A kinetic investigation of the flocculation of alumina with polyacrylic acid

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Abstract

Using a model colloidal system of alumina and polyacrylic acid (PAA), the kinetics of flocculation was investigated at low polymer concentrations and short durations (on the order of seconds). The polymer-induced flocculation processes obeyed Von Smoluchowski’s bimolecular rate equation. Increases in the concentration of the polymer resulted in higher rate constants for the flocculation process. At a fixed concentration (say 50 ppb, parts per billion), the rate constant values showed a maximum value for 250,000 g mol⁻¹ polyacrylic acid. At this polymer concentration, calculations of the surface coverage of alumina by PAA molecules of different molecular weights show that for all the cases the coverage is nearly the same, ∼1 × 10⁻³, but the flocculation response and the rates are significantly different. This trend in flocculation characteristics is attributed to the critical polymer number density requirement for effective flocculation (at least partial charge neutralization and initiation of flocculation). The mechanism governing the flocculation at ultralow concentrations (50 ppb) is the synergistic effect of partial patch neutralization and bridging.

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1. Introduction

In spite of significant progress made in recent years, the understanding of the mechanism of flocculation is far from complete. Several factors involving the particles, reagents, and medium influence flocculation process [1–16]. The kinetic aspects of flocculation are very important because they manifest several concurrent processes and thus have been the focus of attention of several researchers. It is well documented in the literature that polymer-induced flocculation of particulate suspensions involves many steps, namely mixing of polymer molecules among the particles; adsorption of polymer chains on the particles; rearrangement of the adsorbed chains; collisions between polymer-adsorbed particles; and floc formation and also break-up of flocs [17–19].

Among other factors, adsorption and its effectiveness depend on the polymer characteristics, such as its molecular weight, conformation, dosage, and mode of addition, the ionic strength of the medium, and the mixing time, i.e., the time taken by the polymer molecules to adsorb and reorient themselves on the particle surfaces. We have recently shown that flocculation of alumina could be achieved at ultralow concentrations of a low-molecular-weight polyacrylic acid polymer, usually used as a dispersant [20]. For effective flocculation, concentration regimes were established for each molecular weight samples to act either as a flocculant or as a dispersant. This aspect of flocculation has been discussed elsewhere [21].

Flocculation also depends on the mode of polymer addition and the conditioning time, i.e., the time taken by the polymer molecules to adsorb and reorient themselves on the particle surfaces. In an interesting experiment reported in the literature, excess polymer was added to one portion of a particulate suspension and sufficient time was allowed for adsorption and reconformation to take place. When a second portion of the suspension is added, flocculation between the coated and uncoated portions was observed [22,23]. Flocculation is significantly affected by other process parameters, e.g., salt, which is normally added to maintain a constant ionic strength of the medium. By judiciously manipulating the salt concentration, effective flocculation regimes have been established, e.g., polymer was added to a suspension at...
low ionic strength where no flocculation occurred and then flocculation was induced by adding salt [24].

In essence, the characteristics of the particles and polymer together with the solution chemistry conditions all contribute to the flocculation process. In this work, the effect of molecular weight and dosage of polyacrylic acid polymer as well as the influence of the suspension pH on the kinetics of flocculation has been delineated.

2. Materials and methods

2.1. Materials

An alumina sample (AKP-15) was procured from Sumitomo Chemical Company Limited. The mean particle diameter of this sample was determined to be 0.7 µm and the specific surface area measured by BET method was 3.58 m² g⁻¹. Triply distilled water of specific conductivity less than 1.5 µΩ⁻¹ and free of organics (according to total organic carbon analysis and surface tension measurements) was utilized to prepare all suspensions.

Commercially available polyacrylic acids of a wide range of molecular weights (2000 to 4 million g mol⁻¹) were procured for the flocculation experiments. PAA samples of molecular weights 250,000 and 4 million g mol⁻¹ were obtained from Aldrich Chemicals while the rest were bought from Polysciences. As specified by the manufacturers, the polymer samples had a relatively high polydispersity (Mₘ/Mₙ = 3–4). Reagent grade sodium chloride from Fisher Scientific was used “as received” without any further purification.

