Modeling asymmetric volatility clusters with copulas and high frequency data

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October 11-12, 2013

Columbia University in New York

What is it about?

- In financial markets, we often observe tranquil times and turbulent times. That is, there are periods of time when returns hardly change (market tranquility) and others where changes in returns are followed by further large changes (market turbulence,). This is a common characteristic of financial returns which is referred to as volatility clustering.

- This paper examines volatility clusters in both tranquil times and turbulent times. Tranquil times are the periods that volatilities are extremely low while turbulent times are the times the volatilities are extremely high. Therefore, we investigate the clusters of both low and high volatilities. We ask if the clusters of high and low volatilities the same?

- The clusters of volatilities are the same idea as the dependence between volatilities. Thus we examine the dependence of both extremely high volatilities (volatility clustering at turbulent times) and extremely low volatilities (volatility clustering at tranquil times). That is to investigate the dependence structure and extreme dependence of volatilities.

- Traditional GARCH and SV models are not able to capture the structure of volatility dependence and their extreme dependence.

- Copula approach provides a perfect method to examine the dependence structure and volatility clusters at two different market status.

1. Volatility Measure

- Volatility measure is not trivial. Realized volatility has the problem of choosing proper data frequency and noise contamination effects.

- Based on Barndorff-Nielsen et al. (2008, 2009), realized kernel volatility is a robust estimator for the underlying integrated volatility.

- We follow Barndorff-Nielsen et al. (2009) and use the realized kernel volatility based on tick-by-tick data..

2. Model

- Follow Chen and Fan (2006), we extend a copula approach to model the dependence of a **univariate** variable across time, i.e., volatility across time.

- It's well known that a copula is a multivariate distribution whose marginals are uniform distributions on the interval [0,1].

  $$C(u, v)$$

  where $u = F(x), \quad v = G(y)$. and F and G are cumulative distributions of x and y respectively.

- In our application, $x = vo_t$, $y = vo_{t-1}$, the volatilities at time t and $t-1$ respectively, thus $u = F(vo_t), \quad v = G(vo_{t-1})$

- By **Sklar's Theorem:** a copula is related to the joint distribution function. Let $H(vo_t, vo_{t-1})$ be the joint distribution of VO_t and VO_{t-1}, then

  - $H(vo_t, vo_{t-1}) = C(F(vo_t), G(vo_{t-1}))$

- Copula is equivalent to the joint distribution, so it captures the complete relationship between volatilities, while any correlations only give us a partial picture..

- You can separate the joint distribution into a set of marginal distributions and the dependence structure between the marginals.

In our application, this includes a marginal model for the volatility and a joint model for the dependence of volatilities across time.

- It can accommodate any types of the marginal distributions. This is very useful as volatility distribution is seriously skewed and fat tailed.

- It also allows for any types of dependence structure, linear or nonlinear, symmetric or asymmetric, which may well be the case for the volatility clustering.

3 Marginal model

- Nonparametric approach, i.e., empirical distribution function

$$\widehat{G}(vo_t) = \frac{1}{T+1} \sum_{t=1}^{T} 1\{VO_t < vo\}.$$

where vo_t, is the kernel volatility at time t.

4. Joint model for the dependence

- Applied five copulas with different dependence structure.
  - Normal, no tail dependence
  - Student t, symmetric tail dependence
  - Clayton, left tail dependence, no right tail dependence
  - Survival Clayton, right tail dependence, no left tail dependence
  - Symmetrized Joe Clayton (SJC) copula, both right and left tail dependence and symmetric dependence as a special case.
- Our result indicates a best fit of the SJC copula

- Thus we focus on the SJC copula

$$\begin{aligned} & C_{SJC}(u, v|\lambda_r, \lambda_l) \\ = \; & 0.5 \times (C_{JC}(u, v|\lambda_r, \lambda_l) + C_{JC}(1-u, 1-v|\lambda_l, \lambda_r) + u + v - 1) \end{aligned}$$

where $C_{JC}(u, v|\lambda_r, \lambda_l)$ is the Joe-Clayton copula of Joe (1997) defined as

$$\begin{aligned} & C_{JC}(u, v|\lambda_r, \lambda_l) \\ = \; & 1 - (1 - \left\{ \left[1-(1-u)^k\right]^{-r} + \left[1-(1-v)^k\right]^{-r} - 1 \right\}^{-1/r})^{1/k}. \end{aligned}$$

5. Clusters of volatilities at extremes

- Use tail dependence: measures the probability that both variables are at their left or right extremes:

$$\lambda_l = \lim_{u \longrightarrow 0} Pr[G(vo_t) \leq u | G(vo_{t-1}) \leq u] = \lim_{u \longrightarrow 0} \frac{C(u,u)}{u},$$

$$\lambda_r = \lim_{u \longrightarrow 1} Pr[G(vo_t) \geq u | G(vo_{t-1}) \geq u] = \lim_{u \longrightarrow 1} \frac{1 - 2u + C(u,u)}{1-u},$$

So λ_l represents the clusters of extremely low volatilities while λ_r measures the clusters of extremely high volatilities.

6. Persistency and long memory of volatility clusters

- To investigate how long the extreme volatility clusters last and how slowly they die out, we examine the dependence between volatilities at time t, and t-1, t-2, t-3, ...t-40.

- We obtain the estimated time series of the tail dependence coefficients to address the question above.

Estimation

- Due to Sklar's theorem, we can use a two-step maximum likelihood estimation method (namely Canonical Maximum Likelihood, CML) for estimation.

- That is, to estimate the marginal models first and then the joint model given the estimated marginal models.

