Unit Commitment with AC Transmission Constraints via Sequential Cone Programming Relaxation

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***Abstract*—This paper proposes a sequential convex relaxation method for obtaining feasible and near-globally optimal solutions for unit commitment (UC) with AC transmission constraints. First, we develop a conic relaxation for AC unit commitment. To ensure that the resulting solutions are feasible for the original non-convex problem, we incorporate penalty terms into the objective of the proposed relaxation. We generalize our penalization method to a sequential algorithm which starts from an initial point (not necessarily feasible) and leads to feasible and near-optimal solutions for AC unit commitment. Once a feasible point is attained, the algorithm preserves feasibility and improves the objective value until a near optimal point is obtained. The experimental results on Matpower's Illinois 200-bus, South Carolina 500-bus, and European 1354-bus power system models demonstrate the performance of the proposed method in solving challenging instances of AC unit commitment.**

I. Introduction

The unit commitment (UC) is a classical problem in the area of power systems which involves determining the optimal schedule for power generating units throughout a given planning horizon. The main objective is to meet power demand with minimum production cost while respecting the limitations of generating units and network constraints. Due to the economic importance of the UC problem, it has been heavily investigated for decades and is proven to be computationally hard in general [2], [3]. The reader is referred to [4], [5] and the references therein, for detailed surveys of the conventional formulations and methods for solving unit commitment.

A general unit commitment problem can be formulated as a mixed-integer optimization whose solution specifies the optimal status of generating units as well as voltages and power flows throughout the planning horizon. Additionally, several papers have considered uncertainties of demand and renewable generation into consideration using stochastic and robust optimization frameworks [6]–[13]. The incorporation of several other power system optimization problems into unit commitment has been envisioned as well, such as the optimal power flow [14]–[16], network topology control [17], demand response [18], air quality control [19], and scheduling of deferrable loads [20].

Parts of this paper have appeared in the conference paper [1]. Compared with the conference version, the new additions to this paper are major changes to the algorithm and simulations on larger benchmark systems.
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Various optimization methods have been used to approach the UC problem, such as branch-and-bound techniques [21]–[27] and convex relaxations [28]–[30]. In order to improve the efficiency of branch-and-bound searches, many papers have offered partial convex hull characterizations of UC feasible sets [31]–[34]. Conic inequalities are proposed in [14], [35]–[37] to strengthen convex relaxations in the presence of nonlinear cost functions. In [38], a combination of semidefinite programming relaxation and branch-and-bound is used to solve the day-ahead hydro unit commitment problem. In [39], [40], reformulation-linearization cuts are proposed to strengthen semidefinite programming relaxations of unit commitment. In [41], a decomposition method is developed based on second-order cone programming (SOCP) to solve network constrained unit commitment with AC power flow constraints. In [42], a family of valid inequalities are proposed to improve the quality of SOCP relaxations of unit commitment. In [43], a global search algorithm is proposed which solves a sequence of mixed-integer second-order cone programming (MISOCP) problems, as well as nonlinear non-convex problems to lower- and upper-bound the globally optimal cost of unit commitment. In [44], [45], distributed frameworks on high-performance computing platforms are investigated for solving large-scale UC problems. Nevertheless, the improvements in run-time are reported to diminish with more than 15 parallel workers [46].

In this paper, we introduce a novel sequential convex relaxation for solving unit commitment with AC transmission constraints. We propose a penalization method which is guaranteed to recover feasible solutions for general non-convex optimization problems under certain assumptions [47], [48]. The proposed penalized convex relaxation can be solved sequentially in order to find feasible and near-globally optimal solutions. Our experimental results verify the effectiveness of this procedure in solving AC unit commitment problems on Matpower benchmark systems of size 200-bus, 500-bus, and 1354-bus.

A. Notations

Throughout this paper, matrices, vectors, and scalars are represented by boldface uppercase, boldface lowercase, and italic lowercase letters, respectively. The symbols $\mathbb{R}$, $\mathbb{C}$, and $\mathbb{H}_n$ denote the sets of real numbers, complex numbers, and $n \times n$ Hermitian matrices, respectively. The notation "i" is reserved for the imaginary unit. Notation $|\cdot|$ denotes either the absolute value of a scalar or the cardinality of a set, depending on the context. The symbols $(\cdot)^*$ and $(\cdot)^\top$ represent the conjugate transpose and transpose operators, respectively. For a given matrix $\mathbf{A}$, the notations $\mathbf{A}_{\bullet,k}$, $\mathbf{A}_{j,\bullet}$, and A_{jk}

TABLE I: Unit and network constraints in power system scheduling.

