Capital Unemployment, Financial Shocks, and Investment Slumps*

Job Market Paper

[Preliminary and Incomplete]

updates at: http://www.columbia.edu/~po2171/research.html

Pablo Ottonello†

Columbia University

October 16, 2014

Abstract

Financial crises are characterized by a slow recovery of aggregate investment. Motivated by this empirical regularity, this paper constructs an equilibrium framework in which financial shocks lead to investment slumps. In the model, the key assumption is that trade of physical capital occurs in a decentralized market characterized by search frictions, which generates "capital unemployment." In this framework, the recovery from negative financial shocks is characterized by low production of new capital because absorbing existing unemployed capital improves the intertemporal allocation of consumption. An estimation of the model for the U.S. economy using Bayesian techniques shows that the model can generate the investment persistence and half of the output persistence observed in the Great Recession. Over the business cycle, incorporating search frictions in investment makes financial shocks account for 48% of output fluctuations (versus 5% in the benchmark real model without search frictions in investment). Incorporating heterogeneity in match productivity, the model also provides a mechanism for procyclical capital reallocation and misallocation during recessions as observed in the data.

JEL Classification: E22, E23, E44, E32, D53

Keywords: Financial Shocks, Investment Dynamics, Search Frictions, Business Cycles, Capital Reallocation, Great Recession

---

*I would like to thank my advisors Martin Uribe, Stephanie Schmitt-Grohe, Michael Woodford, and Guillermo Calvo for guidance and support. For very useful comments and suggestions, I would also like to thank Jeniffer La'O, Emi Nakamura, Jaromir Nosal, Ricardo Reis, Jon Steinson, Stefania Albanesi, Sushant Acharya, Robert Barro, Javier Bianchi, Saki Bigio, Patrick Bolton, Ryan Chahrour, Bianca De Paoli, Keshav Dogra, Valentina Duque, Fatih Karahan, Ana Fostel, Mariana Garcia, Marc Giannoni, Linda Goldberg, Thomas Klitgaard, Wataru Miyamoto, Tommaso Monacelli, Thuy Lan Nguyen, Friederike Niepmann, Nate Pattison, Diego Perez, Paolo Pesenti, Laura Pilossoph, Benjamin Pugsley, Aysegul Sahin, Argia Sbordone, Joseph Stiglitz, Ernesto Talvi, and seminar participants at Columbia University and the Federal Reserve Bank of New York. This paper was partly written at the Federal Reserve Bank of New York. I am especially grateful for their hospitality.

†Email: po2171@columbia.edu.

1 Introduction

The U.S. *Great Recession*, which started in 2007, was followed by slow recovery of investment and output: Seven years later, the economy is just recovering its precrisis output level and, by most measures, has not recovered its potential; investment, which has recovered more slowly than output, has been the most important contributor to this slump (see Hall, 2014, and Figure 1). The slow recovery from the Great Recession is challenging from the points of view of real and monetary models (see Kydland and Zarazaga, 2012; Del Negro, Giannoni and Patterson, 2012). Indeed, key financial variables related to the economy's contraction have already returned to relatively normal conditions; models that exhibit a balanced-growth path predict that, in the absence of further shocks, the more output falls below its trend, the faster it should recover.

Although it can be argued that specific factors related to the U.S. economy have played a relevant role in the sluggish recovery, historical and international evidence suggests that the pattern exhibited by the U.S. Great Recession is a salient characteristic of financial-crisis episodes (see, for example, Calvo, Izquierdo and Talvi, 2006; Cerra and Saxena, 2008; Reinhart and Rogoff, 2009, 2014). In particular, as illustrated in Figure 1 for a set of advanced- and emerging-market financial-crisis episodes, recovery from financial crises occurs throughout several years, with investment recovering more slowly than output.

Motivated by this evidence, this paper constructs a general equilibrium framework in which negative financial shocks lead to investment slumps – periods of persistently low investment. The key idea in the model is that the production of new capital is affected by existing "capital unemployment" (i.e., owners of idle units of capital unable to find a firm willing to buy or rent these units to produce); after a negative financial shock, the share of unemployed capital is high and the economy can achieve a better allocation by directing more resources to absorb existing unemployed capital units into the production process and less to the construction of new capital units, leading to low investment rates even after the shock has dissipated. The model's main assumption, which leads to equilibrium capital unemployment, is that trade in physical capital occurs in a decentralized market characterized by search frictions.

The model is developed in a quantitative business-cycle framework to assess the importance of the proposed mechanism. To this end, the model is estimated for the U.S. economy

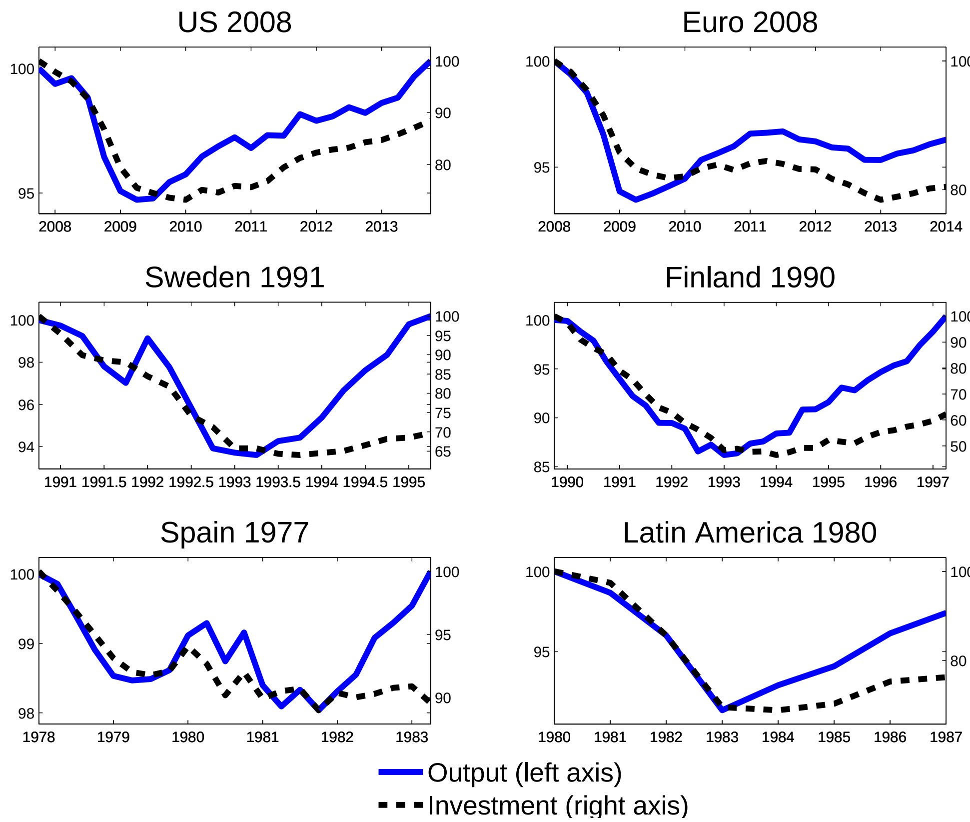

Figure 1: Financial Crises and Investment Slumps.
Note: Output and investment refers to real, per capita, gross domestic product and gross fixed capital formation. See Appendix 7.1 for details and data sources.

using Bayesian techniques, including a rich set of shocks and other relevant frictions (such as financial frictions, investment-adjustment costs, and habit formation). This allows one to ask whether, following a sequence of shocks such as those experienced by the U.S. economy in 2008 – and without any further shock – if the model can predict an investment slump. The answer is yes, the model predicts a path of investment even lower than the one observed in the data and at least half of the output persistence observed during the U.S. Great Recession. Conducting the same exercise in a benchmark model *without* investment search frictions (but *with* the other frictions), the model predicts that both investment and output should be significantly higher than the levels observed in the data, as noted in the previous literature.

Using the estimated model to interpret the sources of U.S. business-cycle fluctuations, the paper shows that search frictions in investment are a relevant propagation mechanism of financial shocks (i.e., shocks that directly affect the net worth of the business sector or

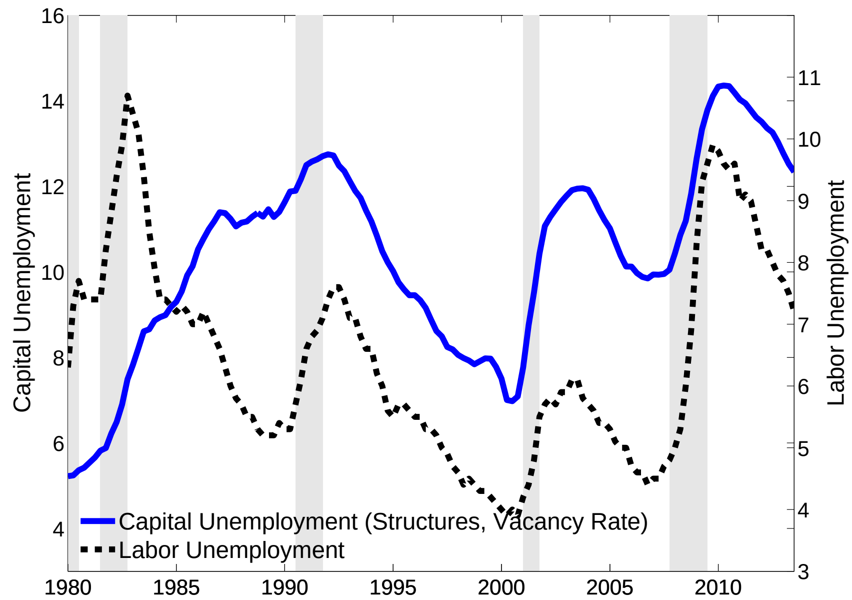

Figure 2: U.S. Unemployment of Physical Capital and Labor, 1980–2013.
Note: Capital unemployment (structures) constructed based on vacancy rates of office, retail and industrial units. Data source: CBRE and REIS. See Appendix 7.1 for details. Labor unemployment refers to the civilian unemployment rate. Data source: Federal Reserve of Saint Louis. Shadow areas denote NBER (peak to trough) recession dates.

credit conditions). While these shocks only account for 5% of output fluctuations in the benchmark real model *without* investment search frictions, they account for 48% of output fluctuations in the model *with* investment search frictions. The role of financial shocks is a key discussion in the business-cycle literature and an important source of discrepancy between real and monetary models, with the latter attributing a much larger effect to these shocks than the former (as discussed in Christiano, Motto and Rostagno, 2014). The present paper shows that an important part of this discrepancy between these two branches of the literature can be reconciled by introducing investment search frictions. In the estimation, the frictions are disciplined with data on vacancy rates in commercial real estate (office, retail and industrial space). As shown in Figure 2, the level and fluctuations in this measure of capital unemployment are comparable to those of unemployment in the U.S. labor market.

The framework developed in this paper can also be used to study capital reallocation. This is done by extending the model to allow for heterogeneity in capital match-specific productivity. This extension allows a characterization not only of the transition of capital from unemployment to employment, but of the transition of capital from employment to employment, since it adds a motive for trading unmatched capital while it remains employed (similar to "on the job search" in the labor-market literature). The paper shows that capital reallocation is procyclical in this framework, as in the data (see Eisfeldt and Rampini, 2006). The reason is that negative shocks are associated with fewer capital purchases by firms, making it harder for sellers of employed capital to find buyers.

On the technical side, in the model search is directed, in the sense that sellers and buyers can search offers at a particular price, and the probability of finding a match depends on this price (see, for example, Shimer, 1996; Moen, 1997). For this reason, the allocation resulting from the mechanism described would be the same as the one chosen by a social planner who faces the same constraints than the private sector, including search effort. Using directed search is specially suitable to study employment-employment transition resulting from heterogeneity match-specific-productivity, as shown in Menzio and Shi (2011) for the labor market.

This paper's *capital unemployment* differs from *capital utilization*.[1] While capital utilization is a variable describing the intensity with which capital is used by firms that own or rent capital (a "consumption decision"), capital unemployment is a variable that describes whether owners of idle capital are unable to be sell or rent it (an "investment decision"). The difference between these two variables is parallel to that between labor unemployment and labor hoarding (see, for example, Burnside, Eichenbaum and Rebelo, 1993; Andolfatto, 1996; Sbordone, 1996). Being two different concepts, capital utilization and capital unemployment should have different empirical measures. For instance, standard empirical measures of capital utilization relate to firms' use of their production capacity.[2] Empirical measures of capital unemployment would instead relate the share of physical capital (owned by either firms or households) that is idle and available in the market for sale or rent, such as the data collected from the commercial real estate market used in this paper (see Figure 2). As illustrated in Appendix 7.2 for recent U.S. recession episodes, these empirical measures of capital unemployment and capital utilization can have significantly different behavior. Being

[1]For surveys on the concept, theory and empirical analysis of capital utilization, see Winston (1974) and Betancourt and Clague (2008).

[2]For the U.S., The Federal Reserve Board estimates capacity utilization for industries in manufacturing (see `http://www.federalreserve.gov/releases/g17/CapNotes.htm` for a description of the methodology). Gorodnichenko and Shapiro (2011) use the Survey of Plant Capacity from the U.S. Census Bureau (`http://www.census.gov/manufacturing/capacity/`) to construct data on capital utilization. Basu, Fernald and Kimball (2006), Basu et al. (2013), and Fernald (2009) provide estimates of factor utilization for the U.S. economy, capturing labor effort and the work week of capital.

two different concepts, whose empirical measure can have a different behavior, capital utilization and capital unemployment can also be modeled differently. Models of capital utilization typically treat it as a control variable whose choice, related to utilization costs, can be described as an intensive margin (e.g., a higher utilization rate causes higher depreciation as in Calvo, 1975; Greenwood, Hercowitz and Huffman, 1988) or as an extensive margin (e.g.: less productive units are left idle, as in Cooley, Hansen and Prescott, 1995; Gilchrist and Williams, 2000). Recent contributions using search frictions in the product market show that this variable can also be related to the probability of a firm finding customers (see, for example, Petrosky-Nadeau and Wasmer, 2011; Bai, Rios-Rull and Storesletten, 2012; Michaillat and Saez, 2013). In the model of capital unemployment developed in this paper this as a state variable; the key margins that affect the flows of unemployed capital to employment are the price of capital posted by sellers and the mass of capital that buyers will be willing to purchase at a given price. For this reason, this paper will show that different factors affect fluctuations in capital utilization and capital unemployment and that different implications follow by explicitly modeling capital unemployment (such as the a low rate of investment when capital unemployment is high). Nevertheless, the concepts of capital utilization and capital unemployment can be seen as complementary. In fact, once the model with capital unemployment is extended to study capital reallocation, changes in the probability of selling capital units will affect firms capital utilization rates.

Related literature. This paper is related to several branches of the literature. First, this paper builds on the growing body of literature that studies the effect of financial shocks on macroeconomic fluctuations. The study of the implications of financial frictions has a long tradition in macroeconomics (for a recent survey, see Brunnermeier, Eisenbach and Sannikov, 2012). Following the Great Recession, a number of studies have shown that shocks that effecting the severity of financial frictions can have a large impact on aggregate fluctuations (see, for example, Arellano, Bai and Kehoe, 2012; Jermann and Quadrini, 2012; Gertler and Kiyotaki, 2013; Christiano, Motto and Rostagno, 2014). The present contibutes to this literature with a novel financial-shock propagation mechanism by introducing the possibility of capital unemployment, whose fluctuations are mostly driven by this type of shock.

By modeling capital unemployment in a search theoretical framework, this paper relates to the extensive literature studying search frictions in assets, labor, and goods markets. Search frictions in the physical capital market were first studied by Kurmann and Petrosky-Nadeau (2007), who show that these frictions are not a *quantitatively* relevant propagation mechanism of TFP shocks.[3] The most important difference from their quantitative framework is the inclusion of financial shocks, that in the present paper account for most of the fluctuations in the market tightness. In fact, if the present paper included only TFP shocks, it would also have concluded that search frictions in investment are not a relevant quantitative propagation mechanism once output fluctuation is matched, a result parallel to that found in Shimer (2005) for the labor market. Two additional differences with respect to the contribution of Kurmann and Petrosky-Nadeau (2007) are the theoretical framework and the empirical strategy. First, the present paper constructs a directed-search model, whereas Kurmann and Petrosky-Nadeau (2007) study a random-search environment. Second, while their study follows a calibration strategy, the present paper conducts an estimation using Bayesian techniques.

The directed-search framework for the physical-capital market developed in the present paper builds on those developed for the labor market in Menzio and Shi (2010, 2011), Schaal (2012) and Kircher and Kaas (2013). Studying these frictions for the physical capital market provides two novel mechanisms: First, it provides a new interaction between the production of new capital and capital unemployment. As shown in Section 2, the existence of high capital unemployment leads to a lower production of new capital goods while existing units are absorbed into production. This mechanism is not present in labor-market models in which that population is generally assumed to be constant or exogenous. Second, as physical capital is not only a factor of production, but can also be used by firms as collateral for loans (see, for example, Kiyotaki and Moore, 1997; Geanakoplos, 2010), fluctuations in capital unemployment interact with financial shocks in a way not seen in the labor market.

Given that physical capital is both a good and an asset, the search frictions studied in this paper are also related to those of goods markets or other asset markets. With regard to goods markets, Bai, Rios-Rull and Storesletten (2012) recently studied search frictions that affect the purchase of investment goods, as in the present paper. Unlike the present paper, these frictions only affect the flow of production and not the stock of existing capital units (which is the main feature of capital unemployment). In other asset markets, a number of contributions

[3] Kim (2012) shows that if it is assumed that capital decisions are not made on marginal units of investment and that new capital does not begin unmatched, search frictions in the physical capital can be a large propagation mechanism of TFP shocks.

have shown how search frictions affect the liquidity and returns of assets (see, for example, Lagos and Rocheteau, 2008, 2009). In the housing market, search frictions have been used to explain fluctuations in prices, trading and vacancy rates (see, for example, Wheaton, 1990; Krainer, 2001; Caplin and Leahy, 2011; Piazzesi, Schneider and Stroebel, 2013). The main difference with respect to these contributions is that the physical capital considered in the present paper is a productive asset, and therefore fluctuations in its unemployment have a direct relationship with economic activity and firms investment.

Organization. The rest of the paper is organized as follows. Section 2 presents the main mechanism in a simple neoclassical growth model. Section 3 builds a quantitative business-cycle model including search frictions in investment. Section 4 presents the model estimation and the quantitative results. Section 5 studies capital reallocation in the framework of the model. Section 6 concludes and discusses possible extensions.

2 Search Frictions in Investment: Basic Framework

This section introduces investment search frictions into a simple neoclassical growth model. The framework abstracts from uncertainty, endogenous labor supply and other frictions – which will be later introduced in the quantitative model – to make the mechanism clear. Policy functions and the dynamic response to unexpected shocks are studied, showing how capital accumulation is affected by existing capital unemployment.

2.1 Environment

Time is discrete and infinite, with four-stage periods. There is no aggregate uncertainty.

Goods. There are consumption and capital goods: Consumption goods are perishable; capital goods depreciate at a constant rate, $\delta > 0$. Capital can be traded in either of two states: matched or unmatched. Only matched capital can be used as input in the production of consumption goods.

