Recipes for the Linear Analysis of EEG
… and applications…

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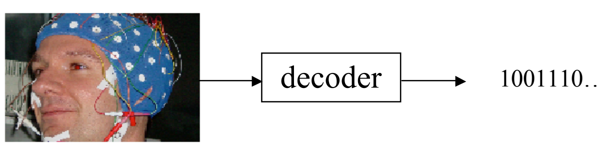
Can we “read” the brain non-invasively and in real-time?



if YES then

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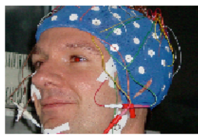
Can we "read" the brain non-invasively and in real-time?



decoder → 1001110…

if YES then

Rehabilitation
Cognitive Neuroscience
Performance Augmentation

requires single-trial analysis

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Outline

- Tutorial on the Linear Analysis of EEG
- Matlab/EEGLab Demo
- Real-time, On-line Applications: Image Triage and Error Correction

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Estimating "Interesting" Components Through Projections

$$y(t) = \mathbf{w}^T \mathbf{x}(t) = \sum_{i=1}^{D} w_i x_i(t)$$

... what is $\mathbf{w}$*?*

Estimating "Interesting" Components Through Projections

<u>Signal summation</u>

$$\text{noise } n_1(t) \text{ and } n_2(t)$$

$$x_1(t) = s(t) + n_1(t)$$

$$x_2(t) = s(t) + n_2(t)$$

choose $\mathbf{w}^T = [1, 1]$

$$y(t) = 2s(t) + n_1(t) + n_2(t)$$

3dB improvement in SNR

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Estimating "Interesting" Components Through Projections

Signal subtraction

$$x_1(t) = s_1(t) + s_2(t)$$
$$x_2 = s_2(t)$$

choose $\mathbf{w}^T = [1, -1]$

$$y(t) = x_1(t) - x_2(t) = s_1(t)$$

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Estimating "Interesting" Components Through Projections

Linear Model for EEG

forward model

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t)$$
$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t)$$

Source Estimation by Linear Projection

backward model

$$\hat{\mathbf{s}}(t) = \mathbf{V}^T\mathbf{x}(t)$$

For Gaussian noise with known correlation structure this is an ML estimator

$$\hat{\mathbf{V}}^T = \mathbf{A}^{\#} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T$$

noise collinear with the source

$$\hat{\mathbf{s}}(t) = \mathbf{s}(t) + \mathbf{V}^T\mathbf{n}(t)$$

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Estimating "Interesting" Components Through Projections

Minimizing Interference via Subtraction

$$\hat{\mathbf{s}}(t) = \mathbf{A}^{\#}\mathbf{x}(t)$$

Estimate interfering source (backward model)

$$\mathbf{x}_{\|}(t) = \mathbf{A}\hat{\mathbf{s}}(t)$$

Estimate contribution to measurements (forward model)

$$\mathbf{x}_{\perp}(t) = \mathbf{x}(t) - \mathbf{x}_{\|}(t) = (\mathbf{I} - \mathbf{A}\mathbf{A}^{\#})\mathbf{x}(t)$$

$\mathbf{x}_{\perp}(t)$ has no activity correlated with $\hat{\mathbf{s}}(t)$

however it has reduced rank--
must deal with appropriately

Estimating "Interesting" Components Through Projections

Forward Model Estimate

$$\mathbf{y} = [y(t_1), ..., y(t_N)], \text{ and } \mathbf{X} = [\mathbf{x}(t_1), ..., \mathbf{x}(t_N)]$$

forward model $\hat{\mathbf{a}}_y$ – one column of the matrix $\mathbf{A}$

$\hat{\mathbf{a}}_y$ can be found by linearly predicting $\mathbf{x}(t)$ from $y(t)$

$$\hat{\mathbf{a}}_y = \mathbf{X}\mathbf{y}^T(\mathbf{y}\mathbf{y}^T)^{-1}$$

"scalp projection"

Some Objectives for Finding Interesting Components

… or how do we estimate $\mathbf{w}$*…*

- Maximum Difference
- Maximum Power
- Statistical Independence

Maximum Difference

$$\Delta\mathbf{x}(\tau) = \frac{1}{N_1}\sum_{t_1}\mathbf{x}(t_1+\tau) - \frac{1}{N_2}\sum_{t_2}\mathbf{x}(t_2+\tau)$$