2.2. Methods

2.2.1. Flocculation experiments

A 1-g alumina sample was dispersed in a 250-ml beaker with 190 ml water. Unless otherwise mentioned, the ionic strength of the suspension was maintained at 0.03 M NaCl. The suspension was ultrasonicated with a probe using a maximum of 75 W power and then mixed for 60 min on a magnetic stirrer. After the pH adjustment, the suspension was further stirred for a period of 60 min. Subsequent to this mixing, the pH was measured again, the suspension was further agitated using an impeller (1 in. in diameter) for better control of the agitation. Desired amount of polymer solutions (usually 10 ml) were then added using a Sage syringe pump for 1 min with the impeller rotating at 600 rpm. After the addition of the polymer, the suspension was further stirred at 300 rpm for 4 min and allowed to settle for predetermined time periods and then a fixed volume was removed by suction, filtered, dried, and weighed. The efficiency of flocculation experiments was expressed in terms of weight percent settled in the beaker. The magnitude of uncertainties in the experimental values was within ±5%.

From the particle–polymer interactions, the minimum time necessary for collision (leading to effective flocculation) was calculated as follows. In a particle suspension, the average time necessary for two particles to collide (tᵣ) is given by

\[ tᵣ = \frac{1}{kₛ N₀} \]

where kₛ is the Smoluchowski rate constant and N₀ is the particle concentration [17–19.25–30]. Using the theoretical value of kₛ as \(12 \times 10^{-12}\) cm³ s⁻¹, tᵣ for AKP-15 alumina sample (average diameter: 0.7 µm; concentration: 1 g alumina in 200 cm³, corresponding to 0.696 × 10¹⁰ particle cm⁻³) has been calculated to be 12 s.

2.2.2. Zeta potential measurements

The zeta potential of the alumina sample at different pH conditions in the absence of polymers was determined by a Zetameter 3.0 system at ~0.01 wt% solids.

3. Results and discussions

The results obtained for the aggregation of alumina particles both in the absence and in the presence of polyacrylic acids are discussed in the following section. The kinetics of alumina flocculation is primarily illustrated with 5000, 50,000 and 250,000 g mol⁻¹ polyacrylic acid samples.

3.1. Kinetics of alumina settling in the absence of polymer

The settling kinetics of alumina at three different pH values in the absence of polymer is illustrated in Fig. 1a. The results show that settling is fastest at the experimentally determined iep of alumina (i.e., at pH ~ 9) due to coagulation induced by attractive van der Waals forces. Settling at pH 3.5 is observed to be slower owing to the higher electrostatic repulsion at this pH as suggested by the larger zeta potential (+60 mV) compared to the zeta potential of ~40 mV obtained at pH 10.5.

The key to successful polymeric flocculation is the effective particle–polymer collisions leading to adhesion. As mentioned earlier, several processes occur to destabilize a polymer-induced particle suspension. The dynamics of the flocculation process has been described [17] by three characteristic time-scales: tₚ (polymer adsorption), tₛ (rearrangement of adsorbed polymer), and tᵣ (particle collision). While the polymer reconformation time (tₛ) may depend on the chemistry of the polymer and the characteristics of the particle and medium; tₛ and tᵣ can be well estimated using the classical second-order reaction kinetics [18,25–27].

From the literature [19.25], it is known that the order of coagulation/flocculation process is mostly bimolecular.
Based on Von Smoluchowski’s classical equation [28–30],

\[ \frac{N_t}{N_0} = \left( 1 + \frac{1}{2}kN_0t \right)^{-2}, \]

where \( N_t \) = concentration of singlets at time \( t \), \( N_0 \) = concentrations of the singlets at \( t = 0 \) and \( k \) = rate constant for collisions between the singlets; a plot of \( (N_0/N_t)\frac{1}{2} \) against \( t \) should give a straight line with an intercept of 1 for a bimolecular process. \( N_0 \) values for a known weight of alumina were calculated by considering the particle diameter (0.7 µm) and density of alumina (3.99 gm cm\(^{-3}\)). For a 1 g alumina sample, \( N_0 \) was calculated to be 1.396 \times 10^{12}. Similarly, knowing the weight of particles flocculated in a given time period, one can calculate \( N_t \) (here the assumption is that the portion of alumina which did not flocculate are singlets).