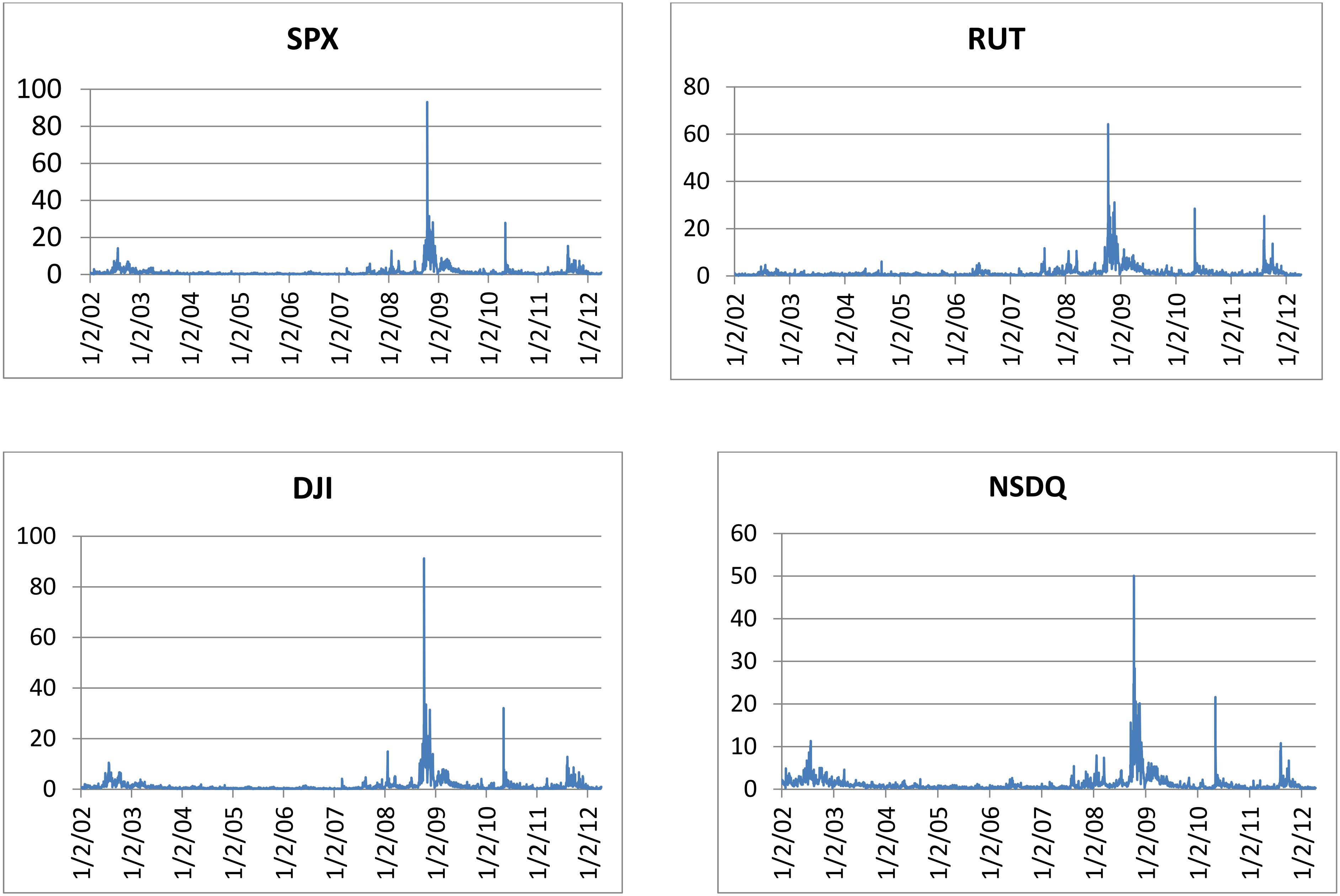
Data and their Adjustments

- We examine both the stock market and FX market. Specifically, we use high-frequency data for the US stock indices including S&P500 (SPX), Russel 2000 (RUT), Dow Jones Industrial Average (DJI), NASDAQ 100 (NSDQ), and the Euro zone blue chip stock index Stoxx50 for the period Jan.2, 2002-April 10.,2012, as well as foreign exchange rates such as British Pound (GBP), Euro (EUR), Swiss Franc (CHF), and Japanese Yen (JPY) from Jan. 3 1999 to March 1, 2009.

- Data were purchased.

- ADF tests indicate stationary of all volatility series.