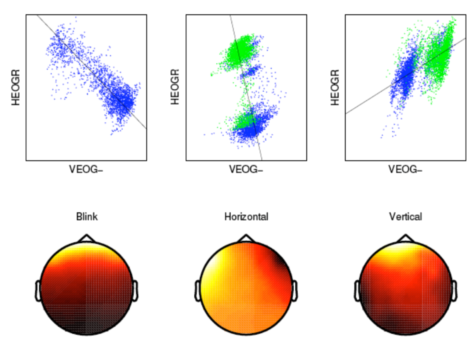
$$\overline{\Delta\mathbf{x}} = \sum_{\tau}\Delta\mathbf{x}(\tau)$$

$$\hat{\mathbf{a}}_y = \frac{\overline{\Delta\mathbf{x}}}{(N_1+N_2)(y_1-y_2)}$$

$$\hat{\mathbf{A}}_{\mathrm{eye}} = [\hat{\mathbf{a}}_{\mathrm{b}}, \hat{\mathbf{a}}_{\mathrm{h}}, \hat{\mathbf{a}}_{\mathrm{v}}]$$

$$\hat{\mathbf{s}}(t) = \hat{\mathbf{A}}^{\#}_{\mathrm{eye}}\mathbf{x}(t)$$

$$\hat{\mathbf{s}}(t) = [\hat{s}_{\mathrm{b}}, \hat{s}_{\mathrm{h}}, \hat{s}_{\mathrm{v}}]^T$$

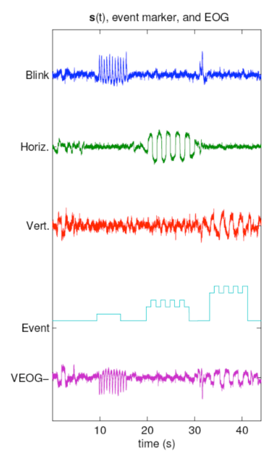


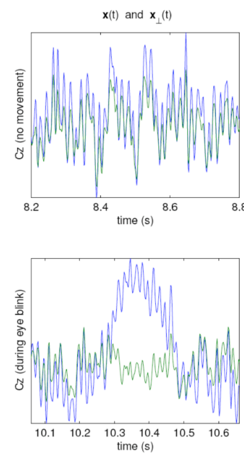
removal of eye-motion artifacts

$$\mathbf{x}_{EBR}(t) = (\mathbf{I} - \hat{\mathbf{A}}_{\mathrm{eye}}\hat{\mathbf{A}}^{\#}_{\mathrm{eye}})\mathbf{x}(t)$$

Use all electrodes in estimation of interference

Maximum Difference





No blink

Blink

Maximum Difference

<u>Maximum Magnitude Difference</u>

$$\mathbf{w}_{\text{erd}} = \mathbf{v} = \mathbf{a}^{\#T} = \overline{\Delta x}/\|\overline{\Delta x}\|^2$$

$$\mathbf{w}_{\text{ml}} = \mathbf{R}^{-1}\overline{\Delta x}$$

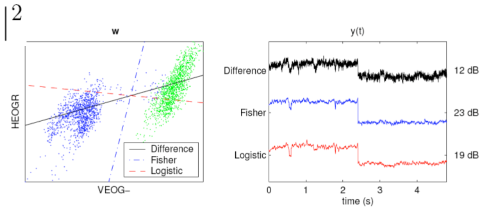
$$\mathbf{w}_{\text{fld}} = (\mathbf{R}_1 + \mathbf{R}_2)^{-1}\overline{\Delta x}$$

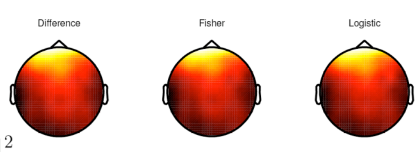
$$\mathbf{w}_{\text{lr}} = \arg\min_{\mathbf{w}} L(\mathbf{w}, b)$$

$$L(\mathbf{w}, b) = -\sum_t \log p(c_t|y_t)$$

$$L(\mathbf{w}, b) = -\sum_t \log p(c_t|y_t) + \frac{\lambda}{2}\|\mathbf{w}\|^2$$

$$p(c = +1|\mathbf{x}) = f(y) = \frac{1}{1+e^{-y}} = \frac{1}{1+e^{-(\mathbf{w}^T\mathbf{x}+b)}}$$





