

Credit Derivatives and the Cost of Capital*

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Abstract

We examine the effects of credit derivatives on equilibrium debt contracts when investors have heterogeneous beliefs, with particular focus on naked credit default swaps. Although such contracts are zero-sum side bets, their existence can have important consequences. They induce investors who are most optimistic about borrower revenues, and would therefore be natural purchasers of debt, to sell credit protection instead. This diverts capital away from potential borrowers and channels it into collateral to support speculative positions. The resulting shift in the cost of debt can result in an increased likelihood of default and the amplification of rollover risk.

Keywords: Speculative side bets, naked credit default swaps, heterogeneous beliefs, cost of capital.

JEL Codes: D53, D84, G12.

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1 Introduction

Credit default swaps (CDS) are contracts in which one party sells protection to another against a failure (by a third party) to make contractual debt repayments; they are said to be **naked** if the protection buyer does not also hold the underlying security. Naked credit default swaps are therefore two-sided directional bets with payoffs that net to zero: one party is betting on default while the other is betting against, and there is no requirement that either has an insurable interest or hedging motive. The notional value of credit default swap contracts prior to the financial crisis of 2007-08 was estimated to be about ten times as great as that of the underlying bonds (Brunnermeier, 2009). Even with the netting out of multilateral positions and the possibility that some naked protection buyers were facing other exposures that were positively correlated with default, there is little doubt that much of this volume was speculative.¹

In May 2010, Germany became the first major economy to prohibit such contracts outright when it announced a unilateral ban on naked credit default swaps on eurozone debt. Other countries are currently considering following suit. Although it is widely accepted that any such restrictions will have major economic repercussions, there is no consensus on whether these effects will be positive or negative on balance. Some have argued that naked credit default swaps should be banned outright on the grounds that they increase volatility, facilitate bear raids, and make default more likely to occur (Soros, 2009; Buiters, 2009; Münchau, 2010; Portes, 2010). Others have countered that they result in more complete markets, better aggregation of information and beliefs, and increased bond market liquidity, making it easier for debt to be issued by distressed borrowers (Carney, 2009; Jones, 2010; Salmon, 2010).

One argument for the benefits of credit derivatives to borrowers hinges on the fact that they facilitate the separation of funding from exposure to credit risk. This allows borrowers to raise funds even from those who are relatively pessimistic about their ability to repay, since this group of investors can shed credit risk by purchasing protection. Meanwhile, those who are most optimistic about future borrower revenues can sell protection, and thereby expose themselves to credit risk on a scale that would not be possible without derivatives. These effects should shift the terms of financing in favor of borrowers while broadening the range of assets available to investors.² But when protection can be purchased by investors who do not

¹Vause (2010) estimates that the elimination of offsetting positions is largely responsible for the decline in the notional amount of outstanding credit default swaps from a peak of \$60 trillion in late 2007 to about \$30 trillion in early 2010, but even this latter amount is consistent with significant speculative interest. Zuckerman (2010) and Lewis (2010) document some spectacular examples of directional bets using naked purchases of protection.

²Geithner (2007) makes this point as follows: “For borrowers, credit market innovation offers the prospect

hold the underlying bonds, those who are most pessimistic about future borrower revenues can also exploit the implicit leverage that derivatives provide. This can shift interest rates and swap spreads in a manner that is damaging to issuers of debt. Our main purpose in this paper is to develop a framework within which such effects can be explored formally, and to examine the impact of credit derivatives on the cost of debt capital, the likelihood of default, and the choice of projects by borrowers.

We begin with a simple model in which a borrower seeks to meet a fixed funding requirement by selling bonds to a group of investors with heterogeneous beliefs concerning the debtor's future income. This belief heterogeneity is due not simply to differences of information applied to a common prior, but to fundamental differences in the interpretation of common information.³ Within this framework, we consider three regimes. The benchmark case is that in which *no credit derivatives exist*. This is compared with a regime, called **covered CDS**, in which bondholders can hedge their risk by purchasing protection against default, but *investors cannot purchase protection if they do not also hold the bonds*. The third regime, called **naked CDS**, allows for *unrestricted contracts*, including naked credit default swaps. The terms of lending and the likelihood of default are compared across these scenarios.

We show that the presence of credit derivatives is beneficial to borrowers if their use is restricted to hedging only, but harmful to borrowers if their use is unrestricted. That is, when credit derivatives exist but protection may be purchased only by those with an insurable interest, the maximum amount that can be funded is greater and the terms of lending more favorable to the borrower relative to the case in which no credit derivatives exist. This is because the most optimistic investors switch from buying bonds to selling protection, thereby increasing the scale of their exposure to credit risk. Since each unit of protection sold corresponds to a unit of bonds purchased by some other investor, each dollar of collateral set aside by protection sellers corresponds to more than a dollar's worth of expenditure on bonds. This leveraging effect results in a higher bond price and a smaller likelihood of default for any given funding requirement.

This effect is reversed entirely, however, when protection can be purchased without an insurable interest. In this regime, borrowers are unambiguously worse off relative to the benchmark case with no credit derivatives: for any given borrowing requirement, the bond

of increased credit supply; better pricing; and a relaxation of financial constraints. For investors, new credit instruments bring the prospect of broader risk and return opportunities; the ability to diversify portfolios; and increased flexibility. And for lenders, innovations can help free up funding and capital for other uses; they can help improve credit risk and asset/liability management; and they can improve the return on capital and provide new and cheaper funding sources.”

³This appears to be a natural assumption in any analysis of speculation, since a zero sum directional bet requires that both parties to the contract *agree to disagree* in the sense of Aumann (1976).

issue is larger and the price of bonds accordingly lower in equilibrium. Investors who are most optimistic again sell protection, using their cash endowments as collateral. Unlike the case of covered CDS, however, the protection is purchased solely as a bet, and not as an insurance, in equilibrium; the demand for naked credit default swaps by the pessimistic investors raises the cost of protection to the point where the use of credit derivatives as insurance is “crowded out.” Bonds must be therefore purchased by investors who are neither optimistic enough to sell protection, nor pessimistic enough to buy naked credit defaults swaps. In order for the bond market to clear, the terms of financing have to be such that these investors are induced to hold bonds. Since the set of bondholders is less optimistic about borrower revenues than in the baseline case, the resulting bond price is less favorable to the borrower. That is, the potential for pessimists to speculate on default swaps the beneficial effects (from the borrower’s perspective) of the separation of funding from credit risk exposure.⁴

While the baseline model sheds some light on the manner in which the terms of financing can be affected by the availability of credit derivatives, it does not deal with one of the major objections to such contracts: the possibility of *self-fulfilling bear raids*. To address this issue, we extend the model to allow for a mismatch between the maturity of debt and the life of the borrower. This raises the possibility that a borrower who is unable to meet contractual obligations because of a revenue shortfall can roll over the residual debt, thereby deferring payment into the future. Multiple equilibria arise naturally in this setting. If investors are confident that debt can be rolled over in the future they accept lower rates of interest on current lending, which in turn implies reduced future obligations and allows the debt to be rolled over if necessary. But if investors suspect that refinancing may not be possible, they demand greater interest rates on current debt, resulting in larger future obligations and an inability to refinance if the revenue shortfall is large.

As in the baseline model, we compare the case of no credit derivatives with that in which naked credit default swaps can be purchased, and uncover two effects. First, the equilibrium in which investors are pessimistic about the ability of the borrower to roll over debt involves higher interest rates when credit derivatives are in use than when they are not. That is, the terms of financing are worse (from the perspective of the borrower) conditional on the selection of the pessimistic equilibrium. Second, the pessimistic equilibrium exists for a larger range of initial borrowing requirements when credit derivatives exist than when they do not. In other words, there is a range of initial borrowing requirements such that fears about the ability of the borrower to repay debt can be self-fulfilling if and only if naked credit default swaps are permitted. It is in this precise sense that the possibility of self-fulfilling bear raids

⁴We also show that qualitatively similar (but quantitatively different) effects arise if one allows for naked short sales instead of credit derivatives.

can be said to arise when the use of credit derivatives is unrestricted.⁵

The fact that the borrower finds it easier to raise funds and obtain better terms when the use of credit derivatives is restricted does not, of course, immediately imply that such restrictions are efficient. The (utilitarian) efficiency of such restrictions will depend on their effects on the funding decision, specifically, how they affect borrower's project selection and the market perception of the revenues from the selected project.

We address these issues by extending the baseline model to allow for an endogenous determination of future revenues, which depend on the borrower's (unobservable) choice between two available projects. One of these is superior to the other with respect to expected revenues, but the inferior project has greater upside potential, which gives rise to a familiar agency problem in the presence of debt financing. The effects identified earlier suggest that naked credit default swaps make it more difficult for the borrower to finance any given project. The presence of such contracts can therefore prevent the funding of the inferior project under certain conditions, but can also prevent the funding of the superior project under others. In addition, credit derivatives can affect the borrower's choice of project, since a greater debt burden increases incentives to choose a riskier distribution of future payoffs. As a result, naked credit default swaps can induce a shift from the superior to the inferior project, resulting in a worse outcome regardless of the beliefs on the basis of which efficiency is being evaluated.

The empirical evidence on the effects of credit derivatives on credit supply and the terms of lending is ambiguous, but broadly consistent with the model presented here. The most comprehensive study of which we are aware is by Ashcraft and Santos (2009), who explore the effects on the cost of debt for borrowers after the onset of CDS trading by comparing firms with and without active credit derivative markets. They report that contrary to the conventional wisdom on the topic, the onset of CDS trading provides no benefit to the average firm in terms of lower bond spreads or lower rates on bank loans. In fact, for relatively risky and informationally opaque firms, they find a significant and robust negative effect. Ashcraft and Santos interpret these findings by arguing that there are reduced incentives for ex-post monitoring by lenders once credit risk has been shed, which would apply even if protection purchases were made only by those with exposure to default risk. Our interpretation is somewhat different, based specifically on the presence of naked credit default swaps and

⁵We do not consider here the possibility that asset prices may be used to infer the beliefs of potentially informed speculators about the distribution of future borrower revenues, resulting in bear raids of a different kind as speculators enter short positions that lead to the abandonment of projects and loss of firm value (Goldstein and Guembel, 2008). Such feedback effects between prices and realized returns can make market manipulation using credit derivatives potentially profitable. But, as we show here, even in the absence of manipulation a maturity mismatch between loans and revenues can result in increased rollover risk when credit derivatives are unrestricted.

heterogeneous investor beliefs.

Shim and Zhu (2010) conduct a similar exercise using data on a sample of Asian firms and find that under normal credit market conditions, those with traded credit default swaps experience a lower cost of borrowing on new bond issues.⁶ This effect is reversed, however, under the stressed credit market conditions following the failure of Lehman in September 2008. They argue that under normal credit market conditions borrowers benefit from the enhanced information flows and more efficient allocation of risk-bearing that credit derivatives facilitate, along lines suggested by Duffie (2008). The reversal of this effect under stressed conditions could be accounted for by our finding that pessimistic equilibria exist for a broader range of funding requirements in the presence of unrestricted credit derivatives than in their absence.