Agents. The economy is populated by a large number of identical households and entrepreneurs. Households consume and produce physical capital. The representative household has a continuum of infinitely lived members of measure one, with a positive fraction of

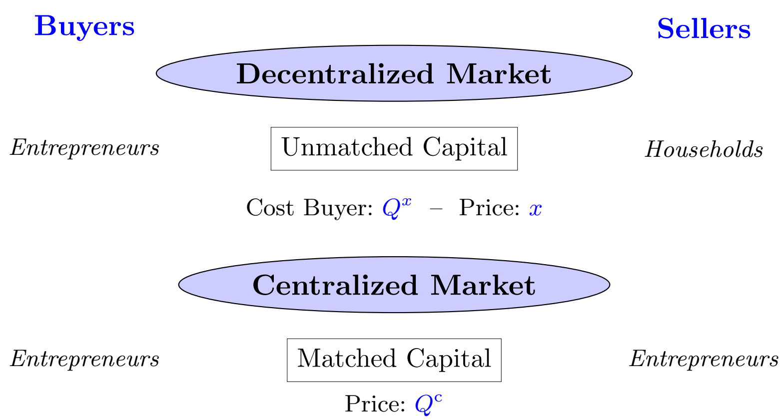

FIGURE 3: STRUCTURE OF CAPITAL MARKETS, BASIC FRAMEWORK.

them being entrepreneurs. Within each household there is perfect consumption insurance.[4] Entrepreneurs have access to a technology to produce consumption goods, using matched capital as input, and to a search technology to transform unmatched capital into matched capital. Capital produced by households begins unmatched. Only entrepreneurs can store matched capital. Capital held by entrepreneurs is denoted *employed capital*, and capital held by households is denoted *unemployed capital*.

Each period, entrepreneurs have a probability $\psi > 0$ of retiring from entrepreneurial activity. The fraction ψ of entrepreneurs who retire from entrepreneurial activity is replaced by a new identical mass of entrepreneurs from the households' members, so the population of entrepreneurs is constant with a measure of one. Retiring entrepreneurs' capital becomes unmatched and is transferred to the household. Dividends from entrepreneurial activity, resulting from capital purchases and production, are transferred each period to the household.

Physical capital markets. Trade of unmatched capital between entrepreneurs and households occurs in a decentralized market with search frictions. In addition to this market, entrepreneurs also have access to a centralized market in which they trade matched capital at price Q^c.[5] Figure 3 summarizes these two markets for capital.

[4]The assumption of large families follows Merz (1995), Andolfatto (1996) and, more recently, Gertler and Karadi (2011) and Christiano, Motto and Rostagno (2014). This assumption facilitates the work in Section 3, when financial frictions are introduced explicitly and entrepreneurs are endowed with net worth. In the current section, this assumption plays no role and is not different from a framework in which a representative firm produces consumption goods.

[5]Including a centralized market where entrepreneurs can trade employed capital is convenient for technical reasons. Particularly, this property will facilitate the study of an equilibrium that does not depend on the distribution of capital among entrepreneurs. In addition, once financial frictions are introduced into the model (Section 3), the centralized market ensures that financing decisions are not influenced by market tightness.

In the decentralized market for unmatched capital, search is directed, following a structure similar to the one developed in Menzio and Shi (2010, 2011) for the labor market and in Menzio, Shi and Sun (2013) for the money market. In particular, this market is organized in a continuum of submarkets indexed by the price of unmatched capital, denoted x, and sellers (households) and buyers (entrepreneurs) can choose which submarket to visit. In each submarket, the market tightness, denoted $\theta(x)$ is defined as the ratio between the mass of capital searched by entrepreneurs and the mass of unemployed capital offered in that submarket. Households face no search cost. Visiting submarket x in period t, they face a probability $p(\theta_t(x))$ of finding a match, where $p : \mathbb{R}_+ \to [0, 1]$ is a twice continuously differentiable, strictly increasing, strictly concave function that satisfies $p(0) = 0$ and $\lim_{\theta\to\infty} p(\theta) = 1$. Entrepreneurs face a cost per unit searched denominated in consumption goods and denoted $c_{\mathrm{s}} > 0$. Visiting submarket x in period t, they face a probability $q(\theta_t(x))$ of finding a match, where $q : \mathbb{R}_+ \to [0, 1]$ is a twice continuously differentiable, strictly decreasing function that satisfies $q(\theta) = \frac{p(\theta)}{\theta}$, $q(0) = 1$ and $\lim_{\theta\to\infty} q(\theta) = 0$. The cost of a unit of capital for entrepreneurs in submarket x is denoted Q^x (which includes two components: the price paid to the seller, x, and the search cost in submarket x).

Timing. Each period is divided into four stages: production, separation, search, and investment. In the *production stage*, entrepreneurs produce consumption goods using capital matched in the previous period; employed and unemployed capital depreciates. In the *separation stage*, a fraction ψ of entrepreneurs retires and their capital becomes unmatched. An identical mass of entrepreneurs begins entrepreneurial activity with no initial capital. In the *search stage*, entrepreneurs who do not retire and new entrepreneurs purchase unmatched capital from households and matched capital from other entrepreneurs, and net dividends in terms of consumption goods are transferred. In the *investment stage*, households produce physical capital and consume, and retired entrepreneurs transfer their capital to households.

2.2 Households

Household preferences are described by the lifetime utility function

$$\sum_{t=0}^{\infty} \beta^t U\left(C_t\right), \tag{1}$$

where C_t denotes household consumption in period t, $\beta \in (0,1)$ is the subjective discount factor, and $U : \mathbb{R}_+ \to \mathbb{R}$ is a twice continuously differentiable, strictly increasing, strictly concave function.

Unemployed capital, held by households, evolves according to the law of motion

$$K^{\mathrm{u}}_{t+1} \;=\; \int_0^{(1-\delta)K^{\mathrm{u}}_t} (1 - p(\theta_t(x_{i,t})))\,\mathrm{d}i + \psi(1-\delta)K^{\mathrm{e}}_t + I_t, \tag{2}$$

where K^{u}_t denotes the stock of unemployed capital at the beginning of period t, K^{e}_t denotes the stock of employed capital at the beginning of period t, I_t denotes the household's investment in period t, and $x_{i,t}$ denotes the submarket in which unemployed capital unit i is listed in period t. The first term of the right-hand side of equation (2) represents the depreciated mass of capital which was unemployed at the beginning of period t and was not sold to entrepreneurs for a given market tightness, $\theta_t(x)$, and submarket choices $x_{i,t}$. The second term of the right-hand side of equation (2) represents the mass of employed capital transferred from retired entrepreneurs to the households. The third term represents the addition (subtraction) to unemployed capital stock from investment.

The household's sequential budget constraint is given by

$$C_t + I_t \;=\; \int_0^{(1-\delta)K^{\mathrm{u}}_t} p(\theta_t(x_{i,t}))x_{i,t}\,\mathrm{d}i + \tilde{\Pi}^{\mathrm{e}}_t, \tag{3}$$

where $\tilde{\Pi}^{\mathrm{e}}_t$ denotes net transfers in terms of consumption goods from entrepreneurs to households in period t – described further in the next section. The left-hand side of equation (3) represents the uses of income: consumption and investment. The right-hand side of the equation represents the sources of income: selling unmatched capital in the decentralized market and transfers from entrepreneurs.

The *household's problem* is then to choose plans for C_t, I_t, K^{u}_{t+1}, and $x_{i,t}$ that maximize utility (1), subject to the sequence of budget constraints (3), the accumulation constraints for unemployed capital (2), given the initial level of capital, $\{K^{\mathrm{u}}_0, K^{\mathrm{e}}_0\}$, the given sequence of net transfers, $\tilde{\Pi}^{\mathrm{e}}_t$, and the given sequence of market-tightness functions, $\theta_t(x)$. Denoting by Λ_t the Lagrange multiplier associated with the budget constraint (3), in an interior solution the optimality conditions are (2), (3), and the first-order conditions

$$\Lambda_t = U'(C_t), \tag{4}$$

$$\Lambda_t = \beta\Lambda_{t+1}(1-\delta)\left[p(\theta_{t+1}(x^{\mathrm{u}}_{t+1}))x^{\mathrm{u}}_{t+1} + (1-p(\theta_{t+1}(x^{\mathrm{u}}_{t+1})))\right], \tag{5}$$

$$-p(\theta(x^{\mathrm{u}}_t)) = p'(\theta_t(x^{\mathrm{u}}_t))\theta'_t(x^{\mathrm{u}}_t)(x^{\mathrm{u}}_t - 1). \tag{6}$$

where x^{u}_t denotes the household's choice of submarket for unmatched capital in period t, and the unit of capital subindex, i, has been dropped because the optimality condition with respect to the choice of submarket, $x_{i,t}$, is the same for all units of capital.

2.3 Entrepreneurs

Entrepreneurs have access to a technology to produce consumption goods that uses matched capital as input:

$$Y_{j,t} = A_t F(K^{\mathrm{e}}_{j,t}), \tag{7}$$

where $Y_{j,t}$ denotes output produced by entrepreneur j in period t, $K^{\mathrm{e}}_{j,t} \geq 0$ denotes the stock of matched capital held by entrepreneur j at the beginning of period t, A_t is an aggregate productivity factor affecting the production technology in period t, and $F : \mathbb{R}_+ \to \mathbb{R}_+$ is a twice continuously differentiable, strictly increasing, strictly concave function satisfying $F(0) = 0$.

The entrepreneur's objective is to maximize the present discounted value of dividends distributed to households:

$$E_t \sum_{s=0}^{\infty} \beta^s \frac{\Lambda_{t+s}}{\Lambda_t} \Pi^{\mathrm{e}}_{j,t+s}, \tag{8}$$

where $\Pi^{\mathrm{e}}_{j,t}$ denotes net dividends paid by entrepreneur j to the household in period t and E_t denotes the expectation conditional on the information set available at time t (the expected value is over the idiosyncratic retirement shock). Net dividends of entrepreneur j are defined by the flow-of-funds constraint:

$$\Pi^{\mathrm{e}}_{j,t} = A_t F(K^{\mathrm{e}}_{j,t}) - (1-\psi_{j,t})\left[\int_x Q^x_t \iota^{\mathrm{e},x}_{j,t}\,\mathrm{d}x + Q^c_t \iota^{\mathrm{e,c}}_{j,t}\right] + \psi_{j,t}(1-\delta)K^{\mathrm{e}}_{j,t}, \tag{9}$$

where $\iota^{\mathrm{e},x}_{j,t} \geq 0$ denotes the mass of capital purchased by entrepreneur j in submarket x in period t, $\iota^{\mathrm{e,c}}_{j,t}$ denotes the mass of capital purchased (sold) by entrepreneur j in the centralized market in period t, and the stochastic variable $\psi_{j,t} \in \{0, 1\}$ takes the value of 1 if entrepreneur j retires from entrepreneurial activity in period t, and 0 otherwise, and satisfies $E_{t-1}(\psi_{j,t}) = \psi \ \forall\ t, j$. The three terms in the right-hand side of equation (9) represent the sources of net dividends transferred from entrepreneurs to households: The first term represents the output in terms of consumption goods produced by entrepreneur j in period t. The second term denotes the net purchase of physical capital, expressed in consumption units, that entrepreneur j makes in case of not retiring in period t. The last term represents the transfer of unmatched capital that entrepreneur j makes to households in case of retiring in period t. The first two terms define the net transfer, in terms of consumption goods, that entrepreneur j makes to households in period t: $\tilde{\Pi}^{\mathrm{e}}_{j,t} \equiv A_t F(K^{\mathrm{e}}_{j,t}) - (1 - \psi_{j,t}) \left[\int_x Q^x_t \iota^{\mathrm{e},x}_{j,t}\, \mathrm{d}x + Q^c_t \iota^{\mathrm{e,c}}_{j,t}\right]$.

By the law of large numbers, the cost per unit of capital, of mass $\iota^{\mathrm{e},x}_{j,t}$, purchased in the submarket x of the decentralized market is given by

$$Q^x_t \quad = \quad x + \frac{c_{\mathrm{s}}}{q(\theta_t(x))}. \tag{10}$$

The right-hand side of equation (10) represents the two components of the cost of a unit of capital in the decentralized market: the price paid to the seller, x, and the search cost, $\frac{c_{\mathrm{s}}}{q(\theta_t(x))}$.

The stock of matched capital for entrepreneur j, who has the opportunity to invest in period t, evolves according to the law of motion

$$K^{\mathrm{e}}_{j,t+1} \quad = \quad (1 - \delta) K^{\mathrm{e}}_{j,t} + \int_x \iota^{\mathrm{e},x}_{j,t}\, \mathrm{d}x + \iota^{\mathrm{e,c}}_{j,t}. \tag{11}$$

Denote by t_{0j} the period in which entrepreneur j enters entrepreneurial activity. It is assumed entrepreneurs enter entrepreneurial activity with no initial matched capital, that is $K^{\mathrm{e}}_{j,t_{0j}} = 0 \ \forall\ t_{0j} \geq 0$.[6]

The *entrepreneur's j problem*, is then to choose plans for $K^{\mathrm{e}}_{j,t+1}$, $\iota^{\mathrm{e},x}_{j,t}$, and $\iota^{\mathrm{e,c}}_{j,t}$ that maximize the present discounted value of dividends (8) subject to the sequence of flow-of-funds constraints (9), the accumulation constraints for matched capital (11), and the nonnegativity constraints for capital purchases in the decentralized market $\left(\iota^{\mathrm{e},x}_{j,t} \geq 0\right)$, given the initial level of matched capital, $K^{\mathrm{e}}_{j,t_{0j}}$, the given sequence of aggregate productivity A_t, the given

[6] A mass one of entrepreneurs starts period 0 with a stock of matched capital K^{e}_0.

sequence of prices, Q_t^{c}, and the given sequence of market-tightness functions, $\theta_t(x)$.

Denoting by $Q_{t+s}\frac{\Lambda_{t+s}}{\Lambda_t}$ the Lagrange multiplier associated with the budget constraint (11) in period $t+s$, and by $\Xi_{t+s}^x \frac{\Lambda_{t+s}}{\Lambda_t}$ the Lagrange multiplier associated with nonnegativity constraint for capital purchases in submarket x in period $t+s$, the optimality conditions (provided $K_{j,t+1}^{\mathrm{e}} > 0$) are (9), (11), $\iota_t^{\mathrm{e},x} \geq 0$, the first-order conditions

$$\Lambda_t Q_t = \beta \Lambda_{t+1} \left[A_{t+1} F'(K_{t+1}^{\mathrm{e}}) + (1-\delta)\left(\psi + (1-\psi) Q_{t+1}\right) \right], \quad (12)$$

$$Q_t^c = Q_t, \quad (13)$$

$$Q_t^x = Q_t + \Xi_t^x, \quad (14)$$

and the complementary slackness conditions

$$\Xi_t^x \geq 0, \quad \iota_t^{\mathrm{e},x} \Xi_t^x = 0, \quad (15)$$

for all x, where the entrepreneur's subindex, j, has been dropped because the first-order conditions of the entrepreneur's problem are the same for all entrepreneurs.

2.4 Equilibrium

Entrepreneurs' optimality conditions (14) and (15) imply that, in equilibrium, any submarket visited by a positive number of entrepreneurs must have the same cost per unit of capital, and entrepreneurs will be indifferent among them. Formally, for all x,

$$\theta_t(x) \left(x + \frac{c_{\mathrm{s}}}{q\left(\theta_t(x)\right)} - Q_t \right) = 0. \quad (16)$$

This condition determines the equilibrium market-tightness function: For all $x < Q_t$,

$$\theta_t(x) = q^{-1} \left(\frac{c_{\mathrm{s}}}{Q_t - x} \right). \quad (17)$$

For all $x \geq Q_t$, $\theta_t(x) = 0$.

Market clearing in the centralized market for matched capital requires $\int_j \iota_{j,t}^{\mathrm{e},c}\,\mathrm{d}j = 0$. Using the definition of market tightness, the law of large numbers, and the fact that a household's choice of submarket, $x_{i,t}$ is the same for all units of capital i, in equilibrium, the flow of capital that transitions from unemployment is given by $p(\theta_t(x_t^{\mathrm{u}}))(1-\delta)K_t^{\mathrm{u}} = \int_j \iota_{j,t}^{\mathrm{e},x^{\mathrm{u}}}\,\mathrm{d}j =$

$\int_j \int_x \iota^{\mathrm{e},x}_{j,t} \, \mathrm{d}j \, \mathrm{d}x$. Aggregating the entrepreneurs' capital-accumulation constraints provides a law of motion for employed capital:

$$K^{\mathrm{e}}_{t+1} \quad = \quad (1-\psi)(1-\delta)K^{\mathrm{e}}_t + p(\theta_t(x^{\mathrm{u}}_t))(1-\delta)K^{\mathrm{u}}_t. \tag{18}$$

From the household's capital-accumulation constraint (2), and using again the law of large numbers and the fact that the choice of submarket, $x_{i,t}$, is the same for all units of capital i, we obtain a law of motion for unemployed capital:

$$K^{\mathrm{u}}_{t+1} \quad = \quad (1-p(\theta_t(x^{\mathrm{u}}_t)))(1-\delta)K^{\mathrm{u}}_t + \psi(1-\delta)K^{\mathrm{e}}_t + I_t. \tag{19}$$

The capital unemployment rate at the beginning of period t can be then defined as

$$k^{\mathrm{u}}_t \quad \equiv \quad \frac{K^{\mathrm{u}}_t}{K_t}, \tag{20}$$

where $K_t \equiv K^{\mathrm{e}}_t + K^{\mathrm{u}}_t$ denotes total aggregate capital stock at the beginning of period t.

Aggregating the households' budget and the entrepreneurs' flow-of-funds constraints and using the law of motion for employed and unemployed capital provides the economy's resource constraint:

$$C_t + I_t + c_{\mathrm{s}}\theta_t(x^{\mathrm{u}}_t)(1-\delta)K^{\mathrm{u}}_t \quad = \quad A_t F(K^{\mathrm{e}}_t). \tag{21}$$

The competitive equilibrium in this economy can then be defined as follows.

Definition 1 (Competitive equilibrium)**.** *Given initial conditions for employed and unemployed capital, K^{e}_0 and K^{u}_0, and sequences of aggregate productivity, A_t, a* competitive equilibrium *is a sequence of allocations $\{C_t, I_t, K^{\mathrm{e}}_{t+1}, K^{\mathrm{u}}_{t+1}, x^{\mathrm{u}}_t\}$, shadow values $\{\Lambda_t, Q_t\}$, prices $\{Q^c_t\}$, and market-tightness functions $\theta_t(x)$ such that.*

(i) The allocations and shadow values solve the households' and entrepreneurs' problems at the equilibrium prices and equilibrium market-tightness functions. They satisfy (4), (5), (6), (12), and (13).

(ii) The market-tightness function satisfies (16) for all x.

