Optimal Liquidity Provision for Decision Makers

Paul Tetlock*

Robert Hahn†

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ABSTRACT

Although prices in financial markets play an important role in improving allocative efficiency in the real economy, few models of securities markets explicitly incorporate resource allocation decisions. In this paper, we study the equilibrium in a securities market when the market price provides valuable information that can improve allocative efficiency. We show that a decision maker will subsidize liquidity in an illiquid securities market to gather valuable information about her decision payoffs. We also show that a decision maker's liquidity subsidy improves expected social welfare by enhancing allocative efficiency, but does not induce the socially optimal level of information acquisition. Finally, we demonstrate that the mere act of linking the allocation decision to the market price will typically enhance liquidity in the securities market. Overall, our results highlight the potential of using securities markets for information to improve decisions.

1 Introduction

There is a growing body of theoretical and empirical research on securities markets for contingent claims on uncertain events, sometimes called prediction or information markets.

*Assistant Professor, Department of Finance, University of Texas at Austin; e-mail: tetlock@mail.utexas.edu.

†Executive Director, AEI-Brookings Joint Center for Regulatory Studies, Washington, DC. The authors would like to thank Aydogan Alti, Andres Almazan, John Griffin, Lorenzo Garlappi, Robin Hanson, John Ledyard, Don Lien, Paul Milgrom, Marco Ottaviani, Sheridan Titman, Justin Wolfers, Eric Zitzewitz, and participants at the London Business School prediction markets conference and in the UT Austin finance seminar series for helpful comments. The usual disclaimer applies.

The main finding of this literature is that the prices in prediction markets can be used to provide forecasts that have a lower mean squared prediction error than conventional alternatives. For example, the prices in prediction markets produce more accurate forecasts of presidential election outcomes than election polls (Berg, Forsythe, Nelson and Rietz (2003)), better forecasts of printer sales than official corporate forecasts (Chen and Plott (2002)), and improved weather forecasts relative to statistical algorithms used by the National Weather Service (Roll (1988)).[1]

Several researchers have noted the potential of prediction markets to improve decisions (e.g., Hanson (2002), Abramowicz (2004), and Sunstein (2006)). In principle, the range of applications is virtually limitless—from helping business make better investments decisions to helping government make better decisions on fiscal or monetary policy. Decision makers already informally condition their policies on market prices as a matter of routine. For example, firms condition their decisions of whether to launch a new product on prevailing prices in related markets. Firms often decide whether to issue equity based on recent equity returns. The Federal Reserve decides whether to cut interest rates based on inflation rates.

In this paper, we explicitly model a decision maker who optimally conditions her decision on a securities market price, and intervenes in this market to obtain better information. For example, suppose the government were interested in managing hurricane disasters. It could provide liquidity in a hurricane futures market to increase the incentives of weather experts to furnish information about the probability of hurricanes, and then act on that information accordingly.

While few would dispute the potential of prediction markets, several scholars have noted that there are fundamental theoretical challenges (see, *e.g.*, Wolfers and Zitzewitz (2006) and Ledyard (2006)). A key issue is whether linking a prediction market to a decision—what we call a decision market—changes the strategies of various agents. For example, a trader who has an interest in a decision based on a prediction market may behave differently in that market if she knew that her behavior in the market could possibly affect the decision. We need a theory to inform how to design such markets that explicitly includes the role of the decision maker. Furthermore, we need a theory that helps to measure the extent to which decision markets can be expected to improve individual decisions and social welfare.

To date, we are not aware of any paper that addresses these problems directly and only one that does so indirectly (Holmstrom and Tirole (1993)). This paper attempts to fill that void, focusing on the economic value that is created when decisions are made based

[1] See Wolfers and Zitzewitz (2004) for a more comprehensive survey of research on prediction markets.

on better information (*e.g.*, Blackwell (1951, 1953), Raiffa and Schlaifer (1961) and Raiffa (1968)). Specifically, we are interested in how to use prediction markets to make better private and public resource allocation decisions.

The prediction market modeled here is a special case of a securities market in which no capital is raised and the security pays off an amount based on a verifiable measure of the decision maker's production revenues. Our motivation is that a decision maker may not require financing for a production task, but could still benefit from decision-relevant information revealed by a securities market price.[2] Because production revenues depend on the general economic environment, knowledgeable experts who cannot productively undertake the endeavor often have useful information about its payoffs.[3] Sometimes it is too costly for the decision maker to find, identify, and write individually tailored contracts with these experts. Furthermore, some experts will be more willing to reveal information if they can remain anonymous.[4] These considerations suggest that a decision maker with limited information about her production revenues should consider using a securities market as a means for aggregating experts' information.

To explore this possibility further, we model the equilibrium in a securities market when the security price provides valuable information that can improve allocative efficiency. In general, we show that a decision maker will provide a liquidity subsidy in a securities market to enhance the informational efficiency of the security price and her production profits. The liquidity subsidy, which equals her trading losses, is more than offset by the increase in her production profits.

We make three contributions to the theory of securities markets. First, to our knowledge, our approach is the first to demonstrate how a rational decision maker will participate in a securities market for decision-relevant information. It is also the first to formally justify a direct liquidity subsidy as a means for improving decision making. Our models show that a decision maker will provide a liquidity subsidy in situations where a competitive market maker would provide very little or no liquidity. Our second contribution is to show that

[2]Although the prices in securities markets play an important role in improving decisions in the real economy, only a few models of securities markets directly address investment and production decisions (*e.g.*, Diamond (1980), Holmstrom and Tirole (1993), Dow and Gorton (1997) and Subramanyam and Titman (1999)). In contrast to our study, all of these models focus on capital markets in which the entrepreneur raises funding for a productive asset.

[3]Similarly, people inside a firm who do not control an allocation decision may be able to provide useful information.

[4]Osband (1989) develops an insightful model of how a principal would write individual contracts with non-anonymous agents who can acquire valuable information.

the decision maker's liquidity subsidy always improves expected social welfare by enhancing allocative efficiency, but does not induce the socially optimal level of information acquisition. Finally, we show how the mere act of linking a resource allocation decision to the securities market price will typically enhance liquidity even when the decision maker does not intervene in the securities market. Well-known theorems in finance that imply trade between rational agents will not occur (*e.g.*, Milgrom and Stokey (1982)) do not apply to our models because these theorems do not consider the possibility that rational agents will use the market price as a basis for decision making.

The plan of the paper is as follows. Section 2 provides a brief overview of related work. Section 3 introduces the modeling framework to be used throughout the paper. To illustrate the structure of our models, we consider the situation of a profit-maximizing farmer, who must choose how much effort to invest in caring for her fields. The farmer can improve her effort decision by learning from the equilibrium price in a rainfall futures market. We formalize the farmer's problem in a general model of a securities market for decision-relevant information when there are only rational informed traders. In section 4, we add further realism to the model by examining the equilibrium when there is an additional securities trader whose payoff depends on the allocation decision. Section 5 highlights key findings and discusses areas for future research.

2 Literature Review

Our treatment of decision market liquidity draws heavily on important results in finance. Milgrom and Stokey's (1982) no-trade theorem is particularly relevant because it provides conditions under which no transactions will occur in securities markets. Our model gives rise to trade because the decision maker has an additional motivation for trading in the securities market. The information she obtains from this market is then used to increase her production profits. In this respect, our model is related to Glosten (1989) who argues that a monopolist specialist may provide more liquidity than a competitive market maker because the competitive market maker must always break even on his trades. The main difference is that our model does not require any liquidity traders to keep the market open because the decision maker in our model can afford to always lose money on her trades.

We consider a securities market that is organized by either a competitive market maker or a decision maker whose trading profits are constrained by competitive entry. We adopt our notion of securities market directly from Kyle (1985), who defines the inverse of liquidity as the price impact of aggregate order flow; however, we extend the Kyle (1985) model to include a new type of trader, the decision maker, who values the information embedded in informed traders' order flow. The introduction of this additional trader is also the key difference between our models and Milgrom and Stokey's (1982) model.

Our model leads to a trade-off between the costs of providing liquidity and the benefits of obtaining information similar to Holmstrom and Tirole (1993). Although both models consider a profit-maximizing risk neutral decision maker, the decision maker in our model directly subsidizes securities market liquidity—and she does so in the presence of a competitive market maker, even when there are no traders with exogenous liquidity motives. In Holmstrom and Tirole (1993), the firm owner indirectly subsidizes liquidity by issuing underpriced capital to traders with exogenous liquidity needs, which would not be possible if either a competitive market maker participates in the initial public offering or there are no liquidity traders. In contrast, our model applies to competitive secondary equity markets where there is frictionless arbitrage. Moreover, our model predicts there will be some trade even when there are no liquidity traders because of the special role of the decision maker.

Our first model consists of a decision maker and a finite number of informed traders, who can acquire information about the production output from the decision maker's effort. There is a market for securities that pay an amount related to the decision maker's production output. Following Grossman and Stiglitz (1980), we assume that informed traders optimally choose how much information to acquire about the payoff of the security based on their expected trading profits and the cost of acquiring information.

Our second model includes the decision maker, an informed trader, and a stakeholder whose payoff depends directly on the effort allocation decision. The results on liquidity in this model build on Kumar and Seppi (1992). Our stakeholder model also resembles the model of price manipulation developed by Hanson and Oprea (2004), except that our model explicitly links allocation decisions to prediction market prices. By making this linkage, we can identify how the decision maker should design a prediction market to maximize her welfare. Our model of price manipulation is also related to a subsequent paper by Ottaviani and Sorensen (2007), which models the interesting case of outcome manipulation where traders can affect the probability that different contract payoffs are realized.

There are several papers in finance that study the interaction between allocative efficiency in the real economy and informational efficiency in securities markets, including Diamond (1980, Chapter 1), Dow and Gorton (1997), and Subrahmanyam and Titman (1999). Our main contribution is to examine the impact of this interaction on securities market liquidity.

Most papers that address liquidity assume exogenous liquidity (or noise) traders, whereas we treat liquidity as an endogenous variable determined by rational agents.

In a somewhat different context and framework, Sherman and Titman (2002) examine a similar question to ours. The authors examine the bookbuilding process for an IPO order book, assuming that an underwriter cares about how accurately he prices an issue. Sherman and Titman (2002) find that underwriters who care most about accuracy will tend to underprice issues by the most. Their idea is similar in spirit to the liquidity subsidy idea embodied in this paper.

Our description of the conditions under which trade will occur in securities markets is related to earlier work by Diamond (1980, Chapter 1) and concurrent work by Bond and Eraslan (2005). Both papers make the point that trade in securities markets can occur when it releases information that is socially valuable for resource allocation, but both models differ from ours in one important respect. A common feature of the models in Diamond (1980) and Bond and Eraslan (2005) is that the efficient owner of the productive asset is unknown in advance. This means that the existence of trade in their models depends crucially on which agent possesses valuable production information and the agents' initial endowments of the asset. Both Diamond (1980) and Bond and Eraslan (2005) interpret their models as describing the transfer of a controlling interest in a corporation.

A concurrent paper by Dow, Goldstein and Guembel (2006) starts from a similar premise to ours that information may be underprovided in a securities market. These authors argue that firms would like to commit to over-investing in certain projects to encourage information production by market participants. In our simple model, the decision maker has no such incentive because the security payoff is independent of the decision maker's choice. In general, however, the Dow, Goldstein and Guembel (2006) idea is a complementary means for enhancing the informational efficiency of securities prices.

3 Markets with Only Rational Informed Traders

To illustrate the structure of the model, we consider the situation of a profit-maximizing farmer, who must choose how much planting effort to put into her fields. We then formalize the farmer's problem by developing a general model of a securities market for production-relevant information when there are only rational informed traders.

3.1 Modeling Framework

The farmer's profit-maximizing quantity of planting effort depends on an uncertain quantity of rainfall in her county this year. For simplicity, suppose that this particular farmer is the only one who knows her land well enough to successfully produce crops, implying she is the efficient land owner.

