NONLINEAR STATE ESTIMATION IN MOBILE ROBOTS USING A FUZZY OBSERVER

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Abstract: The performance of model based fault detection and isolation systems can be improved by designing more accurate estimation methods. This work presents a novel implementation of a nonlinear Kalman filter based on the Takagi–Sugeno (TS) fuzzy structure, for a mobile robot. First, a TS model is derived from the robot kinematic equations, which is optimized through genetic algorithms to obtain an accurate model. Based on this model, several linear Kalman filters are combined using fuzzy logic, designing a nonlinear state estimator. Finally, the resulting fuzzy nonlinear observer is compared with the conventional Extended Kalman Filter, showing an improvement in performance and robustness.

Keywords: Fuzzy modelling, Kalman filters, Mobile robots, State estimation, Robotics.

1. INTRODUCTION

Over the last decade the use of mobile robots has increased, especially in dangerous tasks such as space exploration, land mines extraction, and rescue operations, keeping human operators away from harm. Mainly due to the dynamic and complex environments in which mobile robots work, a large number of different faults may appear, reducing the capabilities or even disabling the robot. Recent studies show that the mean time between failures is less than 20 hours for field robots, after which the robots must be repaired, consuming time and resources (Carlson, 2004). This means that when faults are taken into account, the advantages of using mobile robots are cut back, as repairs are not always possible, like in space exploration, or is very dangerous to retrieve the robot to do them. These facts imply that accurate fault detection and isolation (FDI) systems are required, increasing the reliability of the robots and reducing the costs associated to fault appearance.

The use of system models and system observers is one of the most effective methods for detecting and isolating faults (Basseville and Nikiforov, 1998). As state estimation becomes more accurate, the performance of the associated FDI system increases, detecting faults sooner and reducing confusions during the isolation process.

The Kalman filter is an optimal state estimator for linear systems which uses the model of the system and the measurements to minimize the error in the estimation (Kalman, 1960), making it an excellent tool for implementing a model based FDI system. This filter can be extended to nonlinear systems, such as mobile robots as shown in (Larsen, *et al.*, 1999), by using a linearization of the process on each operating point. Although the Extended Kalman Filter (EKF) gives a good estimation of the state vector of the system, it is very difficult to tune it accurately, as it is very sensitive to modelling errors and noise estimation. The main problem is that the noise covariance matrices needed must be precisely determined to get good results, which is seldom achieved in nonlinear systems.

To solve the problems related with nonlinearities, several authors have used fuzzy logic to tune the EKF dynamically, determining the covariance matrices needed as the operating point changes (Sasiadek and Hartana, 2002; Wang and Goh, 1999).

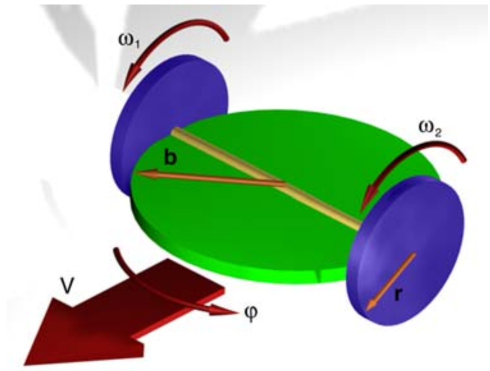

Fig. 1. Mobile robot model and basic parameters.

In this work, a novel implementation of a nonlinear Kalman filter based on the Takagi-Sugeno fuzzy structure is described. First, a kinematic and dynamic model of a mobile robot is presented. Next, a TS model of the robot kinematics is constructed, followed by the design of the fuzzy Kalman filter. Finally, the fuzzy filter is compared with the EKF through simulations, showing improvements in the accuracy and robustness of the estimation.

2. ROBOT MODEL

A simplified model of a mobile robot is presented on figure 1. The body is considered a circular disc of radius *b*, with two independent wheels of radius *r* each. The angular velocity of the right wheel is defined as ω_1, whereas the left one is ω_2. The posture of the robot is defined as a three element vector, containing the coordinates of the position of the robot with respect to some reference point and the heading angle: $\mathbf{X}=[x\ \ y\ \ \varphi]^T$. The equations that describe the behaviour of the robot can be divided into two sets, one for the kinematics and one for the dynamics, which includes the model of the motors used (Angeles, 1997).

