CEE 2005

- Goal: Construct model that replicates dynamic responses to monetary and other shocks and can be used for policy analysis
- Challenge: Replication of impact and dynamic responses
  - Empirical evidence: Persistence and slow adjustment to shocks
  - Standard models: Fast adjustment and cannot reproduce all "impact" responses
- Model features:
  - Sticky prices
  - Demand for liquidity by firms due to working capital requirements
  - Sticky wages
  - Variable capacity utilization
  - Adjustment costs to investment
  - Habit persistence

Review of Empirical Evidence

- Estimating monetary policy shocks- Structural VAR approach
  - Central bank reaction function:

$$R_t = f(\Omega_t) + \varepsilon_t,\ f \text{ linear}, \Omega_t \perp \varepsilon_t$$
$$R_t = \text{Fed funds rate}$$
$$\Omega_t = \text{information set}$$

- Identifying assumptions:
  - Ordering
  - Information structure
- Variables: $Y_t = [Y_{1,t},\ R_t,\ Y_{2,t}]$
  - $Y_{1,t}$ = time t values of variables in Ω_t that do not respond to ε_t
    * GDP, consumption, GDP deflator, investment, real wages, productivity
  - $Y_{2,t}$ = time t values of variables that do respond to ε_t
    * real profits, growth rate of M2

- Statistical Model:

$$Y_t = \sum_{j=1,2,3,4} A_j Y_{t-j} + C\eta_t$$

  – C lower triangular from identifying assumptions
  – Mon. shock is 7th element of η_t
  – $\{A_j\}_{j=1,2,3,4}$ and C estimated by OLS

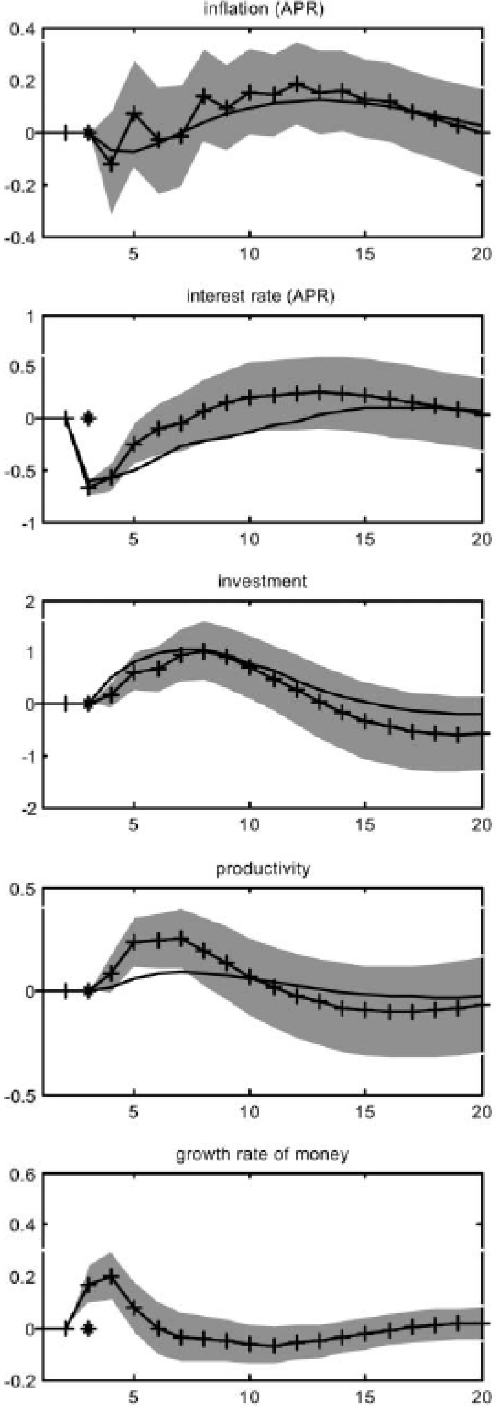


FIG. 1.—Model- and VAR-based impulse responses. Solid lines are benchmark model impulse responses; solid lines with plus signs are VAR-based impulse responses. Grey areas are 95 percent confidence intervals about VAR-based estimates. Units on the horizontal axis are quarters. An asterisk indicates the period of policy shock. The vertical axis units are deviations from the unshocked path. Inflation, money growth, and the interest rate are given in annualized percentage points (APR); other variables are given in percentages.