Unit Constraints:

$$x_{g,t} \in \{0,1\} \tag{1a}$$

$$c_{g,t} = \alpha_g p_{g,t} + \beta_g p_{g,t}^2 + \gamma_g x_{g,t} + \gamma_g^{\uparrow}(1 - x_{g,t\text{-}1}) x_{g,t} + \gamma_g^{\downarrow} x_{g,t\text{-}1}(1 - x_{g,t}) \tag{1b}$$

$$x_{g,\tau} - x_{g,\tau\text{-}1} \leq x_{g,t} \qquad \forall \tau \in \{t - m_g^{\uparrow} + 1, \ldots, t\} \tag{1c}$$

$$x_{g,\tau\text{-}1} - x_{g,\tau} \leq 1 - x_{g,t} \qquad \forall \tau \in \{t - m_g^{\downarrow} + 1, \ldots, t\} \tag{1d}$$

$$\underline{p}_g\, x_{g,t} \leq p_{g,t} \leq \bar{p}_g\, x_{g,t} \tag{1e}$$

$$\underline{q}_g\, x_{g,t} \leq q_{g,t} \leq \bar{q}_g\, x_{g,t} \tag{1f}$$

$$p_{g,t} - p_{g,t\text{-}1} \leq r_g x_{g,t\text{-}1} + s_g(1 - x_{g,t\text{-}1}) \tag{1g}$$

$$p_{g,t\text{-}1} - p_{g,t} \leq r_g x_{g,t} + s_g(1 - x_{g,t}) \tag{1h}$$

AC Network Constraints:

$$\mathbf{d}_{\bullet,t} + \mathrm{diag}\{\mathbf{v}_{\bullet,t}\mathbf{v}_{\bullet,t}^{*}\mathbf{Y}^{*}\} = \mathbf{C}^{\top}(\mathbf{p}_{\bullet,t} + \mathrm{i}\mathbf{q}_{\bullet,t}) \tag{2a}$$

$$\mathrm{diag}\{\vec{\mathbf{C}}\ \mathbf{v}_{\bullet,t}\mathbf{v}_{\bullet,t}^{*}\vec{\mathbf{Y}}^{*}\} \leq \mathbf{f}_{\max;t}^{2} \tag{2b}$$

$$\mathrm{diag}\{\overleftarrow{\mathbf{C}}\ \mathbf{v}_{\bullet,t}\mathbf{v}_{\bullet,t}^{*}\overleftarrow{\mathbf{Y}}^{*}\} \leq \mathbf{f}_{\max;t}^{2} \tag{2c}$$

$$\underline{\mathbf{v}} \leq |\mathbf{v}_{\bullet,t}| \leq \bar{\mathbf{v}} \tag{2d}$$

refer to the k^{th} column, j^{th} row, and $(j,k)^{th}$ entry of the matrix $\mathbf{A}$, respectively. The Notation $\mathbf{A} \succeq 0$ means that $\mathbf{A}$ is symmetric/Hermitian and positive semidefinite.

II. Problem Formulation

The unit commitment (UC) problem aims at finding the most reliable and cost-efficient schedule for a set of generating units throughout a discrete time horizon $\mathcal{T}$, subject to forecasted electricity demands and operational constraints. Let $\mathcal{G}$ denote the set of generating units whose schedule needs to be determined. Define $x_{g,t} \in \{0,1\}$ as a binary variable indicating whether the generating unit $g \in \mathcal{G}$ is committed during the time slot $t \in \mathcal{T}$. If $x_{g,t} = 1$, the unit is active and generates power within its capacity limitations, otherwise, no power is produced by g during the time interval t. Define $p_{g,t}$ and $q_{g,t}$, respectively, as the amounts of active power and reactive power injections of generator g during the time interval t.

Denoted $\mathcal{V}$ and $\mathcal{E}$ as the sets of buses and branches in the network, respectively. For every bus $k \in \mathcal{V}$, the demand forecast at time t is denoted as $d_{k,t} \in \mathbb{C}$, whose real and imaginary parts account for active and reactive power demands, respectively. Let $\mathbf{C} \in \{0,1\}^{|\mathcal{G}|\times|\mathcal{V}|}$ be the incidence matrix whose (g,k) entry is equal to 1, if and only if the generating unit g belongs to the bus k. Define the matrices $\vec{\mathbf{C}}, \overleftarrow{\mathbf{C}} \in \{0,1\}^{|\mathcal{E}|\times|\mathcal{V}|}$ as the *from* and *to* incidence matrices, respectively. The (l,k) entry of $\vec{\mathbf{C}}$ is equal to one, if and only if the line $l \in \mathcal{E}$ starts at bus k, while the (l,k) entry of $\overleftarrow{\mathbf{C}}$ is equal to 1, if and only if the line l ends at bus k. Additionally, define $\mathbf{Y} \in \mathbb{C}^{|\mathcal{V}|\times|\mathcal{V}|}$ as the nodal admittance matrices of the network and $\vec{\mathbf{Y}}$, $\overleftarrow{\mathbf{Y}} \in \mathbb{C}^{|\mathcal{E}|\times|\mathcal{V}|}$ as the *from* and *to* branch admittance matrices.