2.1. Dynamic Equations

To simplify the simulation problem, very often only the kinematic equations are used to model the robot, disregarding the robot dynamics. In this work, the dynamic equations of the robot are also modelled, which helps to analyze the effect they have both on the fuzzy observer and the EKF. These equations relate the torque applied to the wheels by the motors, $\boldsymbol{\tau}$, with the acceleration the robot acquires, $\dot{\boldsymbol{\omega}}$:

$$\mathbf{M}\dot{\boldsymbol{\omega}} + F(\boldsymbol{\omega}) = \boldsymbol{\tau} \tag{1}$$

On equation 1, **M** represents the inertia matrix and *F*( ) is a function that depends on the speed of the wheels, representing the effects of friction. The DC motor equations are also included in this model to determine the torque each motor applies with a given input voltage, v_i. As described on figure 2, a PID controller is used to control the voltages on each motor, achieving the reference angular velocity on each wheel.

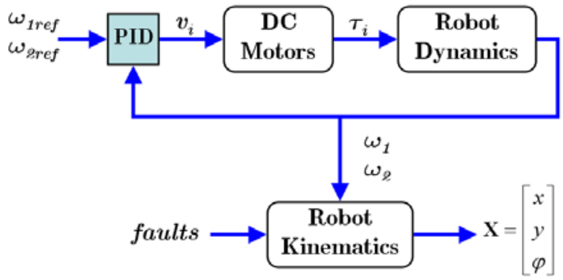

Fig. 2. PID control loop and robot kinematics.

2.2. Kinematic Equations

Using simple geometric relations, the linear speed of the robot at time step *k* can be obtained as equation 2 shows:

$$V_k = \tfrac{r}{2}\left(\omega_{1,k} + \omega_{2,k}\right) \tag{2}$$

The kinematic equations of the robot relate the angular velocity of each wheel with the variation in the posture vector, which is used as state vector through this work:

$$\Delta x_k = x_k - x_{k-1} = V_k \Delta T \cos\left(\varphi_k\right) \tag{3}$$

$$\Delta y_k = y_k - y_{k-1} = V_k \Delta T \sin\left(\varphi_k\right) \tag{4}$$

$$\Delta \varphi_k = \varphi_k - \varphi_{k-1} = \tfrac{r}{2b} \Delta T \left(\omega_{1,k} - \omega_{2,k}\right) \tag{5}$$

3. TAKAGI-SUGENO MODEL GENERATION

The objective of this work is to achieve a more accurate model for using in FDI systems on mobile robots. The first step to achieve this is to design an ideal model that does not consider the effects of noise on the process and the measurements. As the nonlinearities of this system are only on the robot kinematics, the Takagi–Sugeno fuzzy structure is used to model these equations, without considering the dynamic components.

3.1. Model Formulation

The fuzzy structure can be used to fuse together multiple linear models of a system, obtaining a nonlinear model of the process (Lin and Lee, 1996). As there is no single operating point in mobile robots, the whole spectrum of possibilities must be covered. To achieve this, equations 3 and 4 are linearized in five different heading angles: 0, -π/2, π/2, π and, -π. Although π and -π are actually the same angle, both linearizations are needed as the linearization about π will only be valid for values near to it. If it is evaluated near -π the approximation given is very bad, compromising the accuracy of the whole model. Each linear model will have the following structure, where φ_n is the linearization angle:

$$\Delta x_{k,n} = V_k \Delta T \left[\cos\left(\varphi_n\right) - \sin\left(\varphi_n\right)\left(\varphi_k - \varphi_n\right)\right] \tag{6}$$

$$\Delta y_{k,n} = V_k \Delta T \left[\sin\left(\varphi_n\right) + \cos\left(\varphi_n\right)\left(\varphi_k - \varphi_n\right)\right] \tag{7}$$

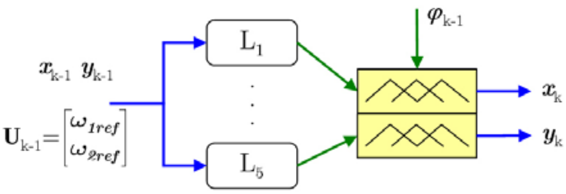

Fig. 3. Takagi–Sugeno nonlinear model.

