Extreme Dependence and Asset Pricing: Returns and Liquidity

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Crash Aversion: Evidence from the Options Literature

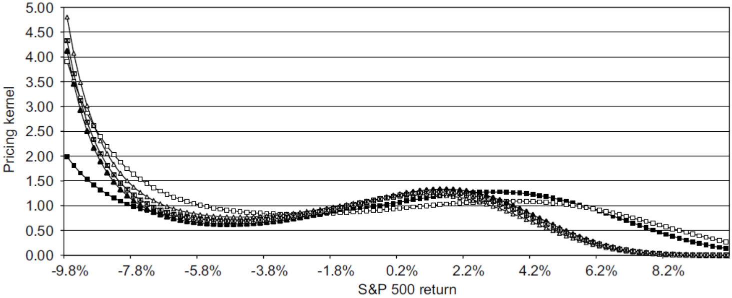

Figure: Engle & Rosenberg (2002, JFE)

- Investors value financial instruments that offer protection against extreme market downturns

Main Findings

...with respect to returns

- Copula-based lower tail dependence (LTD) coefficients between individual stocks and the market can capture crash-sensitivity
- Crash-sensitive stocks deliver higher expected returns

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...with respect to liquidity

- Extreme dependence between individual stock returns/liquidity changes and market returns/market liquidity changes can be captured based on copulas
- Stocks with strong extreme dependence in liquidity deliver significantly higher returns

Related Literature - Returns

- Aggregate tail risk and aggregate market returns (Bali, Demirtas, and Levy (2009, JFQA), Bollerslev and Todorov (2011, JF), Jiang and Kelly (2013))
- Downside beta (Ang, Chen, and Xing (2006, RFS))
- Tail risk exposure and individual stock returns (Jiang and Kelly (2013), Cholette/Lu (2012))

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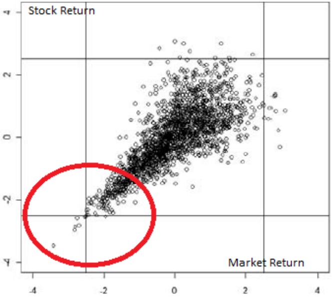
Capturing Crash Sensitivity of Stocks: LTD

Linear correlation = 0.8
LTD = 0
Low crash sensitivity



Linear correlation = 0.8
LTD ≈ 0.8
High crash sensitivity

$$\text{LTD} = \lim_{q \to 0+} P(r_i \leq F_i^{-1}(q) | r_m \leq F_m^{-1}(q))$$

Lower Tail Dependence

Estimation of the Tail Dependence Coefficient

- Estimation approach relies on the semiparametric estimation procedure for Copulas of Genest/Ghoudi/Rivest (1995)
- We estimate 64 convex combinations of basic copulas (for which closed-form solutions for TD exist) to allow for maximum flexibility:

$$C(u_1, u_2, \Theta) = w_1 \cdot C_{\mathsf{LTD}}(u_1, u_2; \theta_1) \\ + w_2 \cdot C_{\mathsf{NTD}}(u_1, u_2; \theta_2) + (1 - w_1 - w_2) \cdot C_{\mathsf{UTD}}(u_1, u_2; \theta_3)$$

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- Estimation procedure (for stock i and year t):
  1. Estimation of a set of copula parameters Θ_j for $j = 1, \ldots, 64$ different parametric copulas $C_j(\cdot, \cdot; \Theta_j)$ between an individual stock return r_i and the market return r_m
  2. Select the appropriate parametric copula $C^*(\cdot, \cdot; \Theta^*)$ by minimizing the distance between the different estimated parametric copulas $C_j(\cdot, \cdot; \widehat{\Theta}_j)$ and the empirical copula $\widehat{C}$
  3. Compute the tail dependence coefficients LTD and UTD implied by the estimated parameters Θ^* of the selected copula $C^*(\cdot, \cdot; \Theta^*)$

Does LTD capture a stock's crash sensitivity?

