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A Primer on Money in DGE Models

This handout illustrates some key properties of several benchmark monetary economies. The effects on anticipated and unanticipated monetary policy on real variables and asset prices are derived.

1 Introduction

There are three ways to introduce money in a DGE model:

- Cash-in-advance (Clower) constraints.
- Shopping-time/Transactions technology.
- Money in the utility function.

This handout illustrates the main properties of cash-in-advance models in economies with infinitely lived agents and briefly comment on the relation of the other types of monetary models to CIA economies in Section 2. Economies with price rigidities and portfolio frictions are presented in Section 3.

2 Classic Monetary Economies

2.1 CIA Models

In cash-in-advance (CIA) economies, agents hold currency because it is required to pay for purchases of goods or securities.

The most basic form of a CIA economy is populated by households, firms and a central bank (CB). There are three markets: a goods market, a labor market and an asset market. A full set of one-period-ahead state contingent claims is traded costlessly on the asset market.

- The central bank sets the money growth rate according to:

$$M_{t+1}^s = M_t^s (1 + x_t), \tag{1}$$

$$X_t = x_t M_t^s. \tag{2}$$

- Households are identical. They have preferences defined over consumption and leisure:
$$E_0 \sum_{t=0}^{\infty} \beta^t U\left(c_t, l_t\right), \tag{3}$$
where U satisfies standard properties.

- Firms are perfectly competitive. They hire labor on a competitive factor market and use it to produce consumption goods with a linear technology:
$$y_t \leq n_t,$$
where y denotes output and n demand for labor.

The properties of this economy depend crucially on the timing of transactions within each time period.

2.1.1 Lucas-Stokey Timing

1. Households enter the period with currency and bond holdings equal to A_t and B_t, respectively.

2. The money shock is realized.

3. Trading in assets takes place. Households are subject to the following constraint in asset market trading:
$$M_t^d + Q_t B_t^d \leq A_t + X_t, \tag{4}$$
where X_t is a transfer of currency from the CB to the household, which takes place on the asset market, and M_t is the amount of currency that household reserve to make goods purchases. Here, Q_t is a vector of prices for one-period ahead bonds that households decide to hold. Bonds are claims to one unit of currency in a subset of states (all states, in case of a risk-free bond, one particular state, in case of a state contingent bond).

4. The labor market meets, goods production takes place and the trading in goods occurs. The households face the following constraint on the goods market:
$$P_t c_t \leq M_t^d. \tag{5}$$

5. The household leaves the period with nominal wealth equal to:
$$A_{t+1} = M_t^d - P_t c_t + W_t\left(1 - l_t\right) + B_t^d, \tag{6}$$
where W_t is the nominal wage.

We will focus on sequence of markets equilibria. Given that markets are complete, we would get the same results analyzing an Arrow-Debreu equilibrium.

To analyze the properties of the equilibrium, we need to write down:

- Market clearing conditions for $t \geq 0$:

$$n_t = 1 - l_t,$$
$$c_t = n_t,$$
$$M^s_{t+1} = A_{t+1},$$
$$B^d_t = 0.$$

- Optimization conditions.

  - Firms:
$$P_t = W_t.$$

  - Households:

They choose c_t, l_t, M^d_t, B^d_t in each period to maximize (3), subject to non-negativity constraints, the CIA constraint and (4)-(6). To solve the household problem, it is useful to set up the Lagrangian. Here, constraint (6) has been substituted into (4):

$$\begin{aligned}\Lambda \;=\; & E_0 \left\{ \sum_{t=1}^{\infty} \beta^t \left[U\left(c_t, l_t\right) - \mu_t \left(P_t c_t - M_t\right) \right.\right. \\ & \left.\left. -\lambda_t \left(M^d_t + Q_t B^d_t - M^d_{t-1} - W_{t-1}\left(1 - l_{t-1}\right) + P_{t-1} c_{t-1} - B^d_{t-1} - X_t\right)\right]\right\} \\ & + U\left(c_0, l_0\right) - \mu_0 \left(P_0 c_0 - M_0\right) - \lambda_0 \left(M_0 + Q_0 B_0 - M_{-1} - X_0\right),\end{aligned}$$

where M_{-1} is exogenous.