Maximum Power

$$\mathbf{w}_{\mathrm{pc}} = \underset{\mathbf{w}, \|\mathbf{w}\|=const.}{\arg\max} \sum_t y^2(t) = \underset{\mathbf{w}}{\arg\max} \frac{\mathbf{w}^T \mathbf{R} \mathbf{w}}{\mathbf{w}^T \mathbf{w}}$$

$$\hat{\mathbf{a}}_{\mathrm{pc}} = \mathbf{R}\mathbf{w}_{\mathrm{pc}} \left(\mathbf{w}_{\mathrm{pc}}^T \mathbf{R} \mathbf{w}_{\mathrm{pc}} \right)^{-1} = \frac{\mathbf{w}_{\mathrm{pc}}}{\|\mathbf{w}_{\mathrm{pc}}\|^2}$$

<u>Maximum Power-Ratio</u>

$$\begin{aligned} \mathbf{w}_{\mathrm{ge}} &= \underset{\mathbf{w}, \|\mathbf{w}\|=1}{\arg\max} \frac{\sum_{t_2} \sum_\tau y^2(t_2+\tau)}{\sum_{t_1} \sum_\tau y^2(t_1+\tau)} \\ &= \underset{\mathbf{w}}{\arg\max} \frac{\mathbf{w}^T \mathbf{R}_2 \mathbf{w}}{\mathbf{w}^T \mathbf{R}_1 \mathbf{w}} . \end{aligned}$$

Maximum Power

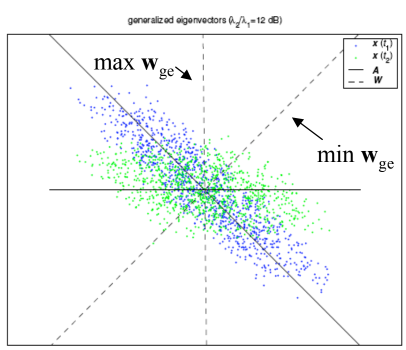


Fig. 5. Generalized eigenvalues and independent components. Dark and light dots indicate (artificial) samples with covariance matrix $\mathbf{R}_1$ and $\mathbf{R}_2$. Dashed lines indicate the projection vectors $\mathbf{w}_{ge}$ that generate the maximum and minimum power-ratio for projected component $y(t)$ on all samples. Solid lines indicate the columns of the corresponding $\hat{\mathbf{A}}_y$.

Maximum Power

ERD/ERS with generalized eigenvalues.

Subject responds to a visual stimulus with a button press.

Prior to the maximum-power ratio analysis, all EEG channels are bandpass filtered between 5-40Hz.

The covariance matrices $\mathbf{R}_1$ and $\mathbf{R}_2$ are computed in a window 200ms before ($\mathbf{R}_1$) and 200ms after ($\mathbf{R}_2$) the button press.

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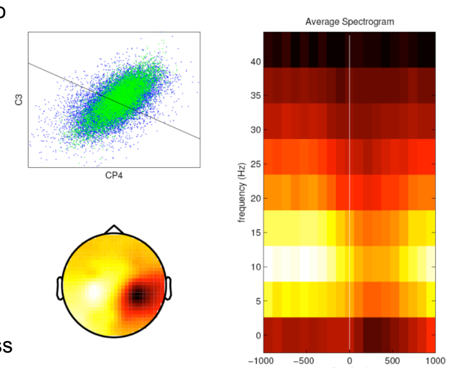
Maximum Power

Top left: Scatter plot of the corresponding activity for two of the 64 EEG sensors. Solid line indicates the orientation, $\mathbf{w}_{ge}$, along with the two distributions having a maximum power (variance) ratio, estimated using generalized eigenvalues.

Bottom left: Estimated forward model corresponding to $\mathbf{w}_{ge}$. Clear is that the source activity originates over motor areas (it is maximal over C3 and CP4) and has opposite sign (180 phase delay) between the hemispheres

Right: Spectrogram computed for the component $\mathbf{y}(t)$ (averaged over 300 button press events) Button press indicated with a vertical white line. Alpha band activity (maximal at 12Hz for this subject) decreases (de-synchronizes) for about 500ms after the button push.



Statistical Independence

Statistical independence implies for all $i \neq j, t, \tau, n, m$:

$$E[s_i^n(t)\, s_j^m(t+\tau)] = E[s_i^n(t)]E[s_j^m(t+\tau)]$$

For M sources and N sensors each t, τ, n, m gives $M(M\text{-}1)/2$ conditions for NM unknowns in $\boldsymbol{A}$.