Table: Daily Excess Returns of LTD Portfolios during Financial Crises

Portfolio	Black Monday	Asia Crisis	Dot-Com Bubble Burst	Lehman Crisis	$r_M^e < -5\%$
1 Weak LTD	-9.5%	-2.4%	-1.7%	-5.9%	-4.4%
2	-13.3%	-4.4%	-3.1%	-6.9%	-6.0%
3	-15.7%	-5.7%	-4.3%	-9.4%	-7.3%
4	-16.3%	-6.3%	-5.9%	-11.2%	-8.4%
5 Strong LTD	-18.7%	-6.8%	-7.3%	-11.8%	-9.2%
5 Strong - Weak	-9.2%	-4.4%	-5.6%	-5.9%	$-4.8\%^{***}$

- During financial crises, strong LTD stocks perform significantly worse than weak LTD stocks

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Main Hypothesis

Stocks with strong LTD exposure have higher expected returns than stocks with weak LTD exposure.

Empirical Research Design

Dataset + Estimation of Extreme Dependence Structures

- Daily returns from US common shares trading on the NYSE or AMEX in the period 1963 - 2009
- Estimation of LTD coefficients for each stock and year
- Final sample: 96767 stock - year observations

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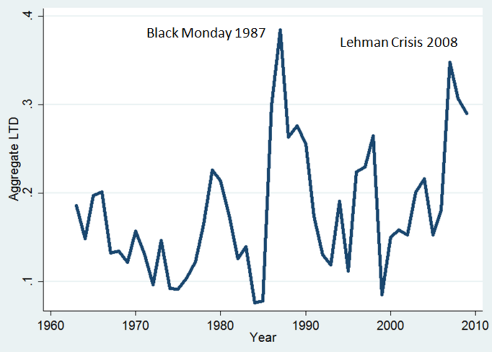
Empirical Strategy

- Focus: Contemporaneous relation between average realized returns and realized LTD (as in Lewellen & Nagel, 2006, JFE)
- Time horizon: Non-overlapping intervals of one year
- Portfolio sorts and Fama-MacBeth regressions on the firm level
- Trading strategy results

Tail Dependence Over Time



Evolution of Aggregate LTD
(Yearly value-weighted, cross-sectional average of individual stock LTD)

Main Result: Univariate Portfolio Sorts

Table: Equal-Weighted Portfolio Sorts: LTD

Portfolio	LTD	Return	CAPM-Alpha	FF-Alpha	CAR-Alpha
1 Weak LTD	0.01	+3.99%	−1.28%	−6.27%	−3.30%
2	0.06	+8.84%	+2.82%	−1.86%	−0.04%
3	0.12	+10.39%	+4.06%	−0.07%	+1.15%
4	0.18	+14.07%	+7.12%	+3.24%	+4.85%
5 Strong LTD	0.29	+19.70%	+12.25%	+9.92%	+10.76%
Strong - Weak	0.28***	15.71%***	13.53%***	16.19%***	14.06%***
		(8.70)	(7.09)	(5.83)	(4.77)

- Monotonically increasing pattern between realized excess returns and realized LTD

Bivariate Portfolio Sorts: Alternative Explanations

Panel A: Beta (β) and Lower Tail Dependence (LTD) [Sharpe (1964, JF), Lintner (1965, RoES)]

β	1 Low	2	3	4	5 High	Average
Weak LTD	4.48%	3.57%	4.53%	5.97%	10.40%	5.79%
⋮	⋮	⋮	⋮	⋮	⋮	⋮
Strong LTD	9.71%	12.04%	13.76%	16.76%	28.69%	16.19%
Strong - Weak	5.23%*** (3.28)	8.48%*** (5.67)	9.23%*** (5.12)	10.79%*** (5.03)	18.29%*** (6.88)	10.40%*** (5.20)

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Panel B: Downside Beta (β^-) and Lower Tail Dependence (LTD) - [Ang, Chen & Xing (2006, RFS)]