Within the farmer's local community, suppose that there are several weather experts who can acquire informative, but costly, signals about future local rainfall. We assume that the experts' payoffs do not directly depend on the farmer's planting effort decision. Furthermore, to motivate the need for an anonymous market, we assume that it is too costly for the farmer to find, recognize, and write individually tailored contracts with these weather experts to obtain their rainfall signals.

Instead, the farmer could set up a market for contingent claims on rainfall that is open to public participation at the local exchange. The owner of a rainfall security receives a payment proportional to realized local rainfall. In this simple example, ignoring other agents with possible hedging motives, only the weather experts would potentially want to participate in such a market. Uninformed agents would fear that they will end up taking the other side of a transaction with the weather experts. Thus, it would seem that a competitive market maker would be unwilling to supply any liquidity in such a market and that the no-trade theorem of Milgrom and Stokey (1982) would apply.

Indeed, the theorem would apply if the farmer were precluded from participating in or making the market. However, even though the farmer is uninformed, she may want to participate in the rainfall market because she can use the market price as the basis for her planting effort decision.[5] Specifically, suppose the farmer acts as a market maker and provides some amount of liquidity in the rainfall market. Anticipating that they could profit by trading on rainfall information, the weather experts would be willing to incur positive costs to acquire such information. After observing their signals, the weather experts would buy rainfall securities in proportion to their signal realizations, obtaining expected profits by trading with the farmer. Because the equilibrium rainfall price will depend on informed traders' orders, the price of rainfall can inform the farmer's effort allocation decision.

Thus, the benefit of trading rainfall securities for the farmer is that the equilibrium price provides valuable information. The farmer is willing to incur expected trading losses in the

[5] The price in the rainfall claims market could be used as an input to the farmer's allocation decision rather than as the sole basis for her decision. In our simple model, it is always optimal for the farmer to rely exclusively on the securities price because she has no private information.

rainfall market up to the point where her expected marginal trading loss equals the expected marginal benefit from obtaining better information through the rainfall market price. Even in the absence of noise traders or other traders whose payoffs depend on the farmer's effort decision, there will be trading in the rainfall market between the farmer and the informed traders.

We formalize this example by considering two models: one with N informed traders and no decision stakeholder, whose payoffs depend on the effort decision; and a second with one informed trader and one stakeholder. The first model with several informed traders illustrates that trade will occur even in the "extreme" case in which only experts participate in the market. In section 4, we present a second model, which shows how adding another decision stakeholder changes the equilibrium properties. We first present those aspects of the decision maker's problem that are common to both models. This is followed by a detailed analysis of the equilibrium properties of both models.

We consider a securities market for contingent contracts, each yielding an uncertain amount y, which is the realized productivity of the decision maker's effort. For our main results, it is only necessary that there is an informative signal about effort productivity that can be verified ex post. In the example above, a farmer's choice of planting effort in a given year may depend on her best forecast of annual rainfall. After a year has elapsed, a verifiable productivity signal would be the annual rainfall measure reported by the National Weather Service for the farmer's county.

We assume that there are at most three types of agents who may participate in the securities market: the decision maker, traders who have private information about the her decision's payoffs, and an uninformed competitive market maker. A decision maker will undertake a certain level of effort (x) based on her best forecast of her effort's productivity (y). In this model, traders and the competitive market maker have no inherent interest in the effort level selected (x).

Initially, all experts and the decision maker have a common prior belief that productivity (y) is normally distributed according to $y \sim N(0, V_0)$ with $V_0 > 0$.[6] Before the market opens, each informed trader i can acquire a productivity signal $s_i = y + \varepsilon_i$, where $\varepsilon_i \sim N(0, V_i)$ independent of y. By incurring a greater cost, the informed trader can improve his signal of s_i in the sense of decreasing the variance of the distribution of s_i. In particular, we assume

[6] A downside of the normal distribution assumption is that negative productivity realizations are possible. When the mean is sufficiently high relative to the standard deviation, such realizations become extremely unlikely.

there is an increasing, convex cost $\frac{c_v}{2}v_i^2$ of acquiring a signal with relative precision v_i, where $v_i = \frac{V_0}{V_0+V_i}$. This assumption is inspired by the classic Grossman and Stiglitz (1980) model of information acquisition. Note that we can think of the informed trader as choosing v_i indirectly through his choice of V_i.

Both the decision maker and the informed traders are risk-neutral, so they will maximize their expected total profits. The decision maker's total profits depend on both her production and trading activities. Her production profits come from two sources: she incurs a quadratic effort cost (x^2) and her production revenues are equal to twice the product of effort and productivity ($2xy$). Thus, her total production profits (π_{P0}) can be expressed as:

$$\pi_{P0} = 2xy - x^2$$

These assumptions imply that the solution to the decision maker's expected profit-maximization problem is:

$$x = E[y|p] \tag{1}$$

where p is the securities market price that she observes. Thus, we assume that the decision maker cannot commit to a suboptimal effort decision after observing the market clearing price. Letting the error in the decision maker's productivity forecast be $y - E[y|p] = \delta$, where $Var(\delta) = V_\delta$ and $E[y\delta] = 0$, we can express the maximized value of her expected production profits (π_P) as:

$$\pi_{P0}^{Max} = E[2y(y-\delta) - (y-\delta)^2] = V_0 - V_\delta \tag{2}$$

Note that the decision maker's maximized production profits are equal to the difference between her expected forecast error before observing the price (V_0) minus her expected forecast error after observing the price (V_δ). We refer to this improvement in her forecast as the informativeness of prices (I):

$$I = V_0 - V_\delta = \pi_{P0}^{Max} \tag{3}$$

Equation (3) says that the decision maker increases her maximized expected production profits by reducing the noise in her productivity information (V_δ). Our general results will hold whenever maximized expected production profits increase with the quality of productivity information. In the example above, the farmer receives greater maximized expected production profits when her forecast is closer to realized rainfall because she can then implement a better production plan.

The decision maker also receives profits from trading in the securities market. We assume that traders are anonymous in the sense that the market maker only sees the aggregate order flow.[7] Traders submit market orders to a market maker who sets a clearing price in a batch auction. We will consider the equilibrium under both a competitive market maker (as in Kyle (1985)) and the decision maker making the market.[8] Both market makers set a securities price p in response to aggregate order flow Q from traders. The decision maker can only make the market if she offers traders a better price than a competitive market maker.

Each informed trader submits his market order q_{Ii} before he knows the equilibrium price, but after viewing his private signal s. Total order flow Q comes from the anonymous traders, which may include informed rational traders and uninformed decision stakeholders. Following Kyle (1985), we only look for a Nash equilibrium in which agents adopt linear strategies. Thus, the price set by the market maker (p) depends linearly on aggregate order flow (Q) according to:[9]

$$p = \lambda Q \tag{4}$$

The expected trading profits of the decision maker (π_{T0}) and informed traders (π_{Ti}) have a similar form:

$$\pi_{T0}(q_0) = E_y \left[q_0(y - p(Q(q_0)) \right] \tag{5}$$

$$\pi_{Ti}(q_i) = E_y \left[q_i(y - p(Q(q_i)) | s_i \right] \tag{6}$$

The only difference lies in the quantity demanded by each trader and the information sets. Thus, the decision maker's total expected profit π_0 is the sum of π_{P0}^{Max} and π_{T0} in equations (2) and (5), respectively. The total profit for each informed trader is the difference between his trading profits, $\pi_{Ti}(q_i)$, and his information acquisition cost, which is described below.

3.2 Formal Model with Only Informed Traders

In our first model, aggregate order flow consists of just the informed traders' orders (q_{Ii}) so

[7] Of course, when only two agents trade in equilibrium, each trader can infer the identity of the other. When there are more than two traders, there is a non-degenerate inference problem.

[8] The decision maker could make the market herself, or contract out that responsibility to a firm that specializes in that task.

[9] In all the linear equilibria in this paper, the constants in the pricing rule, effort decision rule, and trading strategies must be zero as a result of our maintained assumption that the common prior estimate of y is zero.

that $Q = \sum_i q_{Ii}$. We allow the decision maker herself to make the market, but only if she can attract order flow by offering better prices than the competitive market maker. Regardless of the market maker's identity, the pricing rule will be of the form given in equation (4). This rule translates order flow into a publicly observable price, which the decision maker can use. Our main departure from the standard Kyle model is to examine the decision maker's effort choice, which will be given by the linear rule:

$$x = kp \tag{7}$$

where k is determined so that the decision maker's optimal effort rule in equation (1) is satisfied.

In addition, we look for a symmetric linear equilibrium in which all informed traders submit strategies of the form:

$$q_{Ii} = b_I s_i \tag{8}$$

where b_I is the same for all informed traders i. To determine what quality of signal he will acquire, each informed trader i must first solve his profit maximization problem for a given signal quality (v_i) and fixed trading strategies b_{-I} of the other informed traders. After already incurring the cost of acquiring information, the informed trader maximizes expected trading profits ignoring the sunk acquisition cost. Anticipating these maximized expected profits, the informed trader can now select his optimal degree of information acquisition (v_i).

The last step in identifying the equilibrium is to solve for the decision maker's optimal pricing rule. In a Nash equilibrium, her pricing rule and the trading strategies of all informed traders are mutual best responses. Proposition 1 summarizes the equilibrium strategies followed by the decision maker and informed traders:

Proposition 1 *The decision maker sets a linear pricing rule $p = \lambda Q$, which involves expected trading losses ($\pi_{T0} < 0$), whereas a competitive market maker would refuse to operate. The decision maker selects an effort allocation (x) that is proportional to the equilibrium price in the securities market—i.e., $x = kp$, where $1 < k < 2$. Because informed traders choose their demands (q_{Ii}) in the securities market to be proportional to their signal realizations (s_i), the equilibrium securities market price and the decision maker's effort allocation are proportional to the sum of informed traders' signals.*[10]

[10] In a more general model where informed traders may acquire signals with different precisions, the equilibrium securities price would be proportional to a precision-weighted sum of their signals.

Proof: See Appendix.

The most important point in Proposition 1 is that the decision maker is willing to operate at a loss, whereas the competitive market maker would not. Intuitively, with only rational informed traders participating in the market, the competitive market maker's zero profit condition can only be satisfied when he does not trade. This confirms the Milgrom and Stokey (1982) "no trade" result for the case in which the decision maker does not participate in the market. In this equilibrium, anticipating that they cannot profit from their trades, informed agents would acquire zero costly information ($v_i = 0$).

By contrast, the decision maker's liquidity provision leads to an equilibrium with some trading, which allows informed traders to profit from their information acquisition. The decision maker tolerates an expected trading loss in the securities market because she learns valuable information from the equilibrium price. In particular, the decision maker links her effort choice to the equilibrium price, which contains information about her effort's productivity. This decision linkage results in an expected increase in her production profits that outweighs the expected decrease in her trading profits. Thus, there is nothing irrational about the decision maker's securities market trading.

One interpretation of the decision maker's pricing rule is that the equilibrium securities price underreacts to the information contained in insiders' market orders. Consider the following three period interpretation of our model. In period 0, before agents have observed their signals, all agents have a common prior expectation of y equal to 0, implying that $p_0 = 0$ is the initial price that would prevail in period 0. In period 1, after informed agents have observed their signals and submitted their market orders, the securities market clearing price is given by $p_1 = \lambda Q$ as noted in Proposition 1. In period 2, all agents owning the security receive the liquidating dividend equal to y, implying that $p_2 = y$. This means that the covariance between the two changes in price is:

$$Cov(p_2 - p_1, p_1 - p_0) = -\lambda \pi_{T0} > 0 \tag{9}$$

where the equality uses equation (5) and the inequality uses Proposition 1.

In other words, price changes display positive serial correlation. If an outside observer sees the first change in price is positive, he would expect to observe another positive change in price. Prices do not fully adjust to insiders' information as they would in a competitive equilibrium because $p_1 = \lambda Q$ is insufficiently sensitive to order flow relative to the hypothetical price that a competitive market maker would set $p_1 = \lambda_c Q$ with $\lambda_c > \lambda$. This is just a restatement of our main result that the decision maker subsidizes liquidity.