The kinetic plot for the settling of alumina suspension in the absence of any polymer is shown in Fig. 1b. At pH 3.5 the values of \( (N_0/N_t)\frac{1}{2} \) for different \( t \) values are close to 1, indicating that there is no significant settling in this case, whereas the \( (N_0/N_t)\frac{1}{2} \) for pH 9 increases sharply as a function of time. The aggregation rate constants \( k \) were obtained from the slope of \( (N_0/N_t)\frac{1}{2} \) vs \( t \) curve and are given in Table 1. The rate constant of aggregation is many times larger at the iep (−pH 9) than that at the other two pH values.

### 3.2. Kinetics of alumina flocculation in the presence of polymer

In an earlier publication [20], we have reported that alumina can be efficiently flocculated at ultralow concentration (50–100 ppb) of a low-molecular-weight polymer, i.e., 50,000 g mol\(^{-1}\) polyacrylic acid (normally used as a dispersant), at an ionic strength of 0.03 M NaCl. At low polymer concentrations, flocculation was more effective at pH 3.5 though less than 5% of the carboxylic groups are ionized at this pH. In the acidic pH conditions, polyacrylic acid polymers are known to exist as random coils.

Typically tens or hundreds of ppm of polymeric reagents are employed in different industries for effective flocculation. This is in sharp contrast to the ultralow polymer concentrations used in our experiments. Our findings were supplemented by suitably designed kinetic experiments conducted at low polymer concentrations with a view to gaining further insight into the mechanism of polymer–particle interactions under these flocculation conditions.

#### 3.2.1. Kinetics of flocculation with 5000 g mol\(^{-1}\) polyacrylic acid

**Effect of concentration**  Results for the flocculation of alumina at different concentrations of 5000 g mol\(^{-1}\) polyacrylic acid at pH 3.5 are presented in Fig. 2a. It is interesting to note that as low as 5000 g mol\(^{-1}\) polyacrylic acid was also quite effective in causing flocculation of alumina. At 50 ppb concentration, initially the flocculation was slow, but enhanced effectiveness was observed at 250 s. This time lag could be attributed to the relaxation and reconformation of the polymer molecules on the alumina surface, thereby providing favorable spatial environment. As expected, the flocculation efficiency improved with increase in polymer concentration. The plot of \( (N_0/N_t)\frac{1}{2} \) vs \( t \) shows a typical bimolecular rate process for the flocculation (Fig. 2b). The calculated rate constant values at different polymer concentrations are presented in Table 2. At a low dosage of 50 ppb, the rate constant is very small but shows a 100-fold increase at the optimum polymer concentration of 3000 ppb [21].

**Effect of pH**  In a polymer-assisted flocculation process, not only the molecular weight and dosages of the polymer are

<table>
<thead>
<tr>
<th>pH</th>
<th>Rate constant ((k) \times 10(^{-15}) s(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.5</td>
<td>2.1</td>
</tr>
<tr>
<td>9</td>
<td>66.7</td>
</tr>
<tr>
<td>10.5</td>
<td>15.9</td>
</tr>
</tbody>
</table>
important but also other process parameters, namely pH and electrolyte concentrations, assume significance. The flocculation response of alumina with 5000 g mol$^{-1}$ PAA at two widely different pHs, namely 3.5 and 10.5, are shown in Figs. 3a and 3b. The flocculation response is overwhelmly superior at pH 3.5 vis-à-vis pH 10.5. It is interesting to note that in the absence of any polymer, settling at pH 10.5 was appreciably higher than that at pH 3.5 (cf. Fig. 1a). Alumina is negatively charged at pH 10.5 but the magnitude of the electrostatic repulsive forces is not high enough to cause complete stabilization. When polymer (of concentration 3000 ppb) is added, both the alumina and the PAA polymers are negatively charged. Adsorption of negatively charged polymer increases the charge density on the solid and consequently the magnitude of the electrostatic repulsive forces. The molecular weight (5000 g mol$^{-1}$) being rather low (and hence the size being small), the polymer molecules cannot overcome the electrostatic forces to form interparticle bridges. As a result, flocculation at pH 10.5 is poor.