(iii) Market clearing and aggregation: The centralized market for matched capital clears; the

allocations satisfy the laws of motion for employed and unemployed capital (18) *and* (19)*; the resource constraint* (21) *holds.*

2.5 Characterizing Equilibrium

Efficiency. Given the directed-search structure of the decentralized market, it can be shown that the competitive equilibrium is efficient in the sense that its allocation coincides with the solution of the problem of a social planner facing the same technological constraints as those faced by private agents, including search effort. Efficiency is defined and established in the following definition and proposition.

Definition 2 (Efficient equilibrium)**.** *A sequence of allocations,* $\{C_t, I_t, K_{t+1}, k^{\mathrm{u}}_{t+1}, \theta^{\mathrm{u}}_t\}$*, is* efficient *if it solves the following social planner's problem.*

$$\max_{\{C_t, I_t, K_{t+1}, k^{\mathrm{u}}_{t+1}, \theta_t\}} \sum_{t=0}^{\infty} \beta^t U(C_t), \tag{22}$$

$$\begin{aligned} s.t.\ & C_t + I_t + c_{\mathrm{s}}\theta^{\mathrm{u}}_t(1-\delta)k^{\mathrm{u}}_t K_t = A_t F((1-k^{\mathrm{u}}_t)K_t), \\ & K_{t+1} = (1-\delta)K_t + I_t, \\ & (1-k^{\mathrm{u}}_{t+1})K_{t+1} = (1-\delta)[(1-\psi)(1-k^{\mathrm{u}}_t)K_t + A^{\mathrm{e}}_t p(\theta^{\mathrm{u}}_t)(1-\delta)k^{\mathrm{u}}_t K_t]. \end{aligned}$$

Proposition 1. *The competitive equilibrium is efficient.*

Proof. See Appendix 7.3. ■

Policy functions and transitional dynamics. This section studies the policy functions of the social planner's problem (22), and the resulting process of convergence from an initial capital stock and capital unemployment rate to the steady state path, assuming that aggregate technology is constant over time.

Figure 4 shows decision rules for next-period capital stock and next-period capital unemployment rate, as a function of the two state variables: current capital stock and current capital unemployment rate.[7] In each panel, only one state variable varies on the horizontal axis, and the other state variables is fixed at a given specified value. If the current share of unemployed capital is at its steady state level, the planner's decision rules for next-period

[7]Parameter values were set to those used as priors in the quantitative analysis of Section 4. Qualitative findings described in this section are robust to all the range of values considered in Section 4.

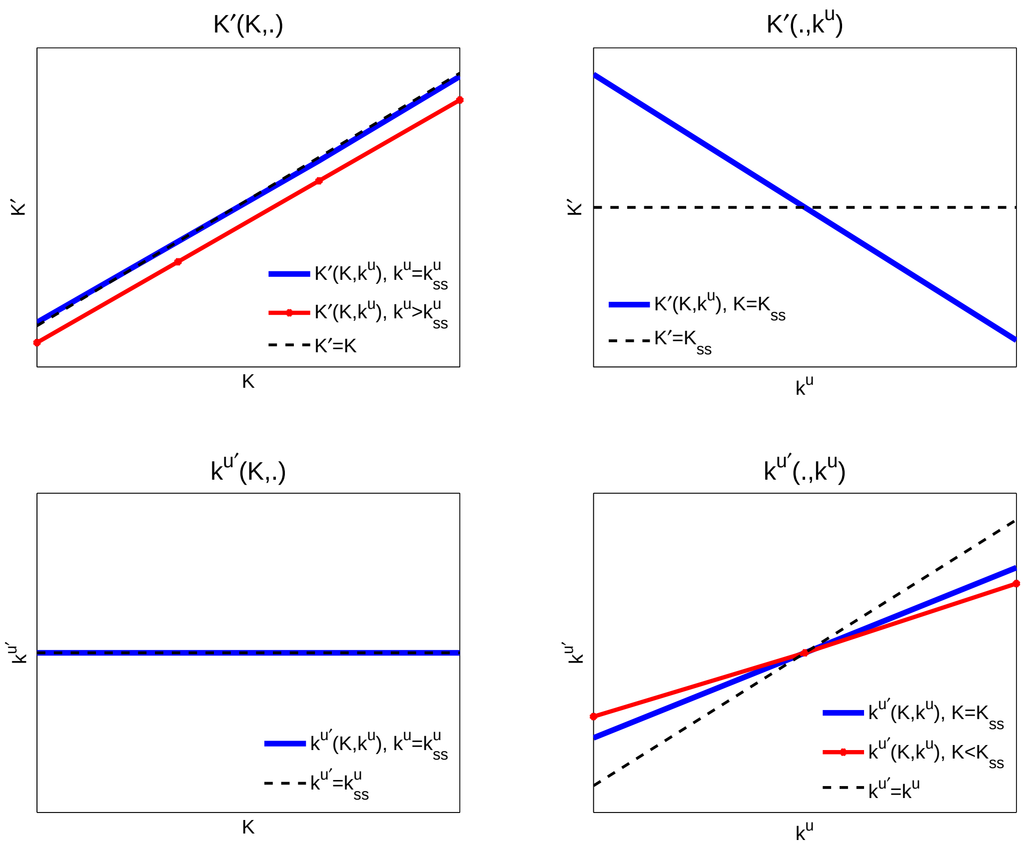

Figure 4: Policy Functions.

Note: Decision rules in the social planner's problem (22). In each panel, only one state variable varies on the horizontal axis, and the other state variables is fixed at a given specified value.

capital are similar to those of the standard neoclassical growth model: increasing the capital stock for levels of current capital stock below the steady-state, decreasing the capital stock for current values of capital above the steady state level, as depicted on the top-left panel of Figure 4. This pattern does no longer hold if the share of unemployment capital is above its steady state level. As also shown on the top-left panel of Figure 4, for a sufficiently high level of the current capital unemployment rate, next-period's optimal capital stock is below its current level even for levels of the current stock of capital below the steady state. The reason for this is that, as depicted on the top-right panel of Figure 4, next period capital stock is a decreasing function of the current share of unemployed capital. For instance, if the stock of capital is at its steady state level, but the share of unemployed capital is above its steady state level, the social planner chooses to decrease the capital stock. This is because, if share of unemployed capital is above its steady state the social planner wants to reduce next-period share of unemployed capital (see bottom-right panel of Figure 4). In the framework of the present paper, the production of new capital goods only increases the stock of unemployed capital (see equation 2). For a given level of consumption, by reducing the stock of capital, the social planner can dedicate more resources to matching, and reduce the share of unemployed capital.

As implied by the policy functions, the transitional dynamics to the steady state, starting from a stock of capital below the steady state depends on the initial share of capital unemployment. As shown in Figure 5, starting from an initial share of unemployed capital equal to the state level, the stock of capital increases monotonically, as it would in the standard neoclassical growth model. However, starting from a sufficiently high share of capital unemployment, the stock of capital first decreases, and then increases to catch-up with the steady state level. For this reason, a model featuring capital unemployment provides a reason why, the recovery from a negative shock can be characterized by an investment slump. The following task is then to study which shocks can lead to a significant increase in the capital unemployment rate. This will be studied quantitatively in Section 4.

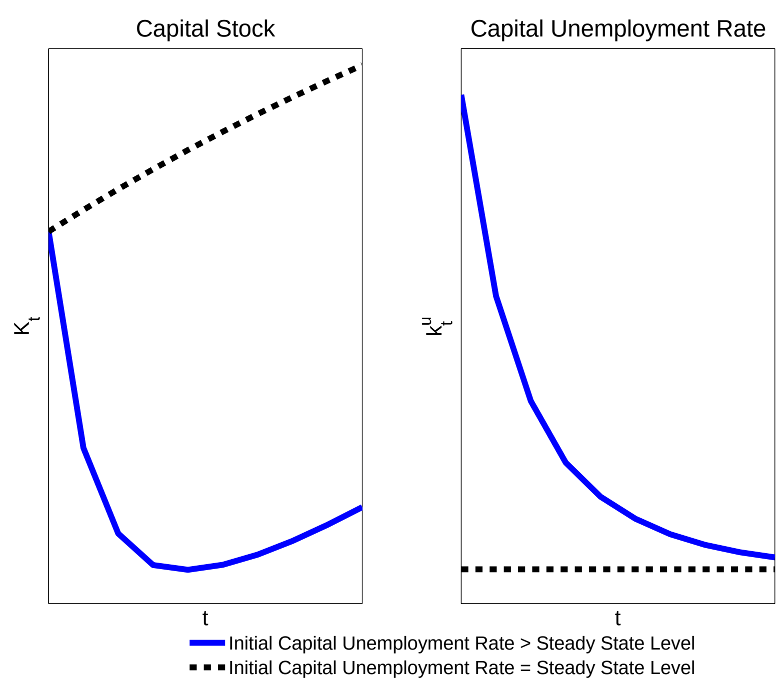

Figure 5: Transitional Dynamics and Initial Capital Unemployment Rate. *Note*: Transitional dynamics from initial capital stock (K_0) below the steady-state level for two alternative values of the initial capital unemployment rate level (k_0^{u}).

3 A Quantitative Business-Cycle Model with Investment Search Frictions

This section extends the basic framework of Section 2 to a stochastic business-cycle environment to conduct a quantitative study of the proposed mechanism. The model includes financial frictions and two shocks related to those financial frictions that have been studied in the literature as having an important role in U.S. business cycles and in the Great Recession: shocks to the cross-sectional idiosyncratic uncertainty and to the business sector's net worth (Christiano, Motto and Rostagno, 2014). The extended model also features other frictions and shocks that the literature has shown to be relevant sources of business-cycle fluctuation in the U.S. economy (see Smets and Wouters, 2007; Justiniano, Primiceri and Tambalotti, 2010, 2011; Schmitt-Grohé and Uribe, 2012). Particularly, the model incorporates investment-adjustment costs, variable capital utilization, internal habit formation in consumption, and four other structural shocks: neutral productivity, investment-specific productivity, government spending and preferences.

3.1 Environment

Goods. As in Section 2, consumption goods are perishable, and capital goods depreciate at a rate $\delta > 0$. Capital goods can be traded in either of two states: matched or unmatched. Only matched capital can be used as input in the production of consumption goods.

Agents. The economy is populated by a large number of identical households, entrepreneurs and financial intermediaries (see Figure 6). Households consume, supply labor, produce physical capital and purchase bonds issued by financial intermediaries. As in Section 2, the representative household has a continuum of infinitely lived members of measure one, with a positive fraction of them being entrepreneurs. Within each household, there is perfect consumption insurance. Entrepreneurs have access to a technology to produce consumption goods, using matched capital and labor as inputs, and to a search technology to transform unmatched capital into matched capital. Capital produced by households begins unmatched. Only entrepreneurs can store matched capital.

Unlike in Section 2, entrepreneurs cannot finance their purchases of capital with direct transfers from households. Instead, entrepreneurs purchase capital each period by borrowing

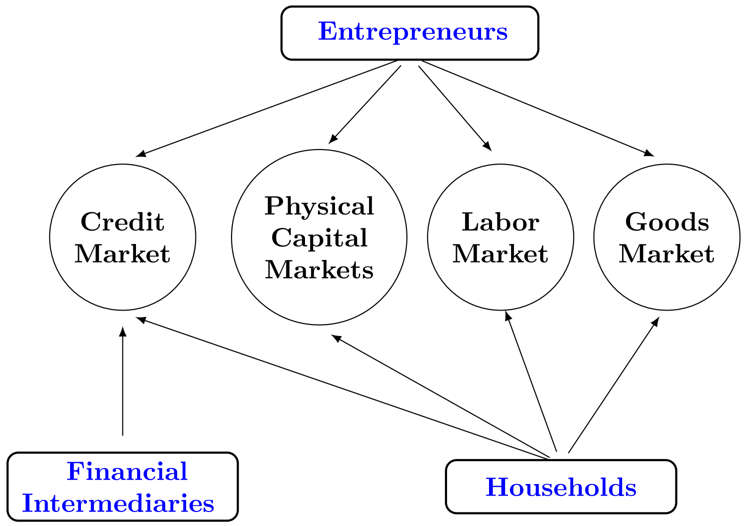

Figure 6: Agents and Markets.

from financial intermediaries and by using their own net worth. Each period, an entrepreneur has a probability $\overline{\psi} > 0$ of retiring from entrepreneurial activity. The fraction $\overline{\psi}$ of entrepreneurs that retires from entrepreneurial activity each period is replaced by a new equal mass of entrepreneurs from the households' members. New entrepreneurs start entrepreneurial activity with an exogenous and stochastic stock of net worth transferred from the households. When an entrepreneur retires, its capital becomes unmatched and is traded with households. Its net worth, after selling the unmatched capital, is transferred to its household.

An unrestricted mass of financial intermediaries can enter the economy each period. They can sell bonds to households and lend to entrepreneurs for capital-good purchases. Additionally, the economy includes a government that creates and implements fiscal policy.

Markets. The economy has four competitive markets: goods, labor, physical capital and credit (see Figure 6). The goods and labor markets are frictionless. The market for physical capital is characterized by search frictions. The credit market is characterized by frictions associated with asymmetric information in lending. Further details on the frictions that characterize the credit and physical-capital markets are provided below.

Credit market. Lending to entrepreneurs is assumed to entail an agency problem associated with asymmetric information and costly state verification (Townsend, 1979). In particular, entrepreneurs face an idiosyncratic shock whose realization is private information and can only be known by the lender through costly verification.

Following Bernanke, Gertler and Gilchrist (1999), it is assumed that the idiosyncratic shock is an i.i.d. shock to the quality of capital, denoted ω, whose realization is known by neither entrepreneurs nor financial intermediaries when lending occurs. Entrepreneurs finance the purchase of capital partly by borrowing and partly from their own net worth. The set of contracts offered to entrepreneurs (Z_{t+1}, D_{t+1}) specifies an aggregate state-contingent interest rate, Z_{t+1}, for each loan amount, D_{t+1}, to be repaid in case of no default. In the case of default, the financial intermediary seizes the entrepreneur's assets, paying a proportional recovery cost, μ_m. It is further assumed that the capital held by the entrepreneur becomes unmatched in the event of default. This form of contract implies in each period that a cutoff value exists for the realization of ω, denoted $\overline{\omega}_t$, below which entrepreneurs default. This formulation implies that all entrepreneurs choose the same level of leverage, leading to an aggregation result by which it is not necessary to track the distribution of net worth among entrepreneurs (which is particularly suitable for quantitative analysis). Each period $t+1$, the realization of ω is drawn from a distribution $F_{\omega,t}(\omega, \sigma_t)$, where σ_t is an exogenous shock to the cross-sectional dispersion of idiosyncratic shocks, introduced in Christiano, Motto and Rostagno (2014).

On the other side of the market, it is assumed that financial intermediaries obtain funds by issuing a one-period, non–state-contingent bond, which is purchased by households (similar to deposits). Financial intermediaries are diversified across idiosyncratic shocks and have free entry.

Physical capital markets. As in Section 2, trade of unmatched capital between entrepreneurs and households occurs in a decentralized market with search frictions; entrepreneurs also have access to a centralized market in which they trade matched capital at price Q^{c}.[8] In addition to the two markets considered in Section 2, this section also includes a centralized market in which unmatched capital can be traded between households, financial intermediaries and retired entrepreneurs at price J^{u}. Including this centralized market facilitates the study of financial intermediaries, who, in the event of default, seize the entrepreneur's capital (recall that the entrepreneur's capital becomes unmatched in the event of default). Figure 7 summarizes these three markets for capital, with the participants and

[8]Including a centralized market where entrepreneurs can trade employed capital is convenient for technical reasons. Particularly, this property allows the analysis to focus on an equilibrium that does not depend on the distribution of capital among entrepreneurs. In addition, the centralized market ensures that entrepreneurs' financing decisions are not influenced by market tightness.

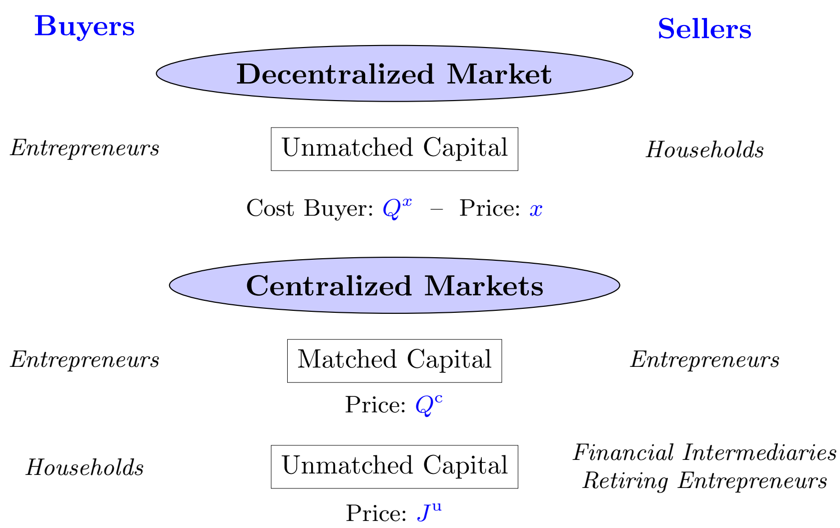

FIGURE 7: STRUCTURE OF CAPITAL MARKETS, QUANTITATIVE MODEL.

forms of trade that characterize each market.

Search frictions that characterize the decentralized market for unmatched capital are identical to those in Section 2. In particular, search is directed: The market is organized in a continuum of submarkets indexed by the price of unmatched capital, denoted x, and sellers (households) and buyers (entrepreneurs) can choose which submarket to visit. In each submarket, the market tightness, denoted $\theta(x)$ is defined as the ratio between the mass of capital searched by entrepreneurs and the mass of unemployed capital offered in that submarket. Households face no search cost. Visiting submarket x, they face a probability $p(\theta(x))$ of finding a match, where $p : \mathbb{R}_+ \to [0, 1]$ is a twice continuously differentiable, strictly increasing, strictly concave function that satisfies $p(0) = 0$ and $\lim_{\theta\to\infty} p(\theta) = 1$. Entrepreneurs face a cost per unit searched, $c_s > 0$, denoted in terms of consumption goods. Visiting submarket x, they face a probability $q(\theta(x))$ of finding a match, where $q : \mathbb{R}_+ \to [0, 1]$ is a twice continuously differentiable, strictly decreasing function that satisfies $q(\theta) = \frac{p(\theta)}{\theta}$, $q(0) = 1$ and $\lim_{\theta\to\infty} = 0$. The cost of a unit of capital for entrepreneurs in submarket x is denoted Q^x (which includes two components: the price paid to the seller, x, and the search cost in submarket x).

Timing. Time is discrete and infinite, with each period divided into six stages: production, repayment, separation, borrowing, search and investment. In the *production stage,* entrepreneurs produce consumption goods using capital matched in the previous period. In the *repayment stage,* entrepreneurs repay their loans from the previous period or default; in case of default their capital becomes unmatched and financial intermediaries monitor and seize the entrepreneur's production and capital. In the *separation stage,* a fraction $\overline{\psi}$ of entrepreneurs that have not defaulted retires and their capital becomes unmatched. A new mass of entrepreneurs begins entrepreneurial activity with no initial capital and with an exogenously determined net worth. In the *borrowing stage,* entrepreneurs who do not retire and new entrepreneurs borrow from financial intermediaries, and financial intermediaries sell bonds to households. In the *search stage,* the remaining entrepreneurs purchase unmatched capital from households and matched capital from other entrepreneurs. In the *investment stage,* households produce physical capital and consume; retired entrepreneurs transfer their net worth, including unmatched capital, to their households; and financial intermediaries sell seized unmatched capital to households.