The importance of considering belief heterogeneity in the analysis of asset pricing was emphasized by Miller (1977), on the grounds that “the very concept of uncertainty implies that reasonable men may differ in their forecasts.” Harris and Raviv (1993) and Kandel and Pearson (1995) suggest that divergence of opinions may result from investors having different economic models which lead them to interpret the same event in different ways. The usefulness of belief heterogeneity as a modeling platform has been recognized by Hong and Stein (2007), who point out that such models “uniquely hold the promise of being able to deliver a comprehensive joint account of stock prices and trading volume.”⁷ More recently, models of debt contracts with collateral and heterogeneous priors have been developed in a general equilibrium framework by Geanakoplos (1997, 2003, 2010). We extend this work in a number of directions. First, we explicitly introduce the funding problem by a third-party borrower, and endogenize the total quantity of the debt issued and its equilibrium likelihood of default. Second, we examine the problem of rolling over debt when the borrower engages in maturity transformation. And third, we examine the efficiency implications of credit derivatives with respect to the borrower’s funding decision. In doing so we draw upon the work of Adrian and Shin (2008) and Holmstrom and Tirole (1997) on agency problems with debt contracts, and that of Calvo (1988), Cole and Kehoe (2000), and Cohen and Portes (2006) on self-fulfilling expectations of default. None of this prior work has explicitly considered the consequences of belief heterogeneity in credit markets, and this constitutes our main contribution to the literature.⁸

⁶Along similar lines, Hirtle (2009) finds that the possibility of hedging by banks results in lower spreads and longer maturities for loans to firms with traded credit derivatives in the US market.

⁷Other examples of asset pricing models based on heterogeneous priors include Harrison and Kreps (1978), Scheinkman and Xiong (2003) and Hong, Scheinkman, and Xiong (2006).

⁸In related research, Bolton and Oehmke (2010) consider the economic consequences of covered (as opposed to naked) credit default swaps. Bondholders who have purchased protection against default have minimal incentives to work with a distressed borrower to restructure debt. While this has often been cited as a source of inefficiency, Bolton and Oehmke argue that the stronger bargaining position of protected

2 Debt Contracts

Consider a borrower who faces an immediate funding requirement of $b > 0$ and chooses to finance this by issuing a quantity $q > 0$ of one-period bonds, each with unit face value. The price of these bonds (to be determined endogenously) is p . The income available to the borrower to pay its creditors when the bonds mature is given by a random variable y . Since its obligation on the maturity date is q , the debtor will repay $\min\{y, q\}$ in the aggregate and each bond will accordingly pay $\min\{y/q, 1\}$. If the realized value of y is at least q , the bondholders are paid in full; otherwise they recover only part of the face value of their bonds.

There exists a unit mass of investors, each endowed with \$1 in cash. Investors have heterogeneous beliefs about the distribution of y : an agent with belief θ perceives that the borrower's future revenue y is distributed according to $G(y|\theta)$ with support $[\eta, 1]$, where $\eta \in [0, 1)$. We adopt the convention that higher values of θ correspond to more optimistic expectations regarding y in the sense of first-order stochastic dominance. Investor beliefs θ are drawn from the interval $[0, 1]$ according to the distribution $F(\theta)$.

We assume throughout that $b > \eta$, which ensures that there is some risk of default even if the borrower were able to meet the funding requirement at a zero interest rate.⁹ Under this assumption, the worst case payoff per bond from the perspective of the lender when the issue size is q is $\eta/q < 1$.

Let

$$\psi(q; \theta) := \int_{\eta}^1 \min \left\{ \frac{y}{q}, 1 \right\} dG(y|\theta) \quad (1)$$

denote the expected payoff per unit bond, as perceived by a bondholder of type θ . Note that ψ is decreasing in q , increasing in θ , and satisfies $\psi(q; \theta) < 1$ for any $q > \eta$. Let

$$\Psi(\theta) := \psi(1, \theta) = \int_{\eta}^1 y dG(y|\theta)$$

denote the expected value of y as perceived by an investor of type θ . Clearly $\Psi(1) < 1$. Note that

$$q\psi(q; \theta) = \int_{\eta}^1 \min \{y, q\} dG(y|\theta) \leq \int_{\eta}^1 y dG(y|\theta) = \Psi(\theta), \quad (2)$$

with strict inequality for $q < 1$ and equality for $q = 1$.

creditors can improve the pledgeability of borrower income, making it easier to raise funds *ex ante*. Since our focus here is on naked credit default swaps, we disregard the issue of bankruptcy reorganization and the effects on fundamentals of covered protection.

⁹Were this not the case, then there would always exist an equilibrium with no possibility of default and no trading in credit derivatives.

Since all investors agree that the borrower’s future income is at most equal to 1, this is also the maximum debt obligation that can be undertaken if there is to be any chance of full repayment. Accordingly, we assume $q \leq 1$. Obligations exceeding this imply certain default *ex ante*. Although it is conceivable that investors would purchase such bonds at a sufficiently low price, we rule it out on practical grounds.

Finally, let $\theta_m \in (0, 1)$ denote a critical investor type such that

$$\Psi(\theta_m) = 1 - F(\theta_m).$$

The left side of the equation is simply the borrower’s expected income—and thus the maximum amount that she can promise to repay—as perceived by type θ_m . The right side is the total cash endowment of agents who are more optimistic than type θ_m . Clearly, this critical value is well-defined, since $\Psi(\theta_m)$ is increasing in θ_m , while $1 - F(\theta_m)$ is decreasing in θ_m and varies from one when $\theta = 0$ to zero when $\theta = 1$. It is intuitive, and will be made precise later, that $\Psi(\theta_m)$ is the maximum amount that the borrower can raise in the benchmark case without credit derivatives.

We now examine the manner in which the terms and limits of borrowing are affected by restrictions on the use of credit default swaps.

2.1 Equilibrium without Credit Derivatives

First consider the case in which no credit default swap contracts are available, so *investors must choose between bonds and cash*. We consider the properties of an equilibrium in which the borrower is able to raise the needed funds, and then identify conditions under which such an equilibrium exists. We assume that agents can convert a dollar of cash into one unit of a divisible consumption good in either period, and choose to maximize their (undiscounted) aggregate consumption.

Consider any equilibrium in which the borrower is able to meet its funding requirement, so that

$$pq \geq b. \tag{3}$$

is satisfied. Each investor can purchase $1/p$ units of the bond with her cash endowment. If the investor has belief θ , her expected payoff when the bond matures is $\psi(q; \theta)/p$. Such an individual will purchase bonds if and only if

$$\psi(q; \theta) \geq p.$$

This expected payoff is monotonic in θ , implying that each investor adopts a cutoff strategy such that she purchases the bond if and only if θ is no less than

$$\hat{\theta}(p, q) := \sup\{\theta \in [0, 1] \mid \psi(q; \theta) \leq p\}, \quad (4)$$

with the convention that $\hat{\theta}(p, q) = 0$ if $\psi(q; \theta) > p$ for all θ . For notational simplicity we suppress the dependence of $\hat{\theta}$ on (p, q) where possible.

Whenever $\hat{\theta} \in (0, 1)$, we must have

$$\psi(q, \hat{\theta}) = p. \quad (5)$$

Observe that $\hat{\theta}$ is continuous and nondecreasing in (p, q) and that $\hat{\theta}(1, q) = 1$ and $\hat{\theta}(0, q) = 0$ for any $q \in (0, 1)$. Since q units of bond are sold, the bond market clearing condition is

$$1 - F(\hat{\theta}(p, q)) = pq. \quad (6)$$

That is, the market clears when the revenue from bond sale (the right side) equals the total cash endowment of those who are more optimistic than the marginal agent (the left side). Since $\hat{\theta}$ is nondecreasing in p , the left side of (6) is nonincreasing in p . The right side is clearly strictly increasing in p . Furthermore, the left side is continuous, is close to one for p sufficiently close to zero and is close to zero for p sufficiently close to one. Hence, for any $q > 0$, there exists a unique price $\hat{p}(q) < 1$ that satisfies (6) and therefore clears the bond market. Moreover, since $\hat{\theta}$ is nondecreasing in (p, q) , it must be the case that $\hat{p}(q)$ is decreasing: a larger bond issue results in a lower price per unit. Note also that $\hat{p}(q)$ is continuous.

Suppose that the borrower issues q units of bond where $p(q)q \geq b$. Then there is clearly an equilibrium in which the bond is sold at $p(q)$ and the borrower indeed meets the requirement. We called this a funding equilibrium. There is another equilibrium in which no bond is sold, supported by an out-of-equilibrium belief that the borrower can never meet the requirement so earns no income, which validates the decision for the investors not to purchase the bond. Throughout the paper, we focus on a funding equilibrium. Given this equilibrium selection, if two different bond issue sizes, q and $q' > q$ can each raise revenue b , the borrower would clearly prefer the smaller issue size since it would result in a lower future debt obligation.

Consequently, the borrower selects the smallest quantity of bonds consistent with a funding equilibrium, assuming it exists. That is, the borrower chooses

$$q^*(b) = \min\{q \in [0, 1] \mid \hat{p}(q)q \geq b\}. \quad (7)$$

Let $p^*(b) := \hat{p}(q^*(b))$ denote the market clearing price corresponding to this optimally chosen issue size, and let $\theta^*(b) := \hat{\theta}(p^*(b), q^*(b))$ denote the belief of the marginal investor at this price and quantity pair.

The following result characterizes the feasible range of funding requirements and the equilibrium bond issue size and price as functions of the funding requirement. (All proofs, unless evident from the discussion in the text, are collected in the appendix.)

Proposition 1. *The maximum revenue that can be raised in equilibrium is $b_m = \Psi(\theta_m)$ at $q = 1$. If $b \leq b_m$, then there exists a unique equilibrium in which the borrower meets the funding requirement of b exactly by issuing $q^*(b)$ bonds, each of which is sold at price $p^*(b)$. The equilibrium issue size rises and the price falls as the funding requirement b rises within the feasible range.*

The manner in which the equilibrium bond contract varies with the funding requirement is illustrated by the following example.

Example 1. *Suppose that $\eta = 0$, $G(y|\theta) = y^{\theta+1}$, and θ is uniformly distributed so $F(\theta) = \theta$ for $\theta \in [0, 1]$. Then for any $q \leq 1$,*

$$\psi(q; \theta) = 1 - \frac{q^{\theta+1}}{\theta + 2}$$

Figure 1 shows how the price and total revenue vary with q . The upper bound for total revenue is $b_m = 0.59$.

2.2 Covered Credit Default Swaps

We now consider equilibrium in the market for debt under the assumption that protection against default can be purchased using credit derivatives, but only with a long position in the underlying bonds. Now agents have four choices: *they can sell protection (using their cash endowment as collateral), they can buy bonds with or without protection, or they can remain in cash.*

Let r denote the (credit default) **swap spread**: the amount paid per unit face value to insure bonds for one period. Suppose an agent of type θ sells protection against default for x units of the bond using as collateral her cash endowment together with the payment received from the protection buyer. The protection seller is obliged to cover the losses of the

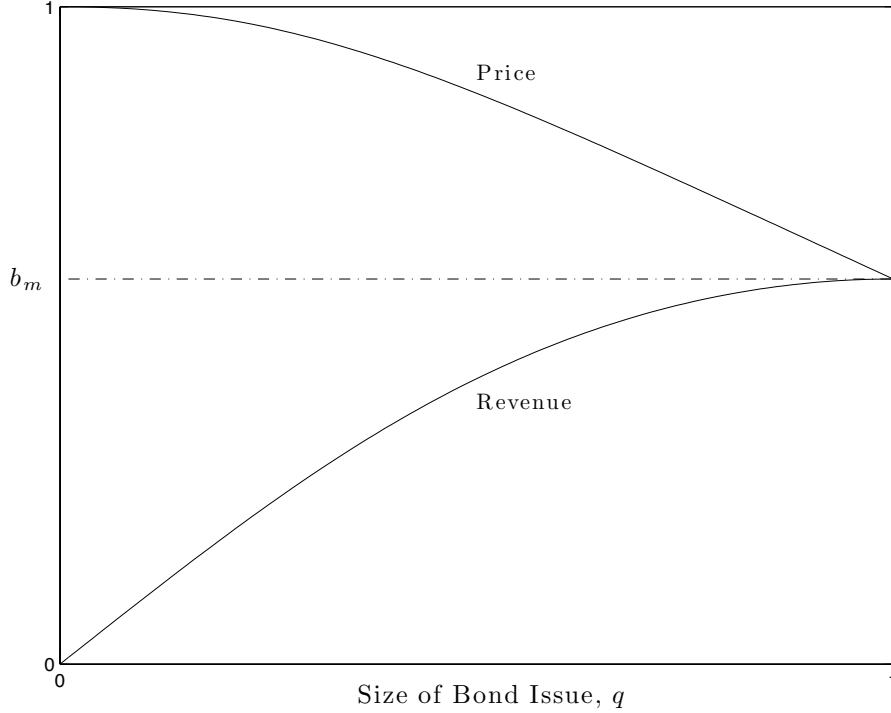


Figure 1 – Bond Price and Total Revenue as Functions of Issue Size

protection buyer, which requires a transfer of

$$\left(1 - \min\left\{\frac{y}{q}, 1\right\}\right) x$$

when the bonds mature. If $y \geq q$ then there is no transfer and the protection seller's payoff (inclusive of the initial cash endowment) is $1 + rx$.