The feasible set of AC unit commitment can be described by unit constraints and AC network constraints. Unit constraints impose the minimum up and down time limits (1c) – (1d), generator capacities (1e) – (1f), as well as ramp limits (1g) – (1h). Define $m_g^{\uparrow}$ and $m_g^{\downarrow}$, respectively, as the minimum up time and minimum down time limits for generating unit g. If the unit g is committed during the interval t, then its, active and reactive power injections must lie within the intervals $[\underline{p}_g, \bar{p}_g]$ and $[\underline{q}_g, \bar{q}_g]$, respectively. Additionally, denote r_g as the maximum variation of active power injection by unit g between two consecutive time slots in which the unit stays committed. Define s_g as the maximum amount of active power injection after start-up and prior to shutdown.

The network constraint (2a) accounts for nodal power balances. The constraint (2d) enforces voltage magnitude limits. Moreover, the constraints (2b) and (2c) enforce the thermal limits of lines.

Given the above definitions, the AC unit commitment problem can be formulated as the optimization problem

$$\text{minimize} \quad \sum_{g,t} c_{g,t} \tag{3a}$$

$$\text{subject to} \quad (\mathbf{x}_{g,\bullet}^{\top}, \mathbf{p}_{g,\bullet}^{\top}, \mathbf{q}_{g,\bullet}^{\top}, \mathbf{c}_{g,\bullet}^{\top}) \in \mathcal{U}_g \qquad \forall g \in \mathcal{G}, \tag{3b}$$

$$(\mathbf{p}_{\bullet,t}, \mathbf{q}_{\bullet,t}, \mathbf{v}_{\bullet,t}) \in \mathcal{N}_t \qquad \forall t \in \mathcal{T}, \tag{3c}$$

with respect to the matrix variables $\mathbf{x} \triangleq [x_{g,t}]$, $\mathbf{p} \triangleq [p_{g,t}]$, $\mathbf{q} \triangleq [q_{g,t}]$, $\mathbf{c} \triangleq [c_{g,t}]$, and $\mathbf{v} \triangleq [v_{k,t}]$. The objective function (3a) is equal the sum of the production costs of all generating units throughout the time horizon $\mathcal{T}$. For any arbitrary generating unit g in time interval t, the production cost consists of the generation cost, start-up cost, shutdown cost, and a fixed cost. The generation cost is a quadratic function with respect to $p_{g,t}$ with nonnegative coefficients α_g and β_g. The start-up cost $\gamma_g^{\uparrow}$ and shutdown cost $\gamma_g^{\downarrow}$ are associated with every time slots at which the unit changes status. The fixed production cost γ_g is enforced if the unit is active.

Definition 1: For every generating units $g \in \mathcal{G}$, define $\mathcal{U}_g \subset \mathbb{R}^{|\mathcal{T}|\times 4}$ to be the set of all quadruplets $(\mathbf{x}_{g,\bullet}^{\top}, \mathbf{p}_{g,\bullet}^{\top}, \mathbf{q}_{g,\bullet}^{\top}, \mathbf{c}_{g,\bullet}^{\top})$ that satisfy the constraints (1a) – (1h) throughout the entire planning horizon.

Definition 2: For every $t \in \mathcal{T}$, define $\mathcal{N}_t \subset \mathbb{R}^{|\mathcal{G}|\times 2} \times \mathbb{C}^{|\mathcal{V}|}$ to be the set of all triplets $(\mathbf{p}_{\bullet,t}, \mathbf{q}_{\bullet,t}, \mathbf{v}_{\bullet,t})$ that satisfy the network constraints (2a) – (2d).

Problem (3a) – (3c) is a mixed-integer nonlinear optimization, due to the presence of binary variables and nonlinearity of the network constraints. In what follows, we will develop a convex relaxation to tackle the non-convexity of this problem.