Figure 1 Kernel Volatilities for indices



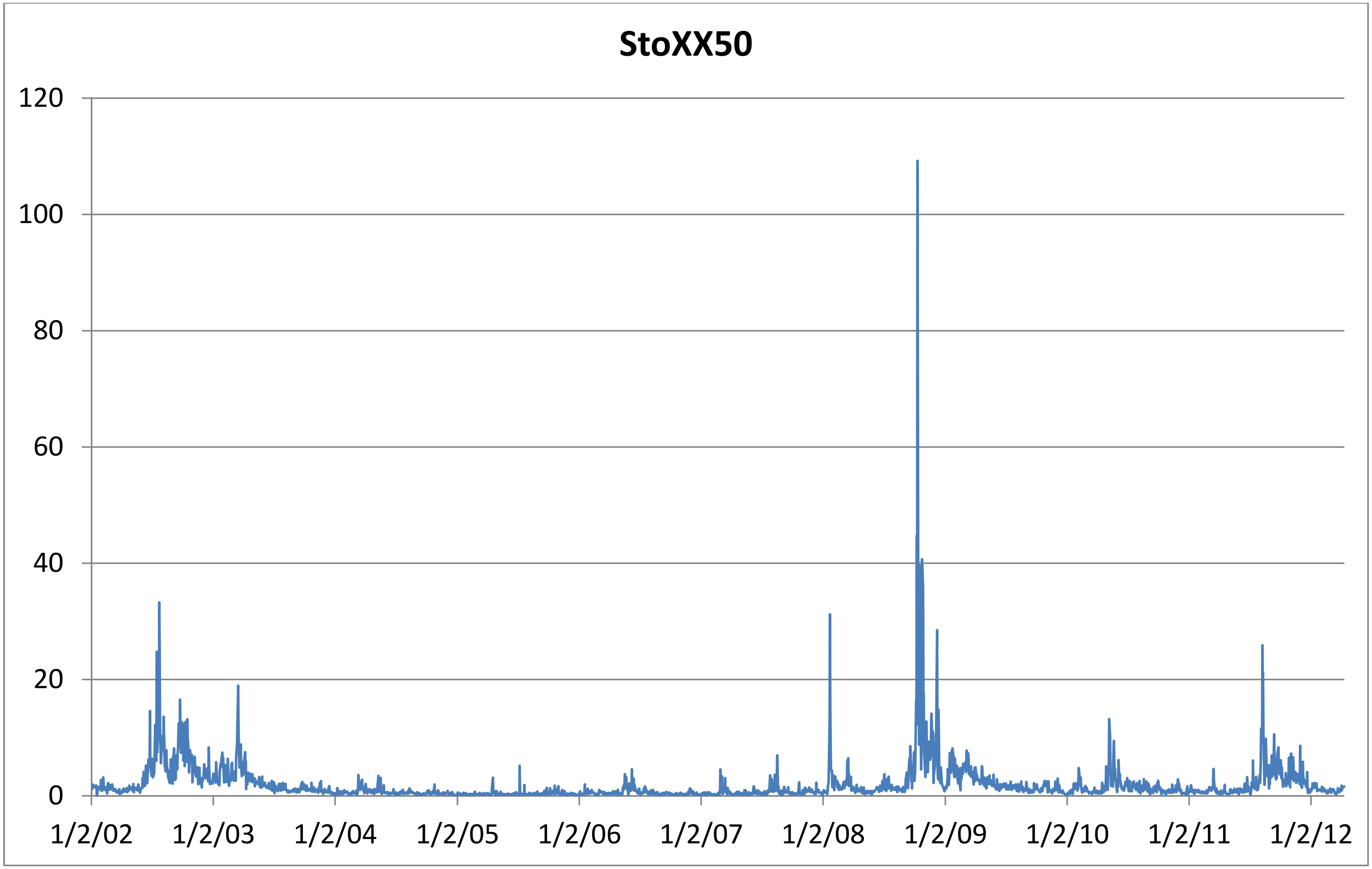

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Figure 1b   Kernel Volatilities for Exchange Rates