We assume that the protection seller is required to hold enough collateral to cover the worst case loss. That is, we rule out the possibility of default by the protection seller. While this assumption is made primarily for simplicity, it can also be derived as an equilibrium phenomenon in a model with multiple feasible margin requirements.¹⁰ More importantly, our results are not sensitive to the particular margin requirement assumed.

Given our assumption, total collateral $1 + rx$ must be large enough to cover the largest

¹⁰Geanakoplos (2010) shows that when agents choose from a rich set of loan contracts differing in margin requirements, only the contract which precludes default is selected in equilibrium. The same reasoning also applies in our context. Geanakoplos also observes that default on contracts with financial assets serving as collateral is extremely rare under normal conditions, although it is clear that in the absence of government intervention such defaults would have occurred during the recent financial crisis. Our focus here is not on crisis conditions, however. Finally, note that we are allowing for default by the bond issuer (which is common, since the bonds are backed by physical assets), but not the swap counterparty.

possible payment $(1 - \eta/q)x$ to bondholders, and the number of contracts sold must therefore satisfy

$$x = \frac{1}{1 - r - \eta/q}. \quad (8)$$

The protection seller's expected payoff (assuming she sells the maximum amount that collateral requirements will permit) is then

$$u^s(\theta) := 1 + rx - x \int_{\eta}^1 \left(1 - \min \left\{ \frac{y}{q}, 1 \right\} \right) dG(y|\theta),$$

which simplifies, using (1) and (8), to

$$u^s(\theta) = \frac{\psi(q; \theta) - \eta/q}{1 - r - \eta/q}. \quad (9)$$

This payoff expression is revealing of what selling protection accomplishes: the payoff equals the risky part of the return from the bond (numerator) multiplied by the notional amount of swaps he sells ($x = 1/(1 - r - \eta/q)$). In other words, selling a unit of protection is equivalent to buying the risky part of the bond. This feature leads to the separation of funding from credit risk exposure, as discussed further below.

Now consider a protection buyer. Such an individual is required (by the assumptions of this section) to have an insurable interest and is therefore also a bondholder. Assuming that protection is purchased to cover the full extent of exposure, such agents receive the promised payment on their bonds regardless of whether default occurs. Entering this position costs $p+r$ per unit of the bond, so the cash endowment is sufficient to purchase $1/(p+r)$ protected bonds. The agent's payoff from a dollar invested in this manner is

$$u^c(\theta) := \frac{1}{p+r}. \quad (10)$$

Notice that this payoff does not depend on the agent's belief type.

Finally consider an investor of type θ who uses her cash endowment to buy bonds without protection. Her payoff is

$$u^b(\theta) := \frac{1}{p} \psi(q; \theta). \quad (11)$$

We are now in a position to consider the swap spread and bond price (r, p) in any equilibrium in which the funding requirement is met, which we refer to as a funding equilibrium. We begin with the following observation:

Lemma 1. *In any funding equilibrium, $r + p \leq 1$.*

This lemma implies that buying the bond with protection is at least weakly preferable to staying in cash in equilibrium. If the former were strictly preferable, no agent would wish to stay in cash. As will be seen later, this cannot occur if b is not too large. We thus consider a funding equilibrium with $r + p = 1$.

Lemma 2. *Suppose $\eta > 0$. In any funding equilibrium with $r + p = 1$, no agent purchases the bond without protection.*¹¹

The simple intuition is that the agents optimistic enough to purchase the bond without protection find it even more profitable to sell protection, for it allows them to leverage. Thanks to Lemma 2, the set of agents can be partitioned into two groups: those who invest safely (in bonds with protection or cash), and those who sell protection. Given $r + p = 1$, an agent is indifferent between the two strategies if and only if her type θ is such that $\psi(q; \theta) = p$. (Such an agent is also indifferent between either of these two strategies and investing in bonds without protection.) Hence the marginal investor's belief satisfies (5), and may be written as $\hat{\theta}(p, q)$ as in the case of no credit derivatives.

For any given $q > 0$, the market clearing condition for the bond price must satisfy

$$\frac{1 - F(\hat{\theta}(p, q))}{1 - r - \eta/q} = q. \quad (12)$$

The left side of this equation is the aggregate quantity of bonds that are insured by protection sellers, and this must equal the total number issued (since bonds are purchased only with protection). Using $r + p = 1$, this condition simplifies to

$$1 - F(\hat{\theta}(p, q)) = pq - \eta. \quad (13)$$

There is a unique market clearing price $\tilde{p}(q)$ that solves (13). Furthermore, it must be the case that $\tilde{p}(q) > \hat{p}(q)$. To see this, suppose to the contrary that $\tilde{p}(q) \leq \hat{p}(q)$. Then, $\hat{\theta}(\tilde{p}(q), q) \leq \hat{\theta}(\hat{p}(q), q)$, so the left side of (13) is not less than that of the corresponding market clearing condition (6). However, the right side of (13) is strictly greater than that of (6), a contradiction. Hence $\tilde{p}(q) > \hat{p}(q)$. For future reference, let $\tilde{r}(q) = 1 - \tilde{p}(q)$ denote the cost of protection when the bond issue size is q .

As before, the borrower selects the smallest issue size that allows for the funding requirement to be met:

$$q^c(b) := \min\{q \in [0, 1] \mid \tilde{p}(q)q \geq b\},$$

¹¹If $\eta = 0$, then agents become indifferent between selling protection and buying the bond without protection. Hence, the size of the bond purchased without protection is not necessarily zero, but it is not uniquely pinned down. Yet, the terms of borrowing is uniquely pinned down.

provided that this set is nonempty. Define $p^c(b) := \tilde{p}(q^c(b))$ and note that $q^c(b) < q^*(b)$ for any $b < b_m$. In equilibrium, agents with type $\theta < \theta^c(b) := \hat{\theta}(p^c(b), q^c(b))$ either remain in cash, or buy bonds together with protection from those with $\theta > \theta^c(b)$. The equilibrium cost of protection is $r^c(b) := \tilde{r}(q^c(b))$.

For such an equilibrium to exist, the total cash endowment of types below the threshold belief must be sufficient to purchase the bonds along with protection. Since the cost per bond is $r + p = 1$, this requirement may be stated as:

$$F(\theta^c(b)) \geq q^c(b) \tag{14}$$

Conversely, if this condition holds, then in any equilibrium (with funding of b taking place) $r + p = 1$.¹² Condition (14) will hold as long as the borrowing requirement is not too great, and we assume that this is indeed the case. The resulting equilibrium may then be characterized as follows.

Proposition 2. *There exists $b_m^c > b_m$ such that for any $b < b_m^c$, there exists a unique equilibrium in which the borrower meets the funding requirement of b by issuing $q^c(b) < q^*(b)$ bonds, each of which is sold at price $p^c(b) > p^*(b)$. The borrower's default probability is lower when bonds can be insured than when they cannot.*

The availability of (covered) credit default swaps therefore benefits the borrower, allowing for more to be raised on better terms. This happens because the credit risk is held by sellers of protection, and the marginal protection seller is more optimistic than the marginal bond buyer in the case of no credit derivatives. In effect, those who are optimistic about the future income of the borrower can hold a larger number of units of credit risk, since the pessimists are financing the safe component of the loan. In other words, derivatives allow the optimists to leverage without actually borrowing to buy bonds.

In fact, it can be shown that the outcome described here is identical to that which would prevail if pessimists were to lend their cash endowments to optimists on a scale that ensured repayment even in the worst case outcome, and these funds were then used by optimists, in combination with their own endowments, in order to make leveraged purchases of bonds. In other words, if leveraging were possible, the availability of covered credit default swaps would be superfluous.¹³

¹²If $r + p < 1$, then all agents will strictly prefer to buying the bond with protection over staying in cash. The condition then implies that there will be excess demand for the bond.

¹³This result depends on our hypothesis that all agents have the same risk attitude (despite heterogeneity in beliefs). With heterogeneity in risk preferences, derivatives could serve as a risk-sharing device, with relatively risk tolerant individuals selling protection to those who are more risk-averse even if they all share the same beliefs about the distribution of borrower revenues.

2.3 Naked Credit Default Swaps

Now suppose that investors may purchase protection without an insurable interest. The payoffs to those who sell protection, buy bonds with protection, and buy bonds with no protection are as before, given by (9), (10), and (11) respectively. If an agent of type θ purchases naked protection, her payoff is given by

$$u^n(\theta) := \frac{1}{r} \int_{\eta}^1 \left(1 - \min \left\{ \frac{y}{q}, 1 \right\} \right) dG(y|\theta) = \frac{1 - \psi(q; \theta)}{r} \quad (15)$$

We argue first that naked protection will be purchased in equilibrium. Suppose, to the contrary, that it is not. Then equilibrium must be the same as in the case of covered CDS. In that equilibrium $r = 1 - p^c$ and $\psi(q^c; \theta^c) = p^c$. Hence for all $\theta < \theta^c$, we have

$$u^n(\theta) = \frac{1 - \psi(q^c; \theta)}{1 - p^c} > \frac{1 - \psi(q^c; \theta^c)}{1 - p^c} = 1,$$

so these agents all strictly prefer to purchase naked protection to buying bonds with protection, contradicting the hypothesis that they make the latter choice in equilibrium.

Next, we show that allowing for naked credit default swaps affects prices in such a manner as to give rise to a safe investment with a positive interest rate.

Lemma 3. *If $\eta > 0$ then $p + r < 1$ in equilibrium.*

Since $p + r < 1$, investing in bonds with protection yields a strictly positive risk-free return, so no investor holds cash except as collateral to support a speculative position. In addition, no investor chooses bonds with protection either:

Lemma 4. *The set of investors who buy bonds with protection in equilibrium has zero measure.*

Intuitively, anyone who prefers buying bonds with protection to selling protection is pessimistic enough to prefer buying naked protection even more. Likewise, anyone who prefers buying bonds with protection to buying naked protection is optimistic enough to prefer selling protection even more. Hence (almost) no investor purchases credit default swaps for the purpose of insuring bonds: all buyers of protection are speculating on default. The availability of naked credit default swaps alters the price of protection in such a manner as to “crowd out” the demand for default insurance.

While the features described in Lemmas 3 and 4 are interesting, they are a consequence of our assumption that investors are risk-neutral. If investors need to hold some portion of

their wealth in the form of a safe asset (because of risk-aversion or a precautionary demand for money) the premium paid by protected bonds over cash may vanish. Nevertheless, our main results are robust to this form of risk aversion. We later introduce the institutional investors with a form of risk aversion that compels them to invest only in risk free asset, and their presence eliminate the extra return on investing with protection. Yet, our results will remain robust to the introduction of such investors (see Section 4.1).