III. Convex Relaxation of the UC Problem

The non-convex sets $\{\mathcal{U}_g\}_{g\in\mathcal{G}}$ and $\{\mathcal{N}_t\}_{t\in\mathcal{T}}$, are the sources of computational complexity. In this paper, we introduce convex surrogates $\{\mathcal{U}_g^{\text{conv}}\}_{g\in\mathcal{G}}$ and $\{\mathcal{N}_t^{\text{conv}}\}_{t\in\mathcal{T}}$, which lead to a class of computationally-tractable relaxations of the problem (3a) – (3c). To this end, define the auxiliary variables $\mathbf{u}, \mathbf{o} \in \mathbb{R}^{|\mathcal{G}|\times|\mathcal{T}|}$, whose components account for monomials $x_{g,t-1}x_{g,t}$ and $p_{g,t}^2$, respectively. Using the defined variables, non-convex constraints (1a) – (1b) can be convexified as (4a),

Unit Constraints:

$$c_{g,t}=\alpha_g p_{g,t}+\beta_g o_{g,t}+\gamma_g x_{g,t} + \gamma_g^{\uparrow}(x_{g,t}-u_{g,t})+\gamma_g^{\downarrow}(x_{g,t\text{-}1}-u_{g,t}), \tag{4a}$$

$$x_{g,\tau}-x_{g,\tau\text{-}1}\leq x_{g,t} \qquad \forall\tau\in\{t-m_g^{\uparrow}+1,\ldots,t\} \tag{4b}$$

$$x_{g,\tau\text{-}1}-x_{g,\tau}\leq 1-x_{g,t} \qquad \forall\tau\in\{t-m_g^{\downarrow}+1,\ldots,t\} \tag{4c}$$

$$\underline{p}_g\, x_{g,t} \leq p_{g,t} \leq \bar{p}_g\, x_{g,t} \tag{4d}$$

$$\underline{q}_g\, x_{g,t} \leq q_{g,t} \leq \bar{q}_g\, x_{g,t} \tag{4e}$$

$$p_{g,t}-p_{g,t\text{-}1} \leq r_g x_{g,t\text{-}1}+s_g(1-x_{g,t\text{-}1}) \tag{4f}$$

$$p_{g,t\text{-}1}-p_{g,t} \leq r_g x_{g,t}+s_g(1-x_{g,t}) \tag{4g}$$

$$\begin{bmatrix} x_{g,t-1} & u_{g,t} \\ u_{g,t} & x_{g,t}\end{bmatrix} - \begin{bmatrix} x_{g,t-1} \\ x_{g,t}\end{bmatrix}\begin{bmatrix} x_{g,t-1} & x_{g,t}\end{bmatrix} \succeq 0, \tag{4h}$$

$$\begin{bmatrix} x_{g,t} & p_{g,t} \\ p_{g,t} & o_{g,t}\end{bmatrix} - \begin{bmatrix} x_{g,t} \\ p_{g,t}\end{bmatrix}\begin{bmatrix} x_{g,t} & p_{g,t}\end{bmatrix} \succeq 0. \tag{4i}$$

AC Network Constraints:

$$\mathbf{d}_{\bullet,t}+\mathrm{diag}\{\mathbf{W}_t\,\mathbf{Y}^*\} = \mathbf{C}^{\top}(\mathbf{p}_{\bullet,t}+\mathrm{i}\mathbf{q}_{\bullet,t}), \tag{5a}$$

$$\mathrm{diag}\{\vec{\mathbf{C}}\;\mathbf{W}_t\,\vec{\mathbf{Y}}^*\} \leq \mathbf{f}^2_{\max;t}, \tag{5b}$$

$$\mathrm{diag}\{\overleftarrow{\mathbf{C}}\;\mathbf{W}_t\,\overleftarrow{\mathbf{Y}}^*\} \leq \mathbf{f}^2_{\max;t}, \tag{5c}$$

$$\underline{\mathbf{v}}^2 \leq \mathrm{diag}\{\mathbf{W}_t\} \leq \bar{\mathbf{v}}^2, \tag{5d}$$

$$\mathbf{W}_t - \mathbf{v}_{\bullet,t}\mathbf{v}^*_{\bullet,t} \succeq_{\mathcal{C}} 0. \tag{5e}$$

TABLE II: Relaxed unit and AC network constraints.

(4h), and (4i). Note that if equality holds for (4h) – (4i) at optimality, then we have:

$$x_{g,t}=x^2_{g,t},\; u_{g,t}=x_{g,t}x_{g,t-1},\; o_{g,t}=p^2_{g,t},\; p_{g,t}=x_{g,t}p_{g,t}. \tag{6}$$

In addition, to relax the non-convexity of AC network constraints, we define the auxiliary variables $\mathbf{W}_t\in\mathbb{H}_{|\mathcal{V}|}$, accounting for $\mathbf{v}_{\bullet,t}\mathbf{v}^*_{\bullet,t}$. Using the above auxiliary variables, the non-convex constraints (2a) – (2d) can be relaxed as (5a) – (5e).

In order to capture the binary requirements of the commitment decisions and enforce the relationship between the auxiliary variables and the corresponding monomials, we strengthen the proposed convex relaxation via conic constraints (4h) – (4i), and (5e), where $\mathcal{C}$ in (5e) is a pointed convex cone. Next, we define the convex surrogates $\{\mathcal{U}_g^{\mathrm{conv}}\}_{g\in\mathcal{G}}$ and $\{\mathcal{N}_t^{\mathrm{conv}}\}_{t\in\mathcal{T}}$.