3.2.2. Kinetics of flocculation with 50,000 g mol$^{-1}$ polyacrylic acid

Effect of concentration In comparison to the somewhat high dosage ($\geq$1000 ppb) required for efficient flocculation in the case of 5000 g mol$^{-1}$ PAA, alumina flocculation with a relatively higher-molecular-weight PAA (e.g., 50,000 g mol$^{-1}$) was significant even at very low concentrations, 50–100 ppb [20]. In a flocculation system, it has been generally observed that when the flocculant concentration is high ($\sim$several ppm), flocculation is typically complete in a relatively short time, thus making it difficult to study the kinetic effect. Therefore, a low-concentration regime of 25–250 ppb was chosen for 50,000 g mol$^{-1}$ PAA to investigate the kinetic effect in detail (Fig. 4). At ultralow concentrations of 25 and 50 ppb, the flocculation was poor during the first 10 s (the weight percent flocculated was $\sim$25–35% in
comparison to ∼5% in the absence of polymer). An increase in the polymer concentration to 250 ppb resulted in flocculation of ∼70% in the first 10 s and nearly 90% flocculation was complete in 20 s. This suggests that increased number density of the partially charged polymer molecules led to enhanced adsorption, thereby facilitating the aggregation of charge-patch-neutralized alumina particles [18,31,32]. In contrast to the 1000–3000 ppb concentration range necessary for flocculation with 5000 g mol$^{-1}$ PAA, the dosage required in the case of 50,000 g mol$^{-1}$ PAA is only 50–250 ppb, primarily due to the larger size of the polymer molecules, which also assist in flocculation by bridging. The plot of $(N_0/N_1)^{1/2}$ vs $t$ (Fig. 4b) also shows a bimolecular rate process for flocculation under these conditions. The calculated rate constant values at different concentrations are given in Table 3. Comparison of Tables 2 and 3 shows that in the case of 50,000 g mol$^{-1}$ PAA, the rate constant $k$ is $132 \times 10^{-15}$ s$^{-1}$ at a concentration of 250 ppb, while the $k$ value for 5000 g mol$^{-1}$ PAA at a much larger concentration of 3000 ppb is comparable at $140 \times 10^{-15}$ s$^{-1}$. This tends to suggest that equivalent flocculation efficiency could be achieved by manipulating the concentration and the molecular weight of the polymer. In the case where the molecular weight is low, higher dosages of polymer would cause flocculation primarily by charge neutralization, while the larger polymer (at relatively lower dosages) enhances flocculation by bridging.

**Effect of pH** The flocculation behavior of alumina at pH 3.5 and 10.5 with 50 ppb of 50,000 g mol$^{-1}$ PAA is illustrated in Fig. 5. As at 5000 g mol$^{-1}$ PAA, the acidic pH condition is also preferred at 50,000 g mol$^{-1}$ PAA for flocculation. However, at pH 10.5, the settling of the particles in the presence of 50 ppb polymer was nearly the same as that in the absence of PAA (cf. Fig. 1), indicating that the concentration of the polymer was too low to overcome the electrostatic repulsive forces at this pH for effective flocculation to occur.

### Table 3

Rate constant for the flocculation of alumina at different concentrations of 50,000 g mol$^{-1}$ PAA at pH 3.5

<table>
<thead>
<tr>
<th>Concentration (ppb)</th>
<th>Rate constant $(k \times 10^{-15} \text{ s}^{-1})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>28.2</td>
</tr>
<tr>
<td>50</td>
<td>42.5</td>
</tr>
<tr>
<td>250</td>
<td>132</td>
</tr>
</tbody>
</table>

**3.2.3. Kinetics of flocculation with 250,000 g mol$^{-1}$ polyacrylic acid**

The results for flocculation kinetics of alumina with a higher-molecular-weight PAA, namely 250,000 g mol$^{-1}$ at pH 3.5, are given in Fig. 6. It may be noted that at 25 ppb concentration, flocculation was poor at short time periods but significant above 100 s. This is also attributed to the relaxation and rearrangement of the limited number of polymer molecules on the alumina surface. It is interesting to note that at the same polymer concentration of 25 ppb, flocculation of alumina with 50,000 g mol$^{-1}$ PAA was appreciable even in a short time period (see Fig. 4). The number of
polymer molecules per alumina particle at 25 ppb concentration is 43 for 50,000 g mol\(^{-1}\) and 9 for 250,000 g mol\(^{-1}\). However, when the concentration of 250,000 g mol\(^{-1}\) PAA was increased to 50 ppb (number density 18), flocculation improved from ~20% to ~80% in 20 s. Thus, it is clear that effective flocculation is the result of an interplay of molecular weight of the polymer (its size) and its concentration (number density per particle).

The flocculation efficiencies of alumina at pH 3.5 with 50 ppb dosages of different-molecular-weight polyacrylic acid samples are compared in Fig. 7. The corresponding number of polymer molecules per particle is also indicated in the same figure.

