John Geanakoplos: The Leverage Cycle

Columbia Finance Reading Group

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Collateral

- Loan contracts specify not just interest but also collateral
- Examples: mortgage, repo
- Leverage: ratio of asset value to down payment (reciprocal of margin)
- Long-term loans backed by physical assets (housing); short-term loans backed by financial assets (repo)
- Can loan market equilibrium determine *both* interest and leverage?
- Why do we have such large variations in leverage over time?
- Should Fed focus on system wide leverage rather than interest rates?

Importance of Leverage: Boom

Leverage dramatically increased in the United States from 1999 to 2006. A bank that in 2006 wanted to buy a AAA-rated mortgage security could borrow 98.4% of the purchase price, using the security as collateral, and pay only 1.6% in cash. The leverage was thus 100 to 1.6, or about 60 to 1. The average leverage in 2006 across all of the US$2.5 trillion of so-called 'toxic' mortgage securities was about 16 to 1, meaning that the buyers paid down only $150 billion and borrowed the other $2.35 trillion. Home buyers could get a mortgage leveraged 20 to 1, a 5% down payment. Security and house prices soared.

Importance of Leverage: Bust

Today leverage has been drastically curtailed by nervous lenders wanting more collateral for every dollar loaned. Those toxic mortgage securities are now leveraged on average only about 1.5 to 1. Home buyers can now only leverage themselves 5 to 1 if they can get a government loan, and less if they need a private loan. De-leveraging is the main reason the prices of both securities and homes are still falling.

Determinants of Leverage

Key idea: housing serves as collateral for (long term) loans, loans serve as collateral in (short-term) repo market

- Repo market: heterogeneous beliefs, optimists borrow from pessimists to buy asset on margin, bad news lowers asset prices, effect amplified through bankruptcy of optimists and endogenous decline in leverage
- Housing market: heterogeneous preferences over asset, those with high values borrow from those with low values, equilibrium contracts allow for possible default
- Both models combined: positive feedback effects between housing and repo markets (double leverage cycle)

Endogenous Collateral with Heterogeneous Beliefs: A Simple Example

Let each agent $h \in H \subset [0,1]$ assign probability h to $s = \mathrm{U}$ and probability $1 - h$ to $s = \mathrm{D}$. Agents with h near 1 are optimists, agents with h near 0 are pessimists.

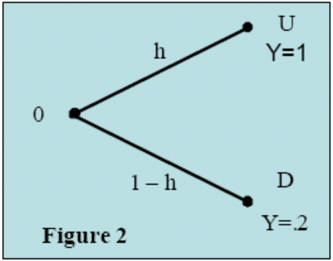

Figure 2

Suppose that 1 unit of Y gives $\$1$ unit in state U and $.2$ units in D.

Preferences and Endowments

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- Asset price (relative to consumption good) is p

Objectives and Constraints

Individual of type h maximizes $c_0 + hc_u + (1-h)\, c_d$ subject to constraints

- Storage plus asset purchases equal endowment of C

$$w_0 + p\,(y_0 - 1) = 1$$

- Second period consumption equals storage plus asset value

$$\begin{aligned} c_u &= w_0 + y_0 \\ c_d &= w_0 + 0.2y_0 \end{aligned}$$

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Aggregate second-period consumption equals storage plus asset value

$$\begin{aligned} \int c_u\, dh &= 1 + \int w_0\, dh \\ \int c_d\, dh &= 0.2 + \int w_0\, dh \end{aligned}$$

Equilibrium without Borrowing

- Without borrowing, those with $h \geq 0.60$ each buy 1.48 units of asset at $p = 0.68$; and those with $h < 0.60$ sell all their asset holdings
- Equilibrium

$$\begin{array}{ll} (c_0, y_0, w_0, c_h, c_d) = (0, 2.48, 0, 2.48, 0.50) & \text{if } h \geq 0.60 \\ (c_0, y_0, w_0, c_h, c_d) = (0, 0, 1.68, 1.68, 1.68) & \text{if } h < 0.60 \end{array}$$

- Pessimist consumption state independent, but not because of risk-aversion

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- Equilibrium: $h \geq 0.69$ each borrow 0.64 at no interest; buy 2.19 units of asset at $p = 0.75$; $h < 0.69$ each sell their asset holdings and lend 0.31.

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- Interest rate is zero because pessimists have excess loanable funds

Menus of Contracts

- Leverage determined in equilibrium (jointly with interest rate)
- Menu of contracts: ordered pairs of promises and collateral
- Suppose collateral is one unit of asset Y
- Let A denote promise (amount to be repaid) and π_a the price of this (amount borrowed)
- Then the interest rate r_a satisfies $1 + r_a = A/\pi_a$

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- Interest rates

$$\begin{aligned}
1 + r_{0.3} &= 0.3/0.269 = 1.12 \\
1 + r_{0.4} &= 0.4/0.338 = 1.18
\end{aligned}$$