Since the purchase of bonds with protection strictly dominates holding cash, and is itself dominated by other strategies for (almost) all belief types, the set of investors can be partitioned into three groups based on their equilibrium actions. Specifically, there are two threshold types $0 < \underline{\theta} < \bar{\theta} < 1$ such that agents with $\theta < \underline{\theta}$ purchase naked protection, those with $\theta \in (\underline{\theta}, \bar{\theta})$ purchase bonds without protection, and those with $\theta > \bar{\theta}$ sell protection.

These threshold types, whose dependence on (p, r, q) we suppress for notational simplicity, are determined as follows. Since type $\underline{\theta}$ must be indifferent between purchasing naked protection and purchasing bonds (without protection), we must have

$$\frac{1 - \psi(q; \underline{\theta})}{r} = \frac{\psi(q; \underline{\theta})}{p}. \quad (16)$$

Similarly, since type $\bar{\theta}$ must be indifferent between purchasing bonds and selling protection, we have

$$\frac{\psi(q; \bar{\theta})}{p} = \frac{\psi(q; \bar{\theta}) - \eta/q}{1 - r - \eta/q}. \quad (17)$$

For any given issue size q , conditions (16-17) determine the two threshold types as functions of the bond price and swap spread (p, r) .

In equilibrium, the bond price and swap spread (p, r) must satisfy a pair of market clearing conditions:

$$\frac{F(\bar{\theta}) - F(\underline{\theta})}{p} = q. \quad (18)$$

$$\frac{F(\underline{\theta})}{r} = \frac{1 - F(\bar{\theta})}{1 - r - \eta/q}. \quad (19)$$

Condition (18) states that the demand for the bond must equal its supply, and follows from the fact that only investor types in the range $[\underline{\theta}, \bar{\theta}]$ buy bonds (the remainder either buy or sell naked protection). Condition (19) states that the demand for protection must equal its supply. Substituting (18) into (19) yields:

$$1 - \left(\frac{1 - \eta/q}{r} \right) F(\underline{\theta}) = pq. \quad (20)$$

Since $1 - \eta/q > r$ in equilibrium from (19), the left side of (20) is strictly less than that of (6), the market clearing condition in the case of no credit derivatives. Hence the terms faced by the borrower are worse than in our benchmark case:

Lemma 5. *For any $q \in (0, 1]$, there exists a bond price $\bar{p}(q)$, a swap spread $\bar{r}(q)$, and threshold types $\underline{\theta}$ and $\bar{\theta}$ that together satisfy (16-19). Furthermore, $\bar{p}(q) < \hat{p}(q)$.*

As before, the borrower chooses the smallest feasible bond issue:

$$q^n(b) = \min\{q \in [0, 1] \mid \bar{p}(q)q \geq b\},$$

provided that this set is nonempty (that is, if the funding requirement b can be met in equilibrium). The associated bond price is $p^n(b) := \bar{p}(q^n(b))$, and the equilibrium swap spread is $r^n(b) := \bar{r}(q^n(b))$. Notice that $q^n(b) > q^*(b)$ for any $b < \bar{p}(1)$. In equilibrium, agents of type $\theta < \underline{\theta}^n(b) := \underline{\theta}(p^n(b), r^n(b), q^n(b))$ buy naked protection from those with $\theta > \bar{\theta}^n(b) := \bar{\theta}(p^n(b), r^n(b), q^n(b))$. Investors with $\theta \in (\underline{\theta}^n(b), \bar{\theta}^n(b))$ purchase bonds without protection.

In comparing the swap spread across the two regimes with credit derivatives, note that for $\eta = 0$, $\bar{p}(q) + \bar{r}(q) = 1$ and so $\bar{r}(q) > \tilde{r}(q)$. Continuity of equilibrium magnitudes in η then implies that for η sufficiently small, $\bar{r}(q) > \tilde{r}(q)$. That is, the cost of protection is greater when naked credit default swaps are allowed than when they are not, holding constant borrower characteristics, provided that the worst case payment to bondholders is sufficiently low.

The following result characterizes equilibrium outcomes with naked credit default swaps in comparison with the cases considered previously.

Proposition 3. *The maximum revenue that can be raised in equilibrium with naked credit default swaps $\bar{p}(1) < b_m$. If $b \leq \bar{p}(1)$, then there is a unique equilibrium in which the borrower issues $q^n(b) > q^*(b)$ bonds at price $p^n(b) < p^*(b)$. For sufficiently small η , the equilibrium swap spread is $r^n(b) > r^c(b)$, the swap spread under covered CDS. The probability of borrower default is higher relative to the case of no credit protection or covered credit default swaps.*

This result establishes that for any given issue size, bond prices (and hence also total revenues) are lower when credit derivatives are unrestricted than when they are absent or restricted to an insurance function. Figure 2 illustrates, using the same specifications as in Example 1.

Not only are the terms on which financing is available worse when protection can be purchased without an insurable interest, but the range of deficits that can be financed is

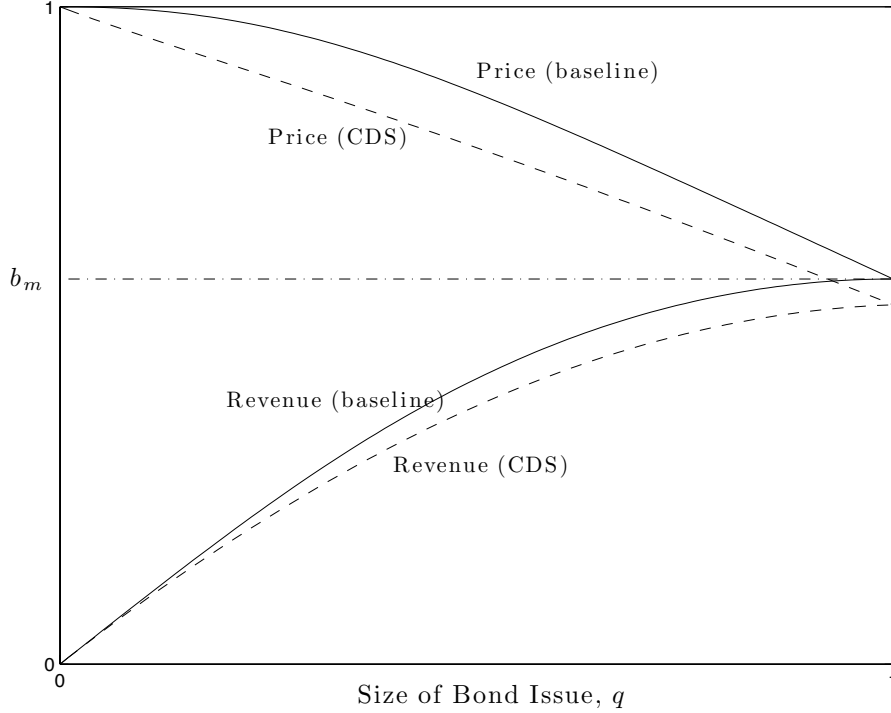


Figure 2 – Comparison of the baseline model with unrestricted protection

itself smaller. This happens because the most optimistic investors prefer to sell protection (when they have an option to do so) rather than to buy bonds, which ties up their capital in the form of collateral and makes the marginal bond buyer more pessimistic than would be the case without credit derivatives. As a result, any borrowing requirement that is feasible without credit derivatives either becomes infeasible, or requires a larger bond issue (and hence a higher interest rate).

3 Rollover Risk

One of the key features of debt contracts is that they frequently involve maturity transformation; the term of the loan is too short to enable full repayment without refinancing. This means that the terms of current financing depend on expectations regarding the ability of the borrower to roll over debt when it comes due. Multiple equilibria arise naturally in this setting, and we are interested in the manner in which the use of credit derivatives affects the cost of debt and the set of equilibria.

Consider three periods $T = 0, 1, 2$. In period $T = 0$, the borrower faces a borrowing requirement $b_0 > 0$, and proposes to finance this by issuing q_0 bonds each with unit face

value. The bond price p_0 is determined by a competitive market in period $T = 0$. In period $T = 1$, the borrower's revenue y_1 is realized. If $y_1 \geq q_0$, then all bonds are paid in full and no refinancing is necessary. If $y_1 < q_0$, then the borrower must issue a quantity q_1 of bonds with unit face value to cover the shortfall of $q_0 - y_1$. Again a competitive market at period $T = 1$ sets the price p_1 of the bonds. In period $T = 2$, the revenue y_2 is realized, and the bond holders are paid $\min\{q_1, y_2\}$ in the aggregate.

To focus on the main idea, we make the simplifying assumption that the borrower's ability to repay is binary: $y_t \in \{0, 1\}$, for $t = 1, 2$. In period $T = 0$, a type θ -agent believes that $y_1 = 1$ with probability θ . As before, θ is drawn from the distribution $F(\theta)$. In period $T = 1$, there is no belief heterogeneity about the distribution of y_2 ; all investors believe that $y_2 = 1$ with probability λ (and $y_2 = 0$ with probability $1 - \lambda$). This common belief assumption plays no essential role; its only purpose is to simplify the analysis. In particular, it implies that in period $T = 1$, there will be no market for credit derivatives. The tree of uncertainty, as faced by an investor of type θ , is shown in Figure 3.

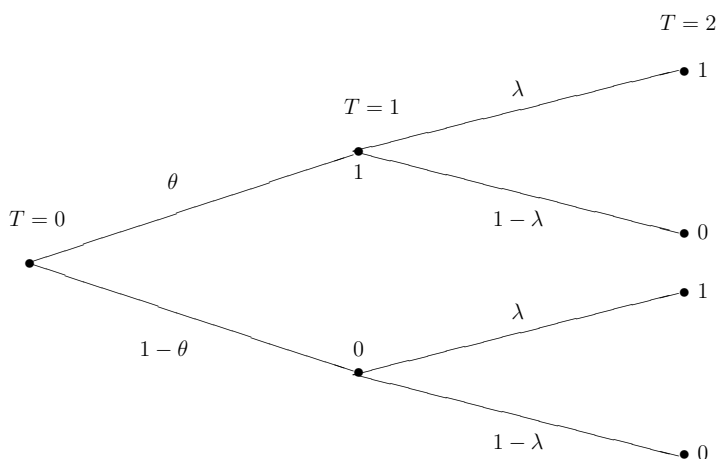


Figure 3 – The tree of uncertainty faced by an investor of type θ

We assume, as before, that the borrower cannot take on greater debt obligations than could be honored even in the highest revenue state. That is, we assume $q_t \leq 1$, for $t = 1, 2$. As we show below, this constraint will not be binding in equilibrium as long as the initial borrowing requirement b_0 is not too large. We also assume that the borrower cannot complete the project if the outstanding debt cannot be rolled over when the low income state is realized at $T = 1$. This rules out a partial rollover of debt, in which earlier investors are not paid in full but the firm is nevertheless able to raise new funds. Such a partial rollover of debt is

rare in practice, given that default entails high fixed costs, a loss of reputation, and severely restricted access to capital markets.

Before proceeding, it is important to consider why the borrower finances via a sequence of short-term obligations rather than a long-term bond that matures at $T = 2$ and therefore avoids rollover risk. There are a number of reasons why firms engage in such maturity transformation, among which is the inability to credibly pledge income that is realized in the interim stage $T = 1$. Income earned well in advance of the maturity date is difficult to monitor, and it is easier for the borrower to divert such resources away from creditors without raising suspicion.¹⁴ This inability to pledge near-term income to service long-term debt implies that the terms available for long-term financing are not generally favorable relative to a sequence of shorter maturity debt. For instance, if the borrower can fully divert her income at $T = 1$ in the presence of a long term contract, the loan is effectively backed only by $T = 2$ income. Such a contract is dominated by the sequence of short term loans that we consider.¹⁵

3.1 Equilibrium without Credit Derivatives

We start by characterizing the set of equilibria without credit derivatives, beginning our analysis at period $T = 1$. If $y_1 = 1$ the initial debt is fully repaid. If $y_1 = 0$, the borrower owes q_0 and must borrow this amount to avoid default. Suppose this is done by issuing an amount q_1 of new one period bonds, each with unit face value. Recall that there is common belief on the part of investors that each such bond will have an expected payoff of precisely λ at $T = 2$. Hence the equilibrium bond price must satisfy $p_1 = \lambda$, and the borrower can therefore borrow $p_1 q_1 = \lambda q_1$. Since $q_1 \leq 1$, the debt can be rolled over if and only if $q_0 \leq \lambda$. In particular, if $q_1 > \lambda$, then no refinancing occurs at all, and bondholders are paid nothing.