Definition 3: For every $g\in\mathcal{G}$, define $\mathcal{U}_g^{\mathrm{conv}}\subset\mathbb{R}^{|\mathcal{T}|\times 6}$ to be the set of all nonuplets $(\mathbf{x}^{\top}_{g,\bullet},\mathbf{p}^{\top}_{g,\bullet},\mathbf{q}^{\top}_{g,\bullet},\mathbf{c}^{\top}_{g,\bullet},\mathbf{u}^{\top}_{g,\bullet},\mathbf{o}^{\top}_{g,\bullet})$ that satisfy the constraints (4a) – (4i) throughout the entire planning horizon.

Definition 4: For every $t\in\mathcal{T}$, define $\mathcal{N}_t^{\mathrm{conv}}\subset\mathbb{R}^{|\mathcal{G}|\times 2}\times\mathbb{C}^{|\mathcal{V}|}\times\mathbb{H}_{|\mathcal{V}|}$ to be the set of all quadruplets $(\mathbf{p}_{\bullet,t},\mathbf{q}_{\bullet,t},\mathbf{v}_{\bullet,t},\mathbf{W}_t)$ that satisfy the constraints (5a) – (5e).

The cone programming relaxation of network constraints can be derived by incorporating the following convex set into the constraint (5e):

$$\hat{\mathcal{C}}\triangleq\left\{\mathbf{H}\in\mathbb{H}_{|\mathcal{V}|}\;\middle|\; H_{ii}\geq 0,\; H_{ii}H_{jj}\geq|H_{ij}|^2,\;\forall(i,j)\in\mathcal{E}\right\},$$

which leads to 3×3 semi-definite programming constraints of the form:

$$\begin{bmatrix} W_{ii} & W_{ij} & v_i \\ W_{ji} & W_{jj} & v_j \\ v_i & v_j & 1\end{bmatrix}\succeq 0 \qquad \forall(i,j)\in\mathcal{E}. \tag{7}$$

This relaxation is not reducible to the well-known second-order cone programming relaxation since we are incorporating the elements of $\mathbf{v}$ into the objective for the purpose of obtaining feasible points. The solution provided by the this relaxation is a lower-bound for the globally optimal solution of AC unit commitment. In general, solutions obtained from convex relaxations are not necessarily feasible for the original non-convex problem. To remedy this shortcoming, we propose a novel penalization method to obtain feasible points. In the next section, we describe the proposed penalization method in details.

IV. Penalization Method

We incorporate a linear penalty term $\kappa(\{\mathbf{W}_t\}_{t\in\mathcal{T}},\mathbf{v},\mathbf{x})$ into the objective of the relaxed problem to enforce feasibility. Given an initial guess $\mathbf{y}^0=(\mathbf{v}^0,\mathbf{x}^0)$ that is sufficiently close to the feasible set of the problem (3a) – (3c), the following choice of penalty function guarantees the feasibility of the resulting solution under the assumptions in [47], [48]:

$$\begin{aligned}&\kappa_{\mathbf{M},\mathbf{y}_0}(\{\mathbf{W}_t\}_{t\in\mathcal{T}},\mathbf{v},\mathbf{x})\triangleq\\ &\sum\nolimits_t \mu_1(\mathrm{tr}\{\mathbf{W}_t\mathbf{M}\}-\mathbf{v}^{0\,*}_{\bullet,t}\mathbf{M}\mathbf{v}_{\bullet,t}-\mathbf{v}^*_{\bullet,t}\mathbf{M}\mathbf{v}^0_{\bullet,t}+\mathbf{v}^{0\,*}_{\bullet,t}\mathbf{M}\mathbf{v}^0_{\bullet,t})+\\ &\mu_2(\mathbf{x}^{\top}_{\bullet,t}\mathbf{1}-2\,\mathbf{x}^{\top}_{\bullet,t}\mathbf{x}^0_{\bullet,t}+\mathbf{x}^{0\top}_{\bullet,t}\mathbf{x}^0_{\bullet,t})\end{aligned} \tag{8}$$

where $\mathbf{M}\in\mathbb{H}_{|\mathcal{V}|}$ is a fixed penalty matrix.