Sufficient conditions if we use multiple:

use	sources assumed	condition	statistic	algorithm
t	non-stationary	$\mathbf{W}\,\mathbf{R}_\mathbf{x}(t)\,\mathbf{W}^T = \text{diag}$	covariance	decorrelation
τ	non-white	$\mathbf{W}\,\mathbf{R}_\mathbf{x}(\tau)\,\mathbf{W}^T = \text{diag}$	cross-correlation	SOBI
n, m	non-Gaussian	$\mathbf{W}\,\mathbf{C}_\mathbf{x}(i,j)\,\mathbf{W}^T = \text{diag}$	4th cumulants	JADE (ICA)

Example: Non-stationary Independent Sources

The independence assumption establishes that the covariance $\mathbf{R}_\mathbf{x}(t)$ is diagonalized by $\mathbf{W}$ for all times t:

$$\mathbf{R}_\mathbf{y}(t_1) = \mathbf{W}\,\mathbf{R}_\mathbf{x}(t_1)\,\mathbf{W}^T = \text{diag}$$
$$\mathbf{R}_\mathbf{y}(t_2) = \mathbf{W}\,\mathbf{R}_\mathbf{x}(t_2)\,\mathbf{W}^T = \text{diag}$$

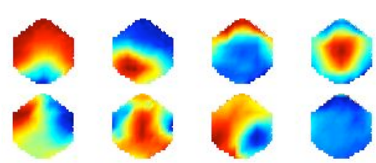
Combining these we obtain the solutions again with the Generalized Eigen-vectors:

$$\mathbf{R}_\mathbf{x}(t_2)^{-1}\mathbf{R}_\mathbf{x}(t_1)\,\mathbf{W} = \mathbf{W}\,\lambda$$

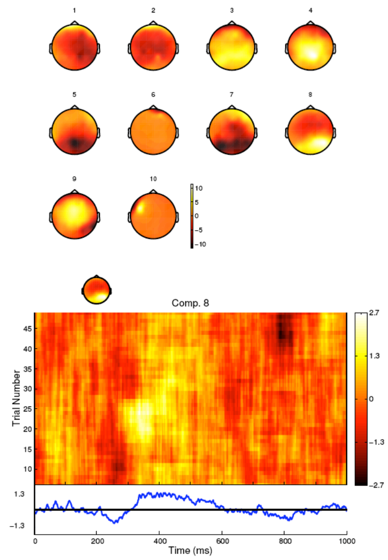
More robust if we use simultaneous diagonalization of multiple covariances.

Example: First 8 independent components that explain 64 observed EEG sensors $\mathbf{x}$ in visual discrimination task 250 ms before and after stimulus presentation

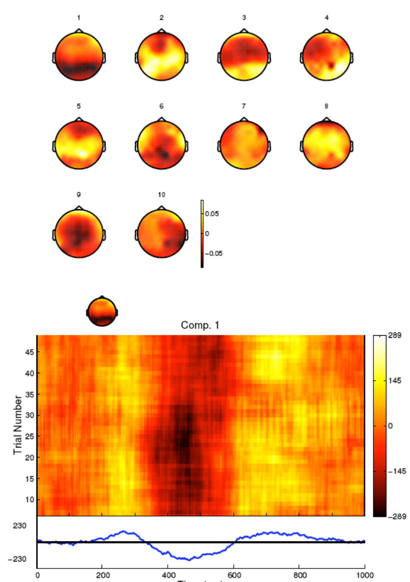
EEG sensor projections $\mathbf{A}=\mathbf{W}^{-1}$



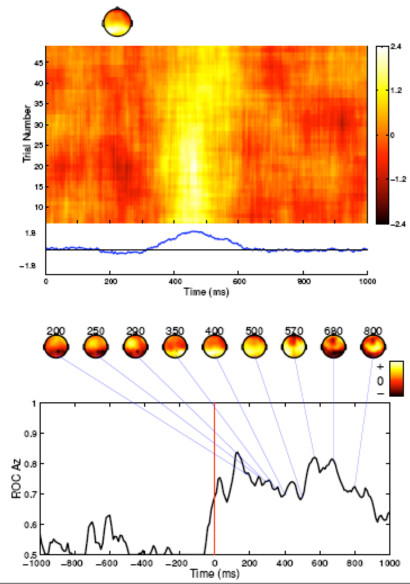
ICA Components



GEVD Components



LR Components



Demo

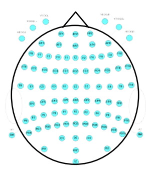
Using Linear Multivariate Processing

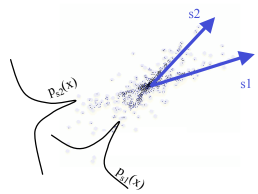
$$Y = WX$$

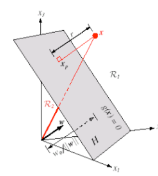
recovered sources → Y; *transformation matrix* → W; *observations* → X