First order necessary conditions for $t > 0$ are:

$$u_{ct} - P_t \left(\mu_t + \beta E_t \lambda_{t+1}\right) = 0, \tag{7}$$

$$\mu_t \left(P_t c_t - M^d_t\right) = 0,\ \mu_t \geq 0, \mathrm{CIA}, \tag{8}$$

$$u_{lt} - W_t \beta E_t \lambda_{t+1} = 0, \tag{9}$$

$$\mu_t - \lambda_t + \beta E_t \lambda_{t+1} = 0, \tag{10}$$

$$-\bar{Q}_t \lambda_t + \beta E_t \lambda_{t+1} = 0, \tag{11}$$

where $\bar{Q}_t$ is the price of a risk-free bond, and

$$\lim_{T \to \infty} \beta^T u_{cT} \left(M^d_T + B^d_T\right) = 0,$$

where u_{ct}, u_{lt} denote the marginal utility of consumption and leisure at time t.

Conditions analogous to (11) can be written for other assets.

Define:
$$R_t = 1/\bar{Q}_t.$$

Then, (11) implies that R_t depends on expected inflation only. Furthermore, from (10) and (11):

$$\mu_t = \lambda_t (1 - 1/R_t) > 0 \text{ for } R_t > 1.$$

Using (7) and (9):

$$\frac{u_{ct}}{u_{lt}} = R_t. \tag{12}$$

To characterize and solve for the equilibrium, note that (12) together with goods and labor market clearing can be used to solve for equilibrium consumption, which is a decreasing function of R_t. This is due to the fact that wages perceived in period t cannot be used for purchases of goods until period $t+1$. Therefore, labor income is "taxed" by expected inflation.

Assuming: $u(c,l) = \ln(c) + \psi \ln(l)\,,\ l = 1 - n$. The equilibrium can be characterized by solving the following system of equations:

$$\begin{aligned} \frac{u_{ct}}{u_{lt}} &= R_t \Longrightarrow c_t = n_t = [\psi R_t + 1]^{-1}, \\ P_t c_t &= (1 + x_t) M_t^s, \end{aligned}$$

$$\lambda_t = R_t \beta E_t \lambda_{t+1} \Longrightarrow \frac{1}{(1+x_t)} = \beta E_t \frac{1}{(1+x_t)(1+x_{t+1})},$$

$$u_{ct} = r_t \beta E_t u_{ct+1} \Longrightarrow [\psi R_t + 1] = r_t \beta E_t [\psi R_{t+1} + 1].$$

In addition, in this model velocity, v_t, is constant and identically equal to 1:

$$v_t = \frac{P_t c_t}{M_{t+1}^S} = 1.$$

We will define the real interest rate as the return on a notional bond that renders agents indifferent between transferring real wealth between the current date and next period. The marginal value of real wealth in this economy is $P_t \lambda_t$. Hence, the real interest rate solves:

$$P_t \lambda_t = \beta r_t E_t P_{t+1} \lambda_{t+1}.$$

Using the household's Euler equation, this implies:

$$u_{ct} = r_t \beta E_t u_{ct+1}.$$

Equilibrium effects of a positive innovation in x at t :

- If money growth is i.i.d.: No effects on consumption, nominal and real interest rates, since the allocation and asset prices depend on expected inflation only. P_t goes up one for one.

- If money growth is a positively autocorrelated, mean reverting process:

- R_t increases.
- Current consumption falls and expected future consumption falls by less, so that real interest rates tend to rise.
- P_t increases more than monetary growth.

- Velocity is constant.

2.1.2 Svensson Timing

This version of the CIA model, due to Svensson (1985), features a different sequencing of transactions within each period.

1. Households enter the period with currency and bond holdings equal to M_t and B_t, respectively.
2. The money shock is realized.
3. The labor market meets, goods production takes place and the trading in goods occurs. The households face the following constraint on the goods market:
$$P_t c_t \leq M_t.$$
4. Trading in assets takes place. Households are subject to the following constraint in asset market trading:
$$M_{t+1} + Q_t B_{t+1} \leq M_t - P_t c_t + W_t (1 - l_t) + B_t + X_t, \tag{13}$$
where X_t is a transfer of currency from the CB to the household, which takes place on the asset market.

We will focus on sequence of markets equilibria. Given that markets are complete, we would get the same results if characterizing an Arrow-Debreu equilibrium.

- Market clearing conditions for $t \geq 0$:
$$n_t = 1 - l_t,$$
$$c_t = n_t,$$
$$M_{t+1}^s = M_{t+1},$$
$$B_{t+1} = 0.$$

- Optimization conditions.
  - Firms:
$$P_t = W_t.$$
  - Households:

They chose c_t, l_t, M_t, B_{t+1} in each period to maximize (3), subject to non-negativity constraints, the CIA constraint and (13). The Lagrangian is given by:

$$\begin{aligned}\Lambda &= E_0\left\{\sum_{t=0}^{\infty}\beta^t\left[U\left(c_t,l_t\right)-\mu_t\left(P_tc_t-M_t\right)\right.\right.\\&\left.\left.-\lambda_t\left(M_{t+1}+Q_tB_{t+1}-M_t-W_t\left(1-l_t\right)+P_tc_t-B_t-X_t\right)\right]\right\},\end{aligned}$$

where M_0 is exogenous.