As these two equations are independent, two different fuzzy structures are used to fuse together the linear models, one for Δx and one for Δy, both using φ as input for the fuzzyfication, as figure 3 shows. The output of each structure is given by the lineal combination of all the linear models, L_n, within:

$$\Delta x_k = \sum_{n=1}^{5} \mu_n \Delta x_{k,n} \tag{8}$$

$$\Delta y_k = \sum_{n=1}^{5} \lambda_n \Delta y_{k,n} \tag{9}$$

The values for the coefficients μ_n and λ_n are determined by the membership function (MF) associated to the linear model n. As there are five linearization angles, there are five MF for each variable: Centre (C) associated to $\varphi_1=0$, Left (L) associated to $\varphi_2=\pi/2$, Right (R) for $\varphi_3=-\pi/2$, Back 1 (B1) associated to $\varphi_4=-\pi$, and Back 2 (B2) associated to $\varphi_5=\pi$.

3.2. Model Optimization

The nonlinearities in the TS model are given by the morphology of the MF associated to each linear model, thus the accuracy of the model can be optimized by modifying each MF. One method that has proved to return good results is genetic optimization, which can be used to modify the morphology of each MF to improve the model.

Genetic optimization is based on the same principles as evolution theories, where a group of possible solutions, called parents, are combined together resulting in new child solutions that tend to be better in solving a certain problem (Lin and Lee, 1996). One recent development in genetic programming (Jung, 2003) uses the same method as bee colonies, where only the best member of the parent population, also called the queen, is used to create the new population, reducing the computational needs and complexity of the optimization.

Several assumptions can help in reducing of the number of free variables in the optimization process. First, as each linearization gives an exact approximation of the nonlinear function at the linearization point, the degree of membership of the associated MF at that point must be 1, whereas the other MF must be 0. Another consideration is that due to the geometric characteristics of the problem, the MF can be assumed to be symmetric about the linearization point. Taking these considerations into account, each MF is coded as a 7 element vector, where each element contains the degree of

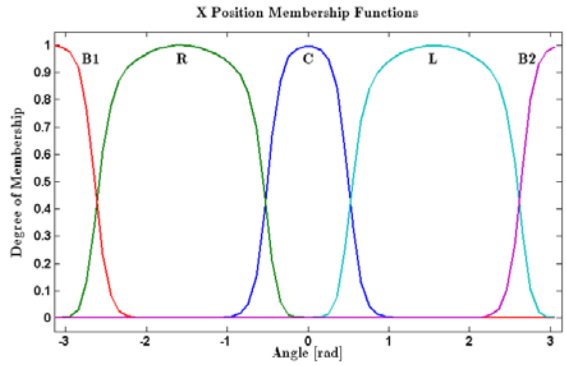

Fig. 4. X position membership functions.

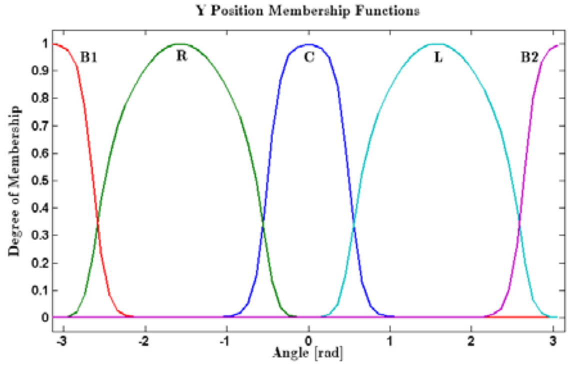

Fig. 5. Y position membership functions.

