# ***Economics of the Internet (W4490)***
## ***Problem set 3-Answer Keys***

**Question 1**

a) What kind of goods are sold through the Internet? What are their characteristics and differences?

b) What is the typical cost structure of the goods sold through the Internet? Does it differ with the type of good sold? Does it differ from the cost structure of the same type of goods not sold through the Internet?

ANSWER: Mainly information goods are sold through the net. They have a relatively large fixed cost but basically marginal cost equal to zero. This need not be true for non-information goods (which may also be sold through the net) that do not have a typical cost structure and the latter does not change whether the goods are sold through the net or not. For a more detailed answer refer to lecture 2 on the class web site.

**Question 2**

a) Give a definition of total cost function. What are the sources of costs we can identify in a total cost function?

ANSWER: the total cost function is the minimum cost at which quantity can be produced at given factor prices. Hence, it is a function of quantity⇒$C=C(q)$. Sources of cost vary depending on whether we are in the short run or in the long run. In the short run some factor quantities may be fixed. In this case we can identify the short run marginal cost (SMC) and the short run average cost (SAC). The latter can be further decomposed in variable average cost (SVAC) and fixed average cost (SFAC). In the long run ALL factors are variable, we have the long run marginal cost (LMC) and the long run average cost (LAC), but since all factors are variable they are exactly the same⇒LMC=LAC.

Give a definition of returns to scale and relate them to the shape of the marginal cost.

ANSWER: returns to scale tell us how quantity varies when ALL the factors vary in same proportion at given factor prices. Consider the following example with two factors, K and L:

$Y=F(K, L)$ and increase both inputs of a proportion $g>1$. Output will increase by a certain proportion given the increase in the inputs. We can write it as:
$g^rY=F(gK, gL)$. Now, if $r<1$, output increase but less than proportionally wrt the increase in factors ($g^r<g$)$\Rightarrow$decreasing returns to scale. If $r=1$, output increases in the same proportion wrt to factors$\Rightarrow$constant returns to scale. If $r>1$, output increases more than proportionally with respect to factors$\Rightarrow$increasing returns to scale.

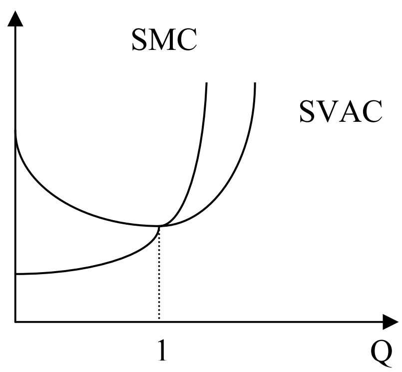
There is a relationship between returns to scale and the long run marginal cost (which is the same as the long run average cost. Also, returns to scale apply only to the long run since ALL factors must be variable). The relationship is the following:

- Increasing returns to scale$\Rightarrow$decreasing LMC
- Constant returns to scale$\Rightarrow$constant LMC
- Decreasing returns to scale$\Rightarrow$increasing LMC

b) The production function for a firm is $Y=10K^{1/2}L^{1/2}$. It pays workers a wage of \$1 and capital a rental rate of \$1.
Does the production function exhibit increasing, constant, or decreasing returns to scale? Justify your answer.
ANSWER: let's check for returns to scale in this case:
$10(gK)^{1/2}(gL)^{1/2}=g(10K^{1/2}L^{1/2})=gY\Rightarrow$constant returns to scale.
This production function is the celebrated Cobb-Douglas production function that can generally be written as: $Y=K^aL^b$. Note that $(gK)^a(gL)^b=g^{a+b}Y$. Hence here $r=a+b$ and you can simply check for returns to scale by looking at the exponents: if $a+b>(<)1$ you have increasing (decreasing) returns to scale. If $a+b=1$ you have constant returns to scale.
In the short run, the firm's capital input is fixed at $K_S=10$ units and its labor input is variable. Find the firm's short run cost function (cost as a function of output).
ANSWER: $C=wL+rK$. Hence in the short run $C=L+10$ (note this is not the short run cost function since it depends on L and not on Y). From the production function: $Y=10*10^{1/2}L^{1/2}\Rightarrow L=Y/1000$. Plug in the short run cost to get the short run cost function: $SCF=Y^2/1000+10$. Now you can also identify all the sources of cost:
$SAC=Y/1000+10/Y$ where $Y/1000$ is the SVAC and $10/Y$ is the SFAC.
$SMC=Y/500$

c) A competitive firm has the short-run cost function: $C(Q)=Q^3-2Q^2+5Q+6$
Give the firm's average cost function, average variable cost function (average cost without including fixed costs), and marginal cost function.
ANSWER: $SAC=Q^2-2Q+5+6/Q$, $SVAC= Q^2-2Q+5$, $SMC=3Q^2-4Q+5$.
At what level of output is average variable cost minimized? Explain how you arrived at your answer.
ANSWER: By minimization of the SVAC we get: $2Q-Q=0\Rightarrow Q^*=1$

Graph the firm's average variable cost function, marginal cost function, and its short-run supply function.
ANSWER:




The graphs look something like this. Note that the SVAC and the SMC meet exactly at the point where the SVAC is minimum. The short run supply function is just the part of the SMC that is above the SVAC (these are standard properties of cost functions that you can find in every book for undergraduate microeconomics).