The results show that the rate constants for the flocculation process are rather small for both the low- and high-molecular-weight polymers and exhibit a maximum value for 250,000 g mol\(^{-1}\) PAA. In the case of 5000 g mol\(^{-1}\) PAA, the number of polymer molecules is large but not enough for both charge neutralization and bridging effects. On the other hand, for 1 and 4 million g mol\(^{-1}\) PAA, the number densities are far too low for effective bridging to occur. Calculations of the surface coverage by the polymer molecules of different molecular weight PAA show that for all the cases the coverage is nearly same ~\(1 \times 10^{-3}\) (Appendix A) but the flocculation efficiency and the rate constants are widely different for the different molecular weights. The cause of efficient flocculation is, thus, attributed to the synergistic effects of the polymer size and the available number of molecules, which are necessary for both charge patch neutralization and bridging. Our results clearly show that such an effect could be observed in the case of 250,000 g mol\(^{-1}\) polyacrylic acid, which at 50 ppb concentration can fulfill all the necessary conditions for efficient flocculation.

3.2.4. Effect of polymer molecular weight on the terminal velocity and size of flocs

Molecular weight and solution pHs have important effects on the flocculation process as well as on the settling characteristics of alumina (PAA-assisted flocculation). Depending on the size, the flocs tend to settle at different terminal velocities \(v_t\). The \(v_t\) of the flocs for a few selected molecular weights are illustrated in Table 4. The corresponding equivalent size of the flocs are also given in the same table (Appendix B). The results show that at 10 s time, pH 3.5, and 50 ppb polymer concentration, the terminal velocity of the flocs increase with the increase in polymer molecular weight. This is due to the extended floc size resulting from the interaction of the alumina particles with increased polymer size. Similarly, it is noticed that pH has a significant effect on the terminal velocity. For example, at 10 s time, the terminal velocities are much higher at pH 3.5 compared to that at pH 10.5 (Tables 5a and 5b). Comparison

<table>
<thead>
<tr>
<th>Polymer MW (g mol(^{-1}))</th>
<th>Flocculation (%)</th>
<th>(v_t) (cm s(^{-1}))</th>
<th>Floc diameter (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5000</td>
<td>2.8</td>
<td>0.014</td>
<td>9.2</td>
</tr>
<tr>
<td>50,000</td>
<td>33.8</td>
<td>0.169</td>
<td>32.1</td>
</tr>
<tr>
<td>250,000</td>
<td>72</td>
<td>0.36</td>
<td>46.9</td>
</tr>
</tbody>
</table>

Time of flocculation = 10 s, polymer concentration = 50 ppb, pH 3.5.

<table>
<thead>
<tr>
<th>pH</th>
<th>Flocculation (%)</th>
<th>(v_t) (cm s(^{-1}))</th>
<th>Floc diameter (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.5</td>
<td>32.8</td>
<td>0.164</td>
<td>31.6</td>
</tr>
<tr>
<td>10.5</td>
<td>2</td>
<td>0.01</td>
<td>7.8</td>
</tr>
</tbody>
</table>

(a) Time of flocculation: 10 s; polymer MW: 5000 g mol\(^{-1}\); polymer concentration: 3000 ppb

(b) Time of flocculation: 10 s; polymer MW: 50,000 g mol\(^{-1}\); polymer concentration: 50 ppb

<table>
<thead>
<tr>
<th>pH</th>
<th>Flocculation (%)</th>
<th>(v_t) (cm s(^{-1}))</th>
<th>Floc diameter (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.5</td>
<td>33.8</td>
<td>0.169</td>
<td>32.1</td>
</tr>
<tr>
<td>10.5</td>
<td>2</td>
<td>0.01</td>
<td>7.8</td>
</tr>
</tbody>
</table>
of Tables 5a and 5b also shows that equivalent terminal velocities could be achieved for a lower molecular weight polymer by increasing the polymer concentration and vice versa. The effects due to pH and polymer concentration, thus, synergistically contribute to the overall efficiency of the system.