The decision maker sets prices to underreact to order flow so that informed traders have incentives to acquire information. Securities must be undervalued when insiders have positive signals; and they must be overvalued when insiders have negative signals. If securities were not mispriced, then insiders would have no incentive to acquire information. This argument is analogous to the Grossman and Stiglitz (1980) idea that informationally efficient markets are an impossibility. In our model, the decision maker deliberately makes the market informationally inefficient in order to motivate information acquisition.[11]

The underreaction of prices applies only to the period in which insiders act on their information. After insiders complete their trades, the market clearing price fully reveals their information because it is proportional to the sum of their signals. This implies that all future prices will incorporate insiders' information. For example, if we added another period just after period 1 in the three-period model above, then the market clearing price in this later period would fully respond to all the information revealed in period 1. The decision maker would have no incentive to subsidize liquidity in this later period unless informed traders had acquired new signals.

The idea of using a liquidity subsidy to obtain better information could also be extended to a dynamic framework. For example, an decision maker could post a narrower bid-ask spread than a competitive market maker to allow informed traders to obtain positive expected profits, which would motivate them to acquire information. Only informed traders with better information than the decision maker would benefit from the liquidity subsidy. No uninformed trader could take advantage of the decision maker's subsidy because she would adjust her quotes immediately after each informed order. Thus, even in a dynamic model, a liquidity subsidy is a viable means for providing incentives to acquire information. Moreover, without a subsidy, it is unclear how any information would ever be revealed in prices (in a model lacking traders with hedging or irrational motives).

The following proposition summarizes the comparative statics of equilibrium information acquisition (v), liquidity (λ^{-1}), and dollar amount of the liquidity subsidy.

Proposition 2 *The size of the liquidity subsidy is larger, prices are more informative, each informed trader acquires more information, and the market is (weakly) less liquid when there*

[11] This inefficiency persists in equilibrium even if we introduce uninformed traders in the model. The nature of the batch auction mechanism in Kyle (1985) only allows simple market order submission strategies. Uninformed traders would always choose to submit zero quantity orders in our model. If we had instead allowed traders to submit price schedules, then this would change the equilibrium. We discuss limit orders and one possible dynamic version of our model later in the paper.

is more uncertainty about productivity. Each informed trader acquires less information and the market is (weakly) more liquid when there are more informed traders and when there are higher costs of acquiring information. In general, the size of the liquidity subsidy may increase or decrease when there are more informed traders and when there are higher information acquisition costs. Price informativeness decreases when there are higher information acquisition costs and may increase or decrease when there are more informed traders.

Proof: See Appendix.

Proposition 2 makes two main points. First, the decision maker subsidizes liquidity to induce informed traders to acquire better signals, which will improve the decision maker's production profits. Consistent with this intuition, the liquidity subsidy is largest when there is more uncertainty about her effort's productivity. Second, equilibrium market liquidity is lower when informed traders acquire better information, which happens when the expected profits from acquiring information are high relative to the costs. In this regard, the decision maker's market making activity is similar to a competitive market maker's activity. The intuition comes from Kyle's (1985) insight that a market maker decreases liquidity in markets with informative order flow to limit her losses. Nevertheless, the decision maker is willing to accept some losses in order to inform her effort choice.

The proposition also describes the properties of equilibrium price informativeness. Recall that price informativeness measures expected production profits because the decision maker implements a production plan based on the equilibrium market clearing price. Thus, in the situations in which information is most important for production (*i.e.*, when uncertainty in productivity is greatest), the decision maker attains greater expected production profits. The intuition for this result comes from a well-known property of maximized expected profit functions: they tend to increase with parameter uncertainty. Because the decision maker can decrease her effort when expected effort productivity is low, she can diminish the impact of negative shocks to expected effort productivity. Moreover, the decision maker can take advantage of favorable expected productivity shocks by increasing her effort. Note that this argument relies on the decision maker having some information about the expected productivity shock. Thus, the decision maker's willingness to subsidize liquidity in the securities market, which informs her effort choice, increases when there is greater uncertainty in effort productivity.

Proposition 2 shows that the decision maker provides more liquidity in the securities market when there are more informed traders. This occurs because each trader acquires less precise information, which mitigates the adverse selection problem faced by the decision maker. However, it is unclear whether the total size of the liquidity subsidy increases or decreases as N increases because the decision maker subsidizes more traders, but each trader acquires less precise information.

We can now assess whether the no-trade equilibrium or the equilibrium with the decision maker as the market maker is more desirable from a social standpoint. We define the social welfare function as equal to the expected production revenues conditional on information revealed by the equilibrium price minus informed traders' costs of acquiring that level of information. We disregard the total trading profits of informed traders and the decision maker because they must sum to zero.

The social welfare maximization problem is almost identical to the maximization problem solved by the decision maker. The only difference comes from the fact that informed traders have some market power, earning abnormal profits above and beyond their information acquisition costs. The decision maker must bear not only the direct acquisition costs, but also the cost of paying the informed traders' expected profits. Proposition 3 summarizes our results on social welfare maximization. The proposition refers to three different equilibria: 1) the social equilibrium is the one selected by a social planner maximizing production profits; 2) the private equilibrium is the equilibrium that prevails when the decision maker maximizes her total production and trading profits; and 3) the competitive equilibrium is the hypothetical situation in which the decision maker does not participate in the securities market.

Proposition 3 *Expected welfare and liquidity are higher at the social optimum than when the decision maker subsidizes liquidity in the securities market. Similarly, welfare and liquidity are higher when the decision maker subsidizes liquidity than in the case of a competitive market maker.*

Proof: See Appendix.

The main point of Proposition 3 is that the decision maker's liquidity subsidy improves social welfare. In one sense, this may seem obvious because the competitive equilibrium without a liquidity subsidy would entail no trade and no acquisition of productivity information, which is clearly undesirable for the economy. This reasoning overlooks the possibility that the decision maker will excessively subsidize information acquisition relative to the social optimum. However, in the simple model here, the decision maker's incentive to increase information acquisition is well-aligned with society's goals because the decision maker is the sole residual claimant of production surplus and there is no consumer surplus.[12]

In fact, the decision maker's subsidy is insufficient to lead to optimal information acquisition. The equilibrium does not attain the social optimum because informed traders have market power in finitely liquid markets. Because each trader with private information faces an inelastic residual supply curve for securities, he restricts the quantity of information below the social optimum to maximize his private profits. Although this effect diminishes as the number of informed traders increases, it does not completely go away because each trader has independent private information that is not perfectly substitutable. More formally, as the number of informed traders approaches infinity, one can use equations (30) and (34) to show that the limit of information acquisition under the decision maker's subsidy is equal to $2^{-1/3} = 79.4\%$ of the limit of the socially optimal information acquisition.

We conclude that the decision maker improves social welfare by subsidizing liquidity and motivating information acquisition when the competitive equilibrium falls short. The decision maker always subsidizes liquidity in this model to improve the informativeness of prices, especially when information is particularly important for her production planning.[13]

4 Markets with Decision Stakeholders

In this section, we investigate the equilibrium in a decision market when there is a stakeholder and an informed trader. To focus our attention on the stakeholder's impact on the equilibrium, we streamline the rest of the model. Specifically, we assume that there is only one informed trader ($N = 1$) who can acquire information at no cost ($c_v = 0$), implying that he will acquire perfect information in equilibrium ($i.e.$, $v = 1$).

We introduce a new trader to the model who has a direct stake in the decision maker's effort choice. Suppose that the stakeholder receives production profits equal to $\pi_{PM}(x) = \theta x$, where θ describes the marginal benefit or cost of a change in the decision maker's effort choice. In general, one can think of the stakeholder as a consumer, competing producer, or even an

[12] This result on the underprovision of liquidity and information may be more general than our simple model suggests. Improved production information will tend to increase the size of producer and consumer surplus. If the farmer neglects the potential increase in consumer surplus or does not receive all of the increase in producer surplus, she may have even less of an incentive to acquire production information.

[13] In related work, we have also shown that adding noise traders to this model does not change the main comparative statics. The decision maker subsidizes liquidity when there are sufficiently few noise traders. Again, the liquidity subsidy improves social welfare, but does not supply enough liquidity to motivate the socially optimal level of information acquisition.

employee. We assume the marginal benefit parameter (θ) is private information for the stakeholder. We also assume for tractability that all other agents have prior beliefs that the stakeholder's decision objective θ is normally distributed as $N(0, V_\theta)$.

We begin by comparing the equilibrium in the model when the decision maker does not intervene in the market to the equilibrium in which the decision maker subsidizes liquidity in the market. In both cases, we look for an equilibrium in which the market maker's price depends linearly on order flow as in equation (4), the decision maker's effort rule depends linearly on the securities market price as in equation (7), and the informed trader's order depends linearly on his private signal as in equation (8). In addition, we conjecture that the stakeholder will submit an order (q_M) that depends linearly on his decision objective (θ):[14]

$$q_M = b_M \theta \tag{10}$$

Before analyzing the equilibria of this model, we offer an intuitive explanation of the two key exogenous parameters in the model: V_θ and V_0. We interpret V_θ as the stakeholder's expected "willingness to trade." When V_θ is larger, other agents believe that the stakeholder is more likely to have a high or a low value of θ. A high or low value of θ means that the stakeholder will receive a larger payoff from moving the price either above or below the security's expected payoff. From the informed trader's perspective, an increase in the stakeholder's (expected) willingness to buy or sell the security at prices that deviate from the security's expected payoff represents a larger profit opportunity. Specifically, the informed trader's expected payoff depends on the value of V_θ.

Just as V_θ determines how much other agents expect to profit from trading with the stakeholder, V_0 determines how much money other agents expect to lose from trading with the informed trader. The parameter V_0 measures the "informational advantage" of the informed trader, who has perfect information in this model—*i.e.*, it captures the difference between the informed trader's expected forecast error (0) and other agents' expected forecast error (V_0). We can think of the ratio of the stakeholder's expected willingness to trade to the informed trader's informational advantage (V_0/V_θ) as a measure of the severity of adverse selection in the securities market. When the informed trader's informational advantage is greater and when the stakeholder is less willing to trade, the adverse selection problem in the market is more severe.

[14] There is also a symmetric linear equilibrium in which there are many identical informed traders and stakeholders. Symmetric refers to the fact that all traders of the same type use the same quantity strategy, but do not necessarily submit the same actual quantities.

First, we analyze the equilibrium for the hypothetical case in which the market maker is competitive and the decision maker does not participate in the securities market (by assumption). However, the decision maker can still use the equilibrium price as an input for her effort rule. Proposition 4 characterizes the competitive market maker's equilibrium pricing rule, the decision maker's effort rule, along with traders' equilibrium strategies:

Proposition 4 *If the decision maker did not participate in the securities market, a competitive securities market maker would set $\lambda_c = \sqrt{V_0/V_\theta}$ when $V_\theta > 0$ and refuse to operate otherwise. The decision maker's equilibrium effort rule would be $x = p$ and the competitive market maker would use a pricing rule of $p = \frac{1}{2}(s + \sqrt{V_0/V_\theta}\theta)$. A competitive market maker would supply more liquidity (λ_c^{-1} is higher) when the stakeholder is more willing to trade (V_θ increases) and when the informed trader has a lower informational advantage (V_0 decreases). The informativeness of the securities price would be $V_0/2$. The informed trader would follow the strategy $q_I = \frac{\sqrt{V_\theta/V_0}}{2}s$ and the stakeholder would follow the trading strategy $q_M = \frac{1}{2}\theta$ when $V_\theta/V_0 > 0$. Otherwise, both traders would not trade.*

Proof: See Appendix.

Unlike the model in section 3, there would always be some trade in the competitive equilibrium unless $V_\theta = 0$. The reason is that the decision stakeholder would trade because he receives a private marginal benefit (θ) from influencing the decision. In equilibrium, the decision maker links her effort to the market price because the price is informative about effort productivity (y). In this sense, the stakeholder in our model can be viewed as an endogenous noise trader as in Kumar and Seppi (1992). Without a trader who benefits from influencing the decision maker's effort choice, there would be no trade in the competitive equilibrium.