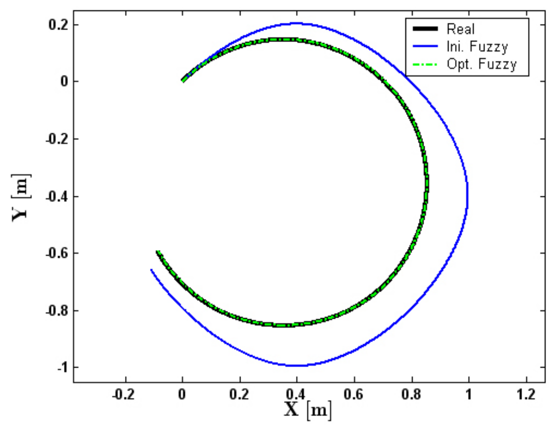

Fig. 6. Comparison between the initial and optimized fuzzy models.

membership or height of the MF at constant intervals in the angle variable. The values in between are obtained through a cubic interpolation of the neighbours.

The optimization is made by comparing the performance of the TS model using each individual in the population, with the result of the exact nonlinear model of the robot, given a certain input vector ***U***. Then, the best individual is selected to produce the next generation of solutions.

Using an initial population of 30 individuals with triangular MF, the system is optimized for 100 generations resulting in the MF of figures 4 for the x variable, and figure 5 for the y variable. Figure 6 shows a comparison between the estimation given by the model using the initial triangular MF, and the optimized model.

The simulation of the exact nonlinear model and the TS model for several different trajectories shows that the fuzzy model is extremely accurate.

4. FUZZY KALMAN FILTER DESIGN

When noise is taken into account, the difference between the estimation of the previous model and the real posture of the robot increases over time, so some correction method is needed. Due to the fact that process disturbances and measurement errors can be characterized as Gaussian white noise, Kalman filters can be used to reduce the deviation caused by these disturbances.

4.1. Kalman Filter

As the TS nonlinear model is based on several linear models, conventional linear tools can be used over each of them. In this case, a Kalman filter can be implemented for each model, obtaining an optimal estimation of the state vector for each of them, and thus reducing the effect of noise over them.

$$\begin{aligned} \boldsymbol{X}_k &= \mathbf{A}\boldsymbol{X}_{k-1} + \mathbf{B}\boldsymbol{U}_{k-1} + \boldsymbol{\varepsilon}_{1,k} \\ \boldsymbol{Y}_k &= \mathbf{C}\boldsymbol{X}_k + \boldsymbol{\varepsilon}_{2,k} \end{aligned} \tag{10}$$

Equation 10 represents a linear system, where $\boldsymbol{X}_k$ is the state vector, $\boldsymbol{U}_k$ the input, and $\boldsymbol{Y}_k$ the measurement vector, at time k. The process and measurement Gaussian noise at time k are denoted by $\boldsymbol{\varepsilon}_1$ and $\boldsymbol{\varepsilon}_2$ respectively.

The Kalman filter is an iterative algorithm that uses both the system equations and actual measurements to correct the prior estimation. First, the state and measurement vectors, and the covariance matrix, $\boldsymbol{P}$, are estimated using the system equations, as shown in (11), where $\mathbf{Q}$ represents the process noise covariance matrix:

$$\begin{aligned} \boldsymbol{X}_k^- &= \mathbf{A}\hat{\boldsymbol{X}}_{k-1} + \mathbf{B}\boldsymbol{U}_{k-1} \\ \boldsymbol{Y}_k^- &= \mathbf{C}\boldsymbol{X}_k^- \\ \boldsymbol{P}_k^- &= \mathbf{A}\boldsymbol{P}_{k-1}\mathbf{A}^T + \mathbf{Q} \end{aligned} \tag{11}$$

Once this estimation is made, the Kalman gain is calculated using $\mathbf{R}$, the measurement covariance matrix:

$$\boldsymbol{K}_k = \boldsymbol{P}_k^- \mathbf{C}^T \left(\mathbf{C}\boldsymbol{P}_k^- \mathbf{C}^T + \mathbf{R} \right)^{-1} \tag{12}$$

Finally, using this gain and the measurement vector $\mathbf{Y}_k$, the estimation is updated:

$$\begin{aligned} \hat{\boldsymbol{X}}_k &= \boldsymbol{X}_k^- + \boldsymbol{K}_k \left(\boldsymbol{Y}_k - \boldsymbol{Y}_k^- \right) \\ \boldsymbol{P}_k &= \left(\mathbf{I} - \boldsymbol{K}_k \mathbf{C} \right) \boldsymbol{P}_k^- \end{aligned} \tag{13}$$