By augmenting the penalty term (8) into the objective function of the relaxed problem, the *penalized convex relaxation* of AC unit commitment can be formulated as:

$$\min \quad \sum_{g,t} c_{g,t}+\kappa_{\mathbf{M},\mathbf{y}_0}(\{\mathbf{W}_t\}_{t\in\mathcal{T}},\mathbf{v},\mathbf{x}) \tag{9a}$$

$$\text{s.t.} \quad (\mathbf{x}^{\top}_{g,\bullet},\mathbf{p}^{\top}_{g,\bullet},\mathbf{q}^{\top}_{g,\bullet},\mathbf{c}^{\top}_{g,\bullet},\mathbf{u}^{\top}_{g,\bullet},\mathbf{o}^{\top}_{g,\bullet})\in\mathcal{U}_g^{\mathrm{conv}} \quad \forall g\in\mathcal{G}, \tag{9b}$$

$$(\mathbf{p}_{\bullet,t},\mathbf{q}_{\bullet,t},\mathbf{v}_{\bullet,t},\mathbf{W}_t)\in\mathcal{N}_t^{\mathrm{conv}} \quad \forall t\in\mathcal{T}, \tag{9c}$$

with respect to decision variables $\mathbf{x}\triangleq[x_{g,t}]$, $\mathbf{p}\triangleq[p_{g,t}]$, $\mathbf{q}\triangleq[q_{g,t}]$, $\mathbf{c}\triangleq[c_{g,t}]$, $\mathbf{o}\triangleq[o_{g,t}]$, $\mathbf{v}\triangleq[v_{k,t}]$, and $\{\mathbf{W}_t\}_{t\in\mathcal{T}}$. The nonnegative penalty parameter $\mu_1,\mu_2>0$ set the trade off between the objective and the penalty functions. The penalized convex relaxation (9a) – (9c) is said to be *tight* if it possesses a unique optimal solution $(\mathbf{x},\mathbf{p},\mathbf{q},\mathbf{c},\mathbf{o},\mathbf{v},\{\mathbf{W}_t\}_{t\in\mathcal{T}})$ such that $x_{g,t}\in\{0,1\}$ and $\mathbf{W}_t=\mathbf{v}_{\bullet,t}\mathbf{v}^*_{\bullet,t}$, for every $g\in\mathcal{G}$ and $t\in\mathcal{T}$. The tightness of the penalization guarantees the recovery of a feasible point for AC unit commitment (3a) – (3c).

A. Choice of Penalty Matrix

Motivated by the previous literatures [49]–[51], we choose $\mathbf{M}$ such that the penalty term $\mathrm{tr}\{\mathbf{W}_t\mathbf{M}\}$ reduces the apparent power loss over the series admittance of every line in the network. Consider the standard π-model of line $l\in\mathcal{E}$, with series admittance $y_{\mathrm{srs},l}\triangleq g_{\mathrm{srs},l}+\mathrm{i}\,b_{\mathrm{srs},l}$ and total shunt susceptance $b_{\mathrm{prl},l}$, in series with a phase shifting transformer

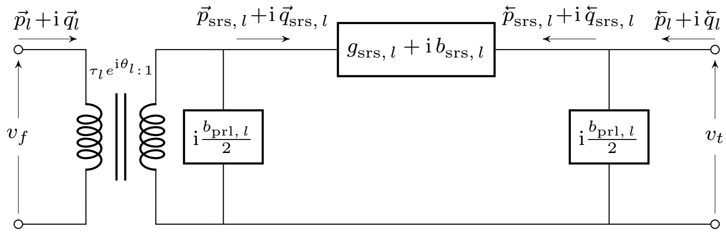

Fig. 1: Branch Model

whose tap ratio has magnitude τ_l and phase shift angle θ_l [52]. The model is shown in Figure 1. In order to penalize the apparent power loss over all lines of the network, we choose matrix $\mathbf{M}$ as,

$$\mathbf{M} = \sum_{(i,j)\in\mathcal{E}} [\mathbf{e}_i, \mathbf{e}_j](\mathbf{M}_{ij} + \alpha\mathbf{I}_2)[\mathbf{e}_i, \mathbf{e}_j]^\top,$$

where $\mathbf{e}_1, \ldots, \mathbf{e}_{|\mathcal{V}|}$ denote the standard basis for $\mathbb{R}^{|\mathcal{V}|}$, and $\alpha = 10^{-8}$. Moreover, each $\mathbf{M}_{ij}$ is a 2×2 positive semidefinite matrix defined as,

$$\mathbf{M}_{ij} = (\vec{\mathbf{Y}}_{\mathrm{p};\,l} + \overleftarrow{\mathbf{Y}}_{\mathrm{p};\,l}) + \eta \times \zeta_{ij}(\vec{\mathbf{Y}}_{\mathrm{q};\,l} + \overleftarrow{\mathbf{Y}}_{\mathrm{q};\,l})$$

where $\eta \geq 0$ sets the trade-off between active and reactive loss minimization, and