Because the competitive market maker would set equilibrium liquidity to increase monotonically with the ratio V_θ/V_0, we interpret this ratio as the "natural liquidity" of the market.[15] This is the level of liquidity that would prevail in the hypothetical benchmark situation where the decision maker did not participate in a securities market with a competitive market maker—*e.g.*, the traditional Kyle (1985) model. The ratio V_θ/V_0 increases as the adverse selection problem becomes less severe, which occurs when either the stakeholder becomes more willing to trade or the informational advantage of the informed trader decreases.

[15] Note that natural liquidity in the securities market is the inverse of the adverse selection measure described above.

In the competitive equilibrium, the decision maker would implement an effort allocation that is solely based on the equilibrium securities price ($x = p$) because this price includes all relevant information about her effort's productivity. By linking her effort to the equilibrium price, the decision maker could incur a loss in production profits from the fact that the stakeholder's trading activity affects her decision. Interestingly, this does not happen because the informed trader trades more aggressively when he expects that the stakeholder is more willing to trade. So the stakeholder's expected willingness to trade would have no impact on price informativeness in the competitive equilibrium. This result will not hold in a more general model in which we allow the decision maker to intervene in the market.

We can compare the hypothetical competitive equilibrium to the equilibrium in the securities market when the decision maker is the market maker. Once again, we allow the decision maker to link her effort allocation to the equilibrium market price. The following proposition summarizes the properties of this equilibrium:

Proposition 5 *The decision maker is willing to subsidize liquidity (i.e., set $\lambda < \lambda_c$) in the securities market in situations where natural liquidity is low ($V_\theta/V_0 < 1$). In these situations, the decision maker incurs expected trading losses and sets her effort rule such that $x = kp$ where $1 < k < 2$. However, the decision maker will not trade in the securities market when natural liquidity is sufficiently large (i.e., $V_\theta/V_0 \geq 1$). In these situations, she incurs no trading losses and sets her effort rule such that $k = 1$. The decision maker's liquidity subsidy induces the informed agent and the stakeholder to trade more aggressively. The informed trader's increased aggressiveness dominates in the sense that price informativeness is greater when the decision maker subsidizes liquidity.*

Proof: See Appendix.

We explain the intuition for the increase in liquidity in the securities market by tracing the impact on the decision maker's production profits. In a more liquid market, the informed trader trades more aggressively on his information because his trades do not move prices (against himself) as much. In contrast, the stakeholder does not trade as aggressively, recognizing that moving the price in a more liquid market would require a larger trade with the informed agent and entail a greater expected loss. As a result, in a liquid market, a greater proportion of aggregate order flow comes from the informed agent, which increases the informativeness of prices. Because the decision maker can use this more informative price to form a better effort rule, her expected production profits are higher when the securities market is more liquid. She is willing to incur some expected trading losses in the securities market to increase her expected production profits.

However, the cost of subsidizing liquidity increases as liquidity and trading volume in the market increase. For a given departure from the equilibrium price, the expected trading loss for the decision maker increases in proportion to the amount of securities traded. This is why the decision maker is only willing to subsidize liquidity when there is less trading volume in the market—*i.e.*, when there is little natural liquidity ($V_\theta < V_0$).

In this case, the decision maker sets prices to underreact to information in order for informed traders to reap extra profits from their trades. However, the decision maker's effort rule ($x = kp$) is not biased. She appropriately adjusts for the underreaction in prices by changing her effort choice more than one-for-one ($k > 1$) in response to changes in prices.

The next proposition summarizes how the decision maker would respond to changes in natural liquidity.[16]

Proposition 6 *The decision maker's liquidity provision in the securities market is greater when natural liquidity is higher—i.e.,* $d\lambda/d(V_\theta/V_0) < 0$. *The decision maker's effort choice becomes less dependent on the securities price when natural liquidity is higher—i.e.,* $dk/d(V_\theta/V_0) < 0$. *When* $V_\theta/V_0 < \overline{V} < 1$, *the size of the decision maker's liquidity subsidy* ($|\lambda V - C|$) *increases when natural liquidity is higher. When* $\overline{V} < V_\theta/V_0 < 1$, *the size of the decision maker's liquidity subsidy* ($|\lambda V - C|$) *decreases when natural liquidity is lower. The liquidity subsidy increases the decision maker's total profits more when natural liquidity is lower.*

Proof: See Appendix.

The first result in Proposition 6 is identical to the competitive market maker case. Analogous to Kyle (1985), when the stakeholder is expected to trade more, adverse selection is less of a problem, which increases securities market liquidity. Similarly, an increase in the informed trader's informational advantage increases the adverse selection problem, which decreases market liquidity. However, there is a cost associated with more stakeholder trade and less informed trade: the price becomes a noisier signal of productivity, so the decision maker relies less on the price when she chooses her effort.

[16] Although we could analyze how liquidity provision changes when both V_θ and V_0 change, this would be somewhat redundant. Because the asset owner's total profits depend only on the ratio of V_θ/V_0, Proposition 6 describes how liquidity and the decision rule vary with this ratio. The dependence of the decision maker's strategy on the ratio V_θ/V_0 completely summarizes the dependence on both of the individual parameters (V_θ and V_0) because both parameters are exogenous—*i.e.*, one is always held fixed when the other one changes.

Proposition 6 also describes a non-monotonic relationship between the size of the liquidity subsidy and natural liquidity in the market. One interpretation is that the decision maker's liquidity subsidy and the stakeholder's willingness to trade are two ways of making the securities market more liquid. The decision maker would like to make the market more liquid when it will not cost her much and it will improve the informativeness of prices. When the stakeholder is quite reluctant to trade (V_θ is low), trading volume in the market is quite low, which means it is not very costly for the decision maker to subsidize liquidity. An increase in the stakeholder's expected willingness to trade leads to a larger liquidity subsidy because this reduces the stakeholder's influence on the price at a reasonable cost (because volume is low). When the stakeholder's expected eagerness to influence the decision becomes strong enough (V_θ is high), trading volume in the market is relatively high, which means that subsidizing liquidity is quite expensive. In this case, the decision maker responds to increases in expected stakeholder trade by allowing the stakeholder to absorb some of the costs of supplying liquidity. In other words, when the decision maker expects more stakeholder trade, the liquidity provided by the stakeholder and the decision maker are substitutes.

Another important point in Proposition 6 is that the decision maker's intervention increases her total profits more when natural liquidity is lower. In other words, the change in her total profits is greatest when the competitive market maker would provide the least liquidity. The intuition is that only a small liquidity subsidy is necessary to enhance price informativeness, which increases the decision maker's production profits. When natural liquidity is low, this means there is very little stakeholder trade. A small liquidity subsidy will increase informed trade by a small amount, which will still have a big effect on the relative amount of informed trade. This is the key variable for price informativeness and the decision maker's total profits. When her expected liquidity subsidy is small, the decision maker receives a very large expected return on her subsidy.

Although it is clear that the decision maker is better off when she is allowed to intervene in the securities market, it is not clear whether her intervention benefits society. We can evaluate social welfare as the sum of the expected production profits of the decision maker and the expected non-trading profits of the stakeholder. In this model, social welfare does not include information acquisition costs because these are assumed to be zero.

We can compare social welfare at the hypothetical competitive equilibrium in which the decision maker does not intervene, at the equilibrium in which the decision maker sets liquidity in the securities market, and at the social optimum when a social planner sets liquidity in the securities market. The next proposition summarizes these social welfare comparisons, focusing on the most interesting cases in which the decision maker would intervene in the market ($V_\theta/V_0 < 1$).

Proposition 7 *Whenever there is little natural liquidity ($V_\theta/V_0 < 1$), expected welfare and liquidity are higher at the social optimum than when the decision maker intervenes in the securities market. Similarly, welfare and liquidity are higher with the decision maker's intervention than in the case of a competitive market maker. In addition, the increase in social welfare from the decision maker's intervention is greater when natural liquidity is lower.*

Proof: See Appendix.

The competitive equilibrium entails the least liquidity because the competitive market maker does not care about price informativeness, which is linked to the decision maker's production profits. The decision maker's liquidity subsidy improves upon social welfare in the competitive outcome because the decision maker is trying to maximize her production profits, which are an important component of social welfare. However, the decision maker is unwilling to subsidize liquidity enough to reach the socially optimal level because she must bear some of the cost of giving the informed trader an incentive to reveal his information. The informed trader restricts his trading because he recognizes his impact on the securities price.

Proposition 7 also shows that the decision maker's intervention improves social welfare more when natural liquidity is lower. The intuition is similar to Proposition 6, which proves the analogous result for the decision maker's total profits. Once again, when natural liquidity is low, only a small liquidity subsidy is necessary to enhance price informativeness, which improves social welfare.

4.1 Equilibrium When the Decision Maker Is a Trader

This section investigates possible strategies for the decision maker as a trader when there is a competitive market maker. This situation could arise, say, if the decision maker were not allowed to be a market maker. In this case, the decision maker may want to submit buy and sell orders in order to make the market more liquid, and thus get better information. In a dynamic setting, this could be accomplished by posting simultaneous bid and ask orders at a narrower spread than the market maker. In a static setting where traders only submit market orders in a batch auction, it is less clear how the decision maker can achieve her liquidity objectives.

We consider the possibility that the decision maker submits a random order to make the market more liquid. Indeed, this is precisely the way that liquidity traders are modeled in the standard Kyle (1985) model. In this "random trading model," we will prove that the amount of liquidity (λ^{-1}) and the informed trading strategies are the *same* whether a rational decision maker optimally trades in the market or optimally makes the market. The proof of this result is quite general and applies to both the model with a stakeholder and the model with no stakeholder.

Formally, consider a model in which the decision maker submits a random order ρ, where $\rho \sim N(0, V_\rho)$, to a competitive market maker. The decision maker selects the distribution of the order, but cannot control the specific realization of her order. Because the single parameter V_ρ is a sufficient statistic for the distribution of ρ, we can think of the decision maker as choosing V_ρ. For simplicity, we suppose that only the decision maker can observe the realization of ρ, which occurs after the order is submitted but before the market clears. This will allow the decision maker to "debias" the market price by accounting for the effect of the realization of ρ. The competitive market maker only observes aggregate order flow, but is aware that the decision maker's random order flow is included in aggregate order flow.[17] Order flow may include orders from an arbitrary number of informed traders and a stakeholder.

Proposition 8 *The amount of liquidity (λ^{-1}), the stakeholder trading strategy, and the informed trading strategy are the same whether a rational decision maker optimally trades in the market or optimally makes the market.*

Proof: See Appendix.

The main point of Proposition 8 is that both models' predictions of how the decision maker will participate in the market do not depend on the assumption that the decision maker can act as a market maker. The proof shows that the decision maker can achieve the

[17] In practice, writing a contract to enforce this random order would not be too difficult. For example, suppose a decision maker submits an order to buy ρ contracts, where ρ is the realization from a third-party random-number generator. The distribution of ρ would need to be public, but the realization could be public (*e.g.*, the point difference in the SuperBowl) or private (*e.g.*, the spin on a roulette wheel with normally distributed values). The decision maker's random order would need to be a binding contract with the market maker that would be verified ex ante and enforced ex post by a neutral third party. Anticipating the distribution of this random order will be a part of total order flow, a competitive market maker would lower the sensitivity of price to order flow.

same expected utility and equilibrium outcome by randomly trading in the market with a competitive market maker.[18]

An interesting implication of Proposition 8 is that the decision maker can choose whether she would like others to benefit from the information provided by her liquidity subsidy. In particular, if the decision maker reveals the realization of her random order flow to other agents, then these other agents can make more accurate inferences about the security's fundamental value (y). On the other hand, if the decision maker would rather keep her information private, then she can do this, too. In our highly stylized model, the decision maker is indifferent to releasing the information to the public. In general, the choice of whether to reveal her information will depend on the decision maker's objectives.