**Question 3**

*(Bertrand Competition).* Consider the price competition model of Problem Set 2 Question 5.

a) Show that the best response to your rival posting a price of 6 dollars is to post the monopoly price of 3 dollars. What is the best response against a price of 4 and 5 dollars?

b) Can you show that the best response to the monopoly price of 3 dollars is to post a price of 2 dollars instead?

c) Show that the Nash equilibrium of this price competition model is for each firm to post a price of 1 dollar. Why is it lower than the monopoly price?

c) Show that if the two firms could cooperate and agree on the prices they post, they could both increase their profits with respect to the Nash equilibrium prices. Why this cooperative solution is not sustainable? (Hint: show that both firms have a profitable deviation from the cooperative solution.)

ANSWER:

| Firm 1/Firm 2 | $ 0 | $ 1 | $ 2 | $ 3 | $ 4 | $ 5 | $ 6 |
|---|---|---|---|---|---|---|---|
| $ 0 | 0,0 | 0,0 | 0,0 | 0,0 | 0,0 | 0,0 | 0,0 |
| $ 1 | 0,0 | 5/2,5/2 | 5,0 | 5,0 | 5,0 | 5,0 | 5,0 |
| $ 2 | 0,0 | 0,5 | 4,4 | 8,0 | 8,0 | 8,0 | 8,0 |
| $ 3 | 0,0 | 0,5 | 0,8 | 9/2,9/2 | 9,0 | 9,0 | 9,0 |
| $ 4 | 0,0 | 0,5 | 0,8 | 0,9 | 4,4 | 8,0 | 8,0 |
| $ 5 | 0,0 | 0,5 | 0,8 | 0,9 | 0,8 | 5/2,5/2 | 5,0 |
| $ 6 | 0,0 | 0,5 | 0,8 | 0,9 | 0,8 | 0,5 | 0,0 |

The matrix is exactly the same as in ps2. It displays all the profits for the two firms depending on the prices they post. I highlighted in yellow the best responses for firm1 and in green those for firm 2 (hope you have a color printer!). Note that you have two Nash equilibria, but the (0,0) one does not matter since p=0 is a dominated strategy for both firms (recall ps2). Hence the important one is (1, 1). For the last question, the firms could decide to agree on prices combination like (2,2), (3,3) and (4,4), where they both get a higher profit than posting (1,1). Anyway, such an agreement is not enforceable since every firm deviates once the other firm has posted a price of 2, 3, or 4.

**Question 4**

*(Cournot Competition).* Now consider the same model of the previous exercise, but suppose that firms compete by choosing one of the quantity levels 0, 1, 2, 3, 4, 5, 6. Hence now the demand function is $p=6-Q$, where $Q=q_1+q_2$, and from the demand function you get a unique price for both firms depending on the quantity they produce

(suppose that *p=0* whenever *Q>6*, so that negative prices are ruled out). Production costs are still zero.

a) Write down the strategic form of this game, i.e. the matrix with the profits for the two firms depending on the quantity levels they produce.

b) What is the best response for firm one to firm two producing a quantity of 4? And to firm two producing a quantity of 2?

d) Find the Nash equilibrium quantity levels and corresponding market price

ANSWER:

| Firm 1/Firm 2 | $ 0 | $ 1 | $ 2 | $ 3 | $ 4 | $ 5 | $ 6 |
|---|---|---|---|---|---|---|---|
| $ 0 | 0,0 | 0,5 | 0,8 | 0,9 | 0,8 | 0,5 | 0,0 |
| $ 1 | 5,0 | 4,4 | 3,6 | 2,6 | 1,4 | 0,0 | 0,0 |
| $ 2 | 8,0 | 6,3 | 4,4 | 2,3 | 0,0 | 0,0 | 0,0 |
| $ 3 | 9,0 | 6,2 | 3,2 | 0,0 | 0,0 | 0,0 | 0,0 |
| $ 4 | 8,0 | 4,1 | 0,0 | 0,0 | 0,0 | 0,0 | 0,0 |
| $ 5 | 5,0 | 0,0 | 0,0 | 0,0 | 0,0 | 0,0 | 0,0 |
| $ 6 | 0,0 | 0,0 | 0,0 | 0,0 | 0,0 | 0,0 | 0,0 |

Again the matrix displays the profits of the two firms depending on the quantities they produce. The profits are now $PR(1)=pq_1$, $PR(2)=pq_2$ where p is given by the demand function $p=6-Q$ and $Q=q_1+q_2$. Best responses are highlighted. You have 7 Nash Equilibria, but you can check that only (2,2) survives after eliminating weakly dominated strategies. For (2,2), the market price is 2 and profits are 4 for both firms.