3.2 Households

Household preferences are described by the lifetime utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t \{U\left(C_t - \rho_{\mathrm{c}} C_{t-1}\right) - V(h_t; \varphi_t)\}, \tag{23}$$

where C_t denotes household consumption in period t, h_t denotes hours worked by the household in period t; $\beta \in (0,1)$ is the subjective discount factor; $\rho_{\mathrm{c}} \in [0,1)$ is a parameter governing the degree of internal habit formation; φ_t denotes an exogenous and stochastic preference shock in period t (labeled a *labor-wedge shock*); for every realization of φ_t, $V(\cdot;\varphi_t) : \mathbb{R}_+ \to \mathbb{R}$ is a twice continuously differentiable, strictly increasing, strictly convex function; $U : \mathbb{R}_+ \to \mathbb{R}$ is a twice continuously differentiable, strictly increasing, strictly concave function; and E_t denotes the expectation conditional on the information set available at time t.

The stock of unemployed capital, held by households, evolves according to

$$K^{\mathrm{u}}_{t+1} = \int_0^{(1-\delta)K^{\mathrm{u}}_t} \left(1 - p(\theta_t(x_{i,t}))\right) \mathrm{d}i + \iota^{\mathrm{h}}_t + A^{\mathrm{I}}_t I_t \left[1 - \Phi\left(I_t, I_{t-1}\right)\right], \tag{24}$$

where K^{u}_t denotes the stock of unemployed capital at the beginning of period t, $x_{i,t}$ denotes the submarket in which unemployed capital unit i is listed in period t, ι^{h}_t denotes the units of unmatched capital purchased by households in the centralized market in period t, I_t denotes the households' investment in period t, A_t^{I} denotes an exogenous aggregate shock that affects the production of capital from investment goods in period t (as in Justiniano, Primiceri and Tambalotti, 2011, labeled an *investment-specific technology shock*), and $\Phi : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is a function that introduces convex investment-adjustment costs of the form proposed by Christiano, Eichenbaum and Evans (2005). The first term of the right-hand side of equation (24) represents the depreciated mass of capital unemployed at the beginning of period t and not sold to entrepreneurs for a given market-tightness function $\theta_t(x)$ and choices of submarket $x_{i,t}$. The second term of the right-hand side of equation (24) represents the mass of employed capital purchased by the households from retired and defaulting entrepreneurs. The third term represents the addition (subtraction) to unemployed capital stock from investment, net of adjustment costs.

Households have access to a one-period, non–state-contingent bond issued by financial intermediaries. The household's sequential budget constraint is given by

$$C_t + I_t + J_t^{\mathrm{u}} \iota_t^{\mathrm{h}} + T_t + B_t \quad = \quad R_{t-1} B_{t-1} + W_t h_t + \int_0^{(1-\delta) K_t^{\mathrm{u}}} p(\theta_t(x_{i,t})) x_{i,t} \, \mathrm{d}i + \Pi_t, \quad (25)$$

where B_t denotes the one-period bond holdings chosen by households at the beginning of period t, which pays a gross non–state-contingent interest rate, R_t; W_t denotes the wage rate; Π_t denotes net transfers from entrepreneurs and financial intermediaries to households in period t – described further in the next sections; and T_t represents a lump-sum government tax (subsidy) in period t.

The *household's problem* is then to choose the state-contingent sequences of C_t, h_t, I_t, ι_t^{h}, K_{t+1}^{u}, B_t and $x_{i,t}$ that maximize the expected utility (23), subject to the sequence of budget constraints (25), the accumulation constraints for unemployed capital (24), for the given initial levels of capital, investment, and consumption, K_0^{u}, K_0^{e}, I_{-1}, and C_{-1}, the given sequence of prices, W_t, J_t^{u} and R_t, the given sequence of dividends and taxes, Π_t and T_t, the given sequence of market-tightness functions, $\theta_t(x)$, and the given sequence of labor wedges, φ_t and investment-specific productivities, A_t^{I}. Denoting by Λ_t the Lagrange multiplier associated with the budget constraint (25), the optimality conditions in an interior solution are (25), (24), and the first-order conditions:

$$\Lambda_t = U'(C_t - \rho_c C_{t-1}) - \beta \rho_c E_t U'(C_{t+1} - \rho_c C_t), \tag{26}$$

$$\Lambda_t J^{u}_t = \beta \Lambda_{t+1}(1-\delta)\left[p(\theta_{t+1}(x^{u}_{t+1}))x^{u}_{t+1} - (1 - p(\theta_{t+1}(x^{u}_{t+1})))J^{u}_{t+1}\right], \tag{27}$$

$$\begin{aligned}\Lambda_t = {} & \Lambda_t J^{u}_t A^{I}_t \left[1 - \Phi\left(I_t, I_{t-1}\right) - I_t \Phi_1\left(I_t, I_{t-1}\right)\right] \\ & + \beta E_t \Lambda_{t+1} J^{u}_{t+1} A^{I}_{t+1} I_{t+1} \Phi_2\left(I_{t+1}, I_t\right),\end{aligned} \tag{28}$$

$$\Lambda_t = \beta R_t E_t \Lambda_{t+1}, \tag{29}$$

$$-p(\theta(x^{u}_t)) = p'(\theta_t(x^{u}_t))\theta'_t(x^{u}_t)(x^{u}_t - J^{u}_t), \tag{30}$$

$$V'\left(h_t; \varphi_t\right) = \Lambda_t W_t. \tag{31}$$

where x^{u}_t denotes the household's choice of submarket for unmatched capital in period t, and the unit of capital subindex, i, has been dropped because the optimality condition with respect to the choice of submarket, $x_{i,t}$, is the same for all units of capital.

3.3 Financial Intermediaries

Financial intermediaries sell one-period non–state-contingent bonds to households and lend to entrepreneurs. The set of contracts offered to entrepreneur j specifies an aggregate, state-contingent interest rate, $Z_{j,t+1}$, for each loan amount, $D_{j,t+1}$, to be repaid in case of no default. In case of default, the financial intermediary seizes the entrepreneur's assets, with a recovery value of $\mathcal{R}_{t+1}(\omega, K^{e}_{j,t+1})$, and pays a proportional monitoring cost, μ_m. Debt schedules, $\mathcal{D}(K^{e}_{j,t+1})$, available for entrepreneur j include all contracts $(Z_{j,t+1}, D_{j,t+1})$ that allow a financial intermediary to repay in all states the risk-free bond sold to households, after diversifying idiosyncratic risk:[9]

$$D_{t+1}R_t = [1 - F_\omega(\overline{\omega}_{j,t+1}; \sigma_t)]Z_{j,t+1}D_{j,t+1} + (1-\mu_m)\int_0^{\overline{\omega}_{j,t+1}} \mathcal{R}_{t+1}(\omega, K^{e}_{t+1})\, \mathrm{d}F_\omega\left(\omega; \sigma_t\right), \tag{32}$$

where $\overline{\omega}_{j,t+1}$ denotes the default threshold in period $t+1$ for entrepreneur j with outstanding debt $D_{j,t+1}$ and stock of matched capital $K^{e}_{j,t+1}$ – to be discussed in detail in the next section. The left-hand side of equation (32) represents the obligations assumed by the financial intermediary selling the risk-free bond to households. The right-hand side of equation (32)

[9]For formulations of debt contracts similar to the one presented in this section, see Arellano, Bai and Zhang (2012) and Christiano, Motto and Rostagno (2014).

represents the resources obtained by the financial intermediary from lending, after diversifying over idiosyncratic risk. It includes two terms representing resources from entrepreneurs who do not default and resources from those who do.

It is assumed that financial intermediaries monitor and so can seize the entrepreneur's production and capital in the default state. Hence,

$$\mathcal{R}_{t+1}(\omega, K^{\mathrm{e}}_{j,t+1}) \;\; = \;\; [r^k_{j,t+1} + (1-\delta)J^{\mathrm{u}}_{t+1}]\omega K^{\mathrm{e}}_{j,t+1}, \tag{33}$$

where $r^k_{j,t+1}$ denotes the net revenues from production per unit of effective capital, $\omega K^{\mathrm{e}}_{j,t+1}$ – to be described in detail in the next section. The second term of the right-hand side of (33) represents the value of selling the depreciated entrepreneur's effective capital in the market of unmatched capital.

3.4 Entrepreneurs

Entrepreneurs have access to technology to produce consumption goods using labor and matched capital as inputs. In particular, the output produced by an effective unit of matched capital i, employing $\tilde{h}_{it}$ hours of work, is given by[10]

$$y_{i,t} \;\; = \;\; A_t \left(\tilde{h}_{i,t}\right)^{1-\alpha}, \tag{34}$$

where $y_{i,t}$ denotes output in units of matched capital i in period t and A_t is an exogenous aggregate productivity shock affecting the production technology in period t (labeled the *neutral technology shock*).

Each period, entrepreneurs face an i.i.d. shock to the quality of their matched capital, denoted ω, drawn from a log-normal distribution with c.d.f. $F_\omega(\omega;\sigma_t)$ and satisfying $E_t(\omega_{t+1}) = 1 \; \forall \; t$ and $\mathrm{Var}_t(\log(\omega_{t+1})) = \sigma_t^2 \; \forall \; t$, where σ_t is an exogenous aggregate shock to the cross-sectional dispersion of idiosyncratic shocks (labeled the *risk shocks*, as in Christiano, Motto and Rostagno, 2014). Output produced by entrepreneur j with a mass of matched capital $K^{\mathrm{e}}_{j,t}$ – with $\tilde{h}_{j,t}$ hours worked in each of these units of capital – is then given by

$$Y_{j,t} \;\; = \;\; A_t \left(\tilde{h}_{j,t}\right)^{1-\alpha} \omega_{j,t} K^{\mathrm{e}}_{j,t}, \tag{35}$$

[10] This production technology is similar to one in which production is carried out in a continuum of plants, as, for example, in Cooley, Hansen and Prescott (1995). In this framework it can be shown that the aggregate production function of the economy displays constant returns to scale.

where $Y_{j,t}$ denotes output produced by entrepreneur j in period t, and $\omega_{j,t}$ denotes the realization of the exogenous and stochastic variable ω for entrepreneur j in period t. The term $\omega_{j,t}K^{\mathrm{e}}_{j,t}$ denotes the effective mass of matched capital held by entrepreneur j at the beginning of period t.

Entrepreneurs pay wage rate W_t per hour worked and face convex costs on the utilization rate. It follows that net revenues from production per unit of effective matched capital for entrepreneur j are given by

$$r^k_t \quad = \quad \left(A_t\tilde{h}_t\right)^{1-\alpha} - W_t\tilde{h}_t, \tag{36}$$

Note that the subindex j has been dropped from equation (36) since r^k_t is independent of the mass of matched capital held by entrepreneur j, $K^{\mathrm{e}}_{j,t}$, and independent of the realization of the idiosyncratic shock for entrepreneur j, $\omega_{j,t}$.

In this setup, entrepreneurs face an expected linear rate of return per unit of capital purchased:

$$R^{k,m}_{t+1} \quad \equiv \quad \frac{r^k_{t+1} + (1-\delta)\left[\overline{\psi}J^{\mathrm{u}}_{t+1} + \left(1-\overline{\psi}\right)Q^{\mathrm{c}}_{t+1}\right]}{Q^m_t}, \tag{37}$$

for $m \in \{x, c\}$. The denominator of the right-hand side of (37) represents the price at which the effective unit of matched capital was purchased. The numerator of the right-hand side of (37) represents the sources of revenue per unit of effective matched capital. The first component of the numerator represents net revenue from production. The second component represents the expected revenue from selling the depreciated unit of effective matched capital. If the entrepreneur retires (with probability $\overline{\psi}$), this effective unit of matched capital is traded unmatched at a price J^{u}_{t+1}. If the entrepreneur does not retire (with probability $1-\overline{\psi}$), this effective unit of matched capital is traded matched at Q^{c}_{t+1}.

Entrepreneurs purchase capital using their net worth and borrowing from financial intermediaries. This means that, at the end of each period t, the following equation, describing the entrepreneur's balance sheet, holds for any entrepreneur j

$$\int_x Q^x_t\tilde{K}^x_{j,t+1}\,\mathrm{d}x + Q^c_t\tilde{K}^c_{j,t+1} \quad = \quad D_{j,t+1} + N_{j,t+1}, \tag{38}$$

where $D_{j,t+1} \geq 0$ denotes debt contracted by entrepreneur j in period t, to be paid in period

$t+1$, $N_{j,t+1} \geq 0$ denotes the net worth of entrepreneur j at the end of period t, $\tilde{K}^{x}_{t+1} \geq 0$ denotes the stock of capital held by entrepreneur j at the end of period t, purchased in submarket x of the decentralized market at a cost per unit Q^{x}_{t}, and $\tilde{K}^{c}_{j,t+1} \geq 0$ denotes the stock of capital held by entrepreneur j at the end of period t purchased in the centralized market at a cost Q^{c}_{t} per unit. The latter case also includes the stock of capital held by entrepreneur j from the previous period, which is equivalent to selling and repurchasing the unit in the centralized market at price Q^{c}_{t}. Note that $\int_x \tilde{K}^{x}_{j,t+1}\,\mathrm{d}x + \tilde{K}^{c}_{j,t+1} = K^{\mathrm{e}}_{j,t+1}$. The left-hand side of equation (38) represents the entrepreneur's assets, given by the value of the matched capital. The right-hand side of equation (38) represents the entrepreneur's liabilities and equity, given by debt with financial intermediaries and net worth.

As in Section 2, by the law of large numbers, the cost per unit of capital, of mass $\tilde{K}^{x}_{t+1}$ purchased in the submarket x of the decentralized market is given by

$$Q^{x}_{t} \quad = \quad x + \frac{c_{\mathrm{s}}}{q(\theta_t(x))}. \tag{39}$$

The right side of equation (39) represents the two components of the cost of a unit of capital in the decentralized market: the price paid to the seller, x, and the search cost, $\frac{c_{\mathrm{s}}}{q(\theta_t(x))}$.

To solve the entrepreneur's problem, it is useful to define the entrepreneur's leverage and "portfolio weights," from the components of the entrepreneurs balance sheet (38). The entrepreneur's leverage in period t is defined as

$$L_{j,t} \quad \equiv \quad \frac{\int_x Q^{x}_{t}\tilde{K}^{x}_{j,t+1}\,\mathrm{d}x + Q^{c}_{t}\tilde{K}^{c}_{j,t+1}}{N_{j,t+1}}. \tag{40}$$

The portfolio weight of each asset considered in the left side of equation (38) is given by

$$w^{m}_{j,t} \quad \equiv \quad \frac{Q^{m}_{t}\tilde{K}^{m}_{j,t+1}}{L_{j,t}N_{j,t+1}}, \tag{41}$$

for $m \in \{x, c\}$. From (38) and the nonnegativity constraint of capital holdings ($\tilde{K}^{x}_{j,t+1} \geq 0$ for $m \in \{x, c\}$), it follows that $w^{m}_{j,t} \in [0, 1]\ \forall m \in \{x, c\}$.

Following the quantitative literature implementing the costly state-verification framework, it is assumed that entrepreneurs are risk neutral and that their objective is to maximize their expected net worth, given at the end of period t by[11]

$$E_t \left\{ \int_{\overline{\omega}_{j,t+1}}^{\infty} \left[\omega R^k_{j,t+1} L_{j,t} N_{j,t+1} - Z_{j,t+1} D_{j,t+1} \right] \mathrm{d}F_\omega(\omega;\sigma_t) \right\}, \tag{42}$$

where the portfolio return, denoted $R^k_{j,t+1}$, is defined by $R^k_{j,t+1} \equiv \int_x w^x_{j,t} R^{k,x}_{t+1}\,\mathrm{d}x + w^c_{j,t} R^{k,c}_{t+1}$, and $(Z_{j,t+1}, D_{j,t+1}) \in \mathcal{D}(K^e_{j,t+1})$, as defined in (32). The first term in the objective function (42) represents the revenues that will be received in period $t+1$ by entrepreneur j. The second term represents debt repayments to financial intermediaries. Given that the entrepreneur receives revenues and performs debt repayment only in case of not defaulting, these terms are integrated over the realizations of $\omega_{j,t}$ above $\overline{\omega}_{j,t+1}$.

From the objective function (42), it follows that the expected value for entrepreneur j of repaying debt $D_{j,t+1}$ in the repayment stage of period $t+1$ is given by

$$V^R_{j,t+1} \quad = \quad \omega_{j,t+1} R^k_{j,t+1} L_{j,t} N_{j,t+1} - Z_{j,t+1} D_{j,t+1}. \tag{43}$$

Given that the expected value of defaulting is equal to zero, equation (43) implies that the optimal default threshold, $\overline{\omega}_{j,t+1}$, is implicitly defined by

$$\overline{\omega}_{j,t+1} R^k_{j,t+1} L_{j,t} N_{j,t+1} = Z_{j,t+1} D_{j,t+1}. \tag{44}$$

Using (40) and (44) in (42), entrepreneur j's objective function can be reexpressed as

$$E_t \left\{ \left[\int_{\overline{\omega}_{t+1}}^{\infty} \omega \,\mathrm{d}F_\omega(\omega;\sigma_t) - \left(1 - F_\omega(\overline{\omega}_{j,t+1};\sigma_t)\right) \overline{\omega}_{j,t+1} \right] R^k_{j,t+1} L_{j,t} N_{j,t+1} \right\}, \tag{45}$$

which is proportional to net worth $N_{j,t+1}$.

Similarly, substituting (44) and (40) into (32) and (33), the financial intermediaries' participation constraint is

$$\frac{L_{j,t}-1}{L_{j,t}} R_t = \left[1 - F_\omega\left(\overline{\omega}_{j,t+1};\sigma_t\right)\right] \overline{\omega}_{j,t+1} R^k_{j,t+1} + (1-\mu_m) \int_0^{\omega_{j,t+1}} \omega \,\mathrm{d}F_\omega\left(\omega;\sigma_t\right) R^{k,\psi}_{j,t+1}, \tag{46}$$

where the portfolio return conditional on separation is defined by $R^{k,\psi}_{j,t+1} \equiv \int_x w^x_{j,t} R^{k,\psi,x}_{t+1}\,\mathrm{d}x + w^c_{j,t} R^{k,\psi,c}_{t+1}$, and $R^{k,\psi,m}_{t+1}$ denotes the return of an effective unit of separated capital, which,

[11] For a recent study relaxing this and other assumptions of the standard implementation of costly state verification used in this paper, see Dmitriev and Hoddenbagh (2013).

similar to (37), is defined by

$$R^{k,\psi,m}_{j,t+1} \equiv \frac{r^k_{t+1} + (1-\delta)J^{\mathrm{u}}_{t+1}}{Q^m_t}, \tag{47}$$

for $m \in \{x, c\}$. The combinations $(\overline{\omega}_{j,t+1}, L_{j,t})$ that satisfy (46) define a menu of $(t+1)$-contingent debt contracts offered to entrepreneurs equivalent to those in the set $\mathcal{D}(K^{\mathrm{e}}_{j,t+1})$ defined in (32).