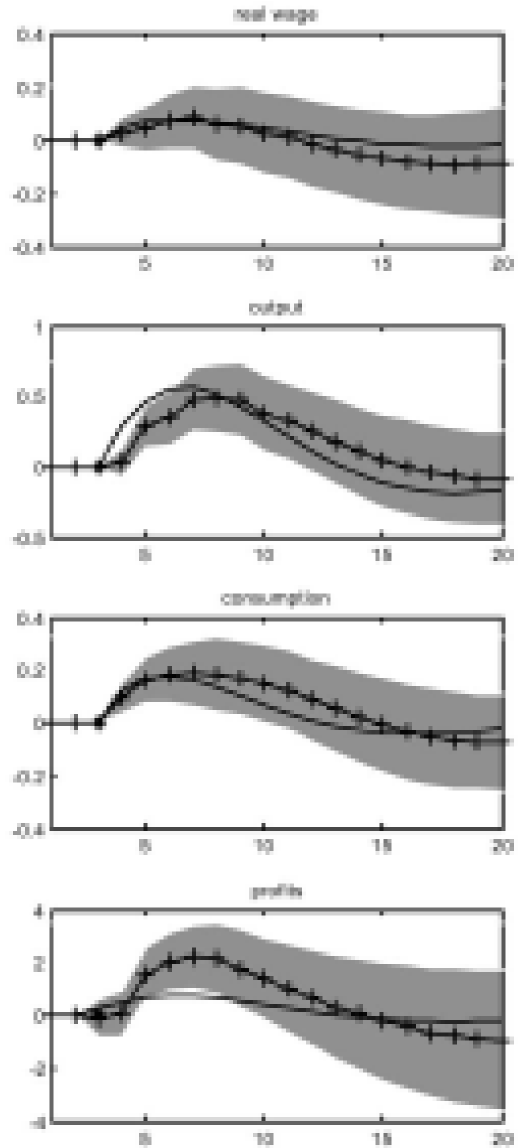


Fig. 1.—Continued

TABLE 1

PERCENTAGE VARIANCE DUE TO MONETARY POLICY SHOCKS

	4 Quarters Ahead	8 Quarters Ahead	20 Quarters Ahead
Output	15	38	27
	(4,26)	(15,48)	(9,35)
Inflation	1	4	7
	(0,8)	(1,11)	(3,18)
Consumption	14	21	14
	(4,26)	(5,37)	(4,26)
Investment	10	26	23
	(2,21)	(7,39)	(6,32)
Real wage	2	2	4
	(0,8)	(0,14)	(0,15)
Productivity	15	14	10
	(3,25)	(3,26)	(3,20)
Federal funds rate	32	19	18
	(18,44)	(8,27)	(5,27)
M2 growth	19	19	19
	(8,29)	(8,26)	(8,24)
Real profits	13	18	7
	(5,25)	(6,31)	(2,20)

NOTE.—Numbers in parentheses are the boundaries of the associated 95 percent confidence interval.

Model

3.1 Firms

- Final good firms:
  - Produce output using a continuum of intermediate goods:

$$Y_t = \left[\int_0^1 Y_{jt}^{1/\lambda_f} dj \right]^{\lambda_f}, \ \lambda_f \geq 1$$

  - Perfectly competitive:

$$Y_{jt} = Y_t \left(\frac{P_t}{P_{jt}} \right)^{\lambda_f/(1-\lambda_f)}$$

Model ...

- Intermediate good firms:
  - Use capital and labor in production:

$$Y_{jt} = \begin{cases} K_{jt}^{\alpha} L_{jt}^{1-\alpha} - \phi \text{ for } K_{jt}^{\alpha} L_{jt}^{1-\alpha} - \phi \geq 0, \; \alpha \in (0,1) \\ 0 \text{ otherwise} \end{cases}$$

  - Borrow to finance working capital
  - Monopolistically competitive
  - Calvo pricing

Model ...

- Cost minimization:

$$S(Y) = \min_{K,L} r^K K + wRL \text{ s.t. } K^\alpha L^{1-\alpha} - \phi \geq Y$$

  – FONCS

$$\begin{aligned} r^K + \lambda\alpha (Y+\phi)/K &= 0, \\ wR + \lambda (1-\alpha)(Y+\phi)/L &= 0, \\ K^\alpha L^{1-\alpha} - \phi &= Y \end{aligned}$$

$\Longrightarrow$

$$Y + \phi = L (K/L)^\alpha = L \left(\frac{\alpha}{1-\alpha} \frac{wR}{r^K} \right)^\alpha$$

Model ...

– Solution:

$$\begin{aligned}
S(Y) &= L\left(r^K K/L + wR\right) = \\
&= (Y+\phi)(K/L)^{-\alpha}\left(r^K K/L + wR\right) \\
&= (Y+\phi)\left(\frac{\alpha}{1-\alpha}\frac{wR}{r^K}\right)^{-\alpha}\left(r^K\frac{\alpha}{1-\alpha}\frac{wR}{r^K} + wR\right) \\
&= (Y+\phi)\left(\frac{r^K}{\alpha}\right)^{\alpha}\left(\frac{wR}{1-\alpha}\right)^{1-\alpha}
\end{aligned}$$

$$s(Y) = \frac{\partial S(Y)}{\partial Y} = \left(\frac{r^K}{\alpha}\right)^{\alpha}\left(\frac{wR}{1-\alpha}\right)^{1-\alpha} = \text{ Marginal Cost}$$

Model ...

- Profit maximization
  - Calvo pricing:
    - $1-\xi_p =$ constant probability of being able to reoptimize price in any period, iid over time and across firms
    - Firms reoptimizing price do so *before* current monetary shock is realized
    - Firms not reoptimizing price have price indexed to inflation:

$$P_{jt} = \pi_{t-1} P_{j,t-1},\ \pi_{t-1} = P_t / P_{t-1}$$

Model ...