4.2. Fuzzy Observer

As described on figure 7, for each of the linear models L_n, a Kalman filter KF_n is implemented, which gives an optimal estimation for that model, reducing the effect of the process and measurement noise. Also, each linear model has its own process noise covariance matrix $\mathbf{Q}_n$, which in the case of linear systems can be easily determined to obtain the best possible performance. Then, each KF_n is specifically tuned with these matrices. This is one of the main advantages of this method, compared with the EKF, where generally it is very difficult to determine the covariance matrices needed to achieve a good performance, resulting in a significant reduction of the tuning process.

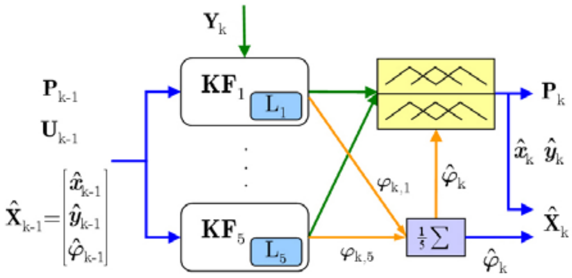

Fig. 7. Takagy-Sugeno based fuzzy observer.

In the same way in which the nonlinear TS fuzzy model fuses together the different linear models, this structure can be used to fuse the estimation given by each Kalman filter, resulting in a fuzzy nonlinear observer.

For each time step k, the previous state estimation $\hat{\boldsymbol{X}}_{k\text{-}1}$ is used together with the input vector $\boldsymbol{U}_{k\text{-}1}$ and measurement vector $\boldsymbol{Y}_k$, to obtain an optimal estimation from each linear Kalman filter KF_n. The different estimations for the heading angle $\varphi_{k,n}$, which are almost the same on each model, are averaged together to determine the best estimation $\hat{\varphi}_k$. This value is used as an input to the fuzzy structure to fuse together the estimations for x and y given by each Kalman filter. The same membership functions determined through the optimization of the nonlinear TS model of the robot kinematics are used to fuse the estimations and obtain $\hat{x}_k$ and $\hat{y}_k$. The covariance matrix $\mathbf{P}_k$ is obtained as a linear combination between the different covariance matrices. The relative weight for each covariance matrix is given by the MF associated to the corresponding model, using the membership functions for the x variable for models 1, 4, and 5 as they are more related to movements on the X axis, whereas for models 2 and 3 the weights are obtained from the y related membership functions.

5. RESULTS

To analyze the robustness and performance of the fuzzy observer, the estimation error is compared with the one obtained from an EKF for several different conditions. The EKF is tuned to obtain the best possible estimation given a certain trajectory, whereas the different linear Kalman filters in the fuzzy observer are tuned using the linear equations, without considering the trajectory.

Due to the fact that these are statistical tools that are used to reduce the effect of noise, for each different

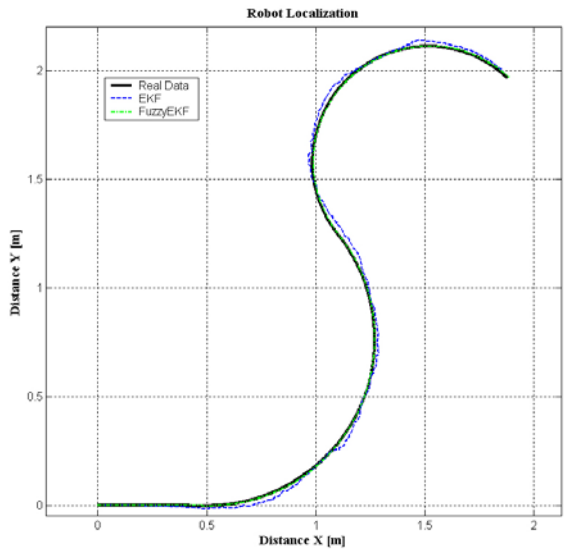

Fig. 8. Robot localization using the EKF and the fuzzy nonlinear observer.