Entrepreneur j's problem is to choose the state-contingent plans $\tilde{h}_{j,t}$, $L_{j,t}$ and $\omega_{j,t+1}$ and $w^m_{j,t}$ (for $m \in \{x, c\}$) that maximize the expected net worth (45) subject to the financial intermediaries' participation constraint (46) and the sequence of technological constraints, (36), return constraints, (37) and (47), and nonnegativity constraint for portfolio weights ($w^m_{j,t} \geq 0$ for $m \in \{x, c\}$) for the given sequence of prices, $W_t, Q^{\mathrm{c}}_t, J^{\mathrm{u}}_t$, market-tightness functions, $\theta_t(x)$, risk, σ_t, and neutral-technology shocks, A_t. With $\Lambda^e_{j,t+1}$ as the Lagrange multiplier on the financial intermediary's participation constraint, and $\Xi^m_{j,t}$ as the Lagrange multiplier associated with nonnegativity constraint for portfolio weights ($w^m_{j,t} \geq 0$), the optimality conditions are (36), (37), (46), (47), and

$$A_t(1-\alpha)\left(\tilde{h}_t\right)^{-\alpha} = W_t, \tag{48}$$

$$E_t\left\{[1-\Gamma_t(\overline{\omega}_{t+1})]\frac{R^k_{t+1}}{R_t} = -\Lambda^e_{t+1}\left[\frac{R^k_{t+1}}{R_t}\left(\Gamma_t(\overline{\omega}_{t+1}) - g_t(\overline{\omega}_{t+1}) + (1-\mu_m)g_t(\overline{\omega}_{t+1})\frac{R^{k,\psi}_{t+1}}{R^k_{t+1}}\right) - 1\right]\right\}, \tag{49}$$

$$\Lambda^e_{t+1} = \frac{\Gamma'_t(\overline{\omega}_{t+1})}{\Gamma'_t(\overline{\omega}_{t+1}) - \mu g'_t(\overline{\omega}_{t+1}) + (1-\mu_m)g'_t(\overline{\omega}_{t+1})\left(\frac{R^{k,\psi}_{t+1}}{R^k_{t+1}} - 1\right)}, \tag{50}$$

$$Q^m_t = Q_t + \Xi^m_t, \tag{51}$$

and the complementary slackness conditions

$$\Xi^m_t \geq 0, \quad w^m_t \Xi^m_t = 0, \tag{52}$$

for $m \in \{x, c\}$, where $\Gamma_t(\overline{\omega}_{t+1}) \equiv [1 - F_\omega(\overline{\omega}_{t+1}; \sigma_t)]\overline{\omega}_{t+1} + g_t(\overline{\omega}_{t+1})$, $g_t(\overline{\omega}_{t+1}) \equiv \int_0^{\overline{\omega}_{t+1}} \omega \, \mathrm{d}F_\omega(\omega; \sigma_t)$, and $Q_t \equiv \frac{r^k_{t+1} + (1-\delta)\left[\overline{\psi}J^{\mathrm{u}}_{t+1} + \left(1-\overline{\psi}\right)Q^{\mathrm{c}}_{t+1}\right]}{R^k_{t+1}}$. The entrepreneur's subindex, j, has been dropped because the objective function is linear in the net worth of entrepreneur j and does not appear in any of the constraints. Therefore, all entrepreneurs will choose the same plans h_t, L_t and ω_{t+1}, independent of net worth.

3.5 Government

The government is assumed to consume a stochastic amount of consumption goods, financed each period by levying lump-sum taxes on households. The government budget constraint is given by

$$G_t = T_t, \tag{53}$$

where G_t is government spending in period t (labeled the *government spending shock*).

3.6 Equilibrium

In equilibrium, centralized markets must clear. For the centralized market of unmatched capital, equilibrium then requires that

$$\iota_t^{\mathrm{h}} = \psi_t(1-\delta)K_t^{\mathrm{e}}, \tag{54}$$

where $\psi_t \equiv (1 - g_{t-1}(\overline{\omega}_t))\overline{\psi} + g_{t-1}(\overline{\omega}_t)$ denotes the total share of employed capital that was separated in period t as a result of entrepreneurs' retirement and default. The left-hand side of (54) represents households' purchases in the market of unmatched capital. The left-hand side of (54) represents the mass of capital sold in the market for unmatched capital, from retired entrepreneurs and financial intermediaries that seized capital of defaulting entrepreneurs (see Figure 7).

Replacing (54) in (24) and using the law of large numbers and the fact that the choice of submarket, $x_{i,t}$, is the same for all units of capital i, we obtain a law of motion for unemployed capital:

$$K_{t+1}^{\mathrm{u}} = (1 - p(\theta_t(x_t^{\mathrm{u}})))(1-\delta)K_t^{\mathrm{u}} + \psi_t(1-\delta)K_t^{\mathrm{e}} + A_t^{\mathrm{I}} I_t \left[1 - \Phi\left(I_t, I_{t-1}\right)\right]. \tag{55}$$

Given that matched capital is homogeneous, arbitrage between centralized and decentralized markets of matched capital requires $Q_t = Q_t^{\mathrm{c}}$. Moreover, entrepreneurs' optimality conditions (51) and (52) imply that, in equilibrium, any submarket visited by a positive number of entrepreneurs must have the same cost per unit of capital, and entrepreneurs will be indifferent among them. Formally, for all x,

$$\theta_t(x)\left(x + \frac{c_{\mathrm{s}}}{q\left(\theta_t(x)\right)} - Q_t\right) \quad = \quad 0. \tag{56}$$

This condition determines the equilibrium market-tightness function: For all $x < Q_t$,

$$\theta_t(x) \quad = \quad q^{-1}\left(\frac{c_{\mathrm{s}}}{Q_t - x}\right). \tag{57}$$

For all $x \geq Q_t$, $\theta_t(x) = 0$.

Using the definition of market tightness, the law of large numbers, and the fact that a household's choice of submarket, $x_{i,t}$ is the same for all units of capital i, in equilibrium, the flow of capital that transitions from unemployment is given by $p(\theta_t(x_t^{\mathrm{u}}))(1-\delta)K_t^{\mathrm{u}} = \int_j \tilde{K}_{j,t+1}^{x^{\mathrm{u}}}\,\mathrm{d}j = \int_j \int_x \tilde{K}_{j,t+1}^{x}\,\mathrm{d}j\,\mathrm{d}x$. Aggregating the entrepreneurs' capital-accumulation constraints and imposing market clearing in the centralized market provides a law of motion for employed capital:

$$K_{t+1}^{\mathrm{e}} \quad = \quad (1-\psi_t)(1-\delta)K_t^{\mathrm{e}} + p(\theta_t(x_t^{\mathrm{u}}))(1-\delta)K_t^{\mathrm{u}}. \tag{58}$$

The capital unemployment rate at the beginning of period t can be then defined as

$$k_t^{\mathrm{u}} \quad \equiv \quad \frac{K_t^{\mathrm{u}}}{K_t}, \tag{59}$$

where $K_t \equiv K_t^{\mathrm{e}} + K_t^{\mathrm{u}}$ denotes total aggregate capital stock at the beginning of period t.

Labor-market clearing requires $\int_j \tilde{h}_{j,t}\omega_{j,t}K_{j,t}^{\mathrm{e}} = h_t$. Aggregating production functions (10) across entrepreneurs, using the fact that all entrepreneurs choose the same level of hours worked and utilization for each unit of effective capital, imposing the labor-market clearing condition yields the aggregate output,

$$Y_t \quad = \quad A_t(K_t^{\mathrm{e}})^{\alpha}\,(h_t)^{(1-\alpha)}, \tag{60}$$

where Y_t denotes aggregate output in period t.

Let ζ_t denote the exogenous aggregate transfer from households to new entrepreneurs in period t (labeled the *equity shock*). Aggregate net worth then evolves following the law of motion,

$$N_{t+1} \quad = \quad (1-\overline{\psi})[1-\Gamma_{t-1}(\overline{\omega}_t)]R_t^{k,c}Q_{t-1}K_t^{\mathrm{e}} + \zeta_t, \tag{61}$$

where N_{t+1} denotes aggregate net worth at the end of period t, and $R_t^{k,c}$ denotes the return of an effective unit of capital that does not separate in period t, which, similar to (37) and (47), is defined by $R_t^{k,c} \equiv \frac{r_t^k+(1-\delta)Q_t^{\mathrm{c}}}{Q_{t-1}}$. The first term on the right-hand side of (61) is represents the aggregate return obtained from effective matched capital employed in period t by entrepreneurs who did not default in the default stage and did not retire in the separation stage. The second term on the right-hand side of (61) represents the exogenous aggregate transfer from households to new entrepreneurs. The return obtained from effective matched capital employed in period t by entrepreneurs who did not default in the default stage, but did retire in the separation stage is transferred to households. It follows that the net transfer from entrepreneurs to households is given by

$$\Pi_t \quad = \quad \overline{\psi}[1-\Gamma_{t-1}(\overline{\omega}_t)]R_t^{k,\psi}Q_{t-1}K_t^{\mathrm{e}} - \zeta_t. \tag{62}$$

Starting from the households' budget constraint (25) and replacing the government budget constraint (53), the market clearing condition for unmatched capital (54), the market clearing condition for the credit and labor markets, the definition of net revenues from production (36) and the participation constraints of financial intermediaries (32) aggregated across entrepreneurs, the expression for aggregate transfers from entrepreneurs (62) yields the economy's resource constraint,

$$C_t + I_t + G_t \quad = \quad Y_t - c_{\mathrm{s}}\theta_t(1-\delta)K_t^{\mathrm{u}} - \Omega_t, \tag{63}$$

where $\Omega_t \equiv \mu g_{t-1}(\overline{\omega}_t)R_{k,t}^{\psi}Q_{k,t-1}K_t^{\mathrm{e}}$.

4 Quantitative Analysis

This section conducts a quantitative study of the role of search frictions in investment based on the model presented in Section 3. It begins by specifying assumptions for functional forms and stochastic processes contained in the model. It then discusses the empirical methodology for calibration and estimation of the model's parameters for the U.S. economy, presents estimation results, and conducts exercises based on the estimation related to the U.S. Great Recession and business cycles.

4.1 Model Estimation

Functional forms. The assumptions made on functional forms are standard in the related literature. For the households' period utility function,

$$\begin{aligned} U(c) &= \frac{c^{1-\upsilon}}{1-\upsilon} \\ V(h;\varphi) &= \varphi\frac{h^{1+\frac{1}{\phi}}}{1+\frac{1}{\phi}}, \end{aligned}$$

where $\upsilon > 0$ is the inverse of the intertemporal elasticity of substitution and $\phi > 0$ is the Frisch elasticity of labor supply.

Investment-adjustment costs are assumed take a quadratic form:

$$\Phi(I_t, I_{t-1}) = \frac{\kappa}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2,$$

where $\kappa > 0$ is a parameter governing the degree of investment-adjustment costs.

The matching function is assumed to take a CES function, yielding two finding probabilities:

$$\begin{aligned} p(\theta) &= \theta\left(1+\theta^{\xi}\right)^{-1/\xi}, \\ q(\theta) &= \left(1+\theta^{\xi}\right)^{-1/\xi}, \end{aligned}$$

where $\xi > 0$. This functional form has been used in quantitative studies of directed search in the labor market (see, for example, Schaal, 2012).

Stochastic processes. The six aggregate shocks are modeled as first-order autoregressive processes:

$$
\begin{aligned}
\log A_t &= \rho_A \log A_{t-1} + \epsilon_t^A, \\
\log A_t^{\mathrm{I}} &= \rho_{A^{\mathrm{I}}} \log A_{t-1}^{\mathrm{I}} + \epsilon_t^{\mathrm{I}}, \\
\log G_t &= (1-\rho_G)\log \overline{G} + \rho_G \log G_{t-1} + \epsilon_t^G, \\
\log \varphi_t &= (1-\rho_\varphi)\log \overline{\varphi} + \rho_\varphi \log \varphi_{t-1} + \epsilon_t^\varphi, \\
\log \sigma_t &= (1-\rho_\sigma)\log \overline{\sigma} + \rho_\sigma \log \sigma_{t-1} + \epsilon_t^\sigma, \\
\zeta_t &= (1-\rho_\zeta)\overline{\zeta} + \rho_\zeta \zeta_{t-1} + \epsilon_t^\zeta,
\end{aligned}
$$

where $\overline{G} > 0$ denotes steady-state government spending, $\overline{\varphi} > 0$ is a parameter that determines steady-state hours worked, $\overline{\sigma} > 0$ denotes the steady-state cross-sectional dispersion of idiosyncratic shocks, $\overline{\zeta}$ denotes steady-state lump-sum transfers from households to entrepreneurs, and it is assumed that $\epsilon_t^i \sim N(0, \sigma^i)\ \forall\ t$ and $i \in \{A, A^{\mathrm{I}}, G, \varphi, \sigma, \zeta\}$.

Data. The model is estimated using U.S. quarterly data prior to the Great Recession, from 1980:Q1 to 2007:Q4.[12] The data include six time series: real per capita GDP, real per capita consumption, real per capita nonresidential private investment, per capita hours worked, credit spreads and commercial, nonresidential real estate vacancy rates. Data on GDP, consumption and investment were linearly detrended. Credit spreads were measured by the difference between the interest rate on BAA corporate bonds and the three-month U.S. government bond rate. Appendix 7.1 provides more detailed information about the sources and construction of these data.

Including data on GDP, consumption, investment, and hours is standard in the empirical business-cycle literature. Including credit spreads is relevant to discipline the financial friction and financial shocks (see Christiano, Motto and Rostagno, 2014). The counterpart of this variable in the model is the difference between the interest rate paid by entrepreneurs, Z_t, and the risk-free rate R_t. Including data on the commercial-real-estate vacancy rate (see Figure 2) is a novel feature of the present paper and is aimed at disciplining the search friction in investment – specifically, the two parameters related to search frictions, the curvature of the matching function, ξ, and the search cost, c_{s}. The counterpart of this variable in the model

[12]The estimation period begins in 1980 due to the availability of commercial-real-estate vacancy rates.

is the capital unemployment rate, k_t^{u}. Section 4.3 studies an alternative strategy of mapping these data on vacancy rates only to capital structures in the model.

It is assumed that all series are observed with measurement error. Measurement error in output, consumption, investment, hours worked, credit spreads and vacancy rates, denoted $\epsilon_{Y,t}^{\mathrm{me}}$, $\epsilon_{C,t}^{\mathrm{me}}$, $\epsilon_{I,t}^{\mathrm{me}}$, $\epsilon_{h,t}^{\mathrm{me}}$, $\epsilon_{s,t}^{\mathrm{me}}$ and $\epsilon_{k^{\mathrm{u}},t}^{\mathrm{me}}$, are assumed to be i.i.d. innovations with mean zero and standard deviation $\sigma_i^{\mathrm{me}}\ \forall\, i \in \{Y, C, I, h, s, k^{\mathrm{u}}\}$.

Empirical strategy. From the assumed functional forms and stochastic processes in the previous sections, the model features 27 structural parameters. Let Θ be a vector containing all the parameters of the model. We include in this vector also the six nonstructural parameters representing the standard deviations of the measurement errors on the observables, as discussed in the previous section. The model parameters are partitioned into two sets: $\Theta = [\Theta_1, \Theta_2]$. The first set,

$$\Theta_1 \equiv [\beta, \upsilon, \phi, \alpha, \delta, \overline{\psi}, \mu_m, \overline{G}, \overline{\varphi}, \overline{\sigma}],$$

contains 10 calibrated or fixed a priori parameters. The remaining 23 parameters,

$$\Theta_2 \equiv [\rho_{\mathrm{c}}, \kappa, c_u, \xi, c_{\mathrm{s}}, \rho_A, \rho_{A^{\mathrm{I}}}, \rho_G, \rho_\varphi, \rho_\sigma, \rho_\zeta, \sigma_A, \sigma_{A^{\mathrm{I}}}, \sigma_G, \sigma_\varphi, \sigma_\sigma, \sigma_\zeta, \sigma_Y^{\mathrm{me}}, \sigma_C^{\mathrm{me}}, \sigma_I^{\mathrm{me}}, \sigma_h^{\mathrm{me}}, \sigma_s^{\mathrm{me}}, \sigma_{k^{\mathrm{u}}}^{\mathrm{me}}],$$

are estimated using Bayesian methods surveyed in An and Schorfheide (2007). In the following sections, we discuss the values assigned to parameters fixed a priori and the estimation of the remaining parameters.

Benchmark model without investment search frictions. To put the results of the estimated model from Section 3 into perspective, a benchmark model for the U.S. economy is also estimated. This benchmark model, detailed in Appendix 7.4, is identical to the model of Section 3 except for the search friction in investment considered in this paper. The same empirical strategy described in the previous section is used for the benchmark model. The only differences are that the set of parameters Θ_2 does not include the parameters related to the search friction (i.e., ξ and c_{s}), and that the structure vacancy data are not included in the estimation as an observable. Henceforth, we label the model in Section 3 as the "Model with Search Frictions" and the benchmark model as "Model No Search Frictions."

Calibrated parameters. Table I displays the values assigned to the calibrated parameters, contained in the vector Θ_1 or related targets. The subjective discount factor, β, the inverse of the intertemporal elasticity of substitution, υ, the Frisch elasticity of labor supply, ϕ, the aggregate capital share, α, and the depreciation rate, δ, are set to 0.99, 2, 1, 0.4, and 0.025, respectively, standard values in related business cycle literature. The labor disutility parameter $\overline{\varphi}$ is set at a value consistent with a steady-state level of hours worked of one. The value of $\overline{\psi}$ is set to 0.027, which is consistent with the average annual exit rate of establishments in the United States for the period 1980–2007 of 11%. This value is also in line with the death rate of entrepreneurs in quantitative implementations of the costly state-verification framework. The value of the steady-state share of government spending, $\overline{G}$, was set at 0.2, a standard value in business-cycle studies for the U.S. economy.