β^-	1 Low	2	3	4	5 High	Average
Strong - Weak	6.89%*** (5.02)	5.16%*** (3.57)	7.98%*** (4.24)	9.40%*** (4.32)	12.98%*** (3.95)	8.48%*** (4.02)

Alternative Downside Beta Definitions

LTD Portfolio Strong - Weak

Cut-Off	1 Low β^-	2	3	4	5 High β^-	Average
20% Quantile	4.99%***	6.87%***	9.06%***	9.10%***	17.30%***	9.46%***
	(3.98)	(6.36)	(4.89)	(4.06)	(6.21)	(5.10)
10% Quantile	6.96%***	8.82%***	12.57%***	13.69%***	17.84%***	11.97%***
	(4.56)	(7.10)	(7.53)	(7.26)	(6.89)	(6.67)
5% Quantile	10.73%***	10.21%***	12.15%***	16.34%***	17.56%***	13.40%***
	(5.44)	(6.15)	(7.72)	(8.21)	(5.84)	(6.67)
2% Quantile	12.52%***	11.39%***	12.55%***	15.72%***	19.82%***	14.40%***
	(6.14)	(6.15)	(7.92)	(9.85)	(6.45)	(7.30)
1% Quantile	15.93%***	13.31%***	13.15%***	15.81%***	17.97%***	15.23%***
	(7.46)	(6.25)	(7.31)	(8.13)	(8.53)	(7.53)

Main Result: FMB-Regressions

	(1) Return	(2) Return	(3) Return	(4) Return	(5) Return	(6) CAR-Alpha	Econ. Sign.
LTD	0.551***	0.584***	0.555***	0.448***	0.452***	0.441***	5.01%
	(8.44)	(9.11)	(11.59)	(9.89)	(10.16)	(9.84)	
UTD		-0.326***	-0.254***	-0.296***	-0.292***	-0.241***	-2.25%
β^-					0.0375***	0.00292	+0.21%
β^+					0.00960	0.00846	+0.65%
β			0.0748**	0.140***			
size			-0.0121*	-0.0302***	-0.0279***	-0.0220***	- 4.66%
btm			0.0383***	0.0300***	0.0286***	0.0167***	+1.22%
coskew			0.127**	0.0863**	0.114**	0.127**	+3.90%
illiq			0.228***	0.198***	0.172***	0.108*	+1.43%
past ret.				-0.0173	-0.0136	-0.0192	-0.88%
idvol.				-3.758**	-1.682	-2.994**	-3.54%
cokurt				0.0063	0.0057	0.0019	+0.51%
max				-0.250**	-0.222**	-0.214**	- 1.57%
const.	0.0460	0.0678*	0.0558	0.385***	0.325***	0.296***	
	(1.38)	(1.92)	(0.56)	(4.97)	(4.51)	(4.56)	
R^2	0.019	0.024	0.110	0.150	0.146	0.086	

The Impact of How Bad 'Bad' Really Is

- Estimated coefficient for LTD from FMB regression in subsamples

	Std.Dev.		VaR	
	low	high	low	high
LTD	0.241***	0.811***	0.673***	0.269***
(Returns)	(9.88)	(9.31)	(9.07)	(9.64)
LTD	0.242***	0.812***	0.675***	0.270***
(CAPM-α)	(9.86)	(9.27)	(9.01)	(9.64)
LTD	0.239***	0.814***	0.675***	0.269***
(FF93-α)	(9.42)	(9.28)	(9.08)	(9.41)
LTD	0.236***	0.785***	0.654***	0.264***
(CAR97-α)	(9.17)	(9.00)	(8.78)	(9.41)

Momentum and LTD

	(1) Mom	(2) Mom	(3) Mom
market	-0.255	-0.456**	-0.409**
	(-1.54)	(-2.53)	(-2.43)
smb	-0.0880	-0.0766	-0.0416
	(-0.44)	(-0.40)	(-0.22)
hml	-0.361*	-0.104	-0.0580
	(-1.76)	(-0.46)	(-0.25)
LTD^{ew} Strong-Weak		0.676**	
		(2.31)	
LTD^{vw} Strong-Weak			0.757**
			(2.46)
alpha	0.128***	0.0188	0.0391
	(3.97)	(0.33)	(0.82)
N	47	47	47

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- Contemporaneous LTD factor can explain the profits of a momentum strategy.
- Partial explanatory power also based on lagged LTD factor.