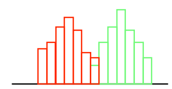
Data → artifact removal and dimensionality reduction → linear discrimination → (1) → P(T)

64-128 channels







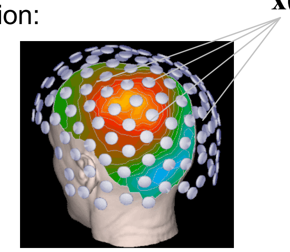


Single-trial Detection with Spatial Integration

Conventional Event Related Potentials (ERP) averages over trials. We substitute trial averaging by spatial integration:

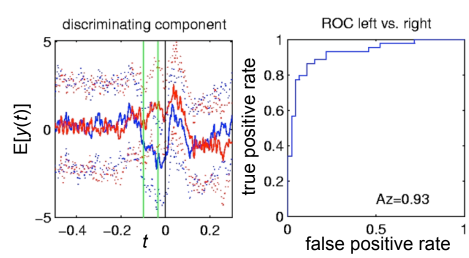
$$s(t) = \mathbf{w}^T\,\mathbf{x}(t)$$

$\mathbf{x}(t)$



Linear discriminants: Compute spatial weighting **w** which maximally discriminates sensor array signals $\mathbf{x}(t)$ for two different conditions.

Ex: Detect motor planning activity Predict button press from 122 MEG sensors with linear discriminator $\mathbf{w}$ such that $s(t)$ differs the most during 100-30 ms window *prior* to button push.



Localization of Discriminating Component

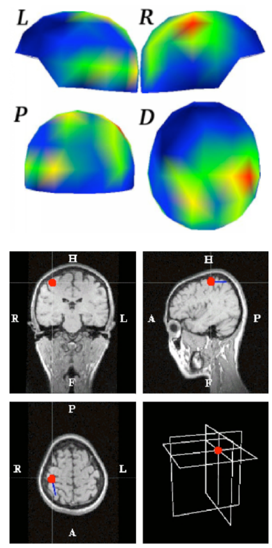
... possible because we have a linear model ...

What is the electrical coupling $\mathbf{a}$ of the hypothetical source s that explains most of the activity $\mathbf{X}$?

Least squares solution:

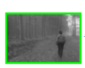
$$\mathbf{a} = \frac{\mathbf{Xs}}{\mathbf{s}^{\mathrm{T}}\mathbf{s}}$$

Strong coupling indicates low attenuation. Intensity on these "sensor projections" $\mathbf{a}$ indicates closeness of the source to the sensors.

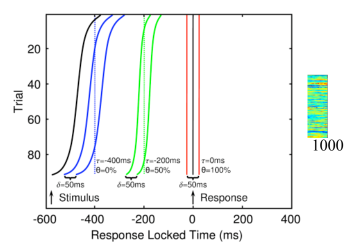


Single-trial Discrimination

Linear discriminants: Compute spatial weighting $\mathbf{w}$ which maximally discriminates sensor array signals $\mathbf{x}(t)$ for two different conditions.

 vs. 

$$\mathbf{y}(t) = \mathbf{w}^{T}_{\tau,\delta,\theta}\mathbf{x}(t) \qquad \tau = \left\{t_k - \frac{\delta}{2} \rightarrow t_k + \frac{\delta}{2}\right\}$$



Localization of Discriminating Component
possible because we have a linear model

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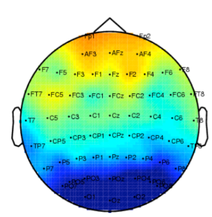
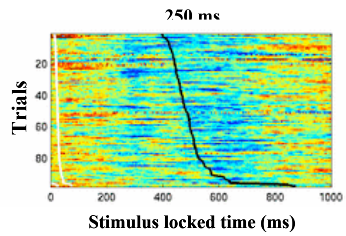
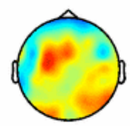


Parra, Sajda et al. Neuroimage, 2002
Parra, Spence, Gerson & Sajda, Neuroimage, 2005

