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MIXED STRATEGY

In the theory of games a player is said to use a mixed strategy whenever he or she chooses to randomize over the set of available actions. Formally, a mixed strategy is a probability distribution that assigns to each available action a likelihood of being selected. If only one action has a positive probability of being selected, the player is said to use a pure strategy.

A mixed strategy profile is a list of strategies, one for each player in the game. A mixed strategy profile induces a probability distribution or lottery over the possible outcomes of the game. A Nash equilibrium (mixed strategy) is a strategy profile with the property that no single player can, by deviating unilaterally to another strategy, induce a lottery that he or she finds strictly preferable. In 1950 the mathematician John Nash proved that every game with a finite set of players and actions has at least one equilibrium.

To illustrate, one can consider the children's game Matching Pennies, in which each of two players can choose either heads (*H*) or tails (*T*); player 1 wins a dollar from player 2 if their choices match and loses a dollar to player 2 if they do not. This game can be represented as follows:

	H	*T*
H	(1, −1)	(−1, 1)
T	(−1, 1)	(1, −1)

Here player 1's choice determines a row, player 2's choice determines a column, and the corresponding cell indicates the payoffs to players 1 and 2 in that order. This game has a unique Nash equilibrium that requires each player to choose each action with probability one-half.

Another example is provided by the Hawk-Dove game, which has been used by evolutionary biologists to model animal conflicts:

	H	*D*
H	(0, 0)	(4, 1)
D	(1, 4)	(2, 2)

In this game any strategy profile in which one player chooses *H* and the other picks *D* is in equilibrium. Hence, there are two pure strategy equilibria, (*H*,*D*) and (*D*,*H*). In addition, there is a mixed strategy equilibrium in which each player selects *H* with probability 2/3.

One feature of a mixed strategy equilibrium is that given the strategies chosen by the other players, each player is indifferent among all the actions that he or she selects with positive probability. Hence, in the matching pennies game, given that player 2 chooses each action with probability one-half, player 1 is indifferent among choosing *H*, choosing *T*, and randomizing in any way between the two. Because randomization is more complex and cognitively demanding than is the deterministic selection of a single action, this raises the question of how mixed strategy equilibria can be sustained and, more fundamentally, how mixed strategies should be interpreted.

In an interpretation advanced in 1973 by John Harsanyi, a mixed strategy equilibrium of a game with perfect information is viewed as the limit point of a sequence of pure strategy equilibria of games with imperfect information. Specifically, starting from a game with perfect information, one can obtain a family of games with imperfect information by allowing for the possibility that there are small random variations in payoffs and that each player is not fully informed of the payoff functions of the other players. Harsanyi showed that the frequency with which the various pure strategies are chosen in these perturbed games approaches the frequency with which they are chosen in the mixed strategy equilibrium of the original game as the magnitude of the perturbation becomes vanishingly small.

A very different interpretation of mixed strategy equilibria comes from evolutionary biology. To illustrate this, consider a large population in which each individual is programmed to play a particular pure strategy. Individuals are drawn at random from that population and are matched in pairs to play the game. The payoff that results from the adoption of any specific pure strategy will depend on the frequencies with which the various strategies are represented in the population. Suppose that those frequencies change over time in response to payoff differentials, with the population share of more highly rewarded strategies increasing at the expense of strategies that yield lower payoffs. Any rest point of this dynamic process must be a Nash equilibrium. In the special case of the Hawk-Dove game any trajectory that begins at a state in which both strategies are present converges to the unique mixed strategy equilibrium of the game. In other words, the long-run population share of each strategy corresponds exactly to the likelihood with which it is played in the mixed strategy equilibrium.

SEE ALSO *Evolutionary Games; Game Theory; Nash Equilibrium; Nash, John; Pure Strategy*

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MODEL SELECTION TESTS

Statistical inference and forecasting, widely used in all of the sciences, are usually based on a statistical model. Yet, the specification of an appropriate statistical model is a difficult problem that has yet to be satisfactorily solved. Model building is as much an art as a science. All statistical models are necessarily false, and their usefulness is their ability to provide the best approximation to the "true" model. A summary of the complementary approaches used in this important problem of model specification is provided below.

THEORY AND SAMPLE DATA

The *conceptual approach* is based on the use of subject matter theory in the specification of a model. However, there may be many competing theories leading to many alternative models. The many different macroeconomic models are an example. In other situations, theory may provide little, if any, information on model specification. For example, dynamic modeling theory typically provides little information on the dynamic relations among variables.

Another source of model specification is the data. Previously unknown relations between variables can be suggested from the data. An example is given by the Phillips curve relation in macroeconomics.

STATISTICAL APPROACHES: HYPOTHESIS TESTING

The problem of model specification can also be addressed using statistical hypothesis testing. In this approach, the comparisons are generally between two competing models. If the two models are nested—that is, one model can be obtained as a special case of the other by specifying appropriate restrictions—standard hypothesis tests can be used to choose between the two models. Although easy to implement, this approach can be problematic. First, the significance level of the test is an arbitrary choice that can affect the conclusion. Also, using the conventional 5 percent level of significance, the null hypothesis has an advantage over the alternative hypothesis.

In situations where two models are nonnested, an artificial compound model can be formulated that includes both rival models as special cases, and then the above nested testing procedure can be applied. Common examples include the *J*-test of Russell Davidson and James MacKinnon (1981) and the D. R. Cox test (1961). Both the *J* and Cox tests can be generalized to situations where there are more than two models. However, these tests include the additional possibility that one accepts or rejects both models. In the hypothesis testing approaches, the order of testing is important and can and usually does affect the final outcome. Thus, two different researchers with exactly the same models and data can arrive at different conclusions based on different orders of testing and significance levels.