Trading Strategy based on past LTD

- Buy stocks with strong past LTD (Top Quintile) and sell stocks with weak past LTD (Bottom Quintile) over the previous year
- Examine equal-weighted returns and alphas on these portfolios over the next month

Portfolio	(1) Return	(2) CAPM-Alpha	(3) FF-Alpha	(4) CAR-Alpha
1 Weak LTD	0.499%	0.097%	−0.333%***	−0.195%**
2	0.671%	0.236%	−0.192%**	−0.029%
3	0.713%	0.256%	−0.129%*	+0.019%
4	0.775%	0.295%	−0.039%	+0.109%*
5 Strong LTD	0.862%	0.350%	+0.122%	+0.187%**
Strong - Weak	0.363%***	0.253%**	0.454%***	0.383%***
	(2.99)	(2.29)	(4.65)	(3.86)
Annualized Alpha	4.34%	3.04%	5.45%	4.57%

Stability

Results are stable if we

- apply value-weighted portfolio sorts (instead of equal-weighted portfolio sorts)
- apply alternative factor models in the asset pricing tests
- use industry-, DGTW-, and risk-adjusted returns
- examine the effect of LTD during different subsamples
- use a longer estimation horizon for LTD
- use alternative LTD estimation procedures
- use different regression methods (instead of FMB-regressions)
- do not pick optimal copula combination

Summary - Extreme Dependence in Returns

- Crash sensitivity in the form of LTD is an important driver of the cross-sectional variation of expected stock returns
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$\longrightarrow$ Investors get a compensation for holding stocks with a strong sensitivity to extreme market downturns

$\longrightarrow$ Implications for risk taking incentives of financial institutions and systemic stability

Extreme Dependence in Liquidity - Main Research Question

Do investors require a liquidity risk premium for holding stocks that are particularly sensitive to liquidity crises or market crashes?

Core Literature: Underlying Theory and Empirical Studies

	returns	returns & illiquidity
symmetric	Sharpe (1964) ...	Acharya/Pedersen (2005) Pastor/Stambaugh (2003)
extreme downside	Ang/Chen/Xing (2006) *Ruenzi/Weigert (2013)*	*THIS STUDY*

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→ Hypothesis: There is a premium for downside liquidity risk.

Linear Risk Measures: Acharya/Pedersen (2005)

	market-return	market-liquidity
security-return	β_{CAPM}	β_{L2}
security-liquidity	β_{L3}	β_{L1}

$\rightarrow$ Acharya/Pedersen (2005) find a small premium for overall linear liquidity risk (i.e. $\sum_i^3 \beta_{Li}$).

$\rightarrow$ Results are driven by β_{L2} and β_{L3}.

Introducing Extreme Downside Liquidity Risk (EDLR)

	market-return	market-liquidity
security-return	β_{CAPM} & $EDRR$	β_{L2} & $EDLR_2$
security-liquidity	β_{L3} & $EDLR_3$	β_{L1} & $EDLR_1$

...where Extreme Downside Liquidity Risk is defined as:

$EDLR_{1,i} = \lim_{t \to 0^+} P(d_i \leq G_i^{-1}(t) | d_M \leq F^{-1}(t))$

and $G_i^{-1}(t)$ and $F^{-1}(t)$ are inverses of d_i's and d_M's CDFs.