– Optimal price $\tilde{P}_t$ solves:

$$\max E_{t-1} \sum_{l=0}^{\infty} \left(\beta \xi_p\right)^l \nu_{t+l} \left(\tilde{P}_t X_{tl} - P_{t+l} s_{t+l}\right) Y_{j,t+l}$$

subject to

$$Y_{jt+l} = Y_{t+l} \left(\frac{P_{t+l}}{P_{jt+l}}\right)^{\lambda_f/(1-\lambda_f)}, l \geq 0$$

$$s_{t+l} = \left(\frac{r_{t+l}^K}{\alpha}\right)^{\alpha} \left(\frac{w_{t+l} R_{t+l}}{1-\alpha}\right)^{1-\alpha}, \ l \geq 0$$

$$X_{tl} = \begin{cases} \pi_t \times \pi_{t+1} \times .... \times \pi_{t+l-1} = P_{t+l-1}/P_{t-1} \text{ for } l > 0 \\ 1 \text{ for } l = 0 \end{cases}$$

$$\nu_{t+l} = \text{ stochastic discount factor}$$

Model ...

– Solution:

$$\begin{aligned}\tilde{P}_t &= \frac{1}{\lambda_f}\frac{E_{t-1}\sum_{l=0}^{\infty}\left(\beta\xi_p\right)^l \nu_{t+l}s_{t+l}\left(P_{t+l}\right)^{\frac{\lambda_f}{(\lambda_f-1)}+1}Y_{t+l}}{E_{t-1}\sum_{l=0}^{\infty}\left(\beta\xi_p\right)^l \nu_{t+l}X_{tl}\left(P_{t+l}\right)^{\frac{\lambda_f}{(\lambda_f-1)}}Y_{t+l}} \\ &= (\text{mark-up})\,\frac{PV\text{ expected marginal cost}}{\text{discounting terms}}\end{aligned}$$

- Aggregate price level: P_t
  - Final goods firms FONC:

$$
\begin{aligned}
Y_{jt} &= Y_t \left(P_t\right)^{\lambda_f/(1-\lambda_f)} \left(P_{jt}\right)^{-\lambda_f/(1-\lambda_f)} \\
\left[\int_0^1 Y_{jt}^{1/\lambda_f} dj\right]^{\lambda_f} &= Y_t \left(P_t\right)^{\lambda_f/(1-\lambda_f)} \left[\int_0^1 P_{jt}^{-1/(1-\lambda_f)} dj\right]^{\lambda_f} \\
P_t &= \left[\int_0^1 P_{jt}^{1/(1-\lambda_f)} dj\right]^{1-\lambda_f}
\end{aligned}
$$

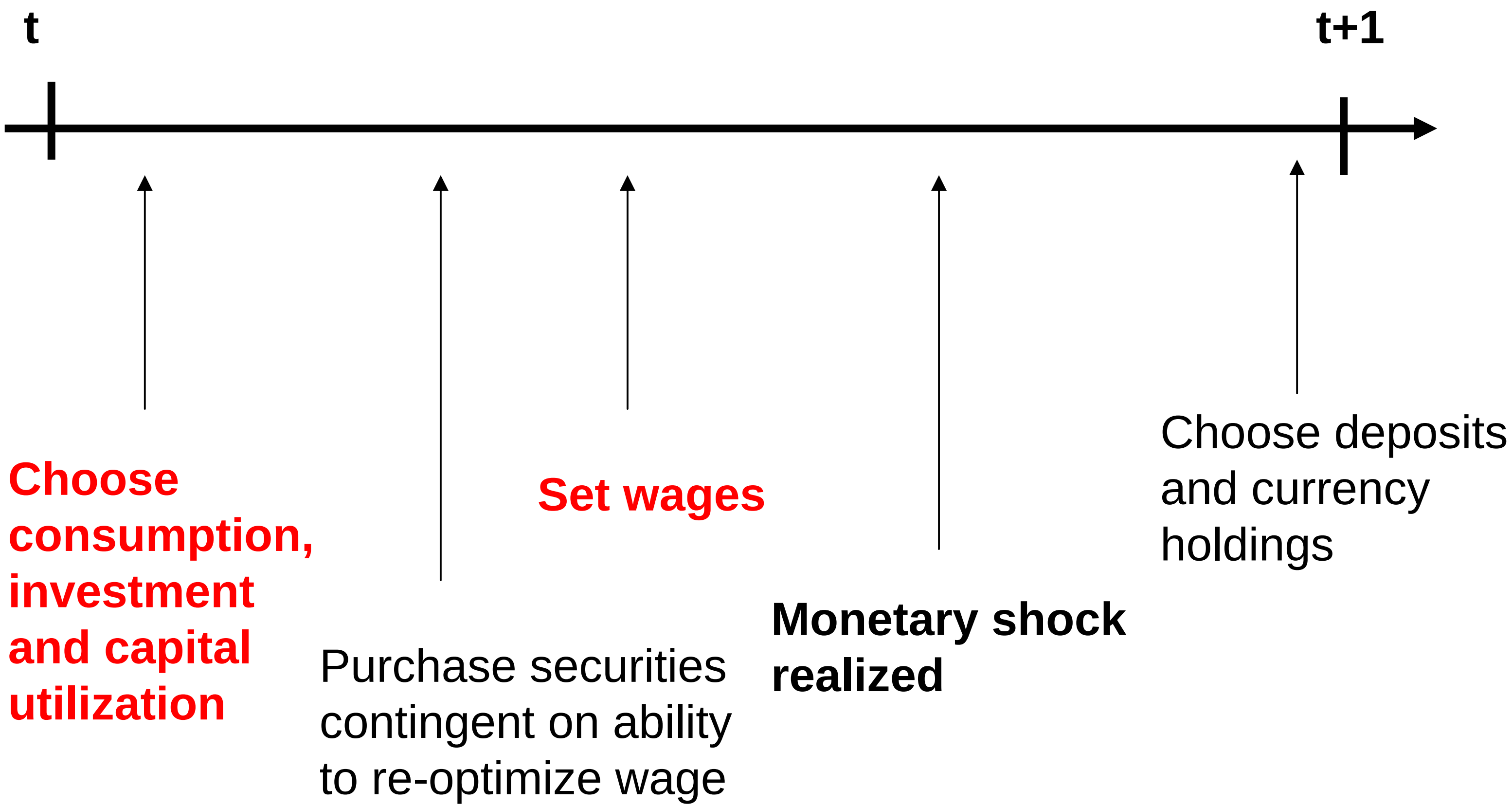
  - Law of motion by *guess and verify*:

$$
P_t = \left[\left(1-\xi_p\right) \tilde{P}_{jt}^{1/(1-\lambda_f)} + \xi_p \left(\pi_{t-1} P_{t-1}\right)^{1/(1-\lambda_f)}\right]^{1-\lambda_f}
$$

3.2 Households

- Assumptions:
  - Money in the utility function
  - Habit persistence
  - Supply labor and accumulate capital
  - Make deposits to financial intermediaries
  - Calvo wage setting (Erceg, Henderson and Levin, 2000)
  - Full insurance via "sticky wage" bonds

Timing of household decisions



Consumption, investment, utilization and wages predetermined

=> L_t also predetermined

- Calvo wage setting
  - Aggregate labor:

$$L_t = \left[\int_0^1 h_{jt}^{1/\lambda_w} dj \right]^{\lambda_w}, \ \lambda_w \geq 1$$

  - Each household supplies specialized labor
    - Labor demand:

$$h_{jt} = L_t \left(\frac{W_t}{W_{jt}} \right)^{\frac{\lambda_w}{\lambda_w - 1}} \tag{1}$$

    - Aggregate wage:

$$W_t = \left[\int_0^1 W_{jt}^{1/(1-\lambda_w)} dj \right]^{1-\lambda_w}$$

Model ...

- Individual household takes L_t and W_t as given
- $1 - \xi_w$ = probability of being able to re-optimize wage, iid across households and over time
- If wage is fixed: $W_{jt} = \pi_{t-1} W_{j,t-1}$
- (1) is a constraint on the household's problem
- Full insurance against "sticky wage" shock
- If wage is flexible: $\tilde{W}_{jt} = \tilde{W}_t$

- Household problem:

$$\max_{\left\{M_{t+1},c_t,Q_t,A_{j,t+1}\tilde{K}_{t+1},u_t,i_t,\tilde{W}_t\right\}} E_{t-1}\sum_{l=0}^{\infty}\beta^l\left[u\left(c_{t+l}-bc_{t+l-1}\right)-z\left(h_{j,t+l}\right)+v\left(q_{t+l}\right)\right]$$

subject to (1) and

$$\begin{aligned} M_{t+1} &= R_t\left[M_t-Q_t+\left(\mu_t-1\right)M_t^s\right]+A_{j,t}+Q_t+W_{j,t}h_{jt} \\ &\quad +R_t^K u_t\tilde{K}_t+D_t-P_t\left[i_t+c_t+a\left(u_t\right)\tilde{K}_t\right] \end{aligned}$$

$$\tilde{K}_{t+1}=\left(1-\delta\right)\tilde{K}_t+F\left(i_t,i_{t-1}\right)$$

$$K_t=u_t\tilde{K}_t$$

  – M_t = nominal wealth, M_t-Q_t = deposits, $\left(\mu_t-1\right)M_t^s$= transfer from central bank
  – $A_{j,t}$ = state contingent payment on "sticky wage" bonds
  – u_t = capital utilization, $\tilde{K}_t$ = physical capital, K_t = capital services
  – $F\left(\cdot\right)$ = adjustment costs
  – $q_t=Q_t/P_t$ = real balances, D_t = dividends

3.3 Monetary and Financial Flows

- Financial intermediaries:

$$W_t L_t = \mu_t M_t^s - Q_t$$

- Money growth

$$M_{t+1}^s = \mu_t M_t^s$$

$$\mu_t = \mu + \theta_0 \varepsilon_t + \theta_1 \varepsilon_{t-1} + \ldots., \text{ MA}(\infty) \text{ represetation}$$

$$\theta_j = \text{ reponse at time } t+j \text{ to a time } t \text{ innovation in } \mu_t$$