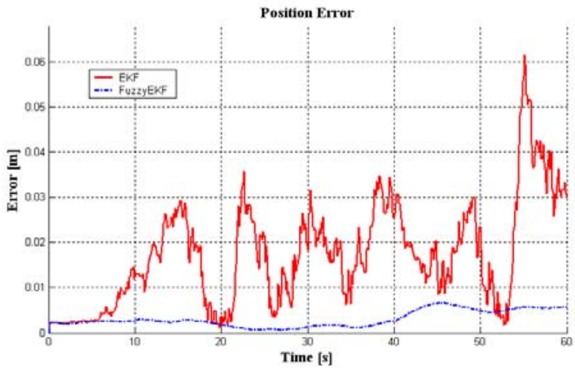

Fig. 9. Estimation error for both methods.

situation 1000 simulations are made choosing random seeds for the noise generation. This allows a more suitable comparison between the two estimations. On each simulation it is assumed that every 0.1 [s] the mobile robot can measure the heading angle (with a magnetic compass for example) and the position (with a GPS or radio beacons).

5.1. Basic Comparison

The basic comparison considers that the value for the standard deviation of both the angle measurement Gaussian noise and process Gaussian noise is 10% of the maximum possible value. The standard deviation for the position measurement noise is considered to be 1 [m]. One of the simulation results is shown on figures 8 and 9. On both figures it can be observed that the fuzzy observer is more accurate than the EKF, even though the tuning for the fuzzy observer is faster and simpler. For these simulations, the average RMS error for the EKF was 0.018 [m], whereas the fuzzy observer presents an average error of 0.006 [m].

The average error of the fuzzy observer in the 1000 simulations was more than 3 times smaller than the error of the EKF. It was also observed that only in 2.6% of the simulations done, the EKF outperformed the fuzzy observer.

5.2. Effect of the Robot Dynamics

The effect that the dynamic equations have on both methods must be also checked, as none of them include the robot dynamics in their structure. To achieve this, instead of using **U**, the reference angular velocities, as input vector for each method, the actual applied angular velocities are used, eliminating the effect of the robot dynamics. Through this test it is observed that both methods improve their estimation, but the improvement of the EKF is higher. This means that the effect of the robot dynamics has less effect on the fuzzy observer compared with the EKF, which needs more accurate equations to achieve a good performance.

5.3. Sensitivity To Measurement and Process Noise

For this test, the standard deviation for the measurements and the process noise were modified by a 30% up and down, analyzing the effect on both methods.

When the measurement noise is incremented, the estimation error increases in both methods, but it affects less the output of the fuzzy observer. The same happens when the process noise is augmented.

5.4. Sensitivity To Noise Estimation

More important than the sensitivity towards the noise is the sensitivity towards the noise estimation. In most cases it is very difficult to estimate the noise level on the measurements and it is even more difficult to do it on the process. This makes important to analyze the effect that a wrong estimation has on both methods.

For these tests the measurements and process noise standard deviation is estimated to be between a 30% more and 30% less than the actual level used in the exact robot model. The results show that when the noise level is estimated incorrectly the estimation error increases for both methods, but in average the improvement of the fuzzy observer over the EKF increases.

It is important to mention that considering all the different tests made, the fuzzy observer gives an estimation error that is at least 50% smaller than the one given by the EKF, whereas the EKF outperformed the fuzzy method in less than a 4% of the simulations.

6. CONCLUSIONS

Through this work a novel nonlinear fuzzy observer is presented. This observer is based on several linear Kalman filters and a Takagi–Sugeno fuzzy model which is used to combine the different estimations, resulting in a more accurate observer for the posture of the robot, compared with the conventional Extended Kalman Filter.

The simulations also show several other interesting advantages of the fuzzy observer. First and more important, as the fuzzy observer is based on linear models, the covariance matrices needed are easily determined, implying that less tuning is required to achieve a good performance, compared with the EKF, which requires more work.

Also, the fuzzy observer is more robust that the EKF as the effect of not considering the robot dynamics and errors in the estimation of the process and measurement noise affects more the performance of the EKF than the performance of the fuzzy observer.

Based on this new nonlinear observer, less noise contaminated residuals can be obtained, which will be used to design a more accurate fault detection and isolation system for mobile robots.

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