First order conditions for $t > 0$ are:

$$u_{ct} - P_t\left(\mu_t + \lambda_t\right) = 0, \tag{14}$$

$$\mu_t\left(P_tc_t - M_t\right) = 0,\ \ \mu_t \geq 0, \mathrm{CIA}, \tag{15}$$

$$u_{lt} - W_t\lambda_t = 0, \tag{16}$$

$$-\lambda_t + \beta E_t\left(\lambda_{t+1} + \mu_{t+1}\right) = 0, \tag{17}$$

$$-\bar{Q}_t\lambda_t + \beta E_t\lambda_{t+1} = 0, \tag{18}$$

$$\lim_{T\to\infty}\beta^T u_{cT}\left(M_T + B_T\right) = 0,$$

where $\bar{Q}_t$ is the price of a risk-free bond.

Define:

$$R_t = 1/\bar{Q}_t.$$

As in the Lucas-Stokey version, R_t depends on expected money growth only. This can be seen from combining (17) and (18) and applying the law of iterated expectations:

$$1/R_t = \beta\frac{E_tu_{ct+2}/P_{t+2}}{E_tu_{c_{t+1}}/P_{t+1}}.$$

There is a shift in timing relative to the Lucas version, however. This feature obtains from the fact that the receipts from a bond purchased at time t can only be used for consumption at time $t+2$. From (14) and (15), the shadow value of cash at t depends on realized inflation at t, i.e. P_t/P_{t-1} and is different from R_t and R_{t-1}. Moreover, the equilibrium level of consumption at time t is given by:

$$\frac{u_{ct}}{u_{lt}} = \frac{\mu_t + \lambda_t}{\lambda_t}. \tag{19}$$

Equilibrium consumption is a decreasing function of $\frac{\mu_t+\lambda_t}{\lambda_t}$. For example, with $u\left(c,l\right) = \log\left(c\right) + \gamma\log\left(l\right):$

$$\frac{1-c_t}{\gamma c_t} = \frac{\mu_t + \lambda_t}{\lambda_t}.$$

Therefore, $c_t = \left(1 + \gamma\frac{\mu_t+\lambda_t}{\lambda_t}\right)^{-1}$. This implies that u_c and u_l are respectively increasing and decreasing in $\frac{\mu_t+\lambda_t}{\lambda_t}$.

A central property of the Svensson timing, from (13), is that money transfers received in the current period are not more liquid than payments on bonds, since they cannot be used for purchasing goods until the following period. In the Lucas version, on the other hand, bonds acquired at time t are less liquid than currency acquired at t, since the latter can be used for purchases at t. However, holding currency overnight does provide liquidity services i.e. the ability to make purchases. Liquidity services can be seen as a dividend on holdings of currency. The realization of this dividend process is uncertain in real terms, since the real purchasing power of currency holdings depends on the realized price level in the following period.

Following Svensson (1985), currency can be priced as any other assets using (17). First, rewrite it as:

$$\lambda_t = \beta E_t[\left(1 + \frac{\mu_{t+1}}{\lambda_{t+1}}\right)\lambda_{t+1}].$$

The term μ_t/λ_t can be interpreted as the *dividend* from holding currency i.e. the shadow value of liquidity services at time t.

Iterating this equation forward over λ_t, one obtains:

$$\lambda_t = \left(\frac{1}{P_t}\right)u_{lt} = \sum_{j=1}^{\infty}\beta^j E_t\left(\mu_{t+j}\right).$$

Notice that the shadow value of nominal wealth increases if the cash in advance constraint is expected to bind in future periods. This is due to the fact that the dividend from holding cash is higher in states which feature a binding CIA constraint.

The price of other assets is also influenced by the expected behavior of the money growth process. For example, the nominal price of a stock, Q_t^s, with dividend process $\{\omega_t\}_t$. The first order condition for this asset is:

$$Q_t^s\lambda_t = \beta E_t[\left(Q_{t+1}^s + P_{t+1}\omega_{t+1}\right)\lambda_{t+1}].$$

Iterating on this equation:

$$Q_t^s\lambda_t = \sum_{j=1}^{\infty}\beta^j E_t\left(P_{t+j}\omega_{t+j}\lambda_{t+j}\right).$$

Since the path of λ_t is affected by the money growth process, the stock price and the return on stocks will also depend on the realization of the money growth process.

