

Economics of the Internet (W4490)
Problem set 4

Due Monday, March 11th

1. *Horizontal differentiation: the linear city.* This is a model, first due to Hotelling in 1929 (so it is him you have to blame). A homogeneous physical good becomes differentiated due to the different location of firms and the presence of transport costs. There is a “linear city” lying on a line of length one where consumers are uniformly distributed with density one along the line (i.e. let’s say there is a consumer for every point of the line, so that at a given point x on the line there are x consumers up to that point). There are also two firms that sell the same physical good. Firm 1 is established at point a and Firm 2 at point $1-b$ and $1-a-b$ is greater or equal then zero (i.e. F1 is at the left of F2, $a=b=0$ means that the two firms are the extremes of the segment (max differentiation) while $a+b=1$ means that the firms have the same location (min differentiation)). The unitary cost for both the firms is equal to c , hence equal to the marginal cost. Consumers face a transport cost t for unit of length and these costs are quadratic. This means that a consumer at point x on the line bears a cost $t(x-a)^2$ to purchase from F1 and a cost $t(x-(1-b))^2$ to purchase from F2. Consumers have unitary demand, i.e. either they buy one unit of the good or they do not buy at all. Every consumer gets a surplus s minus the price they pay and transport costs. If they do not buy their surplus is zero. Firms compete on prices and they post their prices simultaneously (Bertrand competition). Suppose that the difference between the two prices posted is not so high to drive one firm out of the market and that prices are not bigger than s , so that every consumer purchases and all the market is covered.

- a) Show that the demand functions for the two firms are respectively:

$$D_1(p_1, p_2) = a + (1-a-b)/2 + (p_2 - p_1)/2t(1-a-b) \text{ and}$$
$$D_2(p_1, p_2) = b + (1-a-b)/2 + (p_1 - p_2)/2t(1-a-b)$$

- b) Show that the Nash Equilibrium prices are:

$$p_1^c(a, b) = c + t(1-a-b)(1+(a-b)/3)$$
$$p_2^c(a, b) = c + t(1-a-b)(1+(b-a)/3)$$

and find the associated profit levels.

- c) Find the optimal location for every firm given the location of the other firm.

(Hint: for point a), consider that there is a consumer at a certain point x on the line who is indifferent between going to one or the other firm. Such x must satisfy the condition:

$p_1 + t(x-a)^2 = p_2 + t(x-(1-b))^2$, i.e. the consumer at point x is indifferent since given prices he bears the same cost whether he goes to F1 or F2. Hence, all consumers to his left will

prefer to buy from F1 and those to his right will prefer to buy from F2. This means that $D_1=x$ and $D_2=1-x$ since consumers are uniformly distributed on the line with density one.

For point b), maximize for p_i the profit $PR_i=(p_i-c)D_i(p_i, p_j)$ where D_i is the demand function you have derived in point a).

2. Consider again the model of the previous exercise, but now the firms are located at the extremes of the segment (i.e. $a=b=0$) and transport costs are linear. This means that the cost of going to F1 is now tx and that of going to F2 is $t(1-x)$.
 - a) Find the reaction functions (best response functions) $p_i=R_i(p_j)$ for both firms. Find the Nash equilibrium prices and the associated profits.
 - b) Show that the second derivative of the profits with respect to the marginal cost c is negative.