Equilibrium

Definition 1 *A private sector equilibrium is a price system* $\left\{\tilde{W}_t, \tilde{P}_t, P_t, W_t, R_t, R_t^K\right\}_{t\geq 0}$ *, an allocation* $\{c_t, h_{jt}, A_{jt}, Q_t, Y_{jt}, Y_t, L_t, D_t, M_t\}_{t\geq 0}$ *and a monetary growth rate path* $\{\mu_t\}_{t\geq 0}$ *such that:*

1. the households', firms' and financial intermediaries' conditions for optimality are satisfied given the money growth rate path;
2. $R_t \geq 1$ for all $t = 0, 1, 2, ...$
3. at the given allocation, price system and money growth rate path the following market clearing conditions hold:

$$\begin{aligned} W_t L_t &= \mu_t M_t^s - Q_t \\ c_t + i_t + a\left(u_t\right) &\leq Y_t. \end{aligned}$$

Parameterization

5.1 Functional Forms

- $u(\cdot) = \ln(\cdot)$
- $z(\cdot) = \psi_0 (\cdot)^2$
- $v(\cdot) = \psi_q (\cdot)^{1-\sigma_q}$
- $F(i, i_{-1}) = [1 - f(i/i_{-1})]\, i$, $f(1) = f(0) = 0$, $f''(1) = \kappa > 0$
- $a(u)$ satisfies $a(1) = 0$ with $\sigma_a = a''(1)/a'(1)$

5.2 Estimation

- Parameters:
  - $\beta, \phi, \alpha, \delta, \psi_0, \psi_q, \lambda_w, \mu$: calibrated so that steady state values match US long run averages
  - $\gamma = \left\{\lambda_f, \xi_w, \xi_p, \sigma_q, \kappa, b, \sigma_a\right\}$: estimated to minimize distance between model, $\Psi(\gamma)$, and data, $\hat{\Psi}$, impulse response functions

$$J = \min_{\gamma} \left[\hat{\Psi} - \Psi(\gamma)\right]' V^{-1} \left[\hat{\Psi} - \Psi(\gamma)\right]$$

$V =$ diagonal matrix with sample variances of $\hat{\Psi}$ on diagonal

TABLE 2
ESTIMATED PARAMETER VALUES

Model	λ_f	ξ_w	ξ_p	σ_q	κ	b	σ_a	ν
Benchmark	1.20	.64	.60	10.62	2.48	.65	.01	NA
	(.06)	(.03)	(.08)	(.67)	(.43)	(.04)		
Flexible prices	1.11	.65	0	8.63	3.24	.66	.01	NA
	(.05)	(.02)		(.63)	(.47)	(.04)		
Unconditional indexation	1.36	.49	.72	11.09	1.92	.63	.01	NA
	(.09)	(.07)	(.16)	(.67)	(.35)	(.05)		
No variable capital utilization	1.85	.42	.92	10.83	1.58	.62	100	NA
	(.13)	(.05)	(.02)	(.67)	(.28)	(.05)		
No habit formation	1.01	.80	.28	10.12	.91	0	.01	NA
	(.04)	(.02)	(.15)	(.70)	(.18)			
Small adjustment costs in investment	1.06	.76	.64	10.92	.5	.52	.01	NA
	(.04)	(.03)	(.08)	(.70)		(.11)		
Lucas-Prescott investment adjustment costs	1.08	.62	.53	10.60	NA	.71	.01	−.74
	(.06)	(.03)	(.23)	(.60)		(.03)		(.22)
No working capital	1.25	.46	.89	10.85	1.89	.62	.01	NA
	(.06)	(.05)	(.02)	(.67)	(.37)	(.05)		

NOTE.—Standard errors are in parentheses.

Analysis

- Main properties of the estimated model:
  - High persistence in output response
  - Inertial response of inflation
  - Sizeable liquidity effect
- Monetary transmission in the model:
  - Liquidity effect: $W_t L_t$ do not respond to mon. shock in the model$\Longrightarrow$ Monetary injection must be absorbed by households$\Longrightarrow$ R_t must fall$\Longrightarrow$ Marginal cost falls and real profits rise.
  - Habits and adjustment costs$\Longrightarrow$ Slow response of consumption and labor supply
  - Rise in productivity due to increase utilization