The real interest rate is given by: $\lambda_t P_t = r_t\beta E_t\left(\lambda_{t+1}P_{t+1}\right)$

$$r_t = \left(\frac{\beta E_t u_{lt+1}}{u_{lt}}\right)^{-1}.$$

The marginal value of real wealth with this timing corresponds to the marginal utility of leisure, rather than consumption, as in the standard model.

We can characterize the equilibrium by solving the following system of equations. From the household's Euler equation (17):

$$\frac{u_{l,t}}{P_t} = \beta E_t\left(\frac{u_{c,t+1}}{P_{t+1}}\right) \Longrightarrow \frac{\psi}{P_t - P_t c_{t^s}} = \beta E_t \frac{1}{M_{t+1}^S} = \frac{\beta}{\left(1+x_t\right) M_t^S}, \tag{20}$$

where we are assuming that expected nominal spending at time $t+1$ is equal to currency holdings. This will be the case whenever $R_t > 1$. If $R_t = 1$, we select the price sequence for which this condition holds.

We can then use the cash in advance constraint to determine equilibrium consumption. We can define a variable $\bar{c}_t$ which corresponds to planned or desired consumption. This simply corresponds to what agents expect to be able to consume based on time $t-1$ information. This corresponds to:

$$\frac{1-\bar{c}_t}{\psi \bar{c}_t} = R_t.$$

$$c_t = \min\left\{\frac{M_t^s}{P_t}, \bar{c}_t\right\} = \min\left\{\left[1 + \frac{\psi}{\beta}\left(1+x_t\right)\right]^{-1}, \bar{c}_t\right\}. \tag{21}$$

Equations (20) and (21) jointly determine the equilibrium level of P_t and c_t. If the cash in advance constraint is binding, (20) and (21) imply:

$$\begin{aligned} \frac{P_t}{M_t^s} - 1 &= \frac{\psi}{\beta}\left(1+x_t\right), \\ c_t &= \left[1 + \frac{\psi}{\beta}\left(1+x_t\right)\right]^{-1}. \end{aligned}$$

The equilibrium level of R_t satisfies:

$$1 = R_t \beta \frac{E_t\left[\left(1+x_{t+1}\right)\left(1+x_t\right)\right]^{-1}}{E_t\left(1+x_t\right)^{-1}} = \beta E_t\left(1+x_{t+1}\right)^{-1}.$$

If the cash in advance constraint is binding at time t, the equilibrium real interest rate r_t solves:

$$r_t = \left(\frac{\beta E_t\left(1-\bar{c}_{t+1}\right)}{1-\left[1+\frac{\psi}{\beta}\left(1+x_t\right)\right]^{-1}}\right)^{-1}.$$

It is not obvious how velocity should be defined in this model. Until the asset market takes place, the amount of currency in circulation is M_t^S, while end of period currency is M_{t+1}^s. If we define velocity with respect to end of period money supply and the cash in advance constraint is binding, then:

$$v_t = \frac{P_t c_t}{M_{t+1}^s} = \left(1+x_t\right)^{-1}.$$

Even defining velocity with respect to M_t^s, it need not be constant and equal to 1, because the CIA constraint could also be slack. See Svensson (1985) for details.

Effects of a positive innovation in the money growth process:

- i.i.d. money growth
  - Current consumption will be lower than average and the shadow value of liquidity will rise.
  - Real interest rates will fall, due to (16), since equilibrium employment falls.
  - Nominal interest rates will be constant.
- Positively autocorrelated, mean-reverting money growth
  - Future consumption will fall as well, resulting from the increase in expected inflation and therefore R_t.
  - The effect on real interest rates is ambiguous since the marginal utility of wealth in the subsequent periods increases (hence, the marginal utility of leisure at $t+1$ falls). For a mean reverting money growth process it should have the same sign as in the i.i.d. case.
  - Nominal interest rates will increase, but less than with Lucas timing.

2.1.3 Cash-Credit Good Models

In this class of models, introduced by Lucas and Stokey (1983), households preferences are defined over two types of consumption goods, c_1 and c_2. These goods are completely indistinguishable from the standpoint of production. The household faces a CIA constraint on purchases of c_1, while purchases of c_2 give rise to an obligation vis a vis goods producing firms that can be cleared at the end of the period.

We will assume that the timing of the goods and asset market follows Lucas. The economy can also be analyzed under the Svensson timing convention.