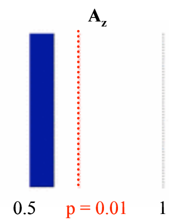
Single-trial Discrimination



$$\mathbf{y}(t) = \mathbf{w}^T_{\tau,\delta,\theta}\mathbf{x}(t)$$



$$\mathbf{a} = \frac{\mathbf{X}\mathbf{y}}{\mathbf{y}^T\mathbf{y}}$$



Discrimination performance

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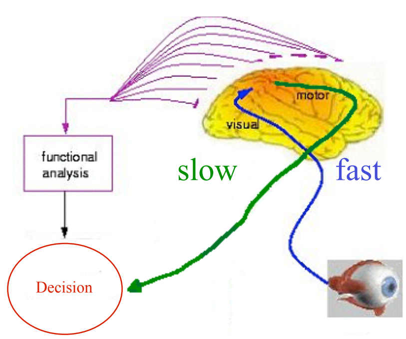
Application: Cognitive User Interface

Hypotheses:

- EEG can be used to detected cognitive events related to visual target detection, discrimination, and perceived error.
- Such cognitive events can be detected more quickly and reliably than overt (motor) responses.



Objective: Use EEG signatures of cognitive events to improve task performance

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Cortically-coupled computer vision (C3 Vision)

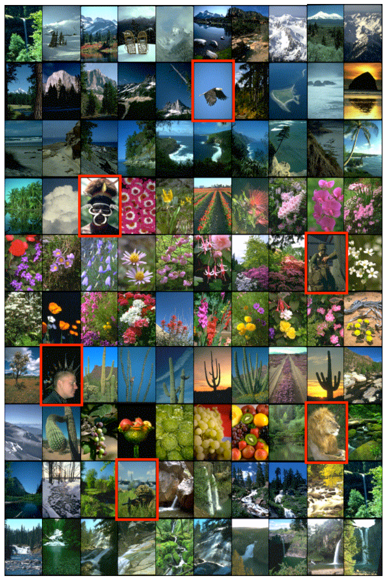
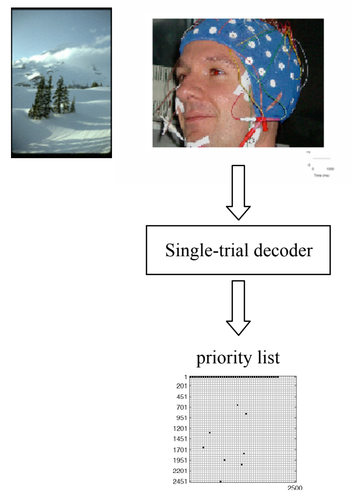


Image Sequence



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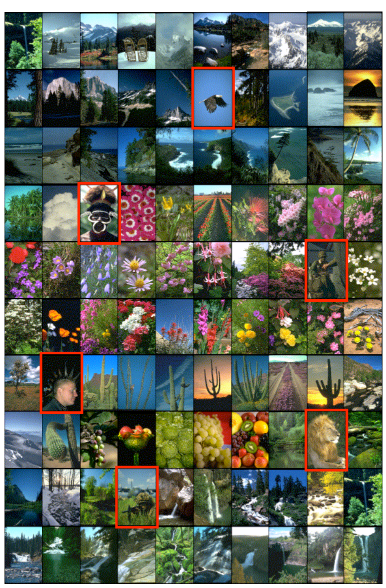
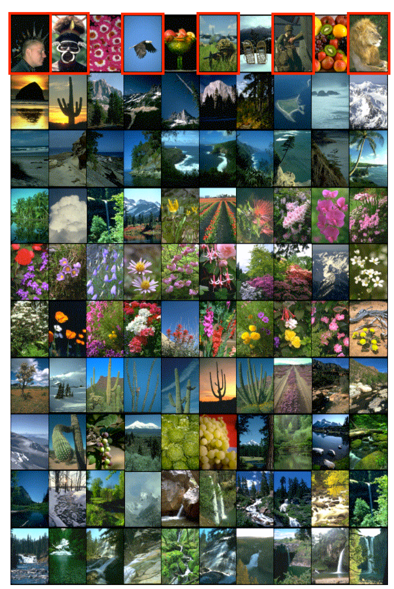
Cortically-coupled computer vision (C3 Vision)



Pre-triage



Post-triage

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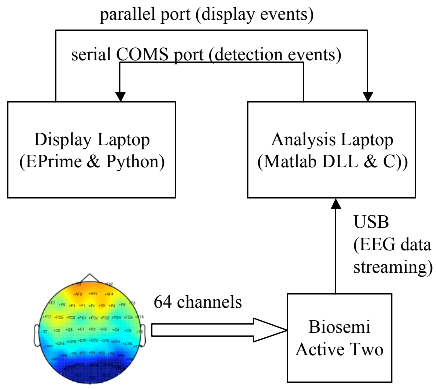
On-line Real-time Portable Image Triage System