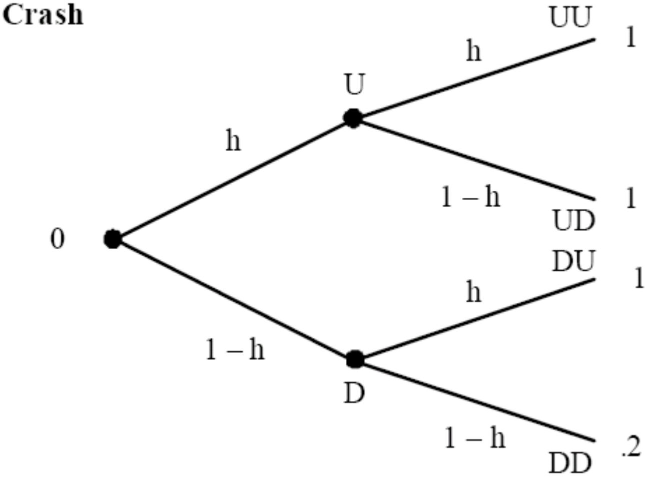
Equilibrium

- Will more optimistic agents pay more for greater leverage?
- No: only $A = 0.2$ will be traded in equilibrium
- All $h > 0.69$ (weakly) prefer to buy $A = 0.2$ at these prices
- All $h < 0.69$ (strictly) prefer to sell $A = 0.2$ at these prices
- Intuition: more leverage raises payments only in good state, optimists consider good state more likely, counteracts effect of higher leverage

Other Issues

- Assets not priced by fundamentals (sensitive to extent of leverage)
- Failure of the law of one price: two identical assets will have different prices if only one can be used as collateral
- Optimists borrow and buy more expensive asset, pessimists lend and sell, remainder buy cheaper asset without leverage
- CDS leads to complete markets, allows pessimists to leverage, lowers prices (or prevents them from rising in the first place)
- Optimists buy all of the asset and consumption good, sell CDS against this collateral.

Extending the Model



Extending the Model

- Three periods: two down moves result in a crash in fundamentals
- Initial endowments and beliefs (at each node) as before
- Agents more optimistic than before; probability of 0.2 is $(1-h)^2$
- Initial short-term borrowing possible; much safer than borrowing at D
- Initial equilibrium leverage depends on anticipated asset price at D
- What are prices in the initial period and following one down move?

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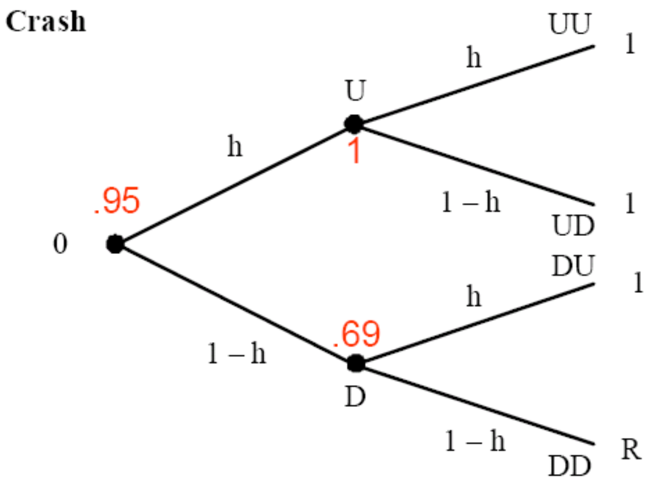
- Equilibrium prices are 0.95 initially, 0.69 at D
- Marginal buyer is $h = 0.87$ initially, top 13% of buyers collectively borrow 0.69 and buy entire asset stock held by the rest
- Total expenditure on asset is $0.13 + 0.69 = 0.82$ to buy 0.87 units of the asset at price $0.82/0.87 = 0.95$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- Leverage: $0.95/(0.95 - 0.69) = 3.7$ to $0.69/(0.69 - 0.20) = 1.4$.
- New marginal buyer is $h = 0.61$ (indifferent between buying and selling at 0.69)

Heterogeneous Preferences and Long-Term Loans

- Housing market: heterogeneous collateral valuations; common priors
- Two goods (perishable food and durable housing)
- Two agent types A and B
- B types value housing more than A types, and have low initial incomes and high future incomes
- Collateral equilibrium with no uncertainty: B agents borrow from A agents to buy housing; repay loans in second period

Collateral Equilibrium with Default

- Suppose that in the down state, B has lower income
- Will loan contract allow for positive default probability?
- Same contract traded as in no-uncertainly case, but at different price
- Free market does not choose levels of collateral that eliminate default
- This is true even with substantial foreclosure costs

The Double Leverage Cycle

- Combine both models; but allow for housing construction
- Labor (instead of housing) endowments
- Type B agents borrow to build, mortgages back loans in repo market
- Most optimistic agents buy mortgages, use these as collateral for short-term loans from pessimists
- In state D, optimists wiped out, collapse in value of mortgage backed securities (but no default)
- Securities now shift to (somewhat) less optimistic agents who hold until maturity, using short-term loans from pessimists to finance

Policy Implications

- Asset prices depend on beliefs of a small minority of agents
- Prices collapse on bad news through bankruptcy and deleveraging
- Major wealth redistribution: optimists win big or get wiped out
- Bankruptcy causes decline in real activity (new construction)
- Anticipated foreclosure diminishes incentives to repair and maintain
- Loan size (endogenously) too large to preclude default
- Curtailing leverage implies smaller declines after bad news, less foreclosure, lower foreclosure costs, smaller decline in new construction
- Contagion across markets possible if individual optimism correlated across asset classes