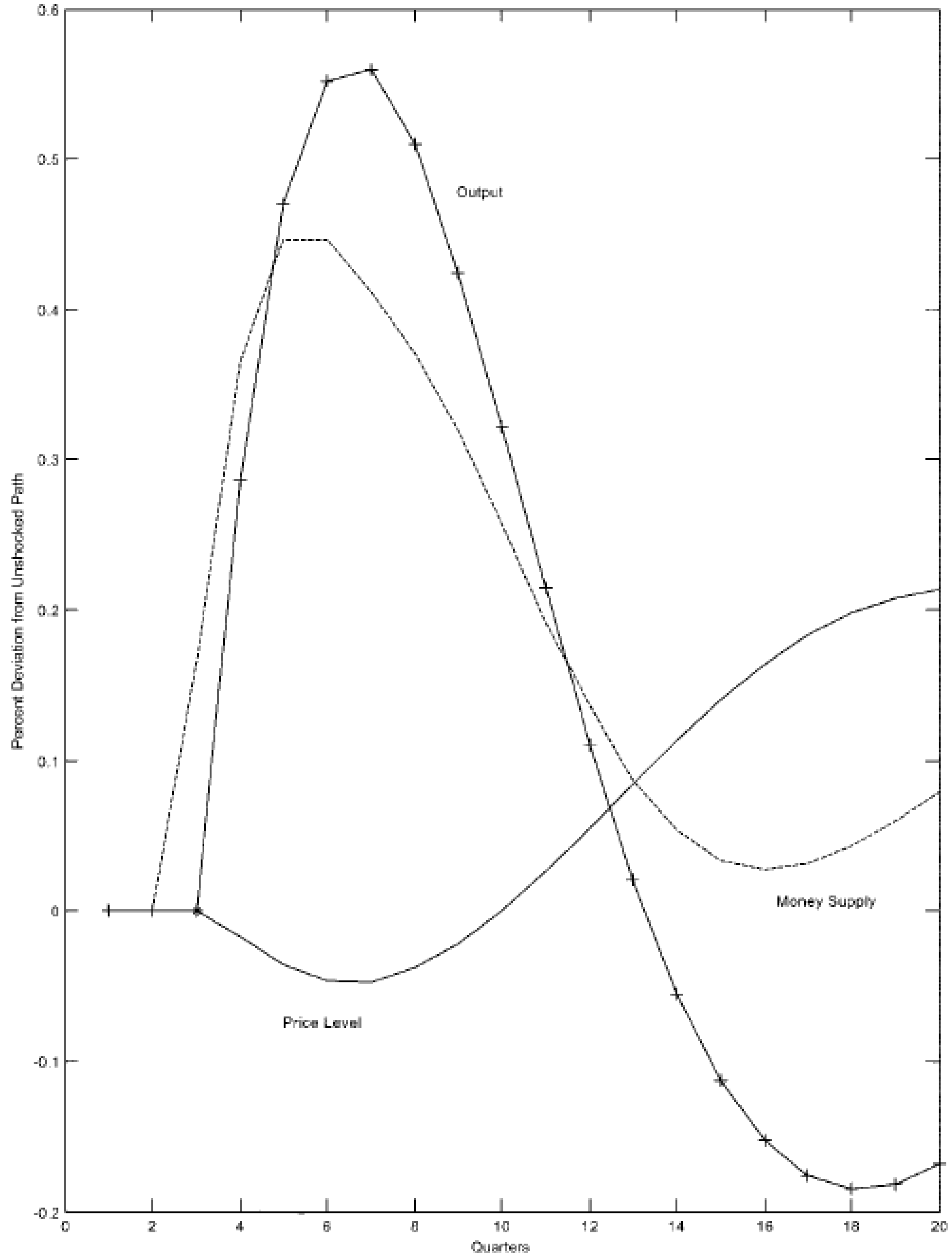


FIG. 2.—Response of price level, output, and money stock to an expansionary monetary policy shock in the benchmark model.

6.1 Inflation Dynamics

- Equilibrium inflation dynamics:
  - Law of motion of intermediate good prices:

$$\tilde{P}_t = \frac{1}{\lambda_f}\frac{E_{t-1}\sum_{l=0}^{\infty}\left(\beta\xi_p\right)^l \nu_{t+l}s_{t+l}P_{t+l}}{E_{t-1}\sum_{l=0}^{\infty}\left(\beta\xi_p\right)^l \nu_{t+l}X_{tl}}$$

$$\tilde{p}_t = \frac{\tilde{P}_t}{P_t} = \frac{1}{\lambda_f}\frac{E_{t-1}\sum_{l=0}^{\infty}\left(\beta\xi_p\right)^l \nu_{t+l}s_{t+l}X_{t,l+1}}{E_{t-1}\sum_{l=0}^{\infty}\left(\beta\xi_p\right)^l \nu_{t+l}X_{tl}} \text{ with } X_{tl} = P_{t+l-1}/P_{t-1}$$

Log-linearizing:

$$\widehat{\tilde{p}}_t = E_{t-1}\sum_{l=0}^{\infty}\left(\beta\xi_p\right)^l \hat{s}_{t+l} + E_{t-1}\sum_{l=0}^{\infty}\left(\beta\xi_p\right)^l \left(\hat{\pi}_{t+l} - \hat{\pi}_{t+l-1}\right)$$

where $\hat{x}_t =$ percentage deviation from steady state x_t and $\hat{\nu}_t$ assumed to be negligeable

$\Longrightarrow$ Acceleration in future inflation generates *front-loading* in price setting.

– Law of motion of aggregate prices:

$$1 = \left[\left(1 - \xi_p\right) \tilde{p}_{jt}^{1/(1-\lambda_f)} + \xi_p \left(\frac{\pi_{t-1}}{\pi_t} \right)^{1/(1-\lambda_f)} \right]^{1-\lambda_f}$$

Log-linearizing:

$$0 = \left(1 - \xi_p\right) \widehat{\tilde{p}_t} - \xi_p \left(\hat{\pi}_t - \hat{\pi}_{t-1}\right)$$

$$\widehat{\tilde{p}_t} = \frac{\xi_p}{1 - \xi_p} \left(\hat{\pi}_t - \hat{\pi}_{t-1}\right)$$

Analysis ...