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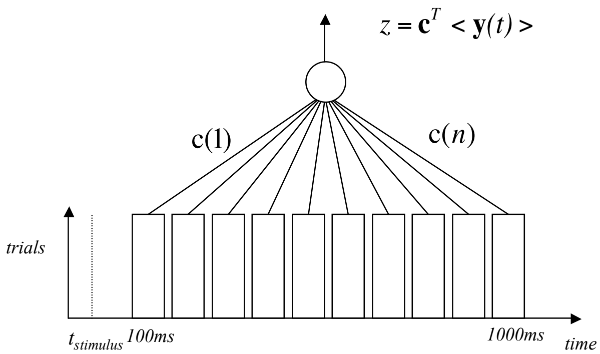
Hierarchical Discriminating Components

…online estimation of all parameters…



$$\mathbf{y}(t) = \mathbf{w}_\tau^T \mathbf{x}(t) \qquad \tau = \left\{ t_k - \frac{\delta}{2} \ldots t_k + \frac{\delta}{2} \right\}$$

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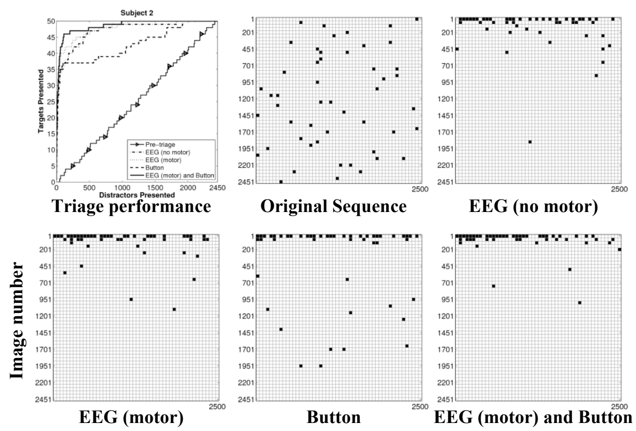
C^3 VISION

Triage results



Gerson, Parra & Sajda, IEEE TNSRE, 2006
Sajda et al., Trends in BCI, 2007

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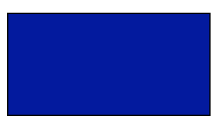
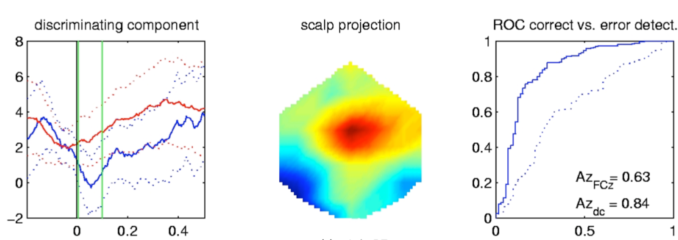
Detection of Error Related Negativity During a Visual Discrimination Event

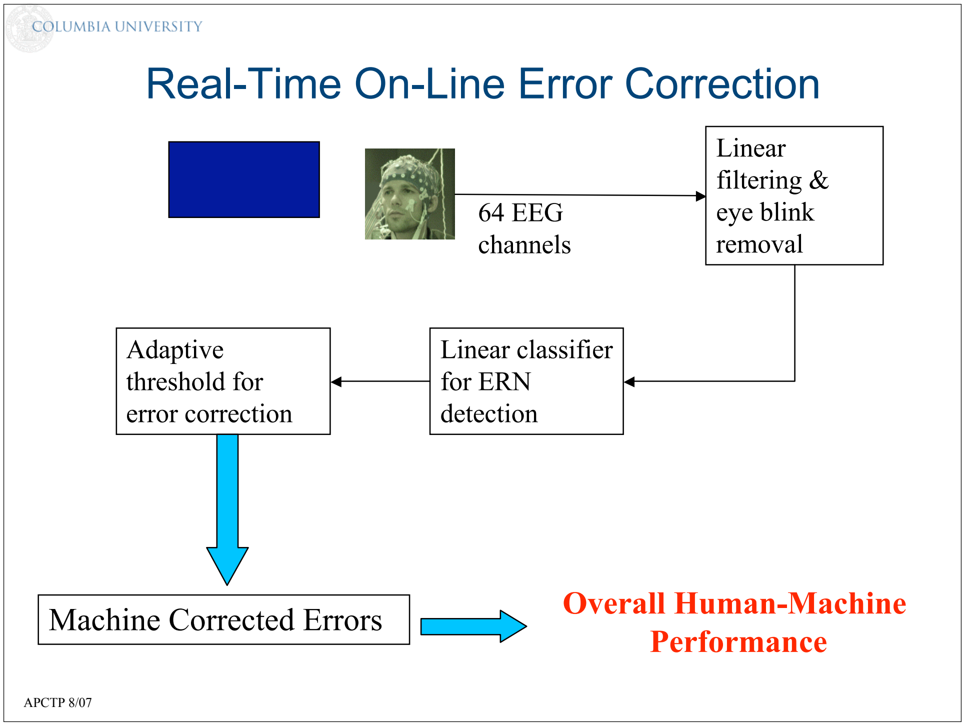
Error Related Negativity (ERN) occurs following perception of errors. It is hypothesized to originate in Anterior Cingulate and to represent response conflict or subjective loss.