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Overall Liquidity-Beta and EDLR measures:

$$\begin{aligned} \beta_L &= \beta_{L1} + \beta_{L2} + \beta_{L3} \\ EDLR &= EDLR_1 + EDLR_2 + EDLR_3 \end{aligned}$$

Data and Variables

Data:

- Daily NYSE & AMEX common stock return and volume, from CRSP 1963-2011
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Measures of dependence (β and tail dependence):

- Estimated based on rolling 3-year-window of weekly data
- Used out-of-sample

Estimation Procedure: Illiquidity Shocks

(1) Compute Amihud's illiquidity ratio for each week and stock
(2) Winsorize and scale illiquidity ratio (following Acharya/Pedersen (2005))
(3) Compute market-liquidity as value-weighted average of stock-liquidity
(4) Estimate illiquidity shocks for each stock and the market via AR(4)-model on a 3-year rolling window basis

Estimation Procedure: Tail Dependence

(1) Use 3-year moving window of weekly returns & liquidity shocks for each stock and the market

(2) Select best-fitting copula based on first three years of data for each risk component

(3) Estimate marginal distributions non-parametrically and parameters for copulas (Genest/Ghoudi/Rivest, 1995)

(4) Compute lower tail dependence coefficients implied by parameters for each stock/moving-window-combination

(5) Use tail dependence (to form portfolios or predict returns) only out-of-sample

Univariate Sort by EDLR

Portfolio-Returns	EDLR
Weak	6.08%
2	7.75%
3	8.84%
4	8.84%
Strong	9.67%
Strong - Weak	3.59%***
	(3.43)

$\rightarrow$ Strong EDLR stocks return significant premium of 3.6% p.a.

Univariate Sort by EDLR & Components

Portfolio-Returns	EDLR	$EDLR_1$	$EDLR_2$	$EDLR_3$
Weak	6.08%	7.75%	6.66%	6.40%
2	7.75%	7.85%	8.11%	7.64%
3	8.84%	8.58%	7.59%	8.16%
4	8.84%	8.74%	8.42%	9.41%
Strong	9.67%	8.27%	9.36%	9.78%
Strong - Weak	3.59%***	0.52%	2.70%***	3.38%***
	(3.43)	(0.62)	(3.38)	(3.24)

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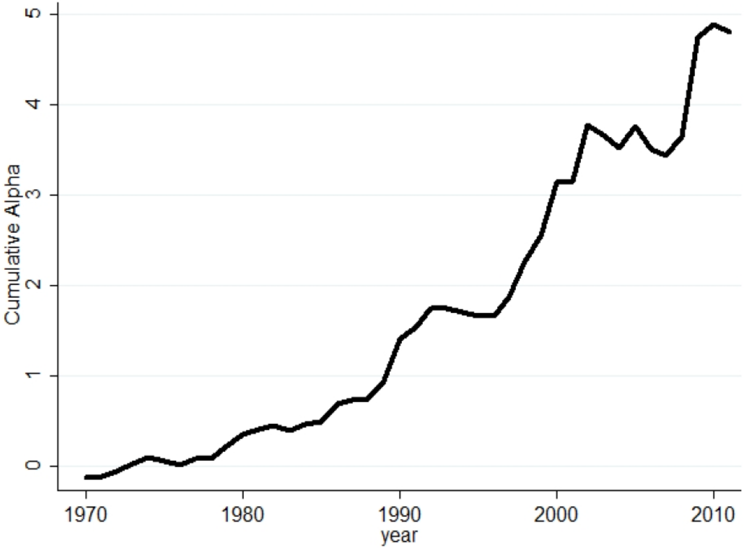
Performance of Strong-Weak Portfolio



Robustness to Choice of Liquidity Proxy

Proxy	Return_{t+2}	CAR
EDLR 5-1	Low Frequency (1969-2011)	
Amihud (2002)	$3.56\%^{***}$	$4.31\%^{***}$
	(3.43)	(4.65)
Corwin/Schultz (2012)	$3.23\%^{***}$	$4.15\%^{***}$
	(2.68)	(3.90)
Zeros (Lesmond, Ogden and Trzcinka, 1999)	$1.20\%^{*}$	$1.04\%^{*}$
	(1.78)	(1.77)
FHT (Fong, Holden and Trzcinka, 2011)	$3.62\%^{***}$	$4.10\%^{***}$
	(4.79)	(5.60)
EDLR 5-1	High Frequency (2000-2010)	
Effective Spread	$3.73\%^{*}$	$4.87\%^{**}$
	(1.69)	(2.35)
Relative Spread	1.08%	1.83%
	(0.43)	(0.76)
Intraday Amihud	$5.59\%^{**}$	$5.88\%^{***}$
	(2.07)	(2.69)
Price Impact	$2.96\%^{***}$	$3.74\%^{*}$
	(1.35)	(1.86)