5 Conclusion

Our model allows new insights into how securities markets are linked to resource allocation decisions. We derive three key results. First, we show that a decision maker will subsidize liquidity in illiquid decision markets to gather valuable decision-relevant information. In our Kyle-type model, the decision maker always provides at least as much liquidity as a competitive market maker and often provides more. There are even some situations in which a competitive market maker would refuse to operate, but a decision maker would be willing to supply some liquidity and incur expected trading losses. From the decision maker's perspective, some trade in a market for decision-relevant information is better than none. We also show that the decision maker can implement her desired liquidity policy either directly as the market maker or indirectly as a noise trader participating in a competitive market.

Second, we show that the decision maker's intervention in the securities market enhances social welfare. In general, without decision stakeholders, noise traders, or hedging demands, there will be little trade in a securities market. In this case, informed traders will have no incentive to acquire costly information or reveal it in prices. As a result, allocative efficiency suffers when there is insufficient trade in the securities market. Fortunately, the decision maker is willing to subsidize liquidity in the securities market to partially remedy this situation. Unfortunately, her subsidy is typically not large enough to motivate the socially optimal level of information acquisition and revelation in prices. The main problem

[18] Proposition 8 raises the question of whether agents who behave as noise traders are doing so to learn more from the market price. Before dismissing such agents as irrational, future theoretical and empirical work may consider this possibility.

is that informed traders recognize the impact of their trades in imperfectly liquid markets. They restrict the quantity of their trades, which increases their private profits but harms social welfare.

Third, our model demonstrates that the mere act of linking the decision to the market price will typically enhance liquidity in the market. Thus, liquidity may be less of a problem in decision markets than in traditional prediction or securities markets. In decision markets, agents may choose to trade because they have strong decision objectives. Moreover, this stakeholder trading is a substitute for the liquidity subsidy provided by a decision maker. This can be compared with the more conventional prediction market case in which even experts with decision objectives will not trade. In both markets, however, liquidity subsidies may be useful for obtaining information. Although we derive special cases of optimal liquidity subsidies in this paper, the form of the optimal subsidies in more general settings requires further investigation.

We believe that information and decision markets are likely to become more prevalent in the future. While we have developed a theory of how such markets could be designed efficiently, it remains to be seen whether such mechanisms will actually work well in practice. More research is needed to understand the properties of different kinds of decision markets. For example, little is known about the theoretical properties of decision markets in which the decision itself has an impact on the realization of decision-relevant information. Applied research, including both laboratory and real-world experiments, can assess how robust different decision markets are in practice. Whereas prediction markets have been found to forecast extremely well, decision markets have not been studied empirically. There are compelling theoretical reasons to expect different behavior in decision markets, but theory alone cannot address this issue.

Appendix

In the appendix, we prove Propositions 1 through 8.

Proof of Proposition 1: After already incurring the cost of acquiring information, the informed trader maximizes expected trading profits ignoring the sunk acquisition cost.

$$q_{Ii} \in \arg\max_{\widehat{q}_{Ii}} E(\widehat{q}_{Ii}(y-p)|s_i) \tag{11}$$

where $p = \lambda Q = \lambda Q_I = \lambda \sum_i q_{Ii}$, which has the solution:

$$q_{Ii} = \frac{v_i(1-\lambda b_{-I}(N-1))}{2\lambda} s_i \tag{12}$$

These are the informed trader's expected trading profits before s_i has been realized, but after the distribution of s_i has been chosen. This expression for q_i can be combined with our symmetry assumption ($b_I = b_{-I}$) to solve for b_I:

$$b_I = \frac{v_i(1-\lambda b_I(N-1))}{2\lambda} \tag{13}$$

where each trader's strategy depends on the degree of information precision v_i acquired by that trader.

Next we assess whether this choice of information precision is consistent with each individual informed trader's profit maximization condition. Substituting the solution to the profit-maximization problem in equation (13) into the original maximand in equation (11), we can compute the maximized expected profits of each informed trader with a signal quality of v_i as:

$$\frac{V_0 v_i}{4\lambda}(1-\lambda b_{-I}(N-1))^2 \tag{14}$$

Anticipating these expected profits, the informed trader can now select his optimal degree of information acquisition:

$$v_i \in \arg\max_{\widehat{v}_i} \frac{V_0 \widehat{v}_i}{4\lambda}(1-\lambda b_{-I}(N-1))^2 - \frac{c_v}{2}\widehat{v}_i^2 \tag{15}$$

which has the solution:

$$v_i = \frac{V_0(1-\lambda b_{-I}(N-1))^2}{4c_v\lambda} \tag{16}$$

when $V_0(1 - \lambda b_{-I}(N-1))^2 < 4\lambda c_v$ and $v_i = 1$ otherwise because $v_i \in [0,1]$. From equation (16), because traders use symmetric trading strategies ($b_I = b_{-I}$), they all optimally choose the same information acquisition parameters ($v_i = v$).

To ensure that each informed trader is choosing a trading strategy that is a best response to the actual trading strategies played by other informed traders, we solve for the value of b_I that satisfies equation (13) and our symmetry result that $v_i = v$:

$$b_I = \frac{v}{\lambda[(N-1)v+2]} \tag{17}$$

Using this equation along with equation (16), we can solve for the degree of information precision that each informed trader will choose as a best response to all other informed traders' strategies:

$$v[(N-1)v+2]^2 = \frac{V_0}{c_v \lambda} \tag{18}$$

Equation (18) establishes a one-to-one mapping between liquidity and the equilibrium level of information precision acquired by informed traders, which we can confirm by showing the derivative of $v(\lambda)$ is always negative:

$$\frac{dv}{d\lambda} = -\frac{V_0/c_v\lambda^2}{[3(N-1)v+2][(N-1)v+2]} < 0 \tag{19}$$

Now we can determine the trading profits (π_{MM}) of a market maker who sets λ. First, we can calculate the variance of order flow (Q) and its covariance with productivity (y):

$$Var(Q) = (N^2V_0 + NV_\varepsilon)b_I^2 \tag{20}$$

$$Cov(y, Q) = NV_0 b_I \tag{21}$$

where we have introduced a new parameter V_ε which corresponds to the common choice of V_i that satisfies $v = \frac{V_0}{V_0+V_i} = v_i$ for all i. For convenience, we define $V = Var(Q)$ and $C = Cov(y, Q)$. From these equations and the equation for b_I above, we can calculate the trading profits (π_{MM}) of a market maker who sets a liquidity parameter λ:

$$\pi_{MM}(\lambda) = E[-Q(y-p)] = \lambda V - C = -\frac{NvV_0}{\lambda[(N-1)v+2]^2} < 0 \tag{22}$$

assuming $V_0 > 0$, $0 < v \leq 1$ and $\lambda > 0$. Thus, a competitive market maker will not be willing to supply any liquidity in this securities market because he would never break even on his trades.

To solve for equilibrium in which the decision maker makes the market, we must express the decision maker's objective function in terms of the model parameters. As shown above, the decision maker's trading profit from choosing the degree of liquidity in the market (λ) is given by $\lambda V - C$, where λ is the price sensitivity parameter she sets. We refer to the trading profits as a liquidity subsidy and focus on this measure throughout the paper.

We can simplify the above expression for trading profits by substituting equation (18) to obtain:

$$\pi_{T0} = \pi_{MM}(\lambda) = -Nc_v v^2 \tag{23}$$

This expression shows that the trading losses incurred by the decision maker are exactly double the total acquisition costs for informed traders. The reason is that each informed trader has private information and recognizes the impact his trades have on the equilibrium securities price. To maximize his profits, he trades less aggressively based on his signal, which increases his profits at the expense of the decision maker.

We can combine these trading losses with the decision maker's maximized production profits (π_{P0}^{Max}) when productivity (y) deviates from her forecast (x). Recall that the expression for production profits in (2) relies on the assumption that the decision maker will implement her best estimate of y after observing aggregate order flow. In particular, the decision maker's equilibrium effort rule is given by:

$$x = E(y|p) = E(y|Q) = E(y) + \frac{C}{V}(Q - E(Q)) = \frac{C}{V}Q = \frac{C}{\lambda V}p \tag{24}$$

which implies the following value for k:

$$x = kp \Rightarrow k = \frac{C}{\lambda V} \tag{25}$$

Note that equation (24) computes the decision maker's conditional expectation of productivity ($E(y|Q)$) by applying the conditional expectation formula for the jointly normally distributed random variables productivity (y) and aggregate order flow (Q).

We can simplify expression (2) for the decision maker's maximized production profits using equation (24) for the effort rule along with equations (20) and (21) for V and C:

$$\begin{aligned}\pi_{P0}^{Max} &= V_0 - V_\delta = V_0 - E(y - \frac{C}{V}Q)^2 \\ &= V_0 - E(y^2) + 2\frac{C}{V}E(yQ) - \frac{C^2}{V^2}E(Q^2) = \frac{C^2}{V} \qquad (26) \\ &= \frac{NvV_0}{(N-1)v+1} \qquad (27)\end{aligned}$$

Using equation (27), we can express the decision maker's total profit maximization problem as:

$$\max_\lambda \pi_0 = \lambda V - C + V_0 - Var(y|Q) = -Nc_v v(\lambda)^2 + \frac{Nv(\lambda)V_0}{(N-1)v(\lambda)+1} \qquad (28)$$

Because equation (18) establishes a one-to-one mapping between λ and v, we can rewrite this maximization in terms of v:

$$\max_v -Nc_v v^2 + \frac{NvV_0}{(N-1)v+1} \qquad (29)$$

The first-order condition for the maximization is:

$$\frac{d\pi_0}{dv} = -2Nc_v v + \frac{NV_0}{[(N-1)v+1]^2} = 0$$

The second-order condition for maximization is clearly satisfied because:

$$\frac{d^2\pi_0}{dv^2} = -2Nc_v - \frac{2N(N-1)V_0[(N-1)v+1]}{[(N-1)v+1]^3} < 0$$

This implies that the first-order condition characterizes a maximum. Rearranging the first-order condition slightly, we obtain a cubic equation in v that can be solved in closed form using standard methods:

$$v[(N-1)v+1]^2 = \frac{V_0}{2c_v} \Rightarrow v = v(N, V_0/c_v) \qquad (30)$$

Equation (30) can be solved in closed form using the cubic equation to obtain at least one positive real root for v. Because the analytical expression for v is quite cumbersome, in the interest of brevity and clarity, we denote the unique positive real root in equation (30) as

$v(N, V_0/c_v)$.[19]

Now we can combine equation (30) and equation (18) to find a solution for λ in terms of v. We can then substitute $v(N, V_0/c_v)$ in the resulting expression to obtain a closed form solution for λ:

$$\lambda = 2\left(\frac{(N-1)v+1}{(N-1)v+2}\right)^2 \Rightarrow \lambda = \lambda(N, V_0/c_v) \tag{31}$$

Using equations (30) and (31), we can also compute the equilibrium trading strategies of informed traders:

$$b_I(N, V_0/c_v) = \frac{v(N, V_0/c_v)}{\lambda(N, V_0/c_v)[(N-1)v(N, V_0/c_v)+2]} \tag{32}$$

From equation (25), the definitions of V and C in (20) and (21), and equation (32) for b_I, we find that the decision maker's equilibrium effort rule is:

$$k = \frac{V_0}{(NV_0 + V_\varepsilon)\lambda b_I} = \frac{(N-1)v(N, V_0/c_v)+2}{(N-1)v(N, V_0/c_v)+1} = k(N, V_0/c_v)$$

Note that $1 \leq k \leq 2$ because $v \in [0, 1]$. QED.

Proof of Proposition 2: We can rewrite equation (30) as $F(v, N) - G(V_0, c_v) = 0$. Applying the implicit function theorem to this equation, we find that:

$$\frac{dv}{dN} = -\frac{\partial F/\partial N}{\partial F/\partial v} = -\frac{2v^2}{3(N-1)v+1} < 0$$

$$\frac{dv}{dV_0} = \frac{\partial G/\partial V_0}{\partial F/\partial v} = \frac{1/2c_v}{[3(N-1)v+1][(N-1)v+1]} > 0$$

$$\frac{dv}{dc_v} = \frac{\partial G/\partial c_v}{\partial F/\partial v} = -\frac{V_0/2c_v^2}{[3(N-1)v+1][(N-1)v+1]} < 0$$

Moreover, for $N > 1$, equation (31) establishes a monotonically increasing relationship between v and λ. This means that the derivatives of λ with respect to the model parameters must the same signs to the derivatives of v, which means that λ^{-1} has the opposite signs. For $N = 1$, λ is equal to a constant, implying that all the derivatives of λ are zero.