Example: Erikson Flanker task



Discrimination of error versus correct response (64 EEG sensors, 100ms)





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Real-Time On-Line Error Correction
Linear filtering & eye blink removal
64 EEG channels
Adaptive threshold for error correction
Linear classifier for ERN detection
Machine Corrected Errors
Overall Human-Machine Performance
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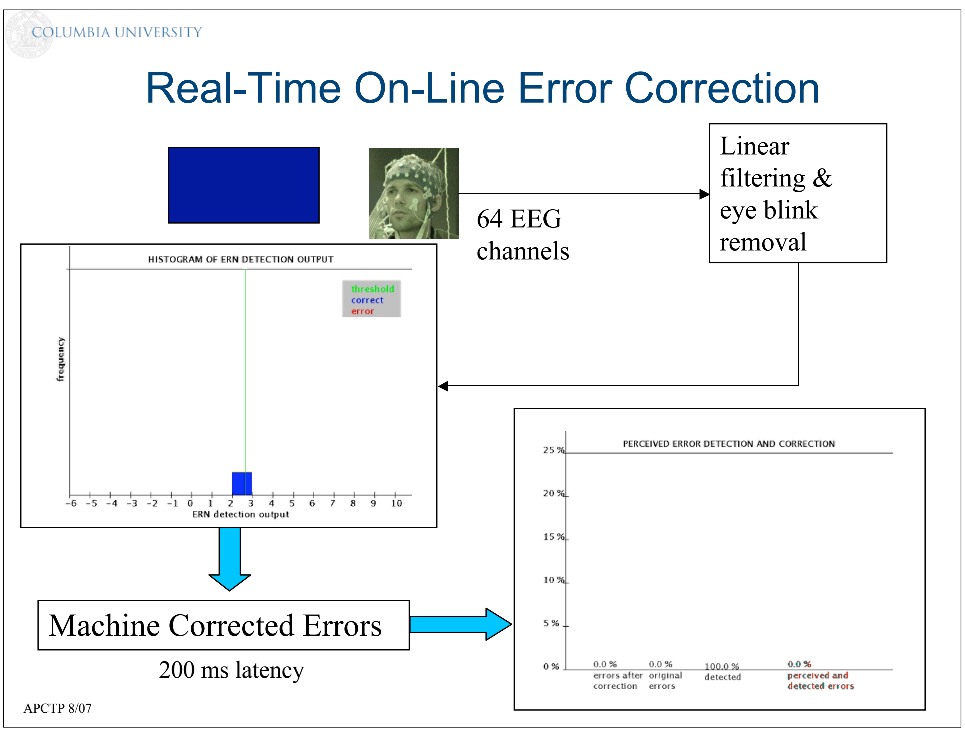

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Real-Time On-Line Error Correction
64 EEG channels
Linear filtering & eye blink removal
HISTOGRAM OF ERN DETECTION OUTPUT
threshold
correct
error
frequency
−6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6 7 8 9 10
ERN detection output
Machine Corrected Errors
200 ms latency
PERCEIVED ERROR DETECTION AND CORRECTION
25 %
20 %
15 %
10 %
5 %
0 %
0.0 % errors after correction
0.0 % original errors
100.0 % detected
0.0 % perceived and detected errors
APCTP 8/07

Further Reading/Info

- Papers and code at http://liinc.bme.columbia.edu