Fama-MacBeth-Regressions: EDLR & Liquidity-CAPM

	(1)	(2)	(3)	(4)
$EDLR$	0.00139***	0.00073**	0.00082***	0.00081***
	(3.02)	(2.35)	(2.89)	(2.37)
β_L		0.00081	0.00137*	
		(1.17)	(1.93)	
$EDRR$		0.00248***	0.00138***	0.00120**
		(5.02)	(2.63)	(2.25)
$Illiq$			0.00105	0.00184
			(0.31)	(0.54)
β_L^-				0.00042
				(0.62)
β_L^+				0.000365
				(0.52)
$Const$	0.00118**	0.00260*	0.00589***	0.00592***
	(2.48)	(1.89)	(4.02)	(4.04)
Controls	No	Yes	Yes	Yes

Controls: Mkt, Size, BM, Mom, EURR, idiovola, coskew

$\rightarrow$ EDLR-premium robust to linear liquidity-risk and liquidity-level.

$\rightarrow$ EDLR-premium robust to firm-specific controls.

Other Robustness Checks

The EDLR premium is robust if we vary:

- the factor model in the asset pricing tests
- the estimation windows for EDLR (1, 2 and 5 years)
- the choice of the copula function
- the weighting-method (value-weighting)
- the lag between EDLR-estimation and return period (1, 3 and 4 weeks)
- the rebalancing frequency (monthly returns)
- or if we industry-adjust returns or compute DGTW-alphas

Conclusion

- Main result: Statistically and economically significant premium for overall extreme downside liquidity risk.
- Effect is different from impact of liquidity level and linear (downside) liquidity risk.
- Impact of Acharya/Pedersen (2005) linear liquidity risk virtually vanishes after including EDLR.

Thank you!

Ruenzi, S./Weigert F. (2013): Crash Sensitivity and the Cross-Section of Expected Stock Returns.

Ruenzi, S./Ungeheuer, M./Weigert, F. (2013): Extreme Downside Liquidity Risk.

Stock Level Correlation Among Dependencies and Liquidity

Correlations	EDLR	β_L	β_L^-	EDRR	Illiq
EDLR	1.00	–	–	–	–
β_L	0.05	1.00	–	–	–
β_L^-	0.11	0.79	1.00	–	–
EDRR	0.24	−0.02	−0.01	1.00	–
Illiq	−0.06	0.05	0.10	−0.15	1.00

Dependent Bivariate Sort by β_L and EDLR

Portfolio	Weak β_L	2	3	4	Strong β_L
Weak EDLR	6.43%	6.37%	6.01%	7.81%	6.34%
2	5.63%	6.53%	7.91%	9.12%	9.86%
3	8.10%	7.31%	7.57%	8.15%	10.37%
4	9.27%	8.32%	8.32%	9.54%	11.11%
Strong EDLR	8.38%	8.42%	8.20%	10.01%	11.12%
Strong-Weak	1.95%	2.04%*	2.19%*	2.20%*	4.77%***
	(1.40)	(1.78)	(1.84)	(1.69)	(2.85)

→ EDLR-premium not due to linear liquidity-risk.

Dependent Bivariate Sort by β_L^- and EDLR

Portfolio	Weak β_L^-	2	3	4	Strong β_L^-
Weak EDLR	6.46%	5.58%	5.78%	8.31%	6.05%
2	7.03%	7.12%	7.60%	9.04%	9.00%
3	7.85%	7.70%	7.38%	8.39%	11.00%
4	9.76%	7.50%	8.58%	9.27%	10.40%
Strong EDLR	10.08%	7.91%	8.74%	9.54%	10.49%
Strong-Weak	3.62%**	2.32%**	2.96%**	1.23%	4.44%***
	(2.40)	(2.11)	(2.43)	(0.94)	(2.73)

→ EDLR-premium not due to simple downside liquidity-risk.

