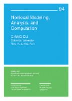
Nonlocal Modeling, Analysis, and Computation



Qiang Du

Publisher:	SIAM
Publication Date:	2019
Number of Pages:	166
Format:	Paperback
Series:	CBMS-NSF Regional Conference Series in Applied Mathematics
Price:	$59.00
ISBN:	978-1-611975-61-1
Category:	Monograph

MAA REVIEW TABLE OF CONTENTS

[Reviewed by Bill Satzer, on 08/16/2020]

This monograph offers a concise introduction to the concepts of nonlocal modeling for continua. It includes both examination of the mathematical foundations and treatment of the challenges of developing numerical solutions.

At its heart, nonlocal modeling attempts to deal with systems that intrinsically involve interactions at a distance. One of the canonical examples is anomalous diffusion. This occurs whenever the mean-squared displacement in a diffusion process is not linear in time. Anomalous diffusion is common in biological systems, such as protein diffusion within cells. Other examples arise in fracture dynamics where local and continuous damage of a structure develops discontinuously into whole-scale fracture. Besides considering systems that are nonlocal in space, nonlocal modeling also addresses systems that are nonlocal in time, such as systems with memory.

Many of the questions that nonlocal modeling investigates have previously been approached through models with partial differential equations (PDEs) that generally represent local information and deal primarily with smooth quantities. Nonlocal modeling, as a complement or alternative to PDE models, can naturally account for nonlocal interactions explicitly and remain valid even when solutions become singular. The cost is significantly greater complexity in both analysis and numerical methods.

Much of prior work on nonlocal modeling focused on scalar equations with an infinite or global range of interactions. The author of this work gives particular attention to systems of nonlocal equations and explores explicit dependence on the finite range of nonlocal interactions. He also considers how to couple local and nonlocal modeling effectively and how to guarantee reliability and robustness of associated numerical methods.

One of the strengths of this book is its approach to establishing an axiomatic framework for vector and tensor fields for nonlocal models and the development of a nonlocal vector calculus. This leads to the beginnings of functional analysis and operator methods for nonlocal systems as well as a corresponding calculus of variations.

The final chapter describes directions for further development of nonlocal modeling. Beyond applications to the physics of diffusion and fracture mechanics the author suggests fruitful research areas in stochastic processes including jump processes and Levy flights. Substantial mathematical work remains to be done in the areas of functional analysis, fractional differential and integrodifferential operators, and numerical methods for particle-based discretization of PDEs.

This is a research level monograph and maybe the only introduction that includes theoretical foundations, pertinent numerical methods, and applications. It would be most appropriate for graduate students and researchers with good backgrounds in PDEs and numerical methods. The book has a very complete bibliography that includes a great deal of the available literature relating to nonlocal modeling and its origins.

Bill Satzer (bsatzer@gmail.com), now retired from 3M Company, spent most of his career as a mathematician working in industry on a variety of applications ranging from speech recognition and network modeling to optical films and ceramic fiber-reinforced composites. Along the way he learned more about ceramics and alloys of aluminum than he could have imagined in graduate school. He did his PhD work in dynamical systems.

Tags: Partial Differential Equations

Log in to post comments