Now consider period $T = 0$. If $b_0 \leq \lambda$, then there exists a trivial equilibrium in which the borrower issues $q_0 = b_0$ at a price $p_0 = 1$. This bond is risk-free (since the debt is certain to be rolled over if necessary) and all investors are therefore willing to pay the face value for each unit regardless of their beliefs.

If $b_0 > \lambda$, then no such equilibrium exists since a debt this large cannot be refinanced if $y_1 = 0$. Hence any bonds sold in the initial period will be repaid if and only if $y_1 = 1$. If

¹⁴For instance, such diversions are unlikely to be regarded as fraudulent by a bankruptcy court.

¹⁵More precisely, a long-term bond with unit face value will be paid in full with probability λ and will pay nothing with probability $1 - \lambda$. Thus the borrower can finance only $b \leq \lambda$, and must issue $q = b/\lambda$ bonds, each of which will be sold at price λ . As we show below, a sequence of short term loans can allow for better terms of financing for the borrower, and for a larger funding requirement to be met.

an agent of type θ spends her unit cash endowment on purchasing bonds, she will expect to earn θ/p_0 . Since this strategy is optimal only when this payoff is no less than a dollar, the agent will purchase bonds if and only if

$$\theta > \hat{\theta} = p_0. \quad (21)$$

Given that the borrower needs to raise $p_0 q_0 = b_0$, the market clearing condition $1 - F(\hat{\theta}) = b_0$ may be written

$$1 - F\left(\frac{b_0}{q_0}\right) = b_0. \quad (22)$$

There is a unique bond issue size that satisfies this, given by

$$\hat{q}_0(b_0) = \frac{b_0}{F^{-1}(1 - b_0)}.$$

Note that $\hat{q}_0(b_0) > b_0$. This means that even when $b_0 \leq \lambda$ (so an equilibrium with $q_0 = b_0$ exists), there can be a second equilibrium if $\hat{q}_0(b_0) > \lambda \geq b_0$ in which investors have pessimistic expectations regarding the borrower's ability to refinance in the low income state. This pessimistic equilibrium has a lower bond price and requires the borrower to incur a larger debt obligation in order to meet its borrowing requirement. Define $\hat{b}_0 := \hat{q}_0^{-1}(\lambda)$. That is, \hat{b}_0 is a critical borrowing requirement that satisfies

$$1 - F\left(\frac{\hat{b}_0}{\lambda}\right) = \hat{b}_0.$$

Clearly, $\hat{b}_0 < \lambda$. The following result identifies a range of values for the initial borrowing requirement such that a multiplicity of equilibria exists.

Proposition 4. *If $b_0 > \lambda$, there exists a unique equilibrium in which the borrower issues $\hat{q}_0(b_0)$ bonds with unit face value at price $p_0 = b_0/\hat{q}_0(b_0)$. Default occurs if and only if $y_1 = 0$. If $b_0 < \hat{b}_0$, there is a unique equilibrium in which the borrower issues $q_0 = b_0$ bonds with unit face value and unit price, never defaults on these bonds, and rolls over the debt if $y_1 = 0$. If $\hat{b}_0 \leq b_0 \leq \lambda$, then both equilibria exist.*

If the initial borrowing requirement is sufficiently low, then investors fully expect that debt will be successfully rolled over in the low income state, and there is a unique equilibrium with zero interest. If the initial borrowing requirement is sufficiently high, there is again a unique equilibrium but one in which default is expected in the low income state, and the interest rate is correspondingly higher. For intermediate values of the initial borrowing requirement, both equilibria can co-exist. If investors believe that the borrower will be

unable to roll over debt in the low income state, they will require higher interest rates as compensation for this risk, and the greater debt burden that results will cause these beliefs to be correct. On the other hand, if they expect that refinancing will be available at either state, this too will be self-fulfilling since the debt burden will be correspondingly lower.

3.2 Credit Default Swaps

Now consider the effects of allowing for naked credit default swaps in this environment. The market for these contracts never materializes in period $T = 1$, and the same is true in period $T = 0$ if investors are confident that the borrower will be able to raise b_0 by issuing $q_0 \leq \lambda$ bonds. These bonds never default, for the debt can be rolled over even in the low income state, and this is known to all agents. But, as in the case without credit derivatives, there can be another equilibrium in which investors are not confident about the borrower's ability to roll over debt in the low income state.

If default protection can be purchased without holding the underlying bond, then, as in the one period model considered earlier, optimistic agents will sell protection or buy bonds without protection in equilibrium, while pessimistic agents will buy naked credit default swaps. As before, the swap spread and bond price must satisfy $p_0 + r_0 = 1$ (recall that the current model corresponds to the case of $\eta = 0$.) Our earlier analysis implies that agents with $\theta > \hat{\theta}$ buy bonds without protection or sell protection, while each agent with $\theta < \hat{\theta}$ purchases protection on bonds with face value $1/(1 - p_0)$. Here the threshold type $\hat{\theta} = p_0$ as in (21).

In equilibrium we must have

$$\frac{1}{p_0} \left(1 - F(\hat{\theta})\right) = q_0 + \left(\frac{1}{1 - p_0}\right) F(\hat{\theta}).$$

Collecting terms and using $\hat{\theta} = p_0$ and $p_0 q_0 = b_0$, we get

$$1 - \left(\frac{q_0}{q_0 - b_0}\right) F\left(\frac{b_0}{q_0}\right) = b_0. \quad (23)$$

One can check that the left side is increasing in q_0 for $q_0 > b_0$, so there is a unique value $\bar{q}_0(b_0) > b_0$ that satisfies the equation. Since the left side of (23) is smaller than that of (22), it also follows that $\bar{q}_0(b_0) > \hat{q}_0(b_0)$. The market clearing bond price is then $\bar{p}_0 = b_0/\bar{q}_0(b_0) < \hat{p}_0$.

Define $\bar{b}_0 := \bar{q}_0^{-1}(\lambda)$. Then, $\bar{b}_0 < \hat{b}_0$. The following result identifies the equilibrium set when naked credit default swaps are permitted.

Proposition 5. *If $b_0 > \lambda$, there exists a unique equilibrium in which the borrower issues $\bar{q}_0(b_0)$ bonds with unit face value at price $\bar{p}_0 < \hat{p}_0$. Default occurs if and only if $y_1 = 0$. If $b_0 < \bar{b}_0$, there is a unique equilibrium in which the borrower issues $q_0 = b_0$ bonds with unit face value and unit price, never defaults on these bonds, and rolls over its debt if $y_1 = 0$. If $\bar{b}_0 \leq b_0 \leq \lambda$, then both equilibria exist.*

The comparison with the case without credit derivatives is instructive. If $b_0 \in (\bar{b}_0, \hat{b}_0)$ then, in the absence of credit derivatives, the no default outcome is the unique equilibrium. Allowing for naked credit default swaps introduces an additional equilibrium in which the borrower defaults in the low income state. Furthermore, even if multiple equilibria exist under both regimes, the terms of financing are worse for the borrower at the equilibrium with the higher interest rate in the presence of naked credit default swaps. The following example illustrates.

Example 2. *Suppose that $\lambda = 0.40$ and $F(\theta) = \theta^2$. In this case $\hat{b}_0 = 0.33$ and $\bar{b}_0 = 0.23$. The range of initial debt levels for which multiple equilibria exist with naked credit default swaps is $[0.23, 0.40]$, but when no such contracts are allowed, this range is $[0.33, 0.40]$. Furthermore, when the more pessimistic equilibrium exists under both regimes, it is more punitive in the presence of naked credit default swaps.*

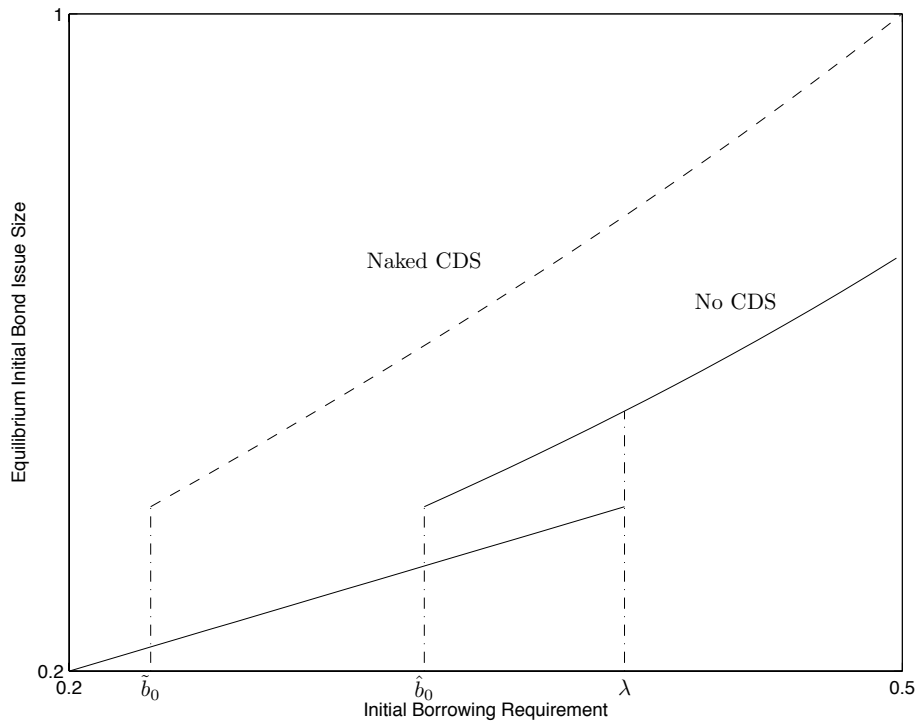


Figure 4 – Equilibrium Bond Issues with and without Naked CDS

The example is illustrated in Figure 4. As is clear from the figure, the presence of naked credit default swaps has two effects. First, it expands the range of initial borrowing requirements for which an equilibrium with default in the low income state exists. And second, conditional on such an equilibrium being selected, interest rates are higher when naked credit default swaps are permitted than when they are not. The latter effect is similar to that identified in the one-period version of the model. And the former effect confirms that self-fulfilling pessimism about the borrower's ability to roll over debt is more likely to arise when naked credit default swaps are permitted than when they are not.

4 Extensions

We now consider two directions in which the results of our static model (without rollover risk) may be extended, by allowing for a demand for safe assets by institutional or risk-averse investors, and for short sales.

4.1 Risk Aversion and Institutional Investors

One interesting feature of equilibrium with naked credit default swaps is the emergence of a positive risk-free interest rate (via purchase of bonds with protection) despite the absence of discounting. However, no investors choose this strategy, preferring instead to buy bonds without protection, or to buy or sell naked protection. This is due to our assumption of investor risk-neutrality.

To allow for the possibility of investor risk-aversion in a simple manner, suppose that in addition to the unit cash endowments that they invest in potentially risky securities, investors also have some wealth in bank deposits (or money market funds), and that these must be held by the financial intermediaries (or fund managers) in the form of safe assets: either cash or bonds with protection. We assume that these institutional investors can only invest in risk free assets and that they hold sufficiently a large cash endowment to purchase the entire bond issue. These investors strictly prefer to purchase bonds with protection if $r + p < 1$, but are indifferent between cash and protected bonds if $r + p = 1$.