Dependent Bivariate Sort by EDRR and EDLR

Portfolio	Weak EDRR	2	3	4	Strong EDRR
Weak EDLR	4.48%	5.60%	5.88%	7.57%	9.58%
2	5.27%	6.17%	7.33%	9.38%	11.19%
3	6.23%	6.86%	9.33%	8.85%	9.82%
4	6.80%	8.06%	8.88%	8.65%	10.45%
Strong EDLR	7.23%	8.15%	9.55%	9.67%	10.39%
Strong-Weak	2.75%*	2.55%*	3.67%***	2.10%*	0.81%
	(1.83)	(1.84)	(2.71)	(1.69)	(0.61)

→ EDLR-premium not due to extreme downside return risk.

Correlations

	LTD	UTD	β	β^-	β^+	size	bookmarket	$illiq$	past retur
LTD	1.00	-	-	-	-	-	-	-	-
UTD	0.12	1.00	-	-	-	-	-	-	-
β	0.38	0.31	1.00	-	-	-	-	-	-
β^-	0.49	0.07	0.77	1.00	-	-	-	-	-
β^+	0.19	0.48	0.78	0.47	1.00	-	-	-	-
size	0.29	0.30	0.05	0.03	0.16	1.00	-	-	-
bookmarket	-0.09	-0.11	-0.11	-0.07	-0.08	-0.34	1.00	-	-
$illiq$	-0.28	-0.28	-0.22	-0.08	-0.19	-0.84	0.30	1.00	-
past return	0.08	-0.05	0.10	0.12	0.05	0.07	0.18	-0.01	1.00
$idiovola$	-0.09	-0.09	0.23	0.27	0.12	-0.42	0.05	0.31	-0.13
$coskew$	-0.37	0.23	0.07	-0.12	0.24	-0.05	0.01	0.07	-0.09
$cokurt$	0.38	0.23	0.21	0.16	0.20	0.21	-0.09	-0.21	-0.00
max	-0.08	-0.10	0.14	0.18	0.07	-0.39	0.14	0.31	0.04

Copulas and Sklar's Theorem

Definition: Bivariate Copula

- A function $C : [0,1]^2 \rightarrow [0,1]$ is called a bivariate copula, if it satisfies the following conditions:
  1. $C(u_1, u_2)$ is increasing in u_1 and u_2
  2. $C(u, 1) = C(1, u) = u$ for all $u \in [0,1]$
  3. $C(x_1, x_2) - C(x_1, y_2) - C(y_1, x_2) + C(y_1, y_2) \geq 0$ for all (x_1, x_2), $(y_1, y_2) \in [0,1]^2$ with $x_1 \leq y_1$ and $x_2 \leq y_2$

Theorem: Sklar (1959)

- Let F be a bivariate distribution function with margins F_1 and F_2. Then there exists a copula $C : [0,1]^2 \mapsto [0,1]$ such that, for all x_1, x_2 in $\overline{\mathbb{R}} = [-\infty, \infty]$,

$$F(x_1, x_2) = C(F_1(x_1), F_2(x_2)).$$

If the margins are continuous, then C is unique.

Copulas and Tail Dependence Coefficients

- Tail dependence coefficients (LTD and UTD) are measures of extreme dependence that depend only on the underlying copula
- Simple expressions for LTD and UTD in terms of the copula C of the bivariate distribution can be derived based on

$$\text{LTD} = \lim_{u \to 0+} \frac{C(u,u)}{u} \tag{1}$$

$$\text{UTD} = \lim_{u \to 1-} \frac{1 - 2u + C(u,u)}{1-u} \tag{2}$$

- Most explicit copula function have closed-form solutions for expressions (1) and (2)