– Combining:

$$\hat{\pi}_t - \hat{\pi}_{t-1} = \frac{\left(1-\beta\xi_p\right)\left(1-\xi_p\right)}{\xi_p} E_{t-1} \sum_{l=0}^{\infty} \left(\beta\xi_p\right)^l \hat{s}_{t+l}$$

- Implications:
  - $\hat{\pi}_t$ does not respond to period t monetary innovations
  - $\hat{\pi}_t$ has unit root leading to inflation inertia
  - $\hat{\pi}_t$ depends on expected future marginal costs
    - ∗ Nominal wage rigidities impart inertia in marginal cost
    - ∗ Variable capital utilization reduces rise in rental rate on capital preventing large rise in marginal cost
    - ∗ R_t persistently declines after an expansionary monetary shock$\Longrightarrow$ prevents rise in marginal cost

6.2 Role of nominal frictions

- Role of sticky prices:
  - Inertial response of output and inflation similar for $\xi_p = 0$
  - Inertia in marginal cost sufficient to generate inertia in prices
- Nominal wage rigidity is essential to generate inertial response of inflation; nominal price rigidity is not.
- Working capital channel
  - Timing assumption on loan market contributes to slow response to of marginal cost:
    - Household fully absorbs monetary injection$\Longrightarrow$ R_t inversely related to sign of monetary injection has large response to shock
    - Habit persistence implies response of R_t is long lived.

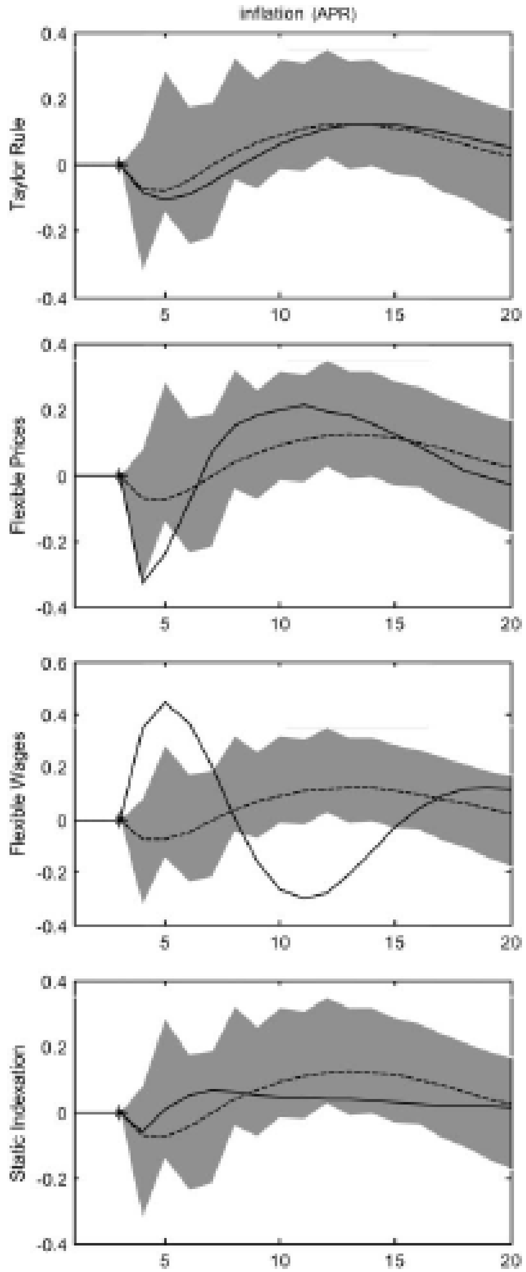


FIG. 4.—Variants of the benchmark model: perturbing the policy and the nominal side. Solid lines are impulse responses for the model on the vertical axis; dashed lines are benchmark model impulse responses. Grey areas are 95 percent confidence intervals about VAR-based impulse responses.