Now we can differentiate equation (23) for trading profits with respect to N to obtain:

$$\frac{\partial(\lambda V - C)}{\partial v}\frac{dv}{dN} + \frac{\partial(\lambda V - C)}{\partial N} = c_v v^2\left(\frac{4Nv}{3(N-1)v+1}\right) - c_v v^2$$

[19] An explicit closed form solution is available from the authors upon request.

Thus, trading profits increase with N if and only if:

$$v > 1/(N+3)$$

This condition may or may not be met depending on the values of V_0 and c_v.

Next, we differentiate equation (23) for trading profits with respect to c_v to obtain:

$$\frac{\partial(\lambda V - C)}{\partial v}\frac{dv}{dc_v} + \frac{\partial(\lambda V - C)}{\partial c_v} = Nv^2\left(\frac{V_0/vc_v}{[3(N-1)v+1][(N-1)v+1]}\right) - Nv^2$$

This implies that trading profits increase with c_v if and only if:

$$V_0/c_v > v[(N-1)v+1]^2 + 2(N-1)v^2[(N-1)v+1]$$

$$v < 1/(N-1)$$

When $N = 1$, this clearly holds, but the condition will sometimes fail depending on the values of c_v and V_0. For example, when $N = 2$ and $V_0/c_v > 9/4$, the reader can verify that $v > 1/2$, which violates the condition above.

Next, we evaluate how trading profits depend on V_0. This is easier because V_0 only indirectly enters the formula for trading profits through v:

$$\frac{\partial(\lambda V - C)}{\partial v}\frac{dv}{dV_0} = -\frac{Nv}{[(N-1)v+1][3(N-1)v+1]} < 0$$

This implies that the decision maker subsidizes trading more when information is more valuable for production.

To assess how price informativeness varies with information acquisition cost, we differentiate equation (27) with respect to c_v:

$$d(V_0 - V_\delta)/dc_v = \frac{-NV_0^2/2c_v^2}{((N-1)v+1)^2(3(N-1)v+1)} < 0$$

This means that prices are less informative when information acquisition costs are higher. Furthermore, by inspecting equation (27), we see that $d(V_0 - V_\delta)/dV_0$ must be positive because price informativeness increases with V_0 directly and indirectly through v.

Finally, the change in price informativeness with respect to the number of informed traders is less clear.

$$d(V_0 - V_\delta)/dN = \frac{-NV_0}{((N-1)v+1)^2} \frac{2v^2}{3(N-1)v+1} + \frac{v(1-v)V_0}{((N-1)v+1)^2} < 0 \text{ if and only if}$$

$$3(N-1)v^2 - (N-4)v - 1 < 0$$

which will sometimes fail depending on the values of N, c_v and V_0. For example, when $N = 2$ and $V_0/c_v > 32/27$, the reader can verify that $v > 1/3$, which violates the condition above. QED.

Proof of Proposition 3: Recall that we have already computed the production profits conditional on acquiring the level of information in equation (27) and total information acquisition costs are given by $Nc_v v^2/2$. Thus, social welfare maximization can be written:

$$\max_v W = -Nc_v v^2/2 + \frac{NvV_0}{(N-1)v+1} \tag{33}$$

First, we look for the socially optimal information acquisition level, which we refer to as v_{soc}. The first-order condition for the social maximization problem is:

$$\frac{dW}{dv} = -Nc_v v_{soc} + \frac{NV_0}{[(N-1)v_{soc}+1]^2} = 0$$

The second-order condition for maximization is clearly satisfied because:

$$\frac{d^2W}{dv^2} = -Nc_v - \frac{N(N-1)V_0}{[(N-1)v+1]^3} < 0$$

This implies that the first-order condition characterizes a social welfare maximum. Rearranging the first-order condition slightly, we obtain a cubic equation in v_{soc} that can be solved in closed form using standard methods:

$$v_{soc}[(N-1)v_{soc}+1]^2 = \frac{V_0}{c_v} \tag{34}$$

Because $V_0 > 0$ and c_v is finite, we can infer that $v_{soc} > 0$. Clearly, this is greater than the level of information acquisition ($v = 0$) when the decision maker does not participate in the securities market and there is no trade.

Now we establish that the decision maker's subsidy is insufficient to motivate the socially optimal level of information acquisition. We know that $dW/dv = 0$ when evaluated at v_{soc}. If we can show that $dW/dv - d\pi/dv > 0$ when both derivatives are evaluated at the social optimum ($v = v_{soc}$), then we can infer that the decision maker's choice is less than the social optimum. The difference between the social and private incentive to increase information acquisition is:

$$\frac{dW}{dv}(v = v_{soc}) - \frac{d\pi}{dv}(v = v_{soc}) = N c_v v_{soc} > 0$$

This means that the socially optimal level of information acquisition will always exceed the decision maker's private optimum.

Finally, recall that equation (18) establishes a one-to-one mapping between liquidity and the equilibrium level of information precision acquired by informed traders. This means that all of the information acquisition comparisons above also apply to securities market liquidity. QED.

Proof of Proposition 4: The first step is to determine the trading strategies of the informed trader and the stakeholder. The informed trader solves the same profit maximization problems as before in equations (11) and (15). The general solution to his problem is given in equation (13). For the model in this section, we have $c_v = 0$, $v = 1$ and $N = 1$. With these values, equation (13) becomes:

$$b_I = (1/2\lambda) \Rightarrow q_I = (1/2\lambda)s \tag{35}$$

The stakeholder's profit maximization problem can be written:

$$q_M \in \arg\max_{\widehat{q}_M} E(\widehat{q}_M(y - p) + \theta x)$$

where $Q = q_I + q_M$. Substituting the equilibrium pricing and decision rules, we obtain:

$$q_M \in \arg\max_{\widehat{q}_M} E(\widehat{q}_M(y - \lambda(q_I + \widehat{q}_M)) + \theta k \lambda(q_I + \widehat{q}_M))$$

which has the solution:

$$q_M = (k/2)\theta \tag{36}$$

Now we can examine how much aggregate order flow varies and covaries with productivity. Using standard definitions, we find:

$$V = \frac{1}{4\lambda^2} V_0 + \frac{k^2}{4} V_\theta \tag{37}$$

$$C = V_0/2\lambda \tag{38}$$

Furthermore, we note that a competitive market maker must set market liquidity (λ_c) to satisfy a zero trading profit constraint, which can be found by setting her profits in equation (22) equal to zero:

$$\lambda_c = C/V \tag{39}$$

Because we maintain the assumption that the decision maker will choose the optimal rule after observing the securities market price, we can use the same expression for k from equation (25) in the earlier model. Combining equations (25) and (39), we find that the decision maker sets $k = 1$, implying that $x = p$.

Next, we can determine an explicit solution for the price sensitivity set by a competitive market maker (λ_c) described in equation (39). Substituting the expressions for V and C in equations (37) and (38) into equation (39), we obtain:

$$\lambda_c \left(\frac{1}{4\lambda_c^2} V_0 + \frac{k^2}{4} V_\theta \right) - V_0/2\lambda_c = 0$$

which has the solution:

$$\lambda_c = \sqrt{V_0/V_\theta} \tag{40}$$

which is identical to the expression in Kyle (1985).

Now we can use the equilibrium values for liquidity (λ_c) and the decision rule (k) to compute traders' equilibrium strategies in equations (35) and (36). The informed trader will follow the strategy:

$$q_I = \frac{\sqrt{V_\theta}}{2\sqrt{V_0}} s$$

and the stakeholder will follow the trading strategy:

$$q_M = \frac{1}{2}\theta$$

Substituting these values in the equilibrium pricing function, we obtain:

$$p = \frac{1}{2}(s + \sqrt{V_0/V_\theta}\theta)$$

In the competitive market making equilibrium, we are examining the hypothetical situation in which the decision maker does not trade. This means that her total profits are equal to her production profits. Using the pricing rule along with the equation for λ_c, we find that the decision maker's expected total profits are given by:

$$\pi_0 = \pi_{P0}^{Max} = V_0 - E(y-p)^2 = V_0/2 \tag{41}$$

In addition, this equation reveals that the mean squared prediction error for prices is given by $E(y-p)^2 = V_0/2$, which is the same value as in Kyle (1985). In equation (3), we also defined this value as the price informativeness. In a competitive equilibrium without decision maker intervention in the securities market, price informativeness is unaffected by the amount of stakeholder trading.

Finally, we note that there is no trade when $V_\theta \to 0$ because this implies that $q_M \to 0$ and $\lambda_c \to \infty$, which implies that $q_I \to 0$. QED.

Proof of Proposition 5: First, we focus on both the decision maker's and the stakeholder's production profits. To determine the decision maker's production profits, we follow the exact same logic that led to equation (26). Then we can substitute equations (37) and (38) for V and C to obtain:

$$\pi_{P0}^{Max}(\lambda) = C^2/V = \frac{V_0^2}{V_0 + \lambda^2 k^2 V_\theta}$$

Using the assumed pricing and decision rules along with equation (36), we find that the stakeholder's payoff from the production effort chosen by the decision maker (x) is given by:

$$\pi_{PM} = E(\theta x) = E(\theta k \lambda Q) = \lambda k^2 V_\theta/2$$

Because the decision maker will choose optimally given the market price she observes, this implies that the constraint on k in equation (25) must hold. Combining this constraint with the expressions for V and C in (37) and (38), we infer that:

$$(\lambda^2 k(\lambda)^2 V_\theta + V_0)k(\lambda) - 2V_0 = 0 \tag{42}$$

We can also combine equation (25) with the non-positive trading profits restriction to show that $k > 1$:

$$\pi_{T0} = \lambda V - C \leq 0 \Rightarrow k(\lambda) = C/\lambda V \geq 1$$

We also note that strictly negative trading profits implies that $\lambda < C/V = \lambda_c$, which means that $k > 1$. The decision maker sets prices to underreact to information in order for informed traders to reap extra profits from their trades. However, the decision maker's effort rule ($x = kp$) is not biased. She appropriately adjusts for the underreaction in prices by changing her effort level more than one-for-one in response to changes in prices.

To show that $k < 2$, we rewrite equation (42) as:

$$\lambda^2 k^3 V_\theta / V_0 = 2 - k$$

Because the left-hand side is always positive (assuming $V_\theta > 0$), the right-hand side must always be positive, implying that $k < 2$. Thus, $1 < k < 2$.