4. Summary and conclusions

Effective polymer-induced flocculation is due to the contributions from several factors representing the characteristics of the particle surface, the polymer and the solution. Some of the factors are more dominant for effecting flocculation under certain conditions. It is possible to judiciously control the polymer concentration to achieve the desired increase in flocculation rate and to optimize the molecular weight of the polymer to leverage the benefits. The mechanism governing the flocculation at ultralow concentrations (say 50 ppb) is the synergistic effect of partial patch neutralization and bridging. Polyacrylic acid of molecular weight 250,000 g mol\(^{-1}\) was found to perform best since it could provide the necessary number of molecules and also had the reasonable size to help bridging flocculation even under ultra-low dosage conditions.

Acknowledgments

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Appendix A

The surface coverage was calculated by considering aluminia particles to be spherical. Since the best flocculation was obtained at pH 3.5 (where the PAA molecules are believed to be in the coiled form), as a rough approximation the polymer molecules were considered to be spherical too. The size of a polymer patch was calculated by using the formula, size = \(\pi R_g^2\), where \(R_g\) is the radius of gyration of the polymer molecule in nm. The rms length of polymer, \(\langle h^2 \rangle^{1/2}_{\text{rms}}\), at pH 3.5 is calculated using the relationship

\[\langle h^2 \rangle^{1/2}_{\text{rms}} = 1N^{1/2},\]

where \(l\) is the step length (for polyacrylic acid, \(l\) is taken as 2.5 Å) and \(N\) is the number of steps. For a random coiled polymer chain, the radius of gyration \(R_g\) of the polymer molecule was calculated using the relationship

\[\sqrt{R_g^2} = 1/6\sqrt{(h^2)_{\text{rms}}}.\]

From the polymer concentration one can calculate the number of molecules per alumina particle (total weight of alumina taken for each flocculation experiment was 1 g). By multiplying the number of molecules/particle with the polymer patch size, one gets the total patch area on the particle surface. Since the particle surface area is known (the diameter being known), one can calculate the fractional surface coverage as total patch area/particle area \((4\pi r^2)\). Some typical values are given in the following table:

<table>
<thead>
<tr>
<th>Polymer MW (g mol(^{-1}))</th>
<th>(R_g) (nm)</th>
<th>Patch size (nm(^2)) ((= \pi R_g^2))</th>
<th>No. of molecules/particle at 50 ppb concentration</th>
<th>Fractional coverage ((\times 10^{-3}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>5000</td>
<td>0.85</td>
<td>2.26</td>
<td>865.5</td>
<td>1.27</td>
</tr>
<tr>
<td>50,000</td>
<td>2.69</td>
<td>22.73</td>
<td>86.53</td>
<td>1.28</td>
</tr>
<tr>
<td>250,000</td>
<td>6.02</td>
<td>113.78</td>
<td>17.3</td>
<td>1.28</td>
</tr>
<tr>
<td>1000,000</td>
<td>12.03</td>
<td>454.73</td>
<td>4.3</td>
<td>1.27</td>
</tr>
<tr>
<td>4000,000</td>
<td>24.07</td>
<td>1819.52</td>
<td>1.08</td>
<td>1.28</td>
</tr>
</tbody>
</table>

For PAA molecular weights 50,000 and 250,000, excellent flocculation (>80%) could be obtained at a very low concentration, namely 50 ppb, and the corresponding surface coverage turns out to be \(\sim 0.12\%\).

Appendix B. Calculation of terminal velocity and floc size

The flocs have been assumed to be spherical in shape, falling in a viscous medium. Equating the force of gravity on the solid in the direction of fall, and the buoyant and kinetic forces acting in the opposite direction,

\[
\frac{4}{3}\pi R^3 \rho_s g = \frac{4}{3}\pi R^3 \rho_l g + 6\pi \mu R v_t, \tag{B.1}
\]

(Gravitational force) (Buoyancy force) (Drag force)

where \(R\) is the radius of the particle, \(\rho_s\) and \(\rho_l\) are the densities of the solid sphere and the fluid, \(\mu\) is the viscosity of the medium, \(g\) is the acceleration due to gravity. Solving this equation for the terminal velocity gives

\[
v_t = \frac{2}{9} \frac{R^2 (\rho_s - \rho_l)}{g / \mu} = \frac{1}{18} a^2 (\rho_s - \rho_l) / g / \mu, \tag{B.2}
\]

where \(a\) is the diameter of the floc.

The terminal velocity obtained from (B.2) has been equated with that given below:

Amount flocculated

\[= (vt \times \text{initial weight/height of the suspension} (h)) \tag{B.3} \]

\((h = 5 \text{ cm s})\).
References