$$\vec{\mathbf{Y}}_{\mathrm{p};\,l} \triangleq \begin{bmatrix} \frac{g_{\mathrm{srs},\,l}}{\tau_l^2} & \frac{e^{\mathrm{i}\theta_l}\, y_{\mathrm{srs},\,l}}{-2\tau_l} \\ \frac{y^*_{\mathrm{srs},\,l}}{-2\tau_l e^{\mathrm{i}\theta_l}} & 0 \end{bmatrix}, \quad \vec{\mathbf{Y}}_{\mathrm{q};\,l} \triangleq \begin{bmatrix} \frac{b_{\mathrm{srs},\,l}}{-\tau_l^2} & \frac{e^{\mathrm{i}\theta_l}\, y_{\mathrm{srs},\,l}}{2\tau_l \mathrm{i}} \\ \frac{y^*_{\mathrm{srs},\,l}}{-2\tau_l \mathrm{i} e^{\mathrm{i}\theta_l}} & 0 \end{bmatrix},$$

$$\overleftarrow{\mathbf{Y}}_{\mathrm{p};\,l} \triangleq \begin{bmatrix} 0 & \frac{e^{\mathrm{i}\theta_l}\, y^*_{\mathrm{srs},\,l}}{-2\tau_l} \\ \frac{y_{\mathrm{srs},\,l}}{-2\tau_l e^{\mathrm{i}\theta_l}} & g_{\mathrm{srs},\,l} \end{bmatrix}, \quad \overleftarrow{\mathbf{Y}}_{\mathrm{q};\,l} \triangleq \begin{bmatrix} 0 & \frac{e^{\mathrm{i}\theta_l}\, y^*_{\mathrm{srs},\,l}}{-2\tau_l \mathrm{i}} \\ \frac{y_{\mathrm{srs},\,l}}{2\tau_l \mathrm{i} e^{\mathrm{i}\theta_l}} & -b_{\mathrm{srs},\,l} \end{bmatrix}.$$

Each $\zeta_{ij} \in \{-1, +1\}$ is determined based on the inductive or capacitive behavior of the line $l \in \mathcal{E}$. More precisely, we set $\zeta_{ij} = 1$ if the series admittance $y_{\mathrm{srs},\,l}$ is inductive (i.e., $b_{\mathrm{srs},\,l} \leq 0$), and $\zeta_{ij} = -1$, otherwise.

B. Sequential Penalized Relaxation

The penalized relaxation (9a) – (9c) is guaranteed to produce a feasible solution for AC unit commitment if the initial guess $\mathbf{y}^0$ is sufficiently close to the feasible set of the original problem (3a) – (3c). If a high quality initial point is not available, the proposed penalized relaxation can be solved sequentially until a feasible point for problem (3a) – (3c) is obtained. Once feasibility is attained, the sequential procedure improves the objective function while preserving the feasibility at each round until a near-optimal point is achieved. This sequential procedure is detailed by Algorithm 1.

Algorithm 1 Sequential Penalized SOCP Relaxation.

Input: μ_1, μ_2, $\mathbf{M}$, $(\mathbf{v}_0, \mathbf{x}_0)$

repeat
    Solve problem (9a) – (9c) to obtain $(\mathbf{v}, \mathbf{x})$
    $(\mathbf{v}_0, \mathbf{x}_0) \leftarrow (\mathbf{v}, \mathbf{x})$
until stopping criteria satisfied

Output: $(\mathbf{v}, \mathbf{x})$

V. Experimental Results

In this section, we present the results of our experiment on Illinois 200-bus and South Carolina 500-bus power system models from MATPOWER [52]. The numerical experiments are performed in MATLAB using a 64-bit computer with an Intel 3.0 GHz, 12-core CPU, and 256 GB RAM. Note that the experiments are all performed on a workstation with a single CPU. The CVX package version 3.0 [53] and MOSEK version 7.0 [54] are used to solve the proposed convex relaxations.

For each experiment, the ramp limits of each generating unit are set to $r_g = s_g = (\bar{p}_g - \underline{p}_g)/2$. For each generating unit, the minimum up and down limits $m_g^{\uparrow}$ and $m_g^{\downarrow}$ are set to 3 and 2 hours, respectively. Hourly load changes for the day-ahead at all buses are considered proportional to the numbers reported in [55]. The changes in demand throughout the 24-hour planning horizon are reported in Table III. For every time epoch, the corresponding demand factor at that time is multiplied by all loads in the system. Lastly, for the case PEGASE 1354, due to missing quadratic and constant cost coefficient data, we have considered the cost coefficients $\alpha_g = 0.01$, $\beta_g = 1$, and $\gamma_g = 100$ for all generators.