Differentiating the constraint on k with respect to λ, we see that:

$$\frac{dk}{d\lambda} = -\frac{2\lambda k^3 V_\theta / V_0}{3\lambda^2 k^2 V_\theta / V_0 + 1} < 0 \tag{43}$$

Now we can write the maximization problem for the decision maker as:

$$\max_\lambda \lambda V - C + C^2/V$$

subject to $k = k(\lambda)$ and $\lambda V - C \leq 0$. The second constraint ensures that the uninformed decision maker will not make positive trading profits as a market maker. We will solve the maximization problem by first ignoring this second constraint, but verifying later that it is satisfied in our proposed solution. Applying the equations for C and V, we can rewrite the maximization as:

$$\max_\lambda -\frac{1}{4\lambda} V_0 + \frac{k^2 \lambda}{4} V_\theta + \frac{V_0^2}{V_0 + \lambda^2 k^2 V_\theta} \tag{44}$$

The derivative of the decision maker's expected profits with respect to λ is:

$$\begin{aligned}
\frac{d\pi}{d\lambda} &= \left(\frac{\partial \pi}{\partial \lambda}\right) + \left(\frac{\partial \pi}{\partial k}\frac{\partial k}{\partial \lambda}\right) \\
&= \left[\frac{V_0}{4\lambda^2} + \frac{k^2}{4} V_\theta - \frac{2\lambda k^2 V_\theta}{(1 + \lambda^2 k^2 V_\theta / V_0)^2}\right] - \left(\frac{k\lambda}{2} V_\theta - \frac{2\lambda^2 k V_\theta}{(1 + \lambda^2 k^2 V_\theta / V_0)^2}\right) \frac{2\lambda k^3 V_\theta / V_0}{3\lambda^2 k^2 V_\theta / V_0 + 1}
\end{aligned}$$

which can be rewritten by repeatedly substituting the $k(\lambda)$ constraint in equation (42):

$$\frac{d\pi}{d\lambda} = \frac{V_0}{4\lambda^2} + \frac{k^2}{4} V_\theta - \frac{\lambda k^3 V_\theta (1 + \lambda k V_\theta / V_0)}{3\lambda^2 k^2 V_\theta / V_0 + 1} \tag{45}$$

Further substitution of the $k(\lambda)$ constraint in equation (42) and simplification gives:

$$\frac{d\pi}{d\lambda} = \frac{V_0}{2\lambda^2 k(3-k)} \left[3 - k - (2-k)^2 - \lambda k^2(2-k)\right]$$

The first-order condition for the decision maker's optimal liquidity provision is equivalent to:

$$\frac{d\pi}{d\lambda} \propto -\lambda k^2(2-k) - k^2 + 3k - 1 = 0 \tag{46}$$

which can be solved for λ in terms of k:

$$\lambda = \frac{3k - (1+k^2)}{k^2(2-k)} \tag{47}$$

Using this solution for $\lambda(k)$ in the $k(\lambda)$ in equation (42), we obtain a solution for k:

$$[3k - (1+k^2)]^2 V_\theta / V_0 - (2-k)^3 k = 0 \tag{48}$$

Equation (48) provides an explicit decision rule k as a function of natural liquidity (V_θ/V_0), which can be solved in closed form using the quartic formula identified by Ferrari and Cardano (1545). Because the analytical expression for k is quite cumbersome, in the interest of brevity and clarity, we denote the unique positive real root for k in equation (48) as $k(V_\theta/V_0)$ to emphasize its dependence on natural liquidity (V_θ/V_0).[20]

This closed form solution can be combined with equation (47) to find a closed form liquidity rule:

$$\lambda(V_\theta/V_0) = \sqrt{\frac{2 - k(V_\theta/V_0)}{k(V_\theta/V_0)^3}} \sqrt{V_0/V_\theta}$$

We can evaluate the first-order condition in equation (46) at $k = 1$ and $\lambda = \sqrt{V_0/V_\theta}$, which is the competitive equilibrium.

$$\frac{d\pi}{d\lambda}(\lambda_c = \sqrt{V_0/V_\theta}) \propto 1 - \sqrt{V_0/V_\theta}$$

This equation shows that $d\pi/d\lambda < 0$ if and only if $V_\theta < V_0$. In other words, the competitive level of price sensitivity is too high when there is little manipulative (endogenous noise) trading activity. Thus, we conclude that the decision maker is willing to subsidize liquidity provision (*i.e.*, set $\lambda < \lambda_c$) in the securities market in situations where the stakeholder

[20] An explicit closed form solution is available from the authors upon request.

(endogenous noise trader) provides the least liquidity. However, the decision maker will not intervene in the securities market when the amount of stakeholder trade is sufficiently large (*i.e.*, $V_\theta \geq V_0$).

Furthermore, we can easily show that the decision maker's liquidity subsidy induces the informed agent to trade more aggressively (*i.e.*, set b_I higher) based on his information. By definition, the decision maker's liquidity subsidy decreases λ relative to λ_c. Using equation (35) for the informed trader's aggressiveness (b_I), we see that the informed trader uses his signal more when liquidity is greater.

In addition, we note that the stakeholder trades more aggressively (*i.e.*, sets b_M higher) based on his decision objective (θ) when the decision maker subsidizes liquidity. The stakeholder's trading aggressiveness (b_M) is given by $k/2$ according to equation (36), which clearly increases in k. We have already shown above that the decision maker selects a greater k value when she sets prices than she would when the competitive market maker sets prices—*i.e.*, $k > 1$. As a result, the stakeholder trades more aggressively based on his decision objective (θ) when the decision maker sets prices.

Lastly, we note that total price informativeness is equal to the decision maker's maximized production profits, which we showed explicitly in equation (3). When the decision maker chooses to subsidize liquidity, she does so only because it increases her total profits. Because her trading profits decrease when she subsidizes liquidity, we can infer that her production profits must increase. This implies that price informativeness is greater when the decision maker subsidizes liquidity. QED.

Proof of Proposition 6: First, we examine how natural liquidity (V_θ/V_0) affects the decision maker's effort rule k by differentiating equation (48) with respect to V_θ/V_0 and solving for $\frac{dk}{d(V_\theta/V_0)}$:

$$\frac{dk}{d(V_\theta/V_0)} = -\frac{[3k - (1+k^2)]^2/2}{[3k-(1+k^2)](3-2k)V_\theta/V_0 + (2k-1)(2-k)^2}$$

which can be simplified using the solution for $k(V_\theta/V_0)$ implicitly defined by equation (48):

$$\frac{dk}{d(V_\theta/V_0)} = \frac{-(2-k)^{-2}[3k-(1+k^2)]^3/2}{(3-2k)(2-k)k + [3k-(1+k^2)](2k-1)} \tag{49}$$

Because the numerator above is always negative, we infer that $\frac{dk}{d(V_\theta/V_0)}$ is negative if and only if the denominator is positive. After some simplification, this condition is equivalent to:

$$dk/d(V_\theta/V_0) < 0 \text{ if and only if } k + 1 > 0$$

which is always true because $1 < k < 2$. We conclude that $dk/d(V_\theta/V_0) < 0$.

By differentiating the $\lambda(k)$ solution in equation (47) with respect to k, we can show that price is more responsive to order flow when the effort decision is more responsive to the price:

$$d\lambda/dk = \frac{4 - 9k + 6k^2 - k^3}{k^3(2-k)^2} = \frac{(k-1)^2(4-k)}{k^3(2-k)^3} > 0$$

where the last inequality holds because we have already shown that $1 < k < 2$. This means that the comparative statics results for k also hold for λ, implying that $d\lambda/d(V_\theta/V_0) < 0$.

Now we can address what happens to the size of the liquidity subsidy as natural market liquidity (V_θ/V_0) increases. Using equations (22) and (25), the size of the subsidy is given by:

$$\pi_{T0} = \lambda V - C = \lambda V(1-k) \leq 0$$

where the inequality relies on the fact that $k \geq 1$. This equation can be simplified by substituting using equations (37) and (47):

$$\pi_{T0} = (V_0/4)\left[\frac{k^2(2-k)^2 + (3k - (1+k^2))^2 V_\theta/V_0}{(3k-(1+k^2))(2-k)}\right](1-k)$$

Using the solution for $k(V_\theta/V_0)$ implicitly defined by equation (48), we can simplify the liquidity subsidy further:

$$\pi_{T0} = (V_0/2)[3k - (1+k^2)]^{-1}k(2-k)(1-k) \leq 0$$

By inspection, it is clear that $\pi_{T0} = 0$ either when $V_\theta/V_0 = 1$ and $k = 1$ or when $V_\theta/V_0 = 0$ and $k = 2$. In addition, we see that $\pi_{T0} < 0$ for $0 < V_\theta/V_0 < 1$ and $1 < k < 2$. This immediately implies that there is a non-monotonic relationship between the size of the subsidy and the market's natural liquidity. In particular, as natural liquidity increases from $V_\theta = 0$ to $V_\theta > 0$, the size of the liquidity subsidy increases. However, beyond some cut-off point $\overline{V}$ between 0 and V_0, the size of the liquidity subsidy diminishes when natural liquidity increases.

Now we examine how the decision maker's expected total profits depend on natural liquidity (V_θ/V_0). In particular, we will evaluate how the difference between her profits when she subsidizes liquidity and her profits when she does not subsidize liquidity depends on natural liquidity. This measures reveals how the impact of the liquidity subsidy depends on natural liquidity. The difference between her profits when she subsidizes liquidity and her profits when she does not subsidize liquidity is given by equation (44) minus equation (41):

$$\Delta\pi = -\frac{1}{4\lambda}V_0 + \frac{k^2\lambda}{4}V_\theta + \frac{V_0^2}{V_0 + \lambda^2 k^2 V_\theta} - \frac{V_0}{2}$$

Holding V_0 fixed and differentiating this change in profits with respect to natural liquidity (V_θ/V_0), we obtain:

$$\frac{d\Delta\pi}{d(V_\theta/V_0)} = V_0\left[\frac{\lambda k^2}{4} - \frac{\lambda^2 k^2}{(1+\lambda^2 k^2 V_\theta/V_0)^2}\right]$$

where we have ignored the indirect effect of natural liquidity through λ and $k(\lambda)$ by the envelope theorem. We can simplify this expression somewhat by substituting equations (42) and (46):

$$\begin{aligned}\frac{d\Delta\pi}{d(V_\theta/V_0)} &= \frac{V_0\lambda k^2}{4}(1-\lambda k^2)\\ &= -\frac{V_0\lambda k^2}{4}\left[\frac{(3-k)(k-1)}{2-k}\right] < 0\end{aligned}$$

where the final inequality holds because $1 < k < 2$. QED.

Proof of Proposition 7: We can evaluate social welfare (W^{soc}) as the sum of the expected production profits of the decision maker in equation (26) and the expected non-trading profits of the stakeholder:

$$W^{soc}(x) = \pi_{P0}^{Max}(x) + \pi_{PM}(x) = C^2/V + E(\theta x) \tag{50}$$

In the hypothetical competitive equilibrium, the decision maker's expected production profits are given by $V_0/2$ and the stakeholder's non-trading profits are given by $\sqrt{V_0 V_\theta}/2$. This implies that total social welfare in the competitive equilibrium is equal to:

$$(\sqrt{V_0}/2)(\sqrt{V_0} + \sqrt{V_\theta}) \tag{51}$$

We can compare the social welfare at the competitive equilibrium, when the decision maker sets liquidity in the securities market, and when a social planner sets liquidity in the securities market. First, we rewrite total social welfare in equation (50) as a function of λ and k:

$$W(\lambda) = \frac{V_0^2}{V_0 + \lambda^2 k^2 V_\theta} + \lambda k^2 V_\theta / 2$$

where we have used our assumption that the decision maker implements her optimal effort rule after observing the equilibrium price. This expression for social welfare with decision maker intervention can be simplified somewhat using the constraint on k in equation (42):

$$W(\lambda) = (k/2)\, V_0[(1 + \lambda k V_\theta / V_0)] \tag{52}$$

To find the social optimum, we must differentiate the social welfare function identified above with respect to λ subject to the constraint on $k(\lambda)$ in equation (42), which gives:

$$dW/d\lambda = -\frac{\lambda k^3 V_\theta (1 + 2\lambda k V_\theta / V_0)}{3\lambda^2 k^2 V_\theta / V_0 + 1} + \frac{k^2}{2} V_\theta \tag{53}$$

where we have used the differentiated constraint on k in equation (43). By evaluating the derivative at the competitive optimum of $k = 1$ and $\lambda = \sqrt{V_0/V_\theta}$, we can compare the social optimum level of liquidity to liquidity in the competitive equilibrium:

$$\frac{dW}{d\lambda}(\lambda = \lambda_c) = -\frac{\sqrt{V_0 V_\theta}(1 + 2\sqrt{V_\theta / V_0})}{4} + \frac{V_\theta}{2} = -\sqrt{V_0 V_\theta}/4 < 0$$

This means that λ_c is too high from a social standpoint, implying that the competitive level of liquidity (λ_c^{-1}) is lower than the social optimum. To compare the social and private optima, we can evaluate the difference between the first-derivative equations (45) and (53):

$$\frac{d\pi}{d\lambda} - \frac{dW}{d\lambda} = \frac{V_0(1 + \lambda^2 k^2 V_\theta / V_0)^2}{4\lambda^2 (3\lambda^2 k^2 V_\theta / V_0 + 1)} > 0$$

Whenever $\lambda = \lambda_{soc}$ and $dW/d\lambda = 0$, we must have $d\pi/d\lambda > 0$, which implies that the private optimum always entails a greater level of price sensitivity (λ) than the social optimum. In other words, the private optimum always involves less liquidity (λ^{-1}) than the socially optimal level of liquidity (λ_{soc}^{-1}). Combining this result with the earlier result that the private optimum always entails at least as much liquidity provision as the competitive optimum, we infer that the decision maker's liquidity subsidy in the private optimum always improves upon the competitive optimum.