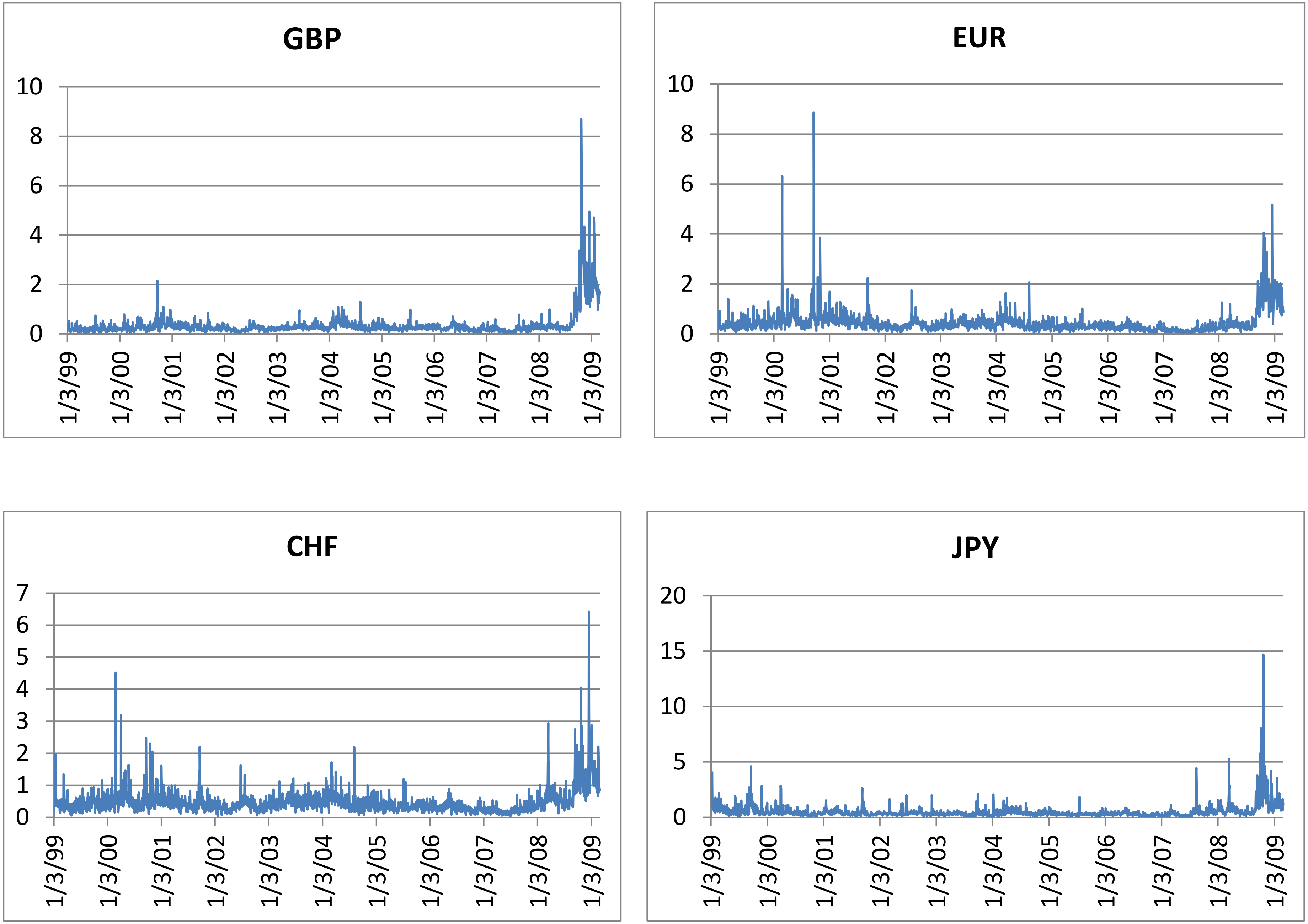


Table 1 Descriptive Statistics

	SPX	*RUT*	*DJI*	*NSDQ*	*Stoxx50*	*GBP*	*EUR*	*CHF*	*JPY*
Mean	1.38	1.43	1.35	1.24	1.91	0.36	0.44	0.47	0.50
Std Deviation	3.16	2.68	3.14	2.14	3.72	0.51	0.45	0.38	0.64
Kurtosis	298.64	149.26	287.43	139.38	290.78	58.76	72.56	41.86	126.25
Skewness	12.89	9.31	12.83	9.03	12.59	6.46	6.23	4.72	8.41
Minimum	0.05	0.05	0.05	0.04	0.01	0.04	0.03	0.04	0.02
Maximum	93.13	64.25	91.26	50.09	109.22	8.70	8.86	6.42	14.68
Count	2567	2567	2567	2567	2592	2555	2555	2555	2555

Table 2 Results from different copulas

Copulas	SPX	RUT	DJI	NSDQ	Stoxx50	GBP	EUR	CHF	JPY
Linear ρ	0.64	0.67	0.65	0.68	0.58	0.88	0.66	0.65	0.70
Normal									
ρ	**0.80**	**0.73**	**0.80**	**0.82**	**0.83**	**0.67**	**0.72**	**0.63**	**0.68**
Std error	0.006	0.007	0.006	0.005	0.005	0.009	0.008	0.010	0.009
AIC	-2614	-1961	-2591	-2886	-3020	-1540	-1831	-1257	-1549
BIC	-2608	-1955	-2585	-2880	-3015	-1534	-1825	-1251	-1543
t Copula									
ρ	**0.81**	**0.74**	**0.80**	**0.83**	**0.84**	**0.67**	**0.72**	**0.63**	**0.68**
Std error	0.006	0.008	0.006	0.006	0.005	0.012	0.009	0.012	0.011
ν	**9.17**	**7.16**	**6.89**	**8.15**	**51.00**	**3.92**	**5.02**	**6.32**	**5.07**
Std error	1.613	1.144	1.027	1.332	0.002	0.452	0.673	0.956	0.636
AIC	-2661	-2016	-2665	-2947	-3059	-1656	-1927	-1322	-1652
BIC	-2649	-2005	-2653	-2936	-3047	-1645	-1915	-1310	-1640
symetric tail	0.32	0.30	0.38	0.38	0.04	0.37	0.36	0.24	0.32
SJC									
λ_L	**0.30**	**0.17**	**0.36**	**0.39**	**0.36**	**0.28**	**0.42**	**0.31**	**0.33**
Std error	0.016	0.023	0.033	0.004	0.002	0.313	0.024	0.027	0.027
λ_R	**0.74**	**0.70**	**0.73**	**0.75**	**0.76**	**0.64**	**0.61**	**0.52**	**0.59**
Std error	0.007	0.007	0.006	0.007	0.001	0.124	0.012	0.015	0.012
AIC	**-2780**	**-2275**	**-2735**	**-2953**	**-3071**	**-1843**	**-1939**	**-1326**	**-1687**
BIC	**-2769**	**-2264**	**-2723**	**-2941**	**-3059**	**-1831**	**-1927**	**-1314**	**-1676**
Clayton - survival									
λ_R	**0.75**	**0.71**	**0.75**	**0.76**	**0.77**	**0.65**	**0.65**	**0.56**	**0.62**
Std error	0.005	0.007	0.006	0.005	0.005	0.008	0.008	0.011	0.009
AIC	-2701	-2241	-2626	-2830	-2991	-1739	-1735	-1207	-1546
BIC	-2695	-2236	-2620	-2824	-2985	-1733	-1729	-1201	-1540
Clayton									
λ_L	**0.61**	**0.52**	**0.63**	**0.65**	**0.66**	**0.50**	**0.57**	**0.47**	**0.52**
Std error	0.010	0.013	0.009	0.008	0.008	0.014	0.011	0.015	0.013
AIC	-1503	-1030	-1562	-1748	-1803	-927	-1241	-815	-1005
BIC	-1497	-1024	-1556	-1742	-1797	-921	-1236	-810	-999