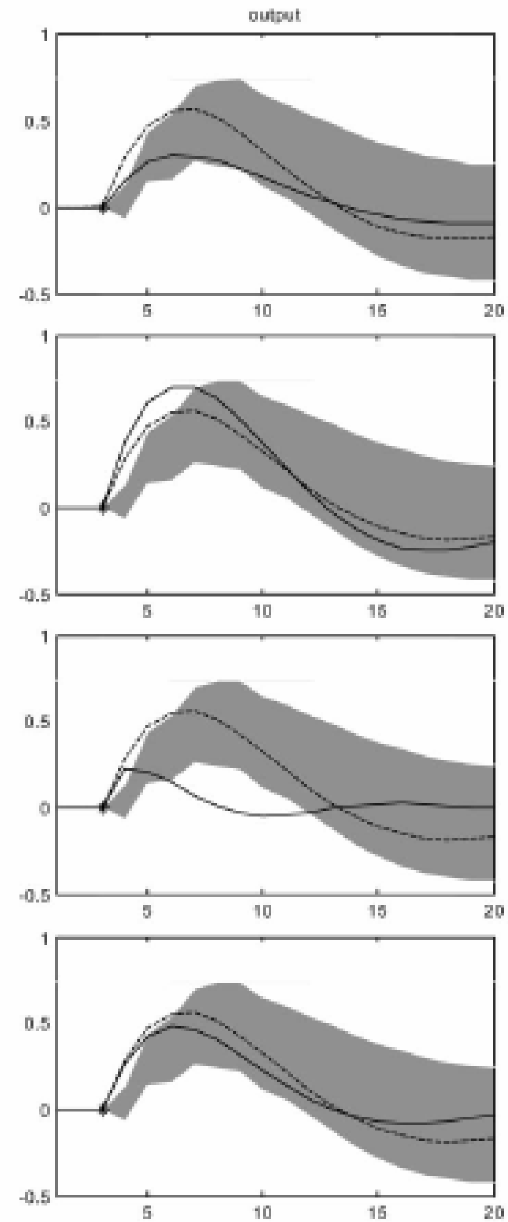
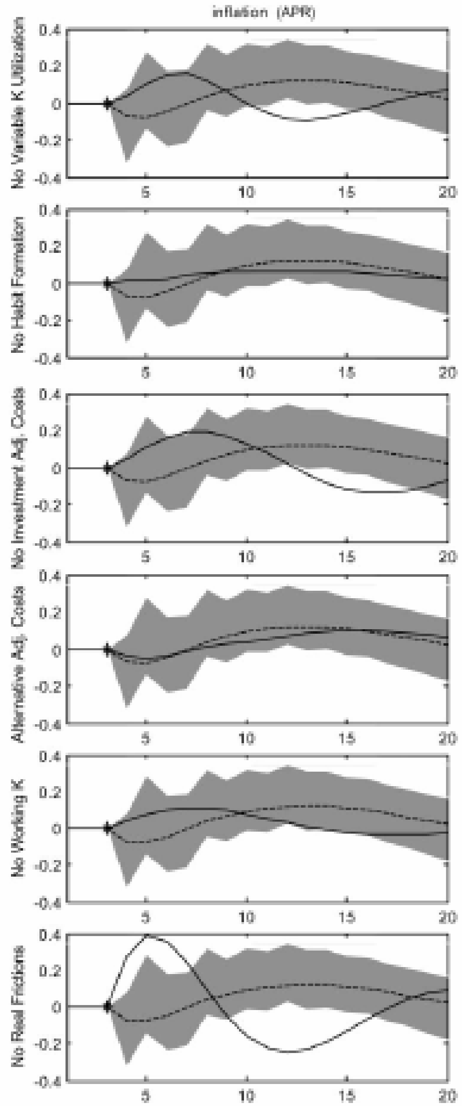


FIG. 4.—*Continued*

6.3 Role of real frictions

- Variable capital utilization
  - Propagation mechanism
- Adjustment costs to investment
  - Slow response of investment implies real interest rate does not fall in response to a monetary contraction
- Habit persistence
  - Growth rate of consumption is related to real interest rate- without habits, level of consumption related to interest rate $\Longrightarrow$ Habits responsible for hump shape response of consumption



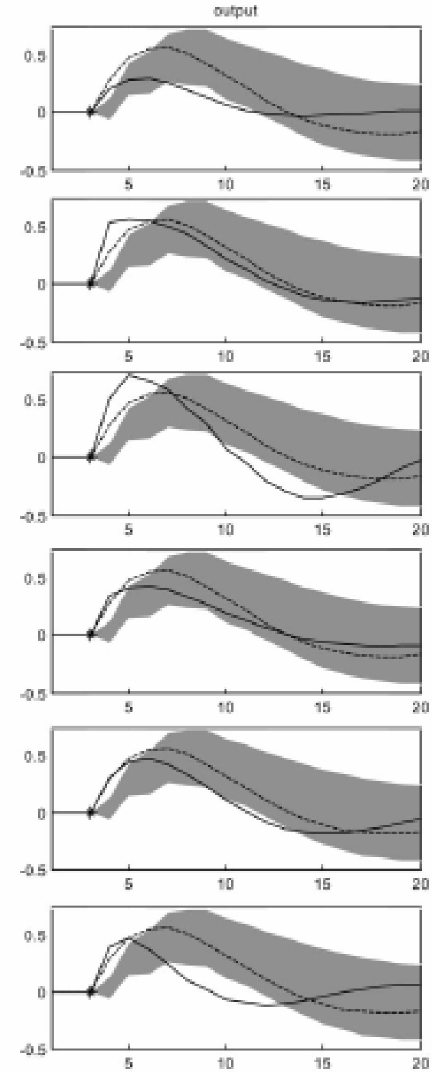


Fig. 6.—*Continued*

Fig. 6.—Variants of the benchmark model: perturbing the real side of the model economy. Solid lines are impulse responses for the model on the vertical axis; dashed lines are benchmark model impulse responses. Grey areas are 95 percent confidence intervals about VAR-based impulse responses.

6.4 Taylor Rule

- Alternative representation of monetary policy:

$$\hat{R}_t = \rho \hat{R}_{t-1} + (1-\rho)\left(a_\pi E_{t-1}\hat{\pi}_{t+1} + a_y \hat{y}_t\right) + \varepsilon_t$$

$$\rho = 0.8,\ a_\pi = 1.5,\ a_y = 0.1$$