The presence of institutional investors has no impact on equilibrium prices in either the case of no credit derivatives or the case of covered credit default swaps. In the former regime, since bonds are risky and protection is unavailable, institutional investors are forced to hold cash. Hence, the equilibrium identified in Proposition 1 continues to hold. In the covered protection regime, again, equilibrium prices remain the same since the condition $p + r = 1$ is

satisfied. The identity of the bondholders may change, however, as agents more optimistic than the threshold belief type $\tilde{\theta}$ sell protection to those who are more pessimistic and/or to institutional investors. Both the group of pessimistic investors as well as the financial intermediaries are indifferent between bonds with protection and cash.

In the regime with naked credit default swaps, however, the presence of institutional investors makes a difference if $\eta > 0$. Extending the reasoning underlying Lemma 5, it can be shown that the entire bond issue must be purchased by institutional investors, with protection purchased from the most optimistic agents. To see this, suppose, by way of contradiction, that some positive measure of (non-institutional) investors also purchase bonds. This implies $r + p < 1$, otherwise all investors would either strictly prefer to sell protection or buy protection, as shown in the reasoning behind Lemma 5. But if $r + p < 1$, then the institutional investors would strictly prefer to purchase the bond with protection to holding cash, a contradiction. We therefore conclude that, in equilibrium, the institutional investors must purchase the entire bond issue with protection, and the arbitrage condition $r + p = 1$ must be satisfied. This in turn means that the set of all other agents may be partitioned into buyers and sellers of naked protection. Specifically, there is a single belief type $\bar{\theta}$ such that agents with $\theta > \bar{\theta}$ sell protection and those with $\theta < \bar{\theta}$ buy protection. Since $r = 1 - p$, we also obtain $\bar{\theta}(p, q) = \hat{\theta}(p, q)$.¹⁶

The most optimistic investors sell protection to the most pessimistic and also to the institutional investors constrained to hold safe assets. Since the institutional investors require one unit of protection for each of the q units of bonds purchased, the equilibrium condition becomes:

$$\frac{F(\hat{\theta})}{r} = \frac{1 - F(\hat{\theta})}{1 - r - \eta/q} - q. \quad (24)$$

Condition (24) states that the demand for naked protection must equal the supply of protection in excess of the q units sold to institutional investors. Rewriting (24) using $r + p = 1$, we get

$$1 - \left(\frac{1 - \eta/q}{1 - p} \right) F(\hat{\theta}) = pq - \eta. \quad (25)$$

Comparing (25) with (13), the left side of the former is less than that of the latter (since $pq > \eta$ in equilibrium), while the expressions of the right side are identical. Hence $\bar{p}(q) < \tilde{p}(q)$. That is, the equilibrium bond price is lower in the regime with naked protection relative to

¹⁶That the type $\bar{\theta}$ must be indifferent between buying and selling protection implies that

$$\frac{1 - \psi(q; \bar{\theta})}{r} = \frac{\psi(q; \bar{\theta}) - \eta/q}{1 - r - \eta/q}$$

which simplifies, using $r = 1 - p$, to $\psi(q; \bar{\theta}) = p$, as in (5).

that with covered protection only, for any given bond issue size.

Next, comparing (25) with (6), we see that as η goes to zero, the right sides of the two equations converge, while the left side of the former remains strictly above that of the latter. For η sufficiently small, therefore, the bond price in the naked protection regime is lower than that in the regime with no credit derivatives. Even with institutional investors induced to purchase the entire bond issue along with the protection sold by the optimists, the presence of naked credit default swaps raises the cost of borrowing. When η is large, however, we cannot rule out the possibility that this effect is reversed.

These results may be summarized as follows.

Proposition 6. *In the presence of institutional investors constrained to hold safe assets, the cost of debt is greater when credit derivatives are unrestricted than when they are restricted to serve an insurance function. When η is sufficiently small, the terms are worse for the borrower under unrestricted credit derivatives than in the absence of credit derivatives.*

Hence the presence of institutional investors constrained to hold safe assets does not substantially alter the main insights obtained earlier, although it does have quantitative effects on equilibrium prices in some cases.

4.2 Short Sales

It is often argued that the availability of credit default swaps acts as a substitute for short sales in the bond market. A traditional short sale requires an investor to borrow (and replace upon demand) the securities that are sold. A naked short sale, in contrast, involves the creation of a synthetic bond that replicates the payments of the underlying security. The buyer pays the sale price of the bond and receives in exchange the promised stream of payments; the seller must post collateral to ensure contract fulfillment. These two forms of short sale are equivalent in the environment considered here, since margin requirements preclude default and investors do not discount future income.

As in the case of naked credit default swaps, short sales constitute speculative side bets that enable investors with opposing views to enter positions as counterparties. Pessimists can sell short to bet on a bond default, just as they can by buying naked protection. Although the qualitative effects of these two forms of speculation on the cost of debt are similar, we show below that their quantitative effects are not the same. The difference arises because the two types of contract entail different degrees of implicit leverage.

Consider our benchmark case of a bond market without credit derivatives, but suppose

that bonds can be sold short by investors using their cash endowment to satisfy margin requirements. In this case there are three options available to investors: buying bonds (from issuers or short sellers), shorting bonds, or remaining in cash.

If an investor shorts z units of the bond, her cash position becomes $1 + zp$, where p is the bond price as before. The worst case outcome for the short seller is that the bond pays its face value in full, in which case she will be required to pay out z . Assuming, as before, that collateral requirements are such as to preclude default by the short seller, the largest position size is then given by

$$z = \frac{1}{1 - p}.$$

The payoff to an agent of type θ who enters this position is then

$$u^h(\theta) := 1 + \frac{p - \psi(q; \theta)}{1 - p} = \frac{1 - \psi(q; \theta)}{1 - p}. \quad (26)$$

The expected payoff $u^b(\theta)$ to a bond buyer of type θ is given by (11) as before. A comparison of (11) and (26) reveals that an investor who is indifferent between buying and shorting bonds must also be indifferent between either of these actions and remaining in cash. The marginal type indifferent between the two options satisfies $\hat{\theta}(p, q)$, as before.¹⁷ It is clear that investors who are more pessimistic than this will short bonds, while those more optimistic will buy bonds (either from the original issuer or from short sellers).

For the bond market to clear, the total supply (by the issuer and short sellers) equals the demand from purchasers:

$$q + \frac{F(\hat{\theta})}{1 - p} = \frac{1 - F(\hat{\theta})}{p},$$

which simplifies to

$$1 - \left(\frac{1}{1 - p} \right) F(\hat{\theta}) = pq. \quad (27)$$

Since $p < 1$ in equilibrium, the left side of (27) is strictly less than that of (6), while their respective right sides are identical. This means that the equilibrium bond price under short sales is $\check{p}(q) < \hat{p}(q)$, the bond price in the absence of credit derivatives. Allowing for short sales worsens the terms faced by the borrower, as might be expected.

A comparison of (27) with (20), the market clearing condition under naked credit default swaps, reveals that the two conditions are identical when $\eta = 0$, but not otherwise. The following example shows that for $\eta > 0$, the cost of debt can be greater under unrestricted derivatives than under short sales.

¹⁷The marginal type satisfies $\frac{1 - \psi(q; \theta)}{1 - p} = 1$, or $\psi(q; \theta) = p$, as in (5).

Example 3. Suppose that θ is uniformly distributed on $[0, 1]$, and

$$G(y|\theta) = \left(\frac{y-\eta}{1-\eta}\right)^{1+\theta}.$$

Then for any $q \in [\eta, 1]$

$$\psi(q; \theta) = \int_{\eta}^q \frac{y}{q} dG(y|\theta) + \int_q^1 dG(y|\theta) = 1 - \frac{(q-\eta)^{2+\theta}}{q(2+\theta)(1-\eta)^{1+\theta}}$$

If $\eta = 0.1$ and $q = 0.6$, then equilibrium with short sales is given by $\check{p} = 0.79$, with threshold type $\check{\theta} = 0.11$. The equilibrium with naked credit default swaps is given by $\bar{p} = 0.78$, $\bar{r} = 0.20$, and thresholds $\underline{\theta} = 0.13$ and $\bar{\theta} = 0.60$.

This example establishes that naked credit default swaps and short sales do not have identical effects on the cost of debt.

Finally, consider the comparison between short sales and naked credit default swaps in the presence of institutional investors constrained to hold safe assets. The presence of such investors has no effect on the equilibrium under short sales, since cash is the only safe asset available to them. Comparing (27) to (25), the following result is immediate.

Proposition 7. *In the presence of institutional investors constrained to hold safe assets, the cost of debt is greater under short sales than under unrestricted credit derivatives if $\eta > 0$, and identical in the two cases if $\eta = 0$.*

Hence short sales and naked credit default swaps are by no means equivalent in terms of their effects on the cost of debt. In the presence of institutional investors constrained to hold safe assets, short sales are worse for the borrower, though the reverse can be true in the absence of such investors.

5 Welfare

Any analysis of welfare in the context of heterogeneous priors faces some conceptual difficulties. In particular, one needs to specify the beliefs on the basis of which the welfare of individuals is to be evaluated, given that these individuals themselves have different and incompatible beliefs. For instance, those who willingly take opposite sides of a zero-sum bet do so because they each believe that their positions have positive expected returns, so the availability of such contracts appears to benefit both parties. But they cannot both benefit from the perspective of a third party, regardless of the beliefs of the latter.

We conduct our analysis of welfare on the basis of a single, exogenously given prior, which we refer to as the policy-maker’s belief. Regardless of what this belief happens to be, the payoffs from the trading of naked credit default swaps sums to zero, so the analysis of (utilitarian) welfare boils down to an assessment of the joint surplus generated by the borrower and the set of investors, that is, the efficiency of the funding decision.

Recall that the presence of naked credit default swaps shifts the terms of financing against borrowers and reduces the range of borrowing requirements that can be met. As a result, some projects that would have been funded in the absence of credit derivatives now fail to attract funding. The efficiency effects of this depend on whether, from the perspective of the policy maker, these projects have positive net present value. A policy maker who considers the population of investors to be excessively optimistic, and therefore willing to fund too great a range of projects, will consider naked credit defaults to be efficiency enhancing. Similarly, if investors are believed to be excessively pessimistic, the range of funded projects will be too small, and this can be corrected by restricting the use of credit derivatives to an insurance function. More generally, policy makers may choose to restrict the use of credit derivatives at precisely those times when they feel that the investor population is too pessimistic about future returns, resulting in counter-cyclical restrictions on the use of credit derivatives.

A more subtle efficiency effect can arise when the borrower is faced with a choice of projects. Suppose that there are two projects with different distributions of returns for each prior belief $\theta \in [0, 1]$. Both projects have the same funding requirement b , but one is unambiguously superior to the other in the sense that it has a higher expected return regardless of θ . Specifically, suppose that the superior project has a return $y \in [0, 1]$ drawn from a distribution $G(y | \theta)$, as assumed in our earlier analysis. The inferior project has a return drawn from the same support according to a distribution $B(\cdot | \theta)$. Let $\Psi_G(\theta)$ and $\Psi_B(\theta)$ respectively denote their expected returns conditional on θ . We assume that

$$\Psi_G(\theta) > \Psi_B(\theta) \tag{28}$$

for all $\theta \in [0, 1]$; this is the sense in which the “good” project is superior. As before, investors have heterogeneous beliefs given by $\theta \in [0, 1]$ drawn from the distribution $F(\theta)$ which admits positive density $f(\theta)$ for $\theta \in (0, 1)$. Higher values of θ correspond to more optimistic beliefs about project returns in the sense of first-order stochastic dominance, and this applies to both projects.