Table 3 Volatility dependence decay over time

<u>Table 3 Panel A: For Stock Indices</u>

	Lag1	Lag2	Lag3	Lag4	Lag5	Lag10	Lag15	Lag20	Lag25	Lag30	Lag35	Lag40
SPX												
λ_L	**0.30**	**0.18**	**0.21**	**0.16**	**0.12**	0.05	0.04	**0.01**	0.02	0.10	0.08	**0.09**
Std_err	0.016	0.035	0.036	0.010	0.036	0.043	0.032	0.000	0.025	0.034	0.032	0.033
λ_R	**0.74**	**0.72**	**0.69**	**0.68**	**0.67**	**0.62**	**0.58**	**0.55**	**0.51**	**0.47**	**0.44**	**0.43**
Std_err	0.007	0.007	0.008	0.009	0.008	0.010	0.012	0.012	0.014	0.016	0.017	0.018
AIC	-2780	-2452	-2175	-2030	-1939	-1508	-1264	-1089	-921	-867	-764	-714
BIC	-2769	-2440	-2164	-2018	-1927	-1496	-1253	-1077	-909	-856	-752	-702
RUT												
λ_L	**0.17**	**0.12**	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**0.01**	**0.01**	**0.01**
Std_err	0.023	0.104	0.000	0.000	2.435	0.000	0.549	0.000	0.004	0.000	0.000	0.000
λ_R	**0.70**	**0.68**	**0.65**	**0.63**	0.62	**0.56**	0.53	**0.47**	0.45	**0.43**	**0.40**	**0.38**
Std_err	0.007	0.020	0.001	0.003	2575	0.011	1868	0.014	1.525	0.016	0.017	0.017
AIC	-2275	-2003	-1655	-1569	-1477	-1132	-977	-785	-688	-639	-574	-524
BIC	-2264	-1991	-1644	-1557	-1465	-1121	-965	-773	-677	-628	-562	-513
DJI												
λ_L	**0.36**	**0.24**	**0.25**	**0.18**	**0.15**	**0.06**	0.03	0.05	0.05	**0.09**	**0.09**	**0.08**
Std_err	0.033	0.014	0.029	0.057	0.027	0.035	0.030	0.032	0.032	0.033	0.032	0.031
λ_R	**0.73**	**0.72**	**0.69**	**0.67**	**0.67**	**0.62**	**0.57**	**0.55**	**0.51**	**0.48**	**0.44**	**0.43**
Std_err	0.006	0.007	0.008	0.009	0.009	0.010	0.012	0.013	0.014	0.016	0.017	0.018
AIC	-2735	-2473	-2164	-2025	-1944	-1516	-1248	-1118	-948	-888	-779	-704
BIC	-2723	-2461	-2153	-2013	-1933	-1504	-1236	-1106	-937	-877	-767	-693
NSDQ												
λ_L	**0.39**	**0.23**	**0.22**	**0.02**	**0.17**	**0.09**	0.06	0.08	0.01	0.04	0.05	0.01
Std_err	0.004	0.021	0.041	0.000	0.038	0.038	0.035	0.036	0.000	0.031	0.030	0.000
λ_R	**0.75**	**0.72**	**0.68**	**0.69**	**0.66**	**0.61**	**0.57**	**0.53**	**0.53**	**0.49**	**0.47**	**0.47**
Std_err	0.007	0.008	0.008	0.001	0.009	0.011	0.012	0.014	0.012	0.015	0.016	0.014
AIC	-2953	-2434	-2091	-2009	-1861	-1472	-1238	-1076	-982	-879	-821	-775
BIC	-2941	-2423	-2080	-1997	-1849	-1461	-1226	-1064	-970	-867	-809	-763
Stoxx50												
λ_L	**0.36**	**0.32**	**0.30**	**0.33**	**0.30**	**0.25**	**0.23**	**0.25**	**0.29**	**0.30**	**0.31**	**0.27**
Std_err	0.002	0.039	0.037	0.034	0.037	0.025	0.036	0.033	0.029	0.028	0.028	0.029
λ_R	**0.76**	**0.71**	**0.68**	**0.67**	**0.66**	**0.61**	**0.56**	**0.52**	**0.48**	**0.46**	**0.43**	**0.42**
Std_err	0.001	0.004	0.009	0.010	0.010	0.011	0.013	0.015	0.017	0.018	0.020	0.020
AIC	-3071	-2489	-2200	-2136	-2052	-1660	-1361	-1195	-1118	-1071	-991	-888
BIC	-3059	-2478	-2188	-2125	-2040	-1648	-1350	-1183	-1106	-1059	-979	-877

Table 3 Panel B: For Major Currencies

	Lag1	Lag2	Lag3	Lag4	Lag5	Lag10	Lag15	Lag20	Lag25	Lag30	Lag35	Lag40
GBP												
λ_L	**0.28**	**0.25**	**0.21**	**0.18**	**0.22**	**0.16**	**0.15**	**0.14**	**0.12**	**0.09**	**0.05**	0.03
Std_er	0.313	0.030	0.030	0.031	0.030	0.030	0.030	0.030	0.029	0.027	0.025	0.021
λ_R	**0.64**	**0.58**	**0.54**	**0.55**	**0.56**	**0.50**	**0.46**	**0.44**	**0.42**	**0.39**	**0.36**	**0.35**
Std_er	0.124	0.012	0.013	0.013	0.013	0.014	0.016	0.017	0.018	0.018	0.019	0.020
AIC	-1843	-1521	-1302	-1268	-1379	-1093	-936	-848	-769	-666	-565	-516
BIC	-1831	-1509	-1291	-1256	-1367	-1081	-925	-836	-757	-654	-553	-504
ERO												
λ_L	**0.42**	**0.41**	**0.39**	**0.39**	**0.45**	**0.39**	**0.39**	**0.37**	**0.39**	**0.36**	**0.31**	**0.31**
Std_er	0.024	0.022	0.022	0.022	0.020	0.021	0.021	0.022	0.021	0.021	0.024	0.023
λ_R	**0.61**	**0.55**	**0.52**	**0.50**	**0.50**	**0.45**	**0.43**	**0.40**	**0.38**	**0.35**	**0.33**	**0.31**
Std_er	0.012	0.014	0.014	0.016	0.016	0.018	0.019	0.020	0.021	0.021	0.023	0.023
AIC	-1939	-1655	-1488	-1410	-1569	-1257	-1189	-1095	-1068	-954	-811	-769
BIC	-1927	-1643	-1476	-1398	-1557	-1245	-1177	-1083	-1056	-942	-800	-757
CHF												
λ_L	**0.31**	**0.30**	**0.28**	**0.27**	**0.35**	**0.29**	**0.25**	**0.26**	**0.28**	**0.25**	**0.19**	**0.19**
Std_er	0.027	0.026	0.026	0.027	0.023	0.025	0.026	0.026	0.025	0.026	0.027	0.028
λ_R	**0.52**	**0.46**	**0.41**	**0.41**	**0.40**	**0.36**	**0.33**	**0.32**	**0.28**	**0.26**	**0.26**	**0.25**
Std_er	0.015	0.017	0.019	0.019	0.019	0.021	0.022	0.023	0.024	0.025	0.024	0.025
AIC	-1326	-1107	-927	-915	-1057	-828	-699	-684	-666	-583	-499	-471
BIC	-1314	-1096	-915	-903	-1045	-816	-687	-673	-654	-571	-487	-459
JPY												
λ_L	**0.33**	**0.26**	**0.22**	**0.18**	**0.21**	**0.18**	**0.15**	**0.12**	**0.09**	**0.10**	0.01	0.00
Std_er	0.027	0.028	0.031	0.031	0.031	0.030	0.030	0.029	0.027	0.026	0.013	0.000
λ_R	**0.59**	**0.53**	**0.48**	**0.47**	**0.46**	**0.39**	**0.36**	**0.32**	**0.27**	**0.23**	**0.24**	**0.26**
Std_er	0.012	0.014	0.016	0.016	0.017	0.020	0.021	0.021	0.023	0.024	0.023	0.000
AIC	-1687	-1291	-1058	-979	-960	-729	-629	-510	-402	-341	-264	-261
BIC	-1676	-1279	-1046	-967	-948	-717	-618	-499	-390	-330	-253	-249