In order to evaluate the resulting feasible solutions from Algorithm 1 we solved an unpenalized convex relaxation of AC unit commitment by setting $\mu_1 = \mu_2 = 0$. The unpenalized relaxation offers a lower bound for the globally optimal cost of AC unit commitment, using which we can calculate the quality of our feasible solutions from Algorithm 1 through the formula

$$\mathrm{GAP}\% = 100 \times \frac{\sum_{g,t}(c_{g,t}^{\mathrm{feasible}} - c_{g,t}^{\mathrm{lower-bound}})}{\sum_{g,t} c_{g,t}^{\mathrm{feasible}}}, \qquad (10)$$

where $c_{g,t}^{\mathrm{feasible}}$ denotes the optimal cost value of the generating unit $g \in \mathcal{G}$ at time $t \in \mathcal{T}$ from the proposed sequential relaxation, and $c_{g,t}^{\mathrm{lower-bound}}$ denotes the cost values obtained from unpenalized relaxation of (3a) – (3c). The initial point of Algorithm 1 for all of the experiments is chosen as $\mathbf{v}^0 = \mathbf{1}_{|\mathcal{V}|\times|\mathcal{T}|}$ and $\mathbf{x}^0$ is set to the output of unpenalized relaxation. For both of the simulations, we set $\mu_1 = 10^3$, $\mu_2 = 10^5$, and $\eta = 0$.

Figures 2, 3, and 4 illustrate the progress of Algorithm 1. As shown by the figures, we arrive to a feasible solutions (less than 10^{-5} per unit constraint violation) after 6, 12 and 5 rounds of Algorithm 1 for cases Illinois 200-bus, South Carolina 500-bus, and PEGASE 1354-bus systems, respectively. Moreover, the resulting optimality gaps are 7.47%, 8.32%, and 10.08%, respectively. Lastly, the resulting commitment keys for the two

Fig. 2: The progress of Algorithm 1 for the case Illinois 200-bus system.

Fig. 3: The progress of Algorithm 1 for the case South Carolina 500-bus system.

Fig. 4: The progress of Algorithm 1 for the case PEGASE 1354-bus system.

TABLE III: Hourly Demand Factor.

Hour	Demand Factor	Hour	Demand Factor
12:00 AM	0.684	12:00 PM	0.946
01:00 AM	0.645	01:00 PM	0.952
02:00 AM	0.620	02:00 PM	0.972
03:00 AM	0.604	03:00 PM	0.999
04:00 AM	0.606	04:00 PM	1.000
05:00 AM	0.627	05:00 PM	0.964
06:00 AM	0.677	06:00 PM	0.961
07:00 AM	0.694	07:00 PM	0.927
08:00 AM	0.730	08:00 PM	0.927
09:00 AM	0.808	09:00 PM	0.909
10:00 AM	0.893	10:00 PM	0.765
11:00 AM	0.922	11:00 PM	0.764

smaller cases are reported in Tables IV and V. The run times of Algorithm 1 for the two smaller cases are less than one hour and for the PEGASE system is less than 4 hours.

VI. Conclusions

In this work, a sequential convex relaxation method is introduced for solving unit commitment with AC transmission constraints. We first, develop a cone programming relaxation to convexity AC unit commitment problems. We then incorporate a penalty term into the objective of the proposed relaxation in order to find feasible solutions for the original non-convex AC unit commitment. The proposed penalized relaxations can be solved sequentially, to find feasible and near-globally optimal points. The experimental results on Matpower's Illinois 200-bus, South Carolina 500-bus, and European 1354-bus power system models demonstrate the effectiveness of the proposed approach in solving challenging instances of AC unit commitment.

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TABLE IV: The resulting commitment key after 7 rounds of Algorithm 1 for the case Illinois 200-bus system.

g	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1																								
2																								
3																								
4																								
5																								
6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
7																								
8																								
9																								
10																								
11																								
12																								
13																								
14																								
15																								
16																								
17																								
18																								
19	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
20	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
21	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
22	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
23																								
24																								
25																								
26																		1	1	1	1	1	1	
27										1	1	1	1	1	1	1	1	1	1					1
28	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
29																								
30																								
31																								
32																								
33																								
34																								
35																								
36																								
37																								
38	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

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TABLE V: The resulting commitment key after 13 rounds of Algorithm 1 for the case South Carolina 500-bus system.

g	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1									1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4										1	1	1	1	1	1	1	1	1	1	1	1	1	1	
5																								
6																								
7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
9	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
11	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
12	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
13	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
14									1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
15	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
16	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
17	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
18											1	1	1	1	1	1	1	1	1	1	1	1		
19														1	1	1	1	1	1					
20	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
21	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
22	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
23	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
24																								
25																								
26																								
27																								
28	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
29	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
30	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
31	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
32	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
33	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
34	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
35																								
36																								
37																								
38																								
39	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
40	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
41	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
42	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
43	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
44	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
45	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
46	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
47																								
48													1	1	1	1	1	1	1					
49	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
50	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
51	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
52	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
53	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
54	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
55	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
56	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

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