor strictly neither weakly dominated strategies for both players, since every strategy does better then the others for some strategies played by the other player.

Tlobbo\Blotto	***1,2***	***1,3***	***1,4***	***2,3***	***2,4***	***3,4***
1	0,1	0,1	0,1	1,2	1,2	1,2
2	0,1	1,2	1,2	0,1	0,1	1,2
3	1,2	0,1	1,2	0,1	1,2	0,1
4	1,2	1,2	0,1	1,2	0,1	0,1

There are instead 12 Nash equilibria that can be written as (T, B)=[a; (b, c)], where $a \neq b, c$. In other worlds all the combination of strategies of the two players which deliver a payoff of (1, 2). This game shows that there is no need for the presence of dominated strategies for a Nash equilibrium to exist.

4. *Hawk-Dove game*.Consider the following game between two spiders:

Spider 1/Spider 2	Concede	Fight
Concede	5,5	0,10
Fight	10,0	X,X

a) By checking for best responses, for both players x has to be compared to 0. Hence:

i) x>0. The Nash equilibria are (C, F) and (F, C).

ii) x<0. The unique Nash equilibrium is (F, F).

b) x is the difference between the expected value of the web (=10) and the physical costs from fighting. If the payoff of winning the web increases over 10, then it is more likely that x will be greater than zero given the costs, i. e. it is more likely that (F, F) will be the Nash equilibrium of the game.

c)

Spider 1/Spider 2	Concede	Fight
Concede	y/2, y/2	0,y
Fight	y,0	x,x

This is a general version of the game where y>0. Note that if one of the two spider concedes, the other will always fight since y>y/2 for every y>0. If one of the two spiders fights, the other will fight or concede depending on x greater or less than zero. Hence what determines if (F, F) is the Nash equilibrium of the game is the difference between the payoffs to fighting and conceding when the other spider chooses to fight. Also, note that with x>0 Concede is strictly dominated by Fight.

d) As noted in point c), y>y/2 for every y>0. Hence, Concede will never be a best response to Concede in this game.

5) *Bertrand price competition.*

Firm 1/Firm 2	$ 0	$ 1	$ 2	$ 3	$ 4	$ 5	$ 6
$ 0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
$ 1	0,0	5/2,5/2	5,0	5,0	5,0	5,0	5,0
$ 2	0,0	0,5	4,4	8,0	8,0	8,0	8,0
$ 3	0,0	0,5	0,8	9/2,9/2	9,0	9,0	9,0
$ 4	0,0	0,5	0,8	0,9	4,4	8,0	8,0
$ 5	0,0	0,5	0,8	0,9	0,8	5/2,5/2	5,0
$ 6	0,0	0,5	0,8	0,9	0,8	0,5	0,0

The matrix displays the profits for both firms given the two prices posted.

a) It easy to check that posting a price of 0 and 6 dollars is weakly dominated by any other strategy of the two players. Also, posting a price of 4 and 5 dollars is weakly dominated by posting 2 and 3 dollars for both players.
b) By profit maximization we get:

$$\frac{d\pi}{dp} = \frac{d[p(6-p)]}{dp} = 6 - 2p = 0 \Rightarrow p^m = 3 \quad \textit{and} \quad \pi^m = 9.$$