G6215, Fall 2011
Columbia University
Prof. Stefania Albanesi

Assignment 1, due at the first recitation session.

You are encouraged to work in groups on the assignment, however, you must turn in an individual solution.

Question 1
Problem 2.2, Lucas and Stokey pp.12

Question 2
Problem 2.4, Lucas and Stokey pp. 15

Question 3

Consider the problem of choosing a consumption sequence c_t to maximize:

$$\sum_{t=0}^{\infty} \beta^t [\log(c_t) + \gamma \log(c_{t-1})],\ 0 < \beta < 1,\ \gamma > 0,$$

subject to $c_t + k_{t+1} \leq Ak_t^{\alpha}$, $c_t \geq 0$, $A > 0$, $0 < \alpha < 1$ and $k_0 > 0$, $c_{-1} > 0$ given. Here, c_t denotes consumption at time t and k_t denotes outstanding capital stock at time t. The form of the current utility function $\log(c_t) + \gamma \log(c_{t-1})$ is designed to represent habit persistence in consumption i.e. past consumption has an impact on current utility.

A) Let $v(k_0, c_{-1})$ be the value of $\sum_{t=0}^{\infty} \beta^t [\log(c_t) + \gamma \log(c_{t-1})]$ for a consumer who begins his life with capital stock k_0 and consumption c_{-1} and behaves optimally. Formulate Bellman's functional equation in $v(k, c_{-1})$.

B) Prove that the solution to Bellman's functional equation is of the form $v(k, c_{-1}) = E + F \log(k) + G \log(c_{-1})$ and that the optimal policy is of the form, $\log(k_{t+1}) = I + H \log(k_t)$, where E, F, M, G, H, I are constants. Give explicit formulas for the constants E, F, G, H in terms of the parameters A, β, α, γ.