Figure 2 Long memory of high volatility

Figure 2A: Long memory of stock indices

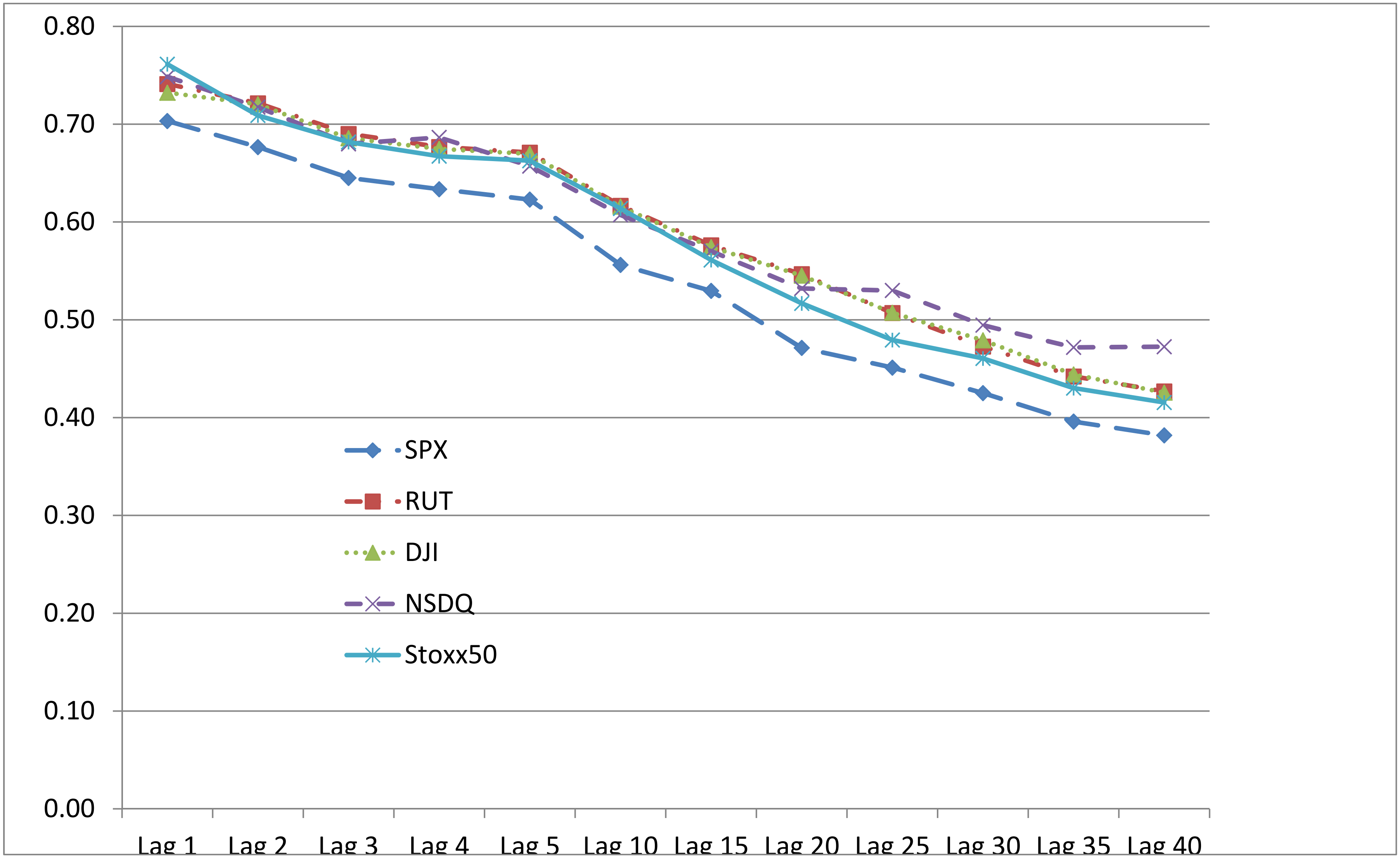


Figure 2B: Long memory of currencies

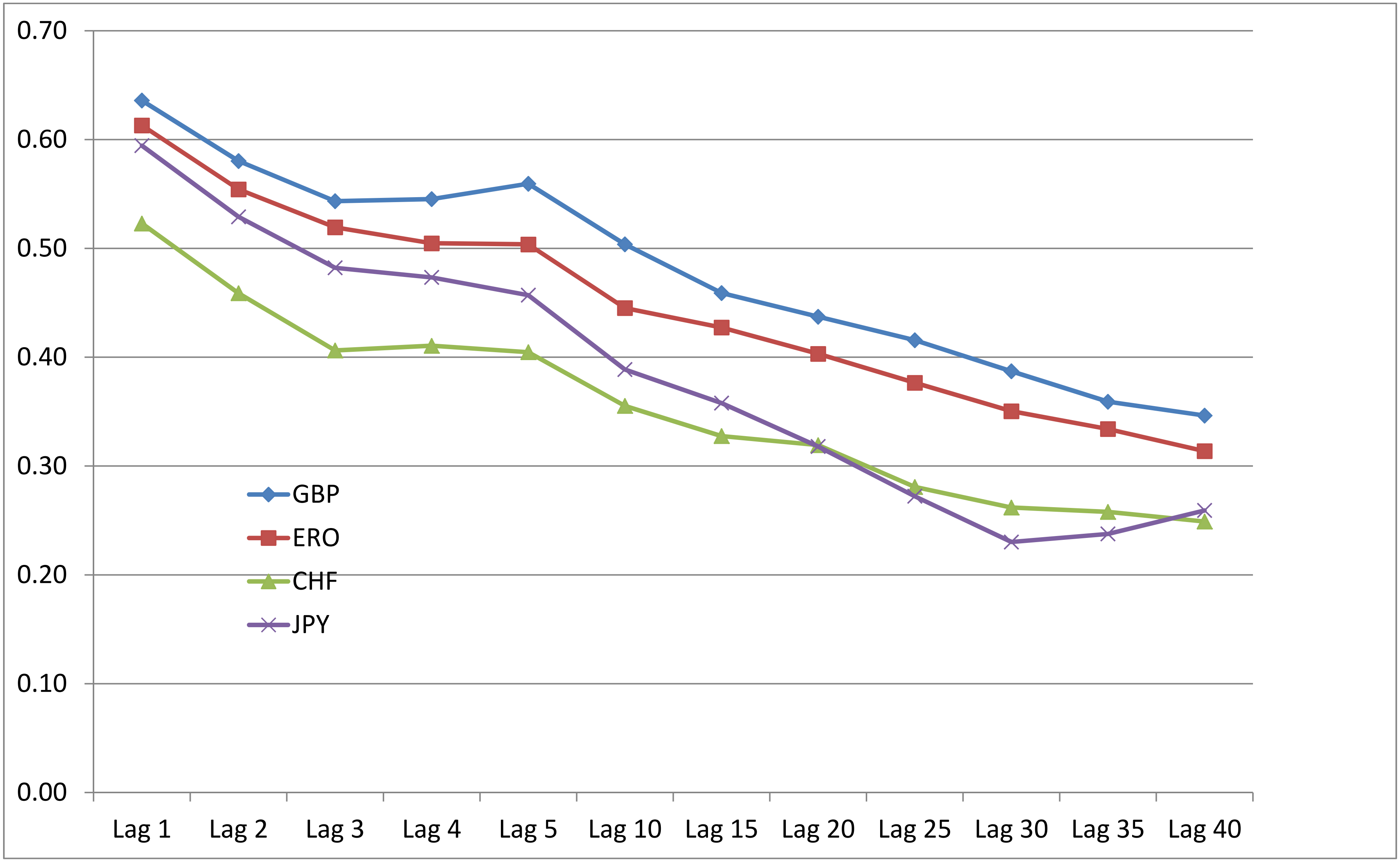


Figure 3. Decay of low volatility clusters

Figure 3A: Decay of clusters of low volatility: stock indices

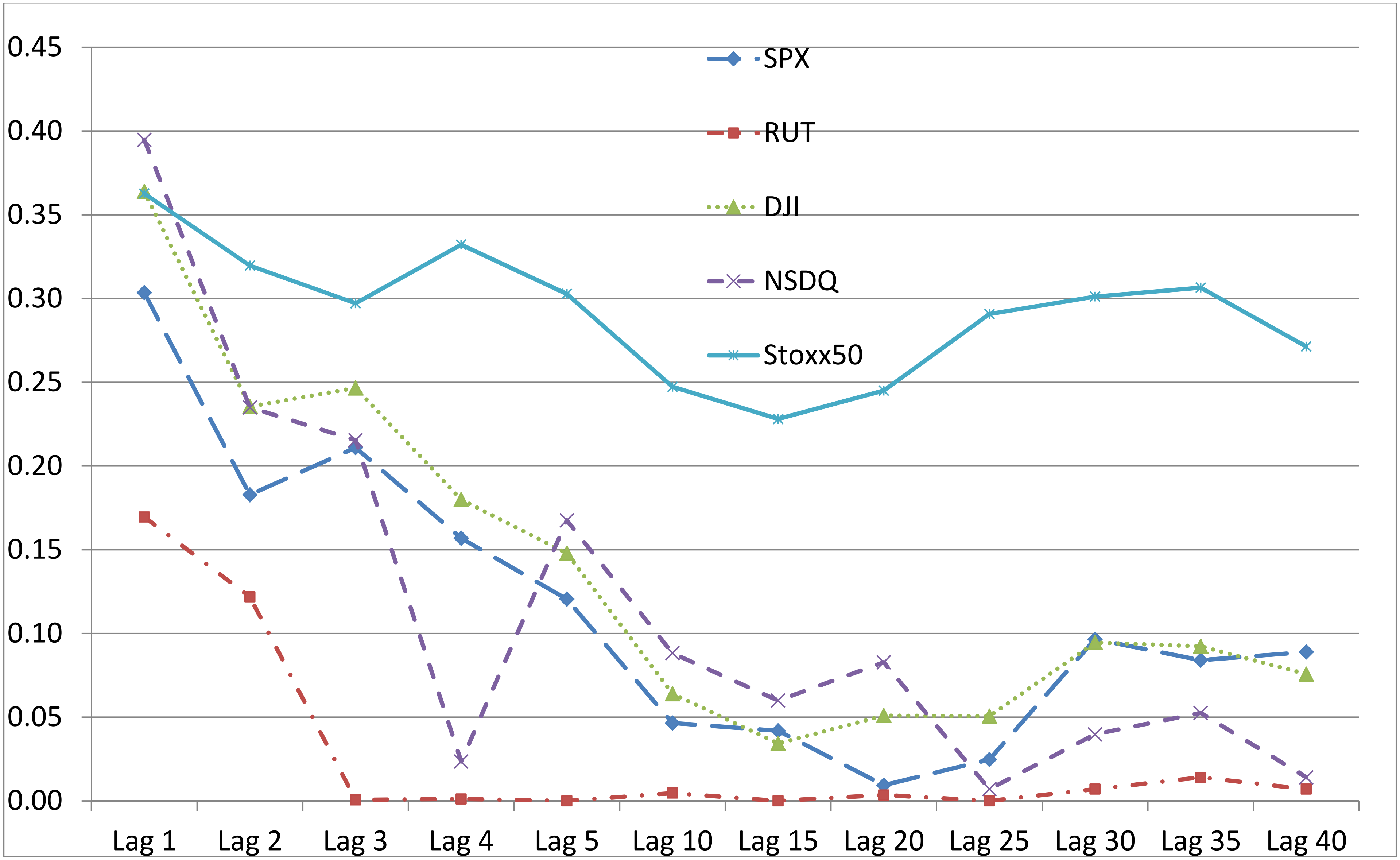