Next, we can evaluate the change in social welfare resulting from the decision maker's intervention. Social welfare under the decision maker's intervention is given by equation (52). Social welfare under the competitive market maker is given by equation (51). Thus, the change in social welfare resulting from decision maker intervention is the difference between these two expressions:

$$\Delta W = (k/2)\, V_0[(1 + \lambda k V_\theta/V_0)] - (V_0/2 + \sqrt{V_0 V_\theta}/2) = (k-1)V_0/2 + (\lambda k^2 - \sqrt{V_0/V_\theta})V_\theta/2$$

using the constraint on k in equation (42). We can simplify this equation using the decision maker's simplified first-order condition in equation (46):

$$\begin{aligned}
\Delta W &= (k-1)V_0/2 + \lambda k^2 - \sqrt{V_0/V_\theta})V_\theta/2 \\
&= (V_0/2)[(k-1)(1 + V_\theta/V_0) + \frac{V_\theta/V_0}{2-k} - \sqrt{V_\theta/V_0}]
\end{aligned}$$

Differentiating this expression with respect to V_θ/V_0, we obtain:

$$\frac{d\Delta W}{d(V_\theta/V_0)} \propto k - 1 + (2-k)^{-1} - \frac{1}{2}(V_\theta/V_0)^{-1/2} + \frac{dk}{d(V_\theta/V_0)}(1 + (V_\theta/V_0) + (V_\theta/V_0)(2-k)^{-2})$$

where $dk/d(V_\theta/V_0)$ is given by equation (49), which can be simplified using the solution for k in equation (48):

$$\frac{dk}{d(V_\theta/V_0)} = -\frac{[3k - (1+k^2)](2-k)k}{2(V_\theta/V_0)(1+k)}$$

Substituting this expression for $dk/d(V_\theta/V_0)$, we obtain:

$$\begin{aligned}
\frac{d\Delta W}{d(V_\theta/V_0)} &\propto -\frac{1}{2}(V_\theta/V_0)^{-1/2} + k - 1 + (2-k)^{-1} \\
&\quad -\frac{[3k - (1+k^2)]^3}{2(2-k)^2(1+k)} - \frac{[3k - (1+k^2)](2-k)k}{2(1+k)}(1 + (2-k)^{-2}) \\
&\propto -\frac{1}{2}\frac{[3k - (1+k^2)]}{\sqrt{(2-k)^3 k}} + \frac{[3k - (1+k^2)]}{(2-k)} \\
&\quad -\frac{[3k - (1+k^2)]^3}{2(2-k)^2(1+k)} - \frac{[3k - (1+k^2)]k}{2(1+k)(2-k)}((2-k)^2 + 1)
\end{aligned}$$

where we have used the solution for k in equation (48) in the simplifying step above. After some additional algebra that relies on the fact that $1 < k < 2$, one can show that $\frac{d\Delta W}{d(V_\theta/V_0)} < 0$

if and only if:

$$-(1+k)\sqrt{(2-k)k}+k(3-2k)<0$$

By inspection, we see that $\frac{d\Delta W}{d(V_\theta/V_0)}<0$ when $\frac{3}{2}\leq k<2$ because both of the terms above are negative. When $k<\frac{3}{2}$, the above expression is negative if and only if:

$$k(3-2k)<(1+k)\sqrt{(2-k)k}$$

which is equivalent to the condition:

$$5k^2(k-\frac{3}{2})-(\frac{3}{2}k-1)(3k-2)<0$$

where both terms must be negative because $1<k<\frac{3}{2}$. Regardless of whether $k\geq\frac{3}{2}$ or $k<\frac{3}{2}$, we see that $\frac{d\Delta W}{d(V_\theta/V_0)}<0$. We conclude that the decision maker's intervention enhances social welfare more when natural liquidity (V_θ/V_0) is low. QED.

Proof of Proposition 8: To prove this, we must verify that the decision maker and all traders are willing to play the same equilibrium strategies (λ, k, b_I, etc.) in both situations—*i.e.*, when the decision maker is trading in the market and when the decision maker makes the market. Because we have already identified all agents' equilibrium strategies when the decision maker makes the market, we only need to verify that these strategies are also optimal for all agents when the decision maker trades in the market.

We can think of the decision maker as indirectly selecting liquidity in the random trading model, subject to the constraint that her selection must satisfy the competitive market maker's zero profit constraint. To show there is an equilibrium in which the decision maker selects the same degree of liquidity as in the market making model, we can assume for now that all other agents behave as they do in the market making equilibrium. If the decision maker's liquidity choice is the same given all other traders' strategies and all other traders' strategies are the same given the decision maker's strategy, then the equilibria in the two models must be the same. The general approach for showing that agents select the same choices in both models is to show that they solve the same maximization problems subject to the same constraints. Throughout the proof, we will denote the variables and parameters in the random trading model by the same notation as in the market making model except for the subscript R—*e.g.*, the equilibrium price is given by p_R rather than p.

To determine the decision maker's optimization problem in the random trading model, we must first examine the liquidity set by the competitive market maker, which places a constraint on the decision maker's liquidity choice. Because he treats the decision maker as a noise trader, the competitive market maker maintains a simple linear pricing rule given by:

$$p_R = \lambda_R(Q_R + \rho) \tag{54}$$

where Q_R is the aggregate order flow of all other traders, which could include informed traders and a decision stakeholder. After she observes the realization of ρ, the decision maker can infer the aggregate order flow of all other traders by inverting the equilibrium price:

$$p_R/\lambda_R - \rho = Q_R \tag{55}$$

The decision maker's inference about the conditional expectation of y is based on the aggregate order flow of all other traders. This order flow is distributed normally because we are examining an equilibrium where strategies are linear and based on normally distributed random variables. Applying the conditional expectation formula for jointly normally distributed variables to the decision maker's inference problem, we obtain:

$$E(y|Q_R) = \frac{C_R}{V_R} Q_R \tag{56}$$

where we have defined $C_R = Cov(y, Q_R)$ and $V_R = Var(Q_R)$. Because we are assuming for now that all other traders follow the same strategies as in the market making model ($Q_R = Q$), the definitions of C_R and V_R imply that $C_R = C$ and $V_R = V$ as long as $\lambda_R = \lambda$, which will be useful later. We suppress the dependence of the covariances and variances of order flow on the market maker's pricing rule because these functional dependences are identical in the random trading and market making models.

As before, we assume the decision maker cannot commit to any decision other than her ex post optimal choice, which is $x_R = E(y|Q_R)$. Using equations (56) and (55), this choice can be expressed as:

$$x_R = \frac{C_R}{V_R}(p_R/\lambda_R - \rho) = k_R Q_R + k_\rho \rho \tag{57}$$

where we have defined two decision rule parameters, $k_R = \frac{C_R}{\lambda_R V_R}$ and $k_\rho = -\frac{C_R}{V_R}$, to facilitate our analogy with the market making model. Moreover, we note that $k_R = \frac{C_R}{\lambda_R V_R}$ is identical to the expression (25) for k in the market making model (after substituting the random trading model parameters). Because $C_R = C$ and $V_R = V$ whenever $\lambda_R = \lambda$, our expression for $k_R = k$ as long as $\lambda_R = \lambda$. However, we still need to show that $\lambda_R = \lambda$.

Recall that the decision maker's total profits come from both her production and her trading activities. We can simplify her production profits using the fact that $k_R = k$, $C_R = C$, and $V_R = V$ whenever $\lambda_R = \lambda$. Following similar reasoning used in the market making model in (27), we find that the production profits expression for the random trading model is:

$$\pi_{P0R}^{Max} = \frac{C_R^2}{V_R} \tag{58}$$

The decision maker also realizes profits or losses from her random trading activity. Using the definition of trading profits and the pricing rule equation (54), her random trading profits are given by:

$$\pi_{T0R} = E(\rho(y - p_R)) = -\lambda_R V_\rho \tag{59}$$

Furthermore, the competitive market maker will set λ_R so that he will attain zero profits conditional on aggregate order flow $(Q_R+\rho)$. This means that $\lambda_R Var(Q_R+\rho) - Cov(y, Q_R+\rho) = 0$, so we can solve for the liquidity parameter λ_R:

$$\lambda_R = \frac{Cov(y, Q_R + \rho)}{Var(Q_R + \rho)} = \frac{C_R}{V_R + V_\rho} \tag{60}$$

Note that equation (60) implies there is a mapping between V_ρ and λ_R, implying that we can treat the decision maker's random trading problem as one where she chooses λ_R. We rearrange equation (60) slightly, solving for the expected trading profits of the decision maker:

$$-\lambda_R V_\rho = \lambda_R V_R - C_R \tag{61}$$

Using (58), (59) and (61), we can write the decision maker's total expected profit maximization with random trading as:

$$\max_{\lambda_R} \lambda_R V_R - C_R + \frac{C_R^2}{V_R} \tag{62}$$

which depends only on λ_R. Because $C_R = C$ and $V_R = V$ whenever $\lambda_R = \lambda$, the random trading objective function (62) above is isomorphic to the objective function for the decision maker when she makes the market. In fact, in both the random trading model with informed traders and the model with a decision stakeholder, the decision maker's objective function is given by the sum of equations (22) and (26), which has the same form of equation (62). Thus, regardless of which traders are included in the random trading model, we infer that $\lambda_R = \lambda$ when all other traders are choosing their equilibrium trading strategies in the market making model. In addition, as noted above, $\lambda_R = \lambda$ implies that $k_R = k$.

Now we must still show that all other traders are willing to choose the same equilibrium strategies given the liquidity and effort decision rule choices, $\lambda_R = \lambda$ and $k_R = k$. That is, we must show that the decision maker's random trading order (ρ) does not alter any trader's trading strategy, holding the liquidity and effort rules constant. Consider a general trader who may or may not have information about the security's payoff (y) and who may or may not care about the decision ($\theta = 0$ or $\theta \neq 0$). We can write the trader's maximization problem as:

$$\max_{\widehat{q}_{iR}} E[\widehat{q}_{iR}(y - p) + \theta x]$$

subject to $p_R = \lambda_R(Q_R + \rho)$ and $x_R = k_R p_R = k_R \lambda_R (Q_R + \rho)$.

To determine whether there is an equilibrium in which all traders adopt the same strategies as in the market making model, we can assume that all traders but one adopt their same strategies ($Q_{-iR} = Q_{-i}$) and ask whether the single trader would have an incentive to deviate from the equilibrium. As mentioned above, we can also assume that the decision maker adopts the same equilibrium strategies for λ and k, such that $\lambda_R = \lambda$ and $k_R = k$. Under these assumptions we can rewrite the trader's maximization problem as:

$$\max_{\widehat{q}_{iR}} E[\widehat{q}_{iR}(y - k\lambda(Q_{-i} + \widehat{q}_{iR} + \rho)) + \theta k\lambda(Q_{-i} + \widehat{q}_{iR} + \rho)]$$

Now we note that, by construction, the decision maker's random order has a zero mean and is uncorrelated with any other trader's order or payoffs. This implies that the trader's maximization problem is:

$$\max_{\widehat{q}_{iR}} E[\widehat{q}_{iR}(y - k\lambda(Q_{-i} + \widehat{q}_{iR})) + \theta k\lambda(Q_{-i} + \widehat{q}_{iR})]$$

which is identical to the problem a trader solves in the market making model. We conclude that all traders would have no incentive to deviate from the identical random trading equilibrium. QED.

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