The values used for the parameters related to the financial friction (μ_m, $\overline{\varpi}$, and $\overline{\zeta}$) are close to those used in previous quantitative studies of the costly state verification. In particular, the values of $\overline{\varpi}$ and $\overline{\zeta}$ and were set to target values of annual default rate and annual spreads of 3% and 200 basis points, respectively, which correspond to the U.S. historical averages (used for example in Bernanke, Gertler and Gilchrist, 1999). To set the value of the parameter μ_m, note that in the framework of the present paper, financial intermediaries in the state of default face a loss of the return of capital R^k_t not only related to monitoring costs (as in previous models with costly state verification, but without search frictions in investment), but also related to the fact that capital becomes unmatched in the event of default (and has a return of $R^{k,\psi}$ instead of R^k_t; see Sections 3.3 and 3.4). To make the loss in default comparable to those of previous studies – e.g., between 0.2 and 0.36 in Carlstrom and Fuerst (1997); 0.12 in Bernanke, Gertler and Gilchrist (1999) – μ_m was set to target a value of steady-state loss in default, $\overline{\mu}$, of 0.2, where the steady-state loss in default is defined as $\overline{\mu} \equiv 1 - (1-\mu_m)\left(\frac{\overline{R}^{k,\psi}}{\overline{R}^k}\right)$, with $\overline{R}^{k,\psi}$ and $\overline{R}^k$ denoting the steady-state values of $R^{k,\psi}_t$ and R^k_t.[13]

Estimated parameters. Table II presents the assumed prior distributions of the estimated parameters contained in the vector Θ_2, denoted $P(\Theta_2)$. For the two parameters related to the search friction in investment – namely, the curvature of the matching function, ξ, and the search cost, c_s, for which, to my knowledge, no estimation is available – uniform distributions

[13] In the benchmark model without search frictions, $\mu_m = \overline{\mu}$.

Table I
Calibrated Parameters

Parameter	Value	Description
β	0.99	Subjective discount factor
υ	2	Intertemporal elasticity of substitution
ϕ	1	Frisch elasticity of labor supply
α	0.4	Capital share
δ	0.025	Depreciation rate of capital
ψ	0.028	Retirement rate of entrepreneurs
$\overline{G}$	0.2	Steady-state share of government spending
$\overline{h}$	1	Steady-state hours worked
$\overline{s}$	0.02	Steady-state annual spreads
$\overline{\mu}$	0.2	Steady-state loss in default
$F(\overline{\varpi})$	0.075	Default rate

Note: The time unit is one quarter.

were chosen. For the other parameters, prior distributions were chosen following the related literature estimating models for the U.S. economy (Smets and Wouters, 2007; Schmitt-Grohé and Uribe, 2012; Christiano, Motto and Rostagno, 2014).

In particular, the standard errors of the innovations are assumed to follow an inverse-gamma distribution with a mean of 0.1 and a standard deviation of 2; the persistence of the autoregressive stochastic processes, a beta distribution with mean 0.5 and standard deviation of 0.2; the parameter that governs internal habit formation (ρ_c), a beta distribution with mean 0.7 and standard deviation of 0.1; the parameter that governs investment adjustment costs (κ), a gamma distribution with mean 4 and standard deviation of 1.5; and the parameter that governs the curvature of capital utilization costs (c_u), a normal distribution with mean 1 and standard deviation of 1. Section 4.3 relaxes these priors and uses instead uniform prior distributions for all parameters. Finally, uniform prior distributions were chosen for the innovations of the measurement error. These variables are restricted to account for at most 6% of the variance of the corresponding observable time series.

Given the prior parameter distribution, $P(\Theta_2)$, the Metropolis–Hastings algorithm was used to obtain draws from the posterior distribution of Θ_2, denoted $\mathcal{L}(\Theta_2|Y)$ where Y is the data sample (see, for example, An and Schorfheide, 2007). Table II presents the posterior estimates of the model parameters with search frictions in investment. The posterior estimated for the benchmark model without search frictions in investment are presented in Appendix 7.4.

Table II
Estimated Parameters on U.S. Data - Model with Search Frictions

Parameter	Description	Prior distribution			Posterior distribution	
		Distribution	Mean	St. dev	Mean	St. dev
A. Economic parameters						
$\rho_{\rm c}$	Habit parameter	Beta	0.7	0.1	0.62	0.04
κ	Investment-adj costs	Gamma	5	1.5	5.7	0.16
ξ	Curvature-matching tech	Uniform	1	0.25	0.61	0.006
$c_{\rm s}$	Search cost	Uniform	0.085	0.04	0.14	0.002
B. Stochastic processes						
Autocorrelations						
ρ_A	Neutral technology	Beta	0.5	0.2	0.97	0.01
$\rho_{A^{\rm I}}$	Investment-specific tech	Beta	0.5	0.2	0.70	0.05
ρ_G	Government spending	Beta	0.5	0.2	0.866	0.03
ρ_φ	Labor wedge	Beta	0.5	0.2	0.986	0.005
ρ_σ	Risk	Beta	0.1	0.5	0.81	0.03
ρ_ζ	Equity	Beta	0.1	0.5	0.90	0.03
Standard deviation innovation						
σ_A	Neutral technology	Inv Gam	0.1	2	0.006	0.0004
$\sigma_{A^{\rm I}}$	Investment-specific tech	Inv Gam	0.1	2	0.069	0.004
σ_G	Government spending	Inv Gam	0.1	2	0.024	0.002
σ_φ	Labor wedge	Inv Gam	0.1	2	0.026	0.002
σ_σ	Risk	Inv Gam	0.1	2	0.077	0.006
σ_ζ	Equity	Inv Gam	0.1	2	0.036	0.003

Note: The time unit is one quarter. Bayesian estimates are based on 200,000 draws from the posterior distribution.

Model fit. The predictions of the model regarding standard deviations, correlation with output and serial correlations of the six time series included in the estimation as observables are presented in Table III, together with their data counterparts. Two sample periods were included for the second moments in the data: The period 1980–2007, which is the estimation period; and the period 1962–2013, which also includes data on the Great Recession and data prior to the *Great Moderation*. The predictions of the benchmark model without search frictions in investment are also presented in Table III for comparison.

Overall the predictions of the estimated models are in line with empirical second moments. The predicted standard deviations of the model with search frictions are in general larger than the one of the model without search frictions, but are in general in line with those observed in the data. The correlations with output and autocorrelations predicted by the estimated models are also in general in line with those observed in the data. It is worth mentioning that the correlation between the observable components of capital unemployment and output (vacancy rates and GDP, respectively) display a positive correlation for the period 1980–2007. This result is mainly driven by the increase in the vacancy rates observed during the late 1980s (see Figure 2). If this correlation is computed for a longer period that includes the Great Recession or if it is computed after detrending the data with an HP filter, a negative correlation is observed, as predicted by the model.

4.2 Quantitative Results

This section conducts two exercises based on the model's estimation aimed at studying the quantitative relevance of the proposed mechanism. The first exercise is related to the Great Recession, which is an example of a deep financial crisis of the sort that motivated this theoretical framework (see Section 1). The second exercise studies the role of financial shocks in U.S. business cycle fluctuations in the presence of search frictions in investment. Both exercises proceed by comparing the results from the model presented in Section 3 to a benchmark model without investment search frictions, as presented in the previous section and detailed in Appendix 7.4.

Recovery from the U.S. Great Recession. The estimated model is used to ask whether, following a sequence of shocks such as those experienced by the U.S. economy in 2008, and without any further shock, the model can predict an investment slump such as the one ob-

TABLE III
MODEL PREDICTIONS

Statistic	Y	C	I	h	s	k_u
			Standard deviations			
Data – 1980–2007	3.4	3.4	9.4	4.2	1.6	2.0
Data – 1962–2013	4.1	3.3	11.5	5.0	1.8	2.3*
Model with search	4.2	3.5	19.5	4.3	3.4	2.4
Model no search	2.5	2.3	12.5	2.9	1.5	
			Correlations with output			
Data– 1980–2007		94.2	5.3	83.0	−28.4	18.2
Data– 1962–2013		89.5	50.5	78.5	−52.6	−27.5*
Model with search		48.7	82.8	52.0	−60.9	−58.1
Model no search		55.1	73.6	52.1	−21.1	
			Autocorrelations			
Data– 1980–2007	97.6	99.1	97.4	98.6	71.8	99.2
Data– 1962–2013	97.9	98.7	98.0	98.7	86.5	99.3*
Model with search	97.4	98.6	98.6	94.5	88.9	91.1
Model no search	96.8	97.8	98.3	97.2	87.0	

Note: Columns labeled Y, C, I, h, s, and k^{u} refer, respectively, to output, consumption, investment, hours worked, credit spreads, and capital unemployment in the model. Data counterparts described in Appendix 7.1. The time unit is one quarter. *Data corresponds to the period 1980–2013.

served following the U.S. Great Recession – that, as discussed in Section 1, is an empirical regularity of financial-crisis episodes. To answer this question, the estimated model is used to smooth the shocks experienced by the U.S. economy through the last quarter of 2008. Beginning in the first quarter of 2009, the predicted response of the economy is computed: All shocks are set to zero, and the driving stochastic processes are only driven by their estimated autoregressive components; states evolve endogenously. Results from this exercise are displayed in Figure 8 and indicate that the model with investment search frictions predicts a slump of investment following the U.S. Great Recession even larger than the one observed in the data. The same exercise in the benchmark model without investment search frictions predicts that both investment and output should be significantly higher than the levels observed in the data, as noted in the previous literature (see Section 1). The right panel of Figure 8 also shows that the proposed model with search frictions in investment can account for 50% of the difference between the observed recovery and the recovery predicted by the benchmark model without search frictions.[14]

[14]It is worth noting that the model's prediction for capital unemployment is in line the data on vacancy rates observed in the Great Recession. This variable and the prediction for the rest of the observables are included in Appendix 7.2, showing that for all variables the model with search frictions in investment predict less recovery than the model without search frictions in investment.

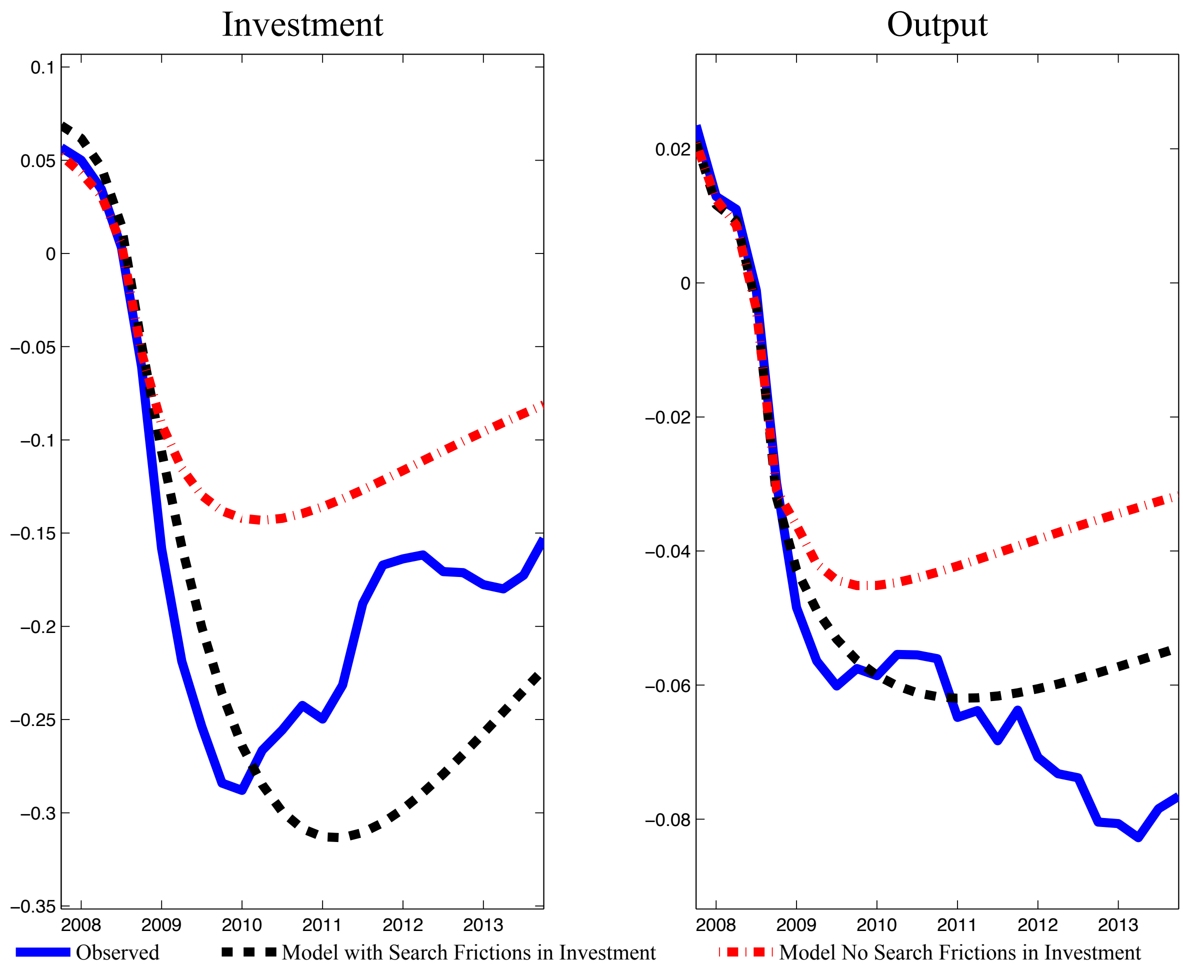

FIGURE 8: U.S. GREAT RECESSION: PREDICTED RECOVERY.

Note: Time-series labeled "Observed" correspond to the data on real per capital investment and output, log linearly detrended (see Appendix 7.1 for details). Time-series labeled "Model with Search Frictions in Investment" and "Model No Search Frictions in Investment" refer, respectively, to predictions from the model presented in Section 3, and to predictions from the benchmark model presented in Appendix 7.4. Model predictions computed since 2009, following the sequence of shocks smoothed from the estimated models for the period 1980–2007. The time unit is one quarter. For details on the models' estimations see Section 4.1 and Appendix 7.4.

The Role of Financial Shocks in U.S. Business Cycles. The estimated model can also be used to interpret the sources of U.S. business-cycle fluctuations. Table IV compares the variance decomposition predicted by the model with investment search frictions to the variance decomposition predicted by the benchmark model without search frictions. The most remarkable result is the difference between the two models in term of the contribution of financial shocks. The benchmark model without investment search frictions assigns a small role to financial shocks, and attributes most of the predicted movements in output and investment to technology shocks (neutral and investment specific) and to labor wedge shocks.

The model with search frictions developed in this paper attributes a relevant role to financial shocks, which account for 48% of output fluctuations and 73% of investment fluctuations.

This result is of interest since the role of financial shocks is a key discussion in the business-cycle literature and an important source of discrepancy between real and monetary models, with the latter attributing a much larger effect to these shocks than the former (as discussed in Christiano, Motto and Rostagno, 2014). The present paper shows that an important part of this discrepancy between these two branches of the literature can be reconciled by introducing investment search frictions. To understand this result, note that in the model with search frictions in investment, 88% of the predicted movements in capital unemployment are explained by financial shocks. Studying impulse-response functions, the next section comes back to this result.

TABLE IV
VARIANCE DECOMPOSITION

Shock		Y	C	I	h	s	k_u
Model no search							
Neutral technology	A	37.5	31.2	28.7	12.7	8.8	
Investment-specific technology	A^{I}	10.0	12.9	19.2	13.5	16.5	
Government spending	G	2.7	4.8	5.1	10.7	2.7	
Labor wedge	φ	45.3	46.0	23.7	57.4	6.5	
Risk	σ	0.1	0.0	0.2	0.2	13.7	
Equity	ζ	4.4	5.1	23.1	5.6	52.1	
Model with search							
Neutral technology	A	14.2	14.6	7.8	5.8	0.5	0.7
Investment-specific technology	A^{I}	0.7	1.8	6.1	3.6	0.7	9.9
Government spending	G	0.5	0.7	0.9	2.3	0.2	0.1
Labor wedge	φ	36.9	49.5	11.7	53.2	0.6	1.1
Risk	σ	0.6	0.6	0.4	1.7	24.2	9.2
Equity	ζ	47.1	32.8	73.1	33.4	73.8	78.9

Note: Columns labeled Y, C, I, h, s, and k^{u} refer, respectively, to output, consumption, investment, hours worked, credit spread, and capital unemployment in the model. Data counterparts described in Appendix 7.1.

Impulse responses. To further study the quantitative findings presented in this section, Figure 9 shows the impulse response of capital unemployment, investment, and output to a one-standard-deviation negative neutral technology shock and a one-standard-deviation negative equity shock. The responses of investment and output in the benchmark model without investment search frictions are also included for comparison.[15] It can be seen that the response of capital unemployment is 10 times larger and much more persistent in response to a one-standard-deviation equity shock than in response to a one-standard-deviation neutral technology shock. For this reason, the responses of investment and output are only significantly different in the case of the financial shock. The impulse-response functions also indicate that investment and output have a large and persistent effect following a negative financial shock that is not present in the benchmark model without investment search frictions.

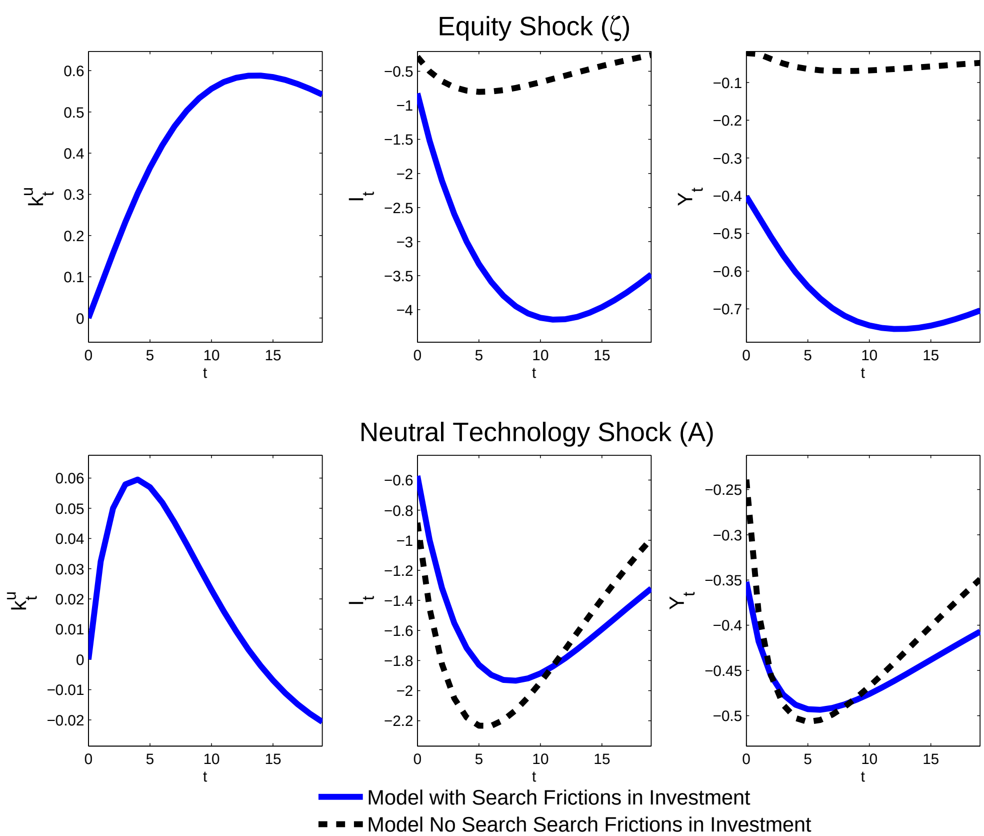

Figure 9: Impulse-Responses to Contractionary Shocks.
Note: Response of capital unemployment, investment, and output to a one-standard-deviation negative equity shock (ζ) and a neutral technology shock (A). Label "Model with Search Frictions in Investment" and "Model No Search Frictions in Investment" refer, respectively, to the model responses presented in Section 3 and the benchmark model in Appendix 7.4. The time unit is one quarter.