The prior belief of the firm (on the basis of which its project choice is made) is given by $\theta_0 \in (0, 1)$, and is commonly known among investors. Although the inferior project has a lower expected return, it carries greater upside potential in the following sense: there exists

$y^* \in (0, 1)$ such that

$$(G(y | \theta_0) - B(y | \theta_0))(y - y^*) \geq 0, \quad (29)$$

with strict inequality for $y \neq y^*$. The highest realizations of y are therefore more likely to occur if the *inferior* project is chosen. This creates a potential agency problem when the firm's project selection is unobservable to investors, which we assume to be the case. As we show below, this agency problem limits the amount of income that the firm can pledge to investors, and thus its capacity to issue debt.¹⁸

Suppose the firm issues q units of debt with unit face value. The expected payment to bondholders when the debt matures (from the perspective of the firm) is $q\psi_j(q; \theta_0)$ if project $j = G, B$ is expected to be chosen, where $\psi_j(q; \theta_0)$ is as defined in (1), but with distribution function $j = G, B$. Assume (for convenience) that a firm that is indifferent between the two projects will choose the superior one. Then, for the firm to have an incentive to choose the better project, it is necessary that

$$\Psi_G(\theta_0) - q\psi_G(q; \theta_0) \geq \Psi_B(\theta_0) - q\psi_B(q; \theta_0). \quad (30)$$

This constraint is satisfied if and only if the size of the bond issue is not too great:

Lemma 6. *There exists $\hat{q} \in (0, y^*)$ such that (30) holds if and only if $q \leq \hat{q}$.*

This result follows from the familiar effect on project choice of the asymmetry of the debt contract. Creditors bear the losses from low revenue realizations, but do not share in the gains from unusually high revenue realizations. The higher the debt burden, the greater is the incentive for the firm to choose projects with higher upside potential, even if they have lower expected returns. Since unrestricted credit derivatives force the firm to issue more debt to meet the same borrowing requirement, the presence of naked credit default swaps can induce the firm to adopt the riskier and less rewarding project.

These effects of credit derivatives on funding efficiency and project choice can be stated more precisely. To do so, it is useful to classify the distribution of investor beliefs into four categories. We say that investors are *highly optimistic* if the funding requirement can be met even when the inferior project is selected and credit derivatives are unrestricted.¹⁹

¹⁸This particular moral hazard problem has been studied by Adrian and Shin (2008), and is also closely related to Holmstrom and Tirole (1997). These authors postulate that the bad project involves a private benefit for management, and assume that (28) holds with the private benefit included. The greater upside potential of the bad project, assumed in (29), makes the private benefit an unnecessary part of the story. The contractual implications of these two approaches are essentially the same: they both limit the pledgeability of income and thus limit the size of debt obligations.

¹⁹The relative degree of investor optimism corresponds to the skewness of the distribution $F(\theta)$ of investor beliefs. If the distribution is skewed toward high value of θ , then there are a large number of optimistic investors, making easy for any project to be funded.

They are *moderately optimistic* if the inferior project can be funded in the absence of credit derivatives but not in their presence. They are *neutral* if, in the absence of credit derivatives, the superior project can be funded but the inferior project cannot. And if neither project can be funded in either policy regime, we say that they are *pessimistic*.

Project choice depends also on the severity of agency problems. Recall that, conditional on the superior project being selected, equilibrium outcomes are (q^*, p^*) in the absence of credit derivatives, and (q^n, p^n) in the presence of naked credit default swaps.²⁰ Conditional on the inferior project being chosen, let (q_B^*, p_B^*) denote the equilibrium outcomes in the absence of credit derivatives, and (q_B^n, p_B^n) the outcomes in the presence of naked credit default swaps. The severity of agency problems can be expressed as follows. If $\hat{q} \geq q^n$, then the incentive condition is *always satisfied*, regardless of the policy regime. In this case the superior project will be selected as long as the funding requirement can be met. If $\hat{q} < q^*$, the incentive condition is *never satisfied*. In this case the inferior project will be selected if the funding requirement can be met, regardless of the policy regime. Finally if $q^* \leq \hat{q} < q^n$ we say that the incentive condition is *conditionally satisfied*. In this case, the superior project will be chosen (provided that it can be funded) if and only if naked credit default swaps are not available. If credit derivatives are unrestricted, then the inferior project will be selected provided that it can be funded.

The combined effect of credit derivatives on funding efficiency and project choice is as follows.

Proposition 8. *Allowing for naked credit default swaps results in*

- *a failure to fund the superior project if investors are neutral or moderately optimistic and the incentive constraint is conditionally satisfied,*
- *a failure to fund the inferior project if investors are moderately optimistic and the incentive constraint is never satisfied, and*
- *a switch from the superior to the inferior project if investors are highly optimistic and the incentive constraint is conditionally satisfied.*

In all other cases the project choice is independent of the policy regime.

This result is summarized in Table 1, where N indicates that neither project is funded, and the arrows represent a change in project selection when credit derivatives are unrestricted.

	Highly Optimistic	Moderately Optimistic	Neutral
$\hat{q} < q^*$	B	$B \rightarrow N$	N
$\hat{q} \in (q^*, q^n]$	$G \rightarrow B$	$G \rightarrow N$	$G \rightarrow N$
$\hat{q} \geq q^n$	G	G	G

Table 1 – Effects of naked CDS on project choice

As noted above, the efficiency implications of naked credit default swaps in this heterogeneous prior framework depend, in general, on the beliefs used for purposes of evaluation. We shall examine these implications from the perspective of a neutral policy maker, although an analogous exercise could be conducted for other cases. A neutral policy maker believes that the superior project should be funded, but the inferior project should not. When investors are moderately optimistic and the incentive constraint is never satisfied, allowing for unrestricted credit derivatives prevents the inferior project from being funded, and is therefore efficiency increasing. In all other cases, allowing naked credit default swaps either has no impact on project selection, or causes a failure of funding for the superior project and is therefore efficiency reducing.

An interesting case with unambiguous efficiency implications arises if the incentive constraint is conditionally satisfied and investors are highly optimistic. In this case funding is available regardless of regulatory regime but the project choice is different: the firm will choose the superior project in the absence of credit derivatives but the inferior project in their presence. Regardless of the beliefs on the basis of which efficiency is evaluated, this entails an efficiency loss, since even the most optimistic investors would prefer that the superior project be selected.

These findings illustrate the difficulty of fine-tuning the regulatory regime in response to evolving market conditions. Unrestricted credit derivatives may be undesirable if investors are neutral or pessimistic, desirable if they are moderately optimistic (depending on whether or not the incentive constraint is easily satisfied) and undesirable again if they are highly optimistic. Given this non-monotonicity, it may be better to decide on the regulatory regime that performs best on average, rather than attempting to track and respond to investor sentiment, especially when there is considerable uncertainty regarding the incentive constraints under which firms are operating.

²⁰We suppress dependence of these magnitudes on b since this is the same for both projects and is held constant throughout this section

6 Conclusions

Since naked credit default swaps are contracts with both a long and a short side and payoffs that net to zero, it is not immediately apparent what (if any) effects their presence has on economic fundamentals. These effects can be nontrivial. The availability of such contracts can shift the terms of debt contracts against borrowers by inducing optimistic investors to divert their capital away from financing real investment and towards the support of collateralized speculative positions. They can result in the emergence of equilibria in which firms are unable to rollover their debt, even when such equilibria would not exist in their absence. And they can induce firms to switch to projects with greater upside potential even if such projects have lower expected returns.

Tobin (1984) observed that the advantages of greater “liquidity and negotiability of financial instruments” come at the cost of facilitating speculation, and that greater market completeness under such conditions could reduce the *functional efficiency* of the financial system, namely its ability to facilitate “the mobilization of saving for investments in physical and human capital... and the allocation of saving to to their more socially productive uses.” Based on our analysis, one could make the case that naked credit default swaps are a case in point. This conclusion, however, is subject to the caveat that there exist conditions under which the presence of such contracts can prevent the funding of inefficient projects. Furthermore, it could be argued that there would be no sellers of protection unless speculators were able to manage risk by trading on both sides of the market.²¹ And legal restrictions on the set of traded contracts may be infeasible in practice due to the emergence of close substitutes through financial engineering. Even so, it is important to recognize that the use of accumulated wealth as collateral to support zero-sum speculative side bets has the consequence that less capital is available to finance productive risky ventures, and that this has effects on economic fundamentals such as the terms of financing and the incidence of default.

While our focus here has been on the cost of debt, a natural extension would be to consider the implications of credit derivatives for capital structure when investor beliefs are heterogeneous. In this case the portfolio choices of investors and the shares of debt and equity in total financing would be determined jointly. Based on the results presented here, it seems likely that the presence of credit derivatives will affect the cost of debt relative to equity and hence the firm’s capital structure, although the precise nature of this effect is as yet unclear. We leave this extension to future research.

²¹Stulz (2010), for instance, claims that “prohibiting naked positions in credit default swaps would essentially destroy this market... hedgers will not find counterparties because the market will have no liquidity. Speculators have to be able to trade on either side of a market for there to be trading in that market.”

Appendix: Proofs

Proof of Proposition 1. Deduce from (5) and (6) that

$$1 - F(\hat{\theta}(\hat{p}(q), q)) = \hat{p}(q)q = q\psi(q; \hat{\theta}(\hat{p}(q), q)) \leq \Psi(\hat{\theta}(\hat{p}(q), q)), \quad (31)$$

where the first equality follows from (6), the second from (5) and the inequality follows from (2). Since the left most term of (31) is decreasing in $\hat{\theta}$ and the right most term is increasing in $\hat{\theta}$, the middle term (total revenue) is bounded above by $\Psi(\theta_m)$. The upper bound is attained at $q = 1$, $\hat{p}(q) = \Psi(\theta_m) = \psi(1, \theta_m)$ and $\hat{\theta}(\hat{p}(1), 1) = \theta_m$. Hence the maximum revenue that can be raised in equilibrium is $b_m = \Psi(\theta_m)$. Any funding requirement $b \leq b_m$ can be met since total revenue varies continuously between 0 and b_m as q varies between 0 and 1. Equilibrium exists and is unique in this case since $q^*(b)$ is unique by definition. To prove the last statement, consider $b < b' \leq b_m$. By definition, $q^*(b) \leq q^*(b')$ and $p^*(b) = \hat{p}(q^*(b)) \geq \hat{p}(q^*(b')) = p^*(b')$. We must have $q^*(b) < q^*(b')$ and $p^*(b) > p^*(b')$, or else $b = p^*(b)q^*(b) = p^*(b')q^*(b') = b'$, a contradiction. *Q.E.D.*

Proof of Lemma 1. Suppose, by way of contradiction, that $p > 1 - r$. Then, for any type θ such that $u^b(\theta) \geq 1$ we must have $\psi(q; \theta) \geq p$ and hence

$$u^s(\theta) = \frac{\psi(q; \theta) - \eta/q}{1 - r - \eta/q} > \frac{\psi(q; \theta) - \eta/q}{p - \eta/q} \geq \frac{\psi(q; \theta)}{p} = u^b(\theta).$$

That is, for all types θ such that $u^b(\theta) \geq 1$ we have $u^s(\theta) > u^b(\theta)$ so no investor buys bonds without protection. Furthermore, if $r + p > 1$ then $u^c(\theta) < 1$ so no investor buys bonds *with* protection either. Hence there can be no equilibrium with $r + p > 1$ in which the funding requirement is met. *Q.E.D.*

Proof of Lemma 2. Suppose to the contrary that the some agents purchase the bond without protection. Then, there must be a type θ such that $u^b(\theta) \geq 1$. For any such type, we must have

$$u^s(\theta) = \frac{\psi(q; \theta) - \eta/q}{1 - r - \eta/q} = \frac{\psi(q; \theta) - \eta/q}{p - \eta/q} > \frac{\psi(q; \theta)}{p} = u^b(\theta).$$

This means that such agents selling protection strictly preferable to buying bonds without protection. This provides the contradiction, which implies that no agent buys bonds without protection. *Q.E.D.*

Proof of Lemma 3. Suppose that $\eta > 0$ and (by way of contradiction) that $p + r \geq 1$. Consider an investor of type θ who buys bonds without protection. Then it must be the case

that $\psi(q; \theta) \geq p$. For any investor of type $\theta' > \theta$, we then have $\psi(q; \theta') > p$ and hence

$$u^s(\theta') = \frac{\psi(q; \theta') - \eta/q}{1 - r - \eta/q} \geq \frac{\psi(q; \theta') - \eta/q}{p - \eta/q} > \frac{\psi(q; \theta')}{p} = u^b(\theta'),$$

where the first inequality follows from $p \geq 1 - r$ and the second from $\eta > 0$. Hence the set of investors who buy the bond without protection has zero measure.