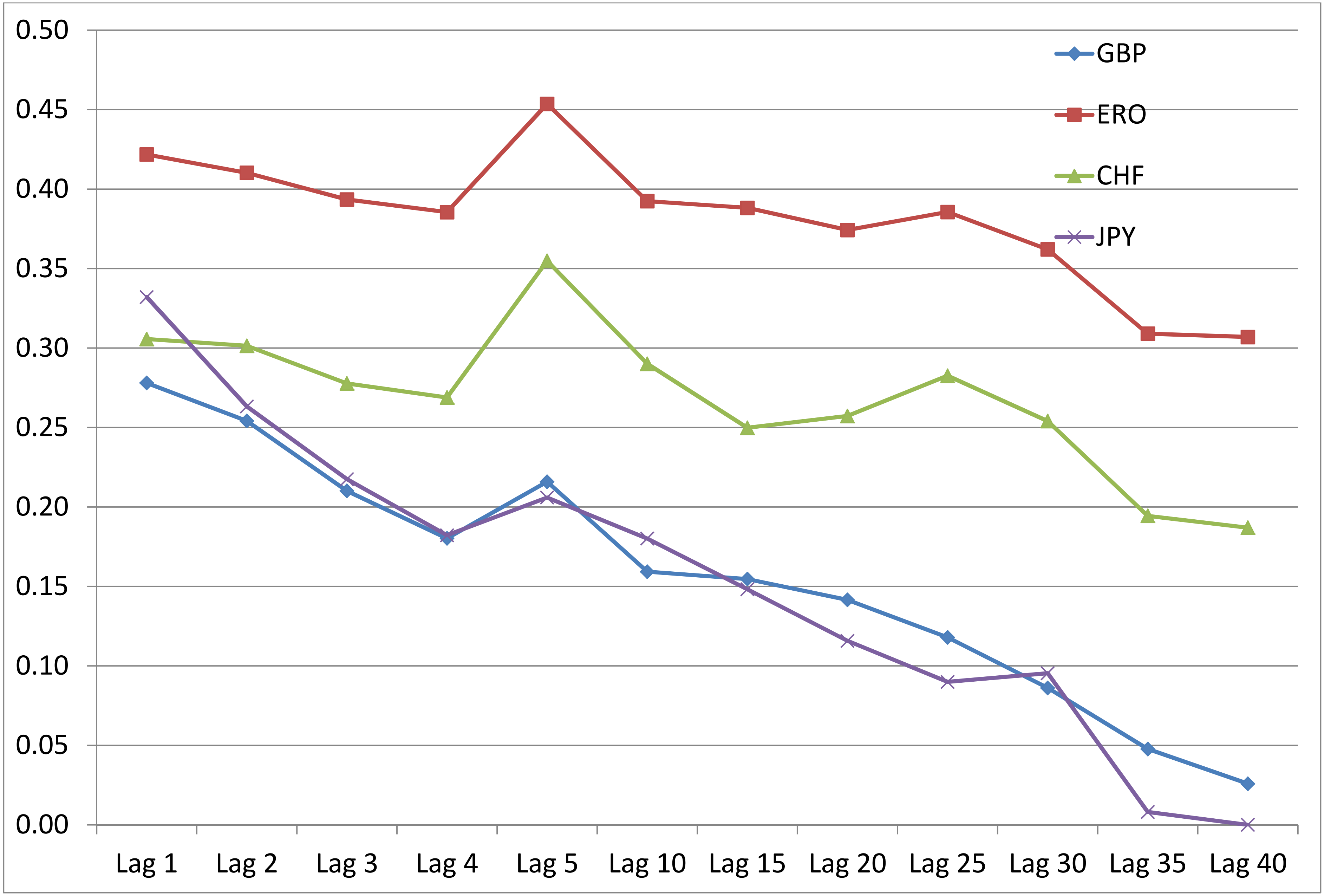


Figure 3B: Low volatility clusters decay: currencies



Conclusion

1. Volatility clusters are nonlinear and strongly asymmetric: clusters of high volatility are much more frequent than the clusters of low volatility.

2. This is consistent with the prolonged asymmetric leverage effect (see Bollerslev et. al (2006) and volatility feedback effect documented in the literature.

$$\ominus R_{t-2} \xRightarrow[L]{} V_{t-1}\uparrow \xRightarrow[F]{} \ominus R_{t-1} \xRightarrow[L]{} V_t \uparrow$$

3. The asymmetruc pattern keeps after 40 days.

4. The clusters of high volatilities remain persistent, indicating long memory of high volatility, low volatility clusters decay fast to 0 for all US stock indices and GBP and JPY.