4.3 Robustness

Maximum likelihood estimation.

[15]Standard deviations refer to those of the model with search frictions in investment. Appendix 7.2 shows the impulse response for all shocks and for the six observables included in the estimation.

Capital structures and equipment. [To be completed.]

5 Capital Reallocation

This section shows that the analytical framework with investment search frictions developed in this paper can also be used to study capital reallocation. It begins by extending the model to allow for heterogeneity in capital match-specific productivity. This extension allows a characterization not only of the transition of capital from unemployment to employment, but of the transition of capital from employment to employment, since it adds a motive for trading capital while it remains employed (similar to "on the job search" in the labor-market literature; see Menzio and Shi, 2011). A quantitative analysis of the extended model shows that the model's predictions regarding capital reallocation are in line with those observed in the data. The model also has predictions regarding misallocation during crises.

5.1 Extended Model with Capital Reallocation

The basis of the analytical framework developed in this section is the quantitative model developed in Section 3. The section begins by describing the extended model's new assumptions regarding production technology and the market structure of physical capital. It then discusses the problem of selling employed capital in the decentralized market, the entrepreneur's problem, and equilibrium in the extended framework. The notation used in this section is the same as that presented in Section 3.

Production technology. As in Section 3, it is assumed that entrepreneurs have access to technology to produce consumption goods using labor and matched capital as inputs. Unlike in Section 3, each unit of employed capital has a match-specific productivity. This match-specific productivity is revealed after an unmatched unit of capital becomes matched, and does not vary until the specific match is destroyed. The output produced by an effective unit of capital i, with match-specific productivity z_i, and employing $\tilde{h}_{it}$ hours of work, is given by

$$y_{i,t} \quad = \quad A_t z_{i,t} \left(\tilde{h}_{i,t}\right)^{1-\alpha}, \tag{64}$$

where $z_i \in \mathcal{Z} = \{z_1, z_2, \ldots, z_{N_z}\}$, $N_z \geq 2$ and $\mathcal{Z} \gg 0$.

Physical capital markets. As in Section 3, capital held by entrepreneurs is denoted employed capital, and capital held by households is denoted unemployed capital. Households can only hold unmatched capital. Trade of unmatched capital between entrepreneurs (buyers) and households (sellers) occurs in a decentralized market with search frictions. Unlike in Section 3, entrepreneurs now also have access to the decentralized market as sellers, where they can sell an employed unit of capital as unmatched capital to other entrepreneurs. When a unit of capital employed with match-specific productivity z_i is traded in the decentralized market, a new match-specific productivity is drawn from the set $\mathcal{Z}$, with a probability mass function $f_Z(z) : \mathcal{Z} \to [0, 1]$, assumed to be the same for all t. Let $\overline{z}$ denote the expected match-specific productivity of a new match (i.e. $\overline{z} \equiv E(z_i)$). It is assumed that $\overline{z} \in \mathcal{Z}$.

As in Section 3, entrepreneurs also have access to a centralized market in which they trade matched capital. When a unit of employed capital is traded in the centralized market it maintains its match-specific productivity. The match-specific productivity of any unit of capital is common knowledge. The difference with respect to Section 3 is that now units of capital matched at different match-specific productivities will be traded at different prices. The price in the centralized market of a unit of capital with match-specific productivity z_i is denoted Q^{z_i}. The price in the centralized market of a unit of capital matched at the average productivity $\overline{z}$ will be denoted $Q^{\overline{z}}$.

Finally, as in Section 3 there is also a centralized market in which unmatched capital can be sold by financial intermediaries and retired entrepreneurs to households at price J^{u}. Figure 10 summarizes these three markets for capital, with the participants and forms of trade that characterize each market.

The search frictions that characterize the decentralized market for unmatched capital are identical to those in Sections 2 and 3. In particular, search is directed: The market is organized in a continuum of submarkets indexed by the price of the unmatched capital, denoted x, and sellers (households and entrepreneurs) and buyers (entrepreneurs) can choose which submarket to visit. Since, after purchasing a unit, the buyer draws a new match-specific productivity, buyers are indifferent between purchasing an unemployed unit of capital or purchasing an employed unit of capital at any given level of match-specific productivity. In each submarket, the market tightness, denoted $\theta(x)$ is defined as the ratio between the mass of capital searched by entrepreneurs and the mass of capital offered in that submarket. Sellers face no search cost; visiting submarket x, they face a probability $p(\theta(x))$ of finding a match,

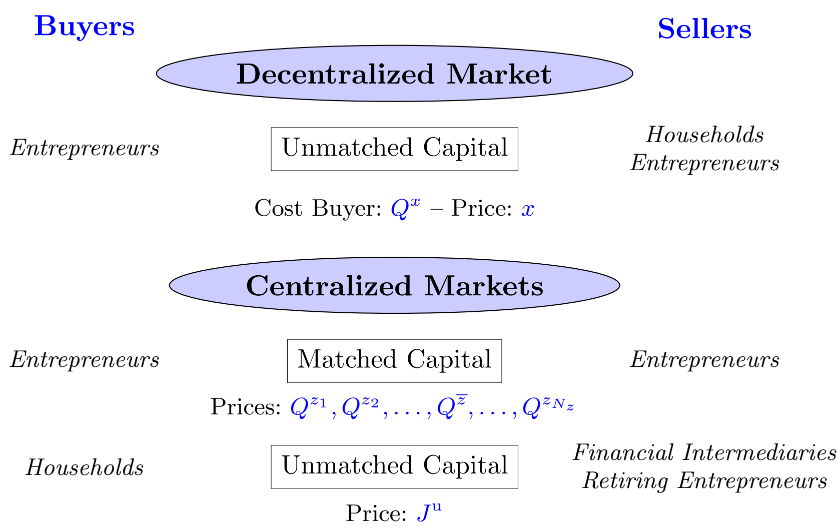

FIGURE 10: STRUCTURE OF CAPITAL MARKETS, MODEL WITH CAPITAL REALLOCATION.

where $p : \mathbb{R}_+ \to [0, 1]$ is a twice continuously differentiable, strictly increasing, strictly concave function that satisfies $p(0) = 0$ and $\lim_{\theta\to\infty} p(\theta) = 1$. Buyers face a cost per unit searched, $c_{\mathrm{s}} > 0$, denoted in terms of consumption goods. Visiting submarket x, they face a probability $q(\theta(x))$ of finding a match, where $q : \mathbb{R}_+ \to [0, 1]$ is a twice continuously differentiable, strictly decreasing function that satisfies $q(\theta) = \frac{p(\theta)}{\theta}$, $q(0) = 1$ and $\lim_{\theta\to\infty} q(\theta) = 0$. The entrepreneur's cost of a unit of capital in submarket x is denoted Q^x (which includes two components: the price paid to the seller, x, and the search cost in submarket x).

Seller's problem for employed capital. An entrepreneur that holds a unit of employed capital matched at productivity z_i can choose to sell this unit in the decentralized market – as an unmatched unit of capital – just as households do with their units of unemployed capital. The only difference between entrepreneurs and households when visiting the decentralized market as sellers is that in the event of not finding a buyer the price of a unit of matched capital is different from the price of a unit of unmatched capital. Therefore, entrepreneurs who visit the decentralized market as sellers and households will typically search in different submarkets. For the same reason entrepreneurs holding units of capital at different match-specific productivities will also search in different submarkets. Formally, the *seller's problem* for an entrepreneur holding a unit of employed capital matched at productivity z_i is given by

$$\max_{x_t^{z_i}} \left\{ p\left(\theta_t\left(x_t^{z_i}\right)\right) x + \left(1 - p\left(\theta_t\left(x_t^{z_i}\right)\right)\right) Q_t^{z_i} \right\}, \tag{65}$$

where $x_t^{z_i}$ denotes the submarket visited by an entrepreneur that holds a unit of capital matched at productivity z_i.

Entrepreneur's problem. As in Section 3, entrepreneurs purchase capital using their net worth and borrowing from financial intermediaries. Including match-specific productivity into the framework developed in Section 3 implies that the entrepreneur's balance sheet now includes different types of assets purchased in the centralized market. At the end of each period, t, equation (66) describes entrepreneur j's balance sheet:

$$\int_x Q_t^x \tilde{K}_{j,t+1}^x \, \mathrm{d}x + \sum_i Q_t^{z_i} \tilde{K}_{j,t+1}^{z_i} \quad = \quad D_{j,t+1} + N_{j,t+1}, \tag{66}$$

where $\tilde{K}_{t+1}^x$ denotes the stock of matched capital held by entrepreneur j at the end of period t, purchased in the submarket x of decentralized market, at a cost Q_t^x per unit of capital; and $\tilde{K}_{j,t+1}^{z_i}$ denotes the stock of capital matched with productivity z_i held by entrepreneur j at the end of period t purchased in the centralized market at price $Q_t^{z_i}$. The latter case also includes the stock of capital matched with productivity z_i held by entrepreneur j from the previous period, which is equivalent to selling and repurchasing the unit in the centralized market at price $Q_t^{z_i}$.

As in Section 3, to solve the entrepreneur's problem, it is useful to define the entrepreneur's leverage and "portfolio weights," from the components of the entrepreneurs balance sheet (66). The entrepreneur's leverage in period t is defined as

$$L_{j,t} \quad \equiv \quad \frac{\int_x Q_t^x \tilde{K}_{j,t+1}^x \, \mathrm{d}x + \sum_i Q_t^{z_i} \tilde{K}_{j,t+1}^{z_i}}{N_{j,t+1}}. \tag{67}$$

The portfolio weight of each asset considered in the left side of equation (66) is given by

$$w_{j,t}^m \quad \equiv \quad \frac{Q_t^m \tilde{K}_{j,t+1}^m}{L_{j,t} N_{j,t+1}}, \tag{68}$$

for $m \in \{x, z_1, z_2, \ldots, z_{N_z}\}$.

As in Section 3, the expected rate of return per unit of matched capital for the assets considered in the left-hand side of equation (66) is defined by

$$R_{t+1}^{k,z_i} \quad \equiv \quad \frac{r_{t+1}^{k,z_i} + (1-\delta)\left[\overline{\psi} J_{t+1}^{\mathrm{u}} + (1-\overline{\psi}) Q_{t+1}^{\mathrm{c}}\right]}{Q_t^{z_i}}, \tag{69}$$

for $i \in \{1, 2, \ldots, N_z\}$, and

$$R_{t+1}^{k,x} \equiv \frac{\sum_i r_{t+1}^{k,z_i} f_z(z_i) + (1-\delta)\left[\overline{\psi} J_{t+1}^{\mathrm{u}} + (1-\overline{\psi}) Q_{t+1}^{\mathrm{c}}\right]}{Q_t^x}, \tag{70}$$

where similar to equation (36) in Section 3, net revenues from production per unit of effective capital matched at productivity z_i are defined by

$$r_t^{k,z_i} = A_t z_i \left(\tilde{h}_t\right)^{1-\alpha} - W_t \tilde{h}_t. \tag{71}$$

The entrepreneurs' objective function (equation (42) in Section 3) can then be expressed as

$$E_t \left\{ \int_{\overline{\omega}_{t+1}}^{\infty} \left[\omega R_{j,t+1}^k L_{j,t} N_{j,t+1} - Z_{j,t+1} D_{j,t+1}\right] \mathrm{d}F_\omega(\omega, \sigma_t) \right\}. \tag{72}$$

where, similar to Section 3, the portfolio return, denoted $R_{j,t+1}^k$, is defined by $R_{j,t+1}^k \equiv \int_x w_{j,t}^x R_{t+1}^{k,x}\,\mathrm{d}x + \sum_i w_{j,t}^{z_i} R_{t+1}^{k,z_i}$.

Similarly, the financial intermediary's participation constraint (equation (46) in Section 3) can be expressed as

$$D_{j,t+1} R_t = \left[1 - F_\omega\left(\overline{\omega}_{t+1}, \sigma_t\right)\right] Z_{j,t+1} D_{j,t+1} + (1-\mu_m) \int_0^{\omega_{t+1}} \omega\, \mathrm{d}F_\omega(\omega, \sigma_t) R_{j,t+1}^{k,\psi} L_{j,t} N_{j,t+1}, \tag{73}$$

where $R_{j,t+1}^{k,\psi}$ denotes the portfolio return of *separated* capital, which, similar to Section 3, is defined by $R_{j,t+1}^{k,\psi} \equiv \int_x w_{j,t}^x R_{t+1}^{k,x,\psi}\,\mathrm{d}x + \sum_i w_{j,t}^{z_i} R_{t+1}^{k,z_i,\psi}$.

From this, the entrepreneur's problem can proceed as in Section 3.

Equilibrium. As in Section 3, any submarket visited by a positive number of buyers must have the same price for capital in equilibrium, and buyers will be indifferent among them. Formally, for all x,

$$\theta_t(x) \left(x + \frac{c_{\mathrm{s}}}{q\left(\theta_t(x)\right)} - Q_t^{\overline{z}} \right) = 0. \tag{74}$$

This condition determines the equilibrium market-tightness function: For all $x < Q_t$,

$$\theta_t(x) = q^{-1}\left(\frac{c_s}{Q_t^{\overline{z}} - x}\right). \tag{75}$$

For all $x \geq Q_t^{\overline{z}}$, $\theta_t(x) = 0$.

The mass of capital that transitions from employment to employment, denoted I_t^{ee}, is defined by

$$I_t^{ee} = \sum_{z_i}(1-\psi_t)p(\theta_t(x_t^{z_i}))(1-\delta)K_t^{z_i},$$

where $K_t^{z_i}$ denotes the stock of employed capital matched at productivity z_i in period t. This object will be the main focus of the next section, when studying the quantitative implications of this model for capital reallocation.

Similar to Section 3, market clearing in centralized markets for capital imply employed capital matched at productivity level z_i evolves then according to the law of motion,

$$K_{t+1}^{z_i} = K_t^{z_i}(1-\psi_t)(1-p(\theta_t(x_t^{z_i}))) + [I_t^{ue} + I_t^{ee}]f_z(z_i),$$

where I_t^{ue} denotes the mass of capital that transitions from unemployment to employment, that using the definition of market tightness, the law of large numbers, and the fact that a household's choice of submarket, $x_{i,t}$ is the same for all units of capital i, is given by $I_t^{ue} = p(\theta_t(x_t^u))(1-\delta)K_t^u$.

The remaining equilibrium conditions are similar to the model presented in Section 3.

5.2 Quantitative Analysis

This section studies some quantitative implications of the model regarding capital reallocation, using the estimated parameters from Section 4.

Procyclical capital reallocation. A well-documented stylized fact is that capital reallocation in the U.S. economy is procyclical (see Eisfeldt and Rampini, 2006). The model presented in this section predicts a correlation between the mass of capital that transitions from employment to employment and output of 51.2%, very close to the 51.1% correlation between capital reallocation and output documented in Eisfeldt and Rampini (2006).

Figure 11 shows that, in response to contractionary shocks, the mass of capital that transitions from employment to employment tends to fall, explaining the procyclical nature of capital reallocation. The explanation of this result through the lens of the model is that contractionary shocks are associated with less demand of capital from entrepreneurs, which leads sellers visit submarkets with less favorable terms, both in terms of price of the units of capital and in terms of the probability of finding a buyer. Therefore, the same factors that lead to a countercyclical capital unemployment lead to a procyclical capital reallocation.

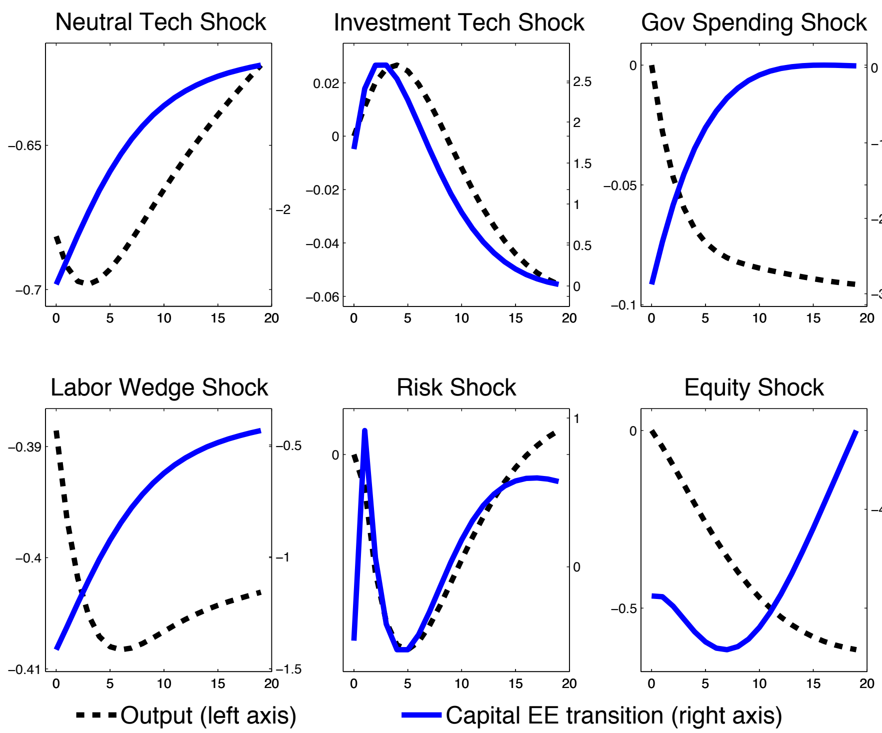

FIGURE 11: IMPULSE RESPONSES TO CONTRACTIONARY SHOCKS.

Note: Response of capital employment-to-employment transitions and output to one-standard-deviation contractionary shocks. Labels *Neutral Tech Shock*, *Investment Tech Shock*, *Gov Spending Shock*, *Labor Wedge Shock*, *Risk Shock*, and *Equity Shock*, refer, respectively, to shocks to the variables A_t, A_t^I, G_t, φ_t, σ_t, and ζ_t presented in Section 3. The time unit is one quarter.

The estimated model can also be used to interpret the sources of fluctuations in capital reallocation. Table V shows the variance decomposition predicted by the model for the transition of capital from employment to employment and shows that most of the predicted capital-reallocation movements (79%) can be accounted by financial shocks. Following the previous argument, since, as shown in Section 4, most of the variation of capital unemployment can be explained by financial shocks, most of the variation in capital reallocation can also be explained by financial shocks. These findings are consistent with those of previous literature explaining procyclical capital reallocation (Cui, 2013).

Table V
Variance Decomposition

Shock		I^{ee}
Neutral technology	A	7.2
Investment-specific technology	A^I	7.1
Government spending	G	3.3
Labor wedge	φ	3.2
Risk	σ	1.1
Equity	ζ	78.1

Misallocation. Empirical evidence points out that recession episodes, and in particular financial crises, are periods of misallocation (see, for example, Midrigan and Xu, 2014).