To complete the proof, we need to show that the set of investors who buy the bond with protection also has zero measure. This is clearly the case if $p + r > 1$, since cash would then be preferred to bonds with protection. Accordingly, suppose that $p + r = 1$ and suppose that an investor of type θ chooses to buy bonds with protection, obtaining payoff 1. For any such investor we must have $\psi(q; \theta) \leq p$, otherwise the investor would prefer to buy bonds *without* protection. Then, for any investor of type $\theta' < \theta$, we have

$$u^n(\theta') = \frac{1 - \psi(q; \theta')}{1 - p} > \frac{1 - \psi(q; \theta)}{1 - p} \geq 1,$$

so any such investor would rather buy naked protection than bonds with protection. Consequently, the set of investors who buy bonds with protection is also of zero measure, and there can be no equilibrium with $p + r \geq 1$ in which the funding requirement is met. *Q.E.D.*

Proof of Lemma 4. Note first that $u^c(\theta) > u^n(\theta)$ implies $u^c(\theta) < u^b(\theta)$, since

$$\frac{1}{p + r} > \frac{1 - \psi(q; \theta)}{r} \iff \frac{1}{p + r} < \frac{\psi(q; \theta)}{p}.$$

Hence any investor who strictly prefers buying bonds with protection to buying naked protection also strictly prefers bonds without protection to bonds with protection. Hence an investor will buy bonds with protection only if she is of a type θ such that $u^c(\theta) = u^n(\theta) = u^b(\theta)$. But then for any investor of type $\theta' > \theta$, $u^c(\theta') < u^b(\theta')$ and for any investor of type $\theta' < \theta$, $u^c(\theta') < u^n(\theta')$. Hence the set of investors who buy bonds with protection has zero measure. *Q.E.D.*

Proof of Lemma 5. Fix $q \in (0, 1]$ and consider any $r < 1 - \eta/q$. Recall the market clearing condition:

$$1 - \left(\frac{1 - \eta/q}{r} \right) F(\underline{\theta}) = pq.$$

Since $\underline{\theta}$ is non-decreasing in p , the left side of this equation is non-increasing in p . The right side is clearly increasing in p . Note that $\underline{\theta} = F(\underline{\theta}) = 0$ when $p = 0$ and (from the proof of Lemma 3) $\underline{\theta} = F(\underline{\theta}) = 1$ when $p = 1 - r$. Hence there exists a unique solution $p(r)$ to (20)

for any $r < 1 - \eta/q$. Note that $p(r)$ is decreasing. Given q and $p(r)$, there is then a unique $r \in (0, 1 - \eta/q)$ that satisfies the market clearing condition (18).

We next show that $\bar{p}(q) < \hat{p}(q)$. Suppose, to the contrary, that $\bar{p}(q) \geq \hat{p}(q)$. Note (from the proof of Lemma 4) that the threshold type $\underline{\theta}$ satisfies $u^b(\underline{\theta}) = u^c(\underline{\theta})$. Using this fact, Lemma 3, the equilibrium condition (5), and the hypothesis that $\bar{p}(q) \geq \hat{p}(q)$ we obtain:

$$\psi(q; \underline{\theta}) = \frac{\bar{p}(q)}{\bar{p}(q) + \bar{r}(q)} > \bar{p}(q) \geq \hat{p}(q) = \psi(q; \hat{\theta}). \quad (32)$$

This implies $\underline{\theta} > \hat{\theta}$. Note that $\bar{r}(q) < 1 - \eta/q$, otherwise the market clearing condition (19) could not hold. Hence, from (20), we obtain

$$F(\underline{\theta}) < 1 - \bar{p}(q)q.$$

Combining this with the market clearing condition (6) and the hypothesis that $\bar{p}(q) \geq \hat{p}(q)$ yields

$$F(\hat{\theta}) = 1 - \hat{p}(q)q \geq 1 - \bar{p}(q)q > F(\underline{\theta}),$$

and hence $\underline{\theta} < \hat{\theta}$, a contradiction. Hence $\bar{p}(q) < \hat{p}(q)$.

Q.E.D.

Proof of Lemma 6. From (1),

$$q\psi_G(q; \theta_0) = \int_0^q y dG(y; \theta_0) + q(1 - G(q)).$$

Integrating the first term by parts, we obtain after simplification

$$q\psi_G(q; \theta_0) = q - \int_0^q G(y; \theta_0) dy$$

Similarly,

$$q\psi_B(q; \theta_0) = q - \int_0^q B(y; \theta_0) dy$$

and hence

$$q\psi_G(q; \theta_0) - q\psi_B(q; \theta_0) = \int_0^q (B(y; \theta_0) - G(y; \theta_0)) dy.$$

Condition (30) can therefore be rewritten as

$$\int_0^q (B(y; \theta_0) - G(y; \theta_0)) dy \leq \Psi_G(\theta_0) - \Psi_B(\theta_0).$$

By (29), the left side is increasing in q for $q < y^*$ and decreasing in q for $q > y^*$. Further

observe that the inequality holds at $q = 0$, and it becomes an equality at $q = 1$. (The latter claim can be verified by integrating the left side of the inequality by parts.) This means that there exists $\hat{q} \in (0, y^*)$ such that (30) holds if and only if $q \leq \hat{q}$. Q.E.D.

Proof of Proposition 8. To prove the statement, we first establish the following:

Lemma 7. *Equilibrium prices and issue sizes satisfy: $q_B^* > q^*$, $p_B^* < p^*$, $q_B^n > \max\{q^n, q_B^*\}$, and $p_B^n < \min\{p^n, p_B^*\}$.*

Proof. Suppose first that there are no credit derivatives. Analogous to the case of the superior project, the equilibrium conditions (conditional on the funding of the inferior project) are described by:

$$1 - F(\theta_B^*(b)) = b = \int_0^1 \min\{y, q_B^*(b)\} dB(y|\theta_B^*(b)). \quad (33)$$

The first equality implies that $\theta_B^*(b) = \theta^*(b)$. The second equality and the corresponding one for the superior project implies that

$$\int_0^1 \min\{y, q_B^*(b)\} dB(y|\theta_B^*(b)) = \int_0^1 \min\{y, q^*(b)\} dG(y|\theta^*(b)),$$

from which it follows that $q_B^*(b) > q^*(b)$, since $\theta_B^*(b) = \theta^*(b)$ and $G(y|\theta) < B(y|\theta)$ for all $(y, \theta) \in (0, 1)^2$. Since $p_B^*(b)q_B^*(b) = b = p^*(b)q^*(b)$, it follows that $p_B^*(b) < p^*(b)$.

Now suppose that naked CDSs are available. Again the equilibrium conditions, assuming the funding of the inferior project, are as follows:

$$1 - \left(\frac{1}{1 - p_B^n(b)} \right) F(\underline{\theta}_B^n(b)) = p_B^n(b)q_B^n(b) = b = \int_0^1 \min\{y, q_B^n(b)\} dB(y|\underline{\theta}_B^n(b)). \quad (34)$$

For convenience, recall the corresponding conditions for the case of the superior project:

$$1 - \left(\frac{1}{1 - p^n(b)} \right) F(\underline{\theta}^n(b)) = p^n(b)q^n(b) = b = \int_0^1 \min\{y, q^n(b)\} dG(y|\underline{\theta}^n(b)). \quad (35)$$

Comparison of the first equation of (34) with that of (33) yields $\underline{\theta}_B^n(b) < \theta_B^*(b)$. Next, suppose $\underline{\theta}_B^n(b) \leq \underline{\theta}^n(b)$. Then, comparison of the last equation of (34) with that of (35) yields $q_B^n(b) > q^n(b)$ since $\underline{\theta}_B^n(b) \leq \underline{\theta}^n(b)$ and $G(y|\theta) < B(y|\theta)$ for all $(y, \theta) \in (0, 1)^2$. Comparison of the second equations of (34) and (35) then yields $p_B^n(b) < p^n(b)$. Comparison of the first equation of (34) and that of (35) then yields a contradiction, since $p_B^n(b) < p^n(b)$ and yet $\underline{\theta}_B^n(b) \leq \underline{\theta}^n(b)$. We thus conclude that $\underline{\theta}_B^n(b) > \underline{\theta}^n(b)$, which in turn implies (via comparison

of the first equations of (34) and (35) respectively) that $p_B^n(b) < p^n(b)$ and $q_B^n(b) > q^n(b)$. Also, since $\theta_B^n(b) < \theta_B^*(b)$, it follows from the last equations of (33) and (34) respectively that $q_B^n(b) > q_B^*(b)$ and $p_B^n(b) < p_B^*(b)$. *Q.E.D.*

To prove the statement of the proposition, suppose first the naked protection purchases are not permitted. If investors are pessimistic, the firm cannot obtain the necessary funding for either project. If investors are not pessimistic, then the equilibrium identified in Proposition 1 holds as long as the issue size q^* does not exceed the critical level \hat{q} required for the incentive constraint (30). In this case, the firm can obtain b and will choose the superior project. Investors understand that if the firm issues $q = q^* \leq \hat{q}$, then it will select the superior project, and given this, it is in the best interests of the firm to issue $q = q^*$. Issuing $q > \hat{q}$ would imply the selection of the inferior project, and in this case even if the firm could raise b it would not benefit from doing so. This can be seen as follows. Suppose the firm indeed picks $q > \hat{q}$ and raises b . Then, the firm's expected payoff is

$$\Psi_B(\theta_0) - q\psi_B(q; \theta_0) < \Psi_B(\theta_0) - q^*(b)\psi_B(q^*(b); \theta_0),$$

where the inequality holds since $q > q^*$; i.e., the firm's debt burden decreases with a smaller issue size. The payoff on the right is smaller than that which the firm would earn by issuing q^* and choosing the superior project, since $q^* \leq \hat{q}$. If $q^* > \hat{q}$, however, the firm will never select the superior project. Given this, investors will not fund the firm unless they are (at least moderately) optimistic.

In summary, in the absence of naked protection, if $q^* \leq \hat{q}$ and investors are not pessimistic, the superior project is chosen; and if investors are (at least moderately) optimistic and $q^* > \hat{q}$, then the inferior project is chosen. In all other cases, the firm cannot obtain funding.

Next suppose that naked protection purchases are permitted. In this case, the superior project is chosen only if investors are not pessimistic and the amount q^n of bonds issued in equilibrium is no greater than \hat{q} . Since $q^n > q^*$, this condition is more difficult to satisfy than in the absence of such derivatives. That is, if $q^n > \hat{q} \geq q^*$, then the firm will choose the superior project only if naked CDSs are disallowed. If such contracts were permitted, the firm would choose the inferior project if it were able to obtain funding, and it will secure funding if and only if investors are highly optimistic.

Since $q_B^* < q_B^n$, the inferior project is less likely to be adopted in the presence of naked credit default swaps than in their absence. That is, conditional on failing the incentive constraint (30), the inferior project will be less frequently chosen in the presence of such contracts. *Q.E.D.*

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