Economics of the Internet (W4490)

Problem Set 4-Solutions

1. *Horizontal differentiation*.

From the figure, it is clear that the consumer at *x'* is indifferent between going to firm 1 or firm 2. Moreover, all consumers at the left of *x'* will prefer to go to firm 1 and all those to the right will prefer to go to firm 2. Hence, the market share of firm 1 is *x'* and those for firm 2 is *1-x'*. To find the value of *x'* we have to solve the equation:

$$p_1 + t(x-a)^2 = p_2 + t(x-1+b)^2$$

This gives:

$$2t(1-a-b)x = p_2 - p_1 + t\left[(1-b)^2 - a^2\right] = p_2 - p_1 + t(1-b-a)(1-b+a), \text{ hence:}$$

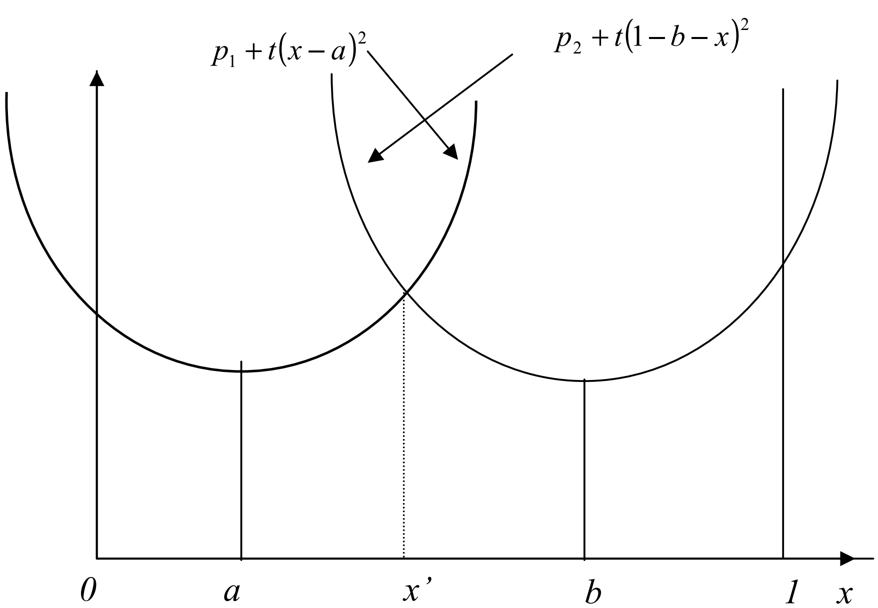
$$x = D_1(p_1, p_2) = \frac{1-b+a}{2} + \frac{p_2 - p_1}{2t(1-a-b)} = a + \frac{1-a-b}{2} + \frac{p_2 - p_1}{2t(1-a-b)}.$$

Demand for firm 2 is simply *1-x*.

a) By maximizing profits: $PR_i = (p_i - c)D_i(p_i, p_j)$ with respect to p_i one gets the best response price functions:

$$p_1 = \frac{1}{2}\left[c + p_2 + t\left((1-b)^2 - a^2\right)\right] \quad , \quad p_2 = \frac{1}{2}\left[c + p_1 + t\left((1-a)^2 - b^2\right)\right]$$

By intersecting the best response functions it is straightforward to get the expressions for the Nash equilibrium prices in the text.



To get the optimal location for F1 just set the derivative of the nash equlibrium profit for F1 with respect to *a* equal to zero.

2.

a) The reasoning here is exactly like in exercise 2. The indifference condition is now:

$$p_1 + tx = p_2 + t(1-x)$$

from which we get:

$$x = D_1(p_1, p_2) = \frac{1}{2} + \frac{p_2 - p_1}{2t}, \qquad 1 - x = D_2(p_1, p_2) = \frac{1}{2} + \frac{p_1 - p_2}{2t}.$$

By maximazing profits one gets the best response functions and from their intersection the Nash equilibrium prices:

$p_1^c = t + c, \quad p_2^c = t + c$ and profits $\pi_1^n = \pi_2^n = \frac{t}{2}$.

b) Since profits do not depend on *c*, the second derivative is equal to zero.