The predictions of the model presented in this section are also consistent with this empirical finding. Figure 12 shows the response to contractionary shocks of the mass of capital employed with a low match-specific productivity and the mass of capital employed at a high match-specific productivity. The share of capital employed in match-specific productivity increases, especially in response to a negative equity shock (ζ_t). This is because reallocation is especially concentrated in units of capital employed at low match-specific productivity. Therefore, through the lens of this model, capital misallocation during crises is the other side of procyclical capital reallocation.

6 Conclusion and Future Research

This paper presented a model with investment search frictions in which financial shocks have a sizable effect in macroeconomic variables though capital unemployment. An estimated version of the model for the U.S. economy shows that the proposed mechanism can lead to investment slumps such as the one observed during the Great Recession. This result is relevant because slow investment recoveries typically characterize financial crisis episodes of advanced and emerging economies.

Using the estimated version of the model to interpret the sources of business-cycle fluctuations in the U.S. economy, the model assigns a large role (48% of output fluctuations) to financial shocks, in the context of a real model that would have assigned a negligible role to these shocks (5% of output fluctuations). This result is relevant because an important source of discrepancy between real and monetary business-cycle models is the role assigned to finan-

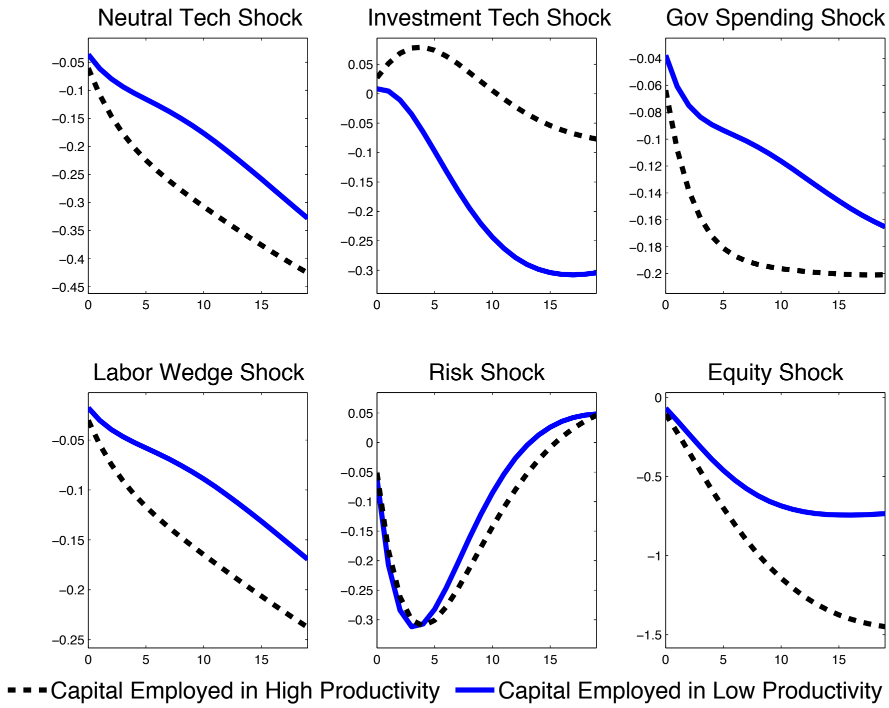

FIGURE 12: IMPULSE RESPONSES TO CONTRACTIONARY SHOCKS.

Note: Response to one-standard-deviation contractionary shocks of the mass of capital employed with a low match-specific productivity ($K_t^{z_1}$, labeled *Low Productivity*) and the mass of capital employed at a high match-specific productivity ($K_t^{z_{N_z}}$ labeled *High Productivity*) predicted by the model presented in Section 5. Labels *Neutral Tech Shock*, *Investment Tech Shock*, *Gov Spending Shock*, *Labor Wedge Shock*, *Risk Shock*, and *Equity Shock*, refer, respectively, to shocks to the variables A_t, A_t^I, G_t, φ_t, σ_t, and ζ_t presented in Section 3. The time unit is one quarter.

cial shocks. This paper shows that incorporating investment search frictions can reconcile an important part of this discrepancy. Finally, the paper shows that the framework can be used to explain capital reallocation and misallocation during crises, as documented by empirical evidence.

The findings of this paper suggest that two related areas of future research could be promising to develop. The first area is normative. As shown in the paper, the directed-search framework studied leads to an efficient allocation. However, combining the search frictions considered in this paper with asymmetric information would lead to a scope for policy related to asset purchases and subsidy programs as shown in Guerrieri and Shimer (2014).

The second area for future research is empirical. In particular, future research could explore more direct evidence of investment search frictions. For instance, it would be possible to investigate the existence of a "Beveridge curve" in the physical-capital market, using data from capital-intermediary firms. It would also be possible to study the testable implications developed from the model in this paper regarding the relationship between capital unemployment, economic activity and investment. This could be done, for instance, using geographical data of the sort used in this paper to measure capital unemployment. These extensions are planned for future research.

References

Andolfatto, David. 1996. "Business Cycles and Labor-Market Search." *American Economic Review*, 86(1): 112–132.

An, Sungbae, and Frank Schorfheide. 2007. "Bayesian Analysis of DSGE Models." *Econometric Reviews*, 26(2–4): 113–172.

Arellano, Cristina, Yan Bai, and Jing Zhang. 2012. "Firm Dynamics and Financial Development." *Journal of Monetary Economics*, 59(6): 533–549.

Arellano, Cristina, Yan Bai, and Patrick Kehoe. 2012. "Financial Frictions and Fluctuations in Volatility." Federal Reserve Bank of Minneapolis. Research Department Staff Report 466.

Bai, Yan, Jose-Victor Rios-Rull, and Kjetil Storesletten. 2012. "Demand shocks as productivity shocks." Federal Reserve Board of Minneapolis.

Basu, Susanto, John Fernald, and Miles Kimball. 2006. "Are Technology Improvements Contractionary?" *American Economic Review*, 96(5): 1418–1448.

Basu, Susanto, John Fernald, Jonas Fisher, and Miles Kimball. 2013. "Sector-Specific Technical Change." National Bureau of Economic Research. Manuscript.

Bernanke, Ben S., Mark Gertler, and Simon Gilchrist. 1999. "The Financial Accelerator in a Quantitative Business Cycle Framework." In *Handbook of Monetary Economics*. Vol. 1C, , ed. John B. Taylor and Michael Woodford, 1341–1393. Amsterdam:North-Holland.

Betancourt, Roger R., and Christopher K. Clague. 2008. *Capital Utilization.* Cambridge, UK:Cambridge Books.

Brunnermeier, M., T. Eisenbach, and Y. Sannikov. 2012. "Macroeconomics with Financial Frictions: A Survey." In *Advances in Economics and Econometrics: Tenth World Congress of the Econometric Society*. Vol. 2, , ed. D. Acemoglu, M. Arellano and E. Dekel, 3–94. Cambridge, UK:Cambridge University Press.

Burnside, Craig, Martin Eichenbaum, and Sergio Rebelo. 1993. "Labor Hoarding and the Business Cycle." *Journal of Political Economy*, 101(2): 245–273.

Calvo, Guillermo A. 1975. "Efficient and Optimal Utilization of Capital Services." *American Economic Review*, 65(1): 181–186.

Calvo, Guillermo A., Alejandro Izquierdo, and Ernesto Talvi. 2006. "Sudden Stops and Phoenix Miracles in Emerging Markets." *American Economic Review*, 96(2): 405–410.

Caplin, Andrew, and John Leahy. 2011. "Trading frictions and house price dynamics." *Journal of Money, Credit and Banking*, 43(s2): 283303.

Carlstrom, Charles T., and Timothy S. Fuerst. 1997. "Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis." *American Economic Review*, 87(5): 893–910.

Cerra, Valerie, and Sweta Chaman Saxena. 2008. "Growth Dynamics: The Myth of Economic Recovery." *American Economic Review*, 98(1): 439–457.

Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans. 2005. "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy." *Journal of Political Economy*, 113(1): 1–45.

Christiano, Lawrence J., Roberto Motto, and Massimo Rostagno. 2014. "Risk Shocks." *American Economic Review*, 104(1): 27–65.

Cooley, Thomas F., Gary D. Hansen, and Edward C. Prescott. 1995. "Equilibrium Business Cycles with Idle Resources and Variable Capacity Utilization." *Economic Theory*, 6(1): 35–49.

Cui, Wei. 2013. "Delayed Capital Reallocation." University College London. Manuscript.

Del Negro, Marco, Marc Giannoni, and Christina Patterson. 2012. "The Forward Guidance Puzzle." Federal Reserve Bank of New York. Staff Report No. 574.

Dmitriev, Mikhail, and Jonathan Hoddenbagh. 2013. "The Robustness of the Financial Accelerator in Costly State Verification Models." Boston College. mimeo.

Eisfeldt, Andrea L., and Adriano A. Rampini. 2006. "Capital reallocation and liquidity." *Journal of Monetary Economics*, 53(3): 369–399.

Fernald, John. 2009. "A Quarterly, Utilization-Adjusted Series on Total Factor Productivity." Federal Reserve Bank of San Francisco. Manuscript.

Geanakoplos, John. 2010. "The leverage cycle." *NBER Macroeconomics Annual 2009*, 24: 1–65.

Gertler, Mark, and Nobuhiro Kiyotaki. 2013. "Banking, Liquidity and Bank Runs in an Infinite-Horizon Economy." National Bureau of Economic Research. No. w19129.

Gertler, Mark, and Peter Karadi. 2011. "A Model of Unconventional Monetary Policy." *Journal of Monetary Economics*, 58(1): 17–34.

Gilchrist, Simon, and John C. Williams. 2000. "Putty–Clay and Investment: A Business Cycle Analysis." *Journal of Political Economy*, 108(5): 928–960.

Gorodnichenko, Yuriy, and Matthew Shapiro. 2011. "Using the Survey of Plant Capacity to Measure Capital Utilization." U.S. Census Bureau Center for Economic Studies. Discussion Paper.

Greenwood, Jeremy, Zvi Hercowitz, and Gregory W. Huffman. 1988. "Investment, Capacity Utilization, and the Real Business Cycle." *American Economic Review*, 78(3): 402–417.

Guerrieri, Veronica, and Robert Shimer. 2014. "Dynamic Adverse Selection: A Theory of Illiquidity, Fire Sales, and Flight to Quality." *American Economic Review*, 104(7): 1875–1908.

Hall, Robert E. 2014. "Quantifying the Lasting Harm to the U.S. Economy from the Financial Crisis." In *NBER Macroeconomics Annual*. Vol. 29, , ed. Jonathan Parker and Michael Woodford, forthcoming. Cambridge, MA:National Bureau of Economic Research.

Jermann, Urban, and Vincenzo Quadrini. 2012. "Macroeconomic Effects of Financial Shocks." *American Economic Review*, 102: 238–271.

Justiniano, Alejandro, Giorgio E. Primiceri, and Andrea Tambalotti. 2010. "Investment Shocks and Business Cycles." *Journal of Monetary Economics*, 57(2): 132–145.

Justiniano, Alejandro, Giorgio E. Primiceri, and Andrea Tambalotti. 2011. "Investment Shocks and the Relative Price of Investment." *Review of Economic Dynamics*, 14(1): 102–121.

Kim, Yong Jin. 2012. "Capital Reallocation and Liquidity with Search Frictions." *Journal of Economic Theory and Econometrics*, 23(1): 1–29.

Kircher, Philipp, and Leo Kaas. 2013. "Efficient firm dynamics in a frictional labor market." Society for Economic Dynamics. 2013 Meeting Papers, no. 160.

Kiyotaki, Nobuhiro, and John Moore. 1997. "Credit cycles." *The Journal of Political Economy*, 105(2): 211–248.

Krainer, John. 2001. "A theory of liquidity in residential real estate markets." *Journal of Urban Economics*, 49(1): 32–53.

Kurmann, André, and Nicolas Petrosky-Nadeau. 2007. "Search Frictions in Physical Capital Markets as a Propagation Mechanism." Centre Interuniversitaire sure le Risque, les Politiques Économiques and l'Emploi. Working Paper 07-12.

Kydland, Finn E., and Carlos E. J. Zarazaga. 2012. "Fiscal Sentiment and the Weak Recovery from the Great Recession: A Quantitative Exploration." Society for Economic Dynamics. Meeting Papers no. 1139.

Lagos, Ricardo, and Guillaume Rocheteau. 2008. "Money and capital as competing media of exchange." *Journal of Economic Theory*, 142(1): 247–258.

Lagos, Ricardo, and Guillaume Rocheteau. 2009. "Liquidity in asset markets with search frictions." *Econometrica*, 77(2): 403–426.

Menzio, Guido, and Shouyong Shi. 2010. "Block Recursive Equilibria for Stochastic Models of Search on the Job." *Journal of Economic Theory*, 145: 1453–1494.

Menzio, Guido, and Shouyong Shi. 2011. "Efficient Search on the Job and the Business Cycle." *Journal of Political Economy*, 119(3): 468–510.

Menzio, Guido, Shouyong Shi, and Hongfei Sun. 2013. "A Monetary Theory with Non-degenerate Distributions." *Journal of Economic Theory*, 148(6): 2266–2312.

Merz, Monika. 1995. "Search in the Labor Market and the Real Business Cycle." *Journal of Monetary Economics*, 36(2): 269–300.

Michaillat, Pascal, and Emmanuel Saez. 2013. "Aggregate Demand, Idle Time, and Unemployment." National Bureau of Economic Research. Working Paper No. 18826.

Midrigan, Virgiliu, and Daniel Yi Xu. 2014. "Finance and Misallocation: Evidence from Plant-Level Data." *American Economic Review*, 104(2): 422–458.

Moen, Espen R. 1997. "Competitive Search Equilibrium." *Journal of Political Economy*, 105: 385–411.

Petrosky-Nadeau, Nicolas, and Etienne Wasmer. 2011. "Macroeconomic Dynamics in a Model of Goods, Labor and Credit Market Frictions." Institute for the Study of Labor. Discussion Paper 5763.

Piazzesi, Monika, Martin Schneider, and Johannes Stroebel. 2013. "Segmented housing search." Stanford University. Working paper.

Reinhart, Carmen M., and Kenneth S. Rogoff. 2009. "The Aftermath of Financial Crises." *American Economic Review, American Economic Association*, 99(2): 466–472.

Reinhart, Carmen M., and Kenneth S. Rogoff. 2014. "Recovery from Financial Crises: Evidence from 100 Episodes." *American Economic Review*, 104(5): 50–55.

Sbordone, Argia M. 1996. "Cyclical Productivity in a Model of Labor Hoarding." *Journal of Monetary Economics*, 38(2): 331–361.

Schaal, Edouard. 2012. "Uncertainty, Productivity and Unemployment in the Great Recession." Federal Reserve Bank of Minneapolis. mimeo.

Schmitt-Grohé, Stephanie, and Martin Uribe. 2012. "What's News in Business Cycles." *Econometrica*, 80(6): 2733–2764.

Shimer, Robert. 1996. "Contracts in a Frictional Labor Market." Massachusetts Institute of Technology. mimeo.

Shimer, Robert. 2005. "The Cyclical Behavior of Equilibrium Unemployment and Vacancies." *American Economic Review*, 95(1): 25–49.

Smets, Frank, and Rafael Wouters. 2007. "Shocks and Frictions in U.S. Business Cycles: A Bayesian DSGE Approach." *American Economic Review*, 97(3): 586–606.

Townsend, Robert M. 1979. "Optimal Contracts and Competitive Markets with Costly State Verification." *Journal of Economic Theory*, 21(2): 265–293.

Wheaton, William C. 1990. "Vacancy, search, and prices in a housing market matching model." *Journal of Political Economy*, 98(6): 1270–1292.

Winston, Gordon C. 1974. "The Theory of Capital Utilization and Idleness." *Journal of Economic Literature*, 12(4): 1301–1320.

7 Appendices

7.1 Data Appendix

7.1.1 Investment and Output during Financial Crisis Episodes

The following data were used to construct the time series shown in Figure 1.

1. Nominal Gross Domestic Product, National Currency.

2. Nominal Gross Fixed Capital Formation, National Currency.

3. Real Gross Domestic Product.

4. GDP Deflator: constructed as (4) = (1)/ (3).

5. Population.

6. Real Per Capita GDP: constructed as (6)=(3)/(5).

7. Real Per Capita Investment: constructed as (7)=((2)/(4))/(5).

The time series used in Figure 1 were (6) and (7). The sources of these data were the following:

(i) Data US: BEA,

(ii) Data Europe: Eurostat,

(iii) Data Sweden, Finland, Spain: OECD,

(iv) Data Latin America: WEO.

7.1.2 Capital Unemployment

The data on capital unemployment for structures in the U.S. economy – used in Figure 2 and in the model estimation of Section 4– were constructed as a weighted average of vacancy rates of office space, retail space, and industrial space. Data were obtained from CBRE and Reis. Weights were defined using data on Current-Cost Net Stock of Private Fixed Assets, Equipment, Structures, and Intellectual Property Products by Type, source BEA. (Table 2.1).

7.1.3 Bayesian Estimation

The following data for the U.S. economy were used to construct the quarterly time series used in the model estimation of Section 4:

1. Nominal GDP: Gross domestic product, billions of dollars, seasonally adjusted at annual rates. Source: BEA, National Income and Product Accounts Tables (Table 1.1.5).
2. Nominal Consumption: Sum of personal consumption expenditures, durable good and services, billions of dollars, seasonally adjusted at annual rates. Source: BEA, National Income and Product Accounts Tables (Table 1.1.5).
3. Nominal Investment: Sum of gross private domestic fixed nonresidential investment in structures, equipment and software Source: BEA, National Income and Product Accounts Tables (Tables 1.1.5 and 5.3.5).
4. Real GDP: Gross domestic product, billions of chained (2009) dollars, seasonally adjusted at annual rates. Source: BEA, National Income and Product Accounts Tables (Table 1.1.6).
5. GDP Deflator: constructed as (5) = (1)/ (4).
6. Nonfarm Business Hours Worked. Source: BLS, Major Sector Productivity and Costs
7. Civilian Noninstitutional Population, 16 years and over. Source: BLS, Labor Force Statistics from the Current Population Survey.
8. 3-Month Treasury Bill, Secondary Market Rate. Source: Federal Reserve Bank of St. Louis.
9. Moody's Seasoned Baa Corporate Bond Yield. Source: Federal Reserve Bank of St. Louis.
10. Real Per Capita GDP: constructed as (10)=(4)/(7)
11. Real Per Capita Consumption: constructed as (11)=((2)/(5))/(7)
12. Real Per Capita Investment: constructed as (12)=((3)/(5))/(7)
13. Per Capita Hours Worked: constructed as (13)=(6)/(7)
14. Credit Spreads: constructed as (14) = (1+(9))/(1+(8))
15. Capital Unemployment: constructed based on data on vacancy rates of nonresidential commercial real estate, office retail and industrial sector. Methodology detailed in 7.1.2

The six time series used in the bayesian estimation were (10), (11), (12), (13), (14) and (15), with (10), (11), (12) log linearly detrended.

7.2 Additional Figures

7.3 Proofs

7.4 Benchmark Model Economies

[To be completed.]