The representative household solves the following problem:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_{1t}, c_{2t}, l_t\right),$$

$$\begin{aligned} M_t^d + Q_t B_t^d \quad \leq \quad & M_{t-1}^d - P_{t-1} c_{1t-1} - P_{t-1} c_{2t-1} \\ & + B_{t-1}^d + W_{t-1}\left(1 - l_{t-1}\right) + X_t \text{ for } t > 0, \end{aligned}$$

$$M_0^d + Q_0 B_0^d \leq B_{-1} + X_0,$$

$$P_t c_{1t} \leq M_t^d \text{ for } t \geq 0,$$

with B_{-1} given.

The Lagrangian for the household problem is given by:

$$
\begin{aligned}
\Lambda &= E_0 \left\{ \sum_{t=1}^{\infty} \beta^t \left[U\left(c_{1t}, c_{2t}, l_t\right) - \mu_t \left(P_t c_{1t} - M_t^d\right) \right.\right. \\
&\quad \left.\left. - \lambda_t \left(M_t^d + Q_t B_t^d - M_{t-1}^d - W_{t-1}\left(1 - l_{t-1}\right) + P_{t-1} c_{1t-1} + P_{t-1} c_{2t-1} - B_{t-1}^d - X_t\right)\right]\right\} \\
&\quad + U\left(c_{10}, c_{20}, l_0\right) - \mu_0\left(P_0 c_{10} - M_0^d\right) - \lambda_0\left(M_0^d + Q_0 B_0^d - B_{-1} - X_0\right).
\end{aligned}
$$

Manipulation of the Euler equations gives rise to the following set of optimality conditions for the households:

$$
\frac{u_{1t}}{u_{2t}} = \frac{\mu_t + \beta E_t \lambda_{t+1}}{\beta E_t \lambda_{t+1}} = \frac{\lambda_t}{\beta E_t \lambda_{t+1}} = \frac{1}{Q_t} \equiv R_t \geq 1,
$$

$$
u_{1t} = \beta E_t \left(u_{1,t+1} R_{t+1} \frac{P_t}{P_{t+1}} \right),
$$

$$
u_{3t} + \frac{W_t}{P_t} u_{2t} = 0.
$$

The firms are perfectly competitive and their technology for goods production is given by:

$$
c_{1t} + c_{2t} \leq n_t^d.
$$

Optimality by firms implies:

$$
\frac{W_t}{P_t} = 1,
$$

in equilibrium.

Market clearing conditions are given by:

$$
n_t^d = 1 - l_t,
$$

$$
\begin{aligned}
M_t^d &= M_t^s, \\
B_t^d &= 0,
\end{aligned}
$$

$$
c_{1t} + c_{2t} = n_t^d,
$$

and must hold for all $t \geq 0$.

The properties of the response of this economy to monetary shocks are the same as in the Lucas version of the cash-in-advance model. However, velocity here responds to changes in the nominal interest rate.

This can be seen from the intratemporal optimization conditions for consumption, which in the steady state can be rewritten as:

$$
\frac{u_1}{u_2} = R. \tag{22}
$$

Assume that utility is of the form:

$$
u\left(c_1, c_2, l\right) = u\left(v\left(c_1, c_2\right), l\right).
$$

As an example consider a linear u, with:

$$v(c_1, c_2) = \frac{c_1^{1-\sigma_1} - 1}{1 - \sigma_1} + \frac{c_2^{1-\sigma_2} - 1}{1 - \sigma_2}.$$

In this economy output is given by $y = c_1 + c_2$, real money balances by $m = c_1$ and velocity by $v = y/m$. Then, we can rewrite (22) as:

$$\frac{m^{-\sigma_1}}{(y - m)^{-\sigma_2}} = R. \tag{23}$$

Therefore:

$$\frac{v^{\sigma_1 - \sigma_2}}{(v - 1)^{-\sigma_2}} y^{\sigma_2 - \sigma_1} = R,$$

determines steady state velocity. For the case $\sigma_1 = \sigma_2$, velocity increases with R.

In addition, the income elasticity of money demand can be obtained by differentiating (23) and is given by:

$$\frac{dm}{dy}\frac{y}{m} = \frac{1}{\frac{\sigma_1}{\sigma_2} + \left(\frac{\sigma_2 - \sigma_1}{\sigma_2}\right)\frac{1}{v}},$$

where v is steady state velocity. For $\sigma_1 = \sigma_2$, the income elasticity of demand for real balances is equal to 1.

2.2 Other models

2.2.1 Shopping Time Economies

It is also possible to introduce money in DGE economies by postulating that it reduces the time required for transactions. The earliest attempt at this formulation is due to McCallum (1983).

For example, consider the following economy, where household preferences are as in (3), firms and the monetary authority behave as in the previous sections. Households must spend time shopping to acquire consumption goods and currency reduces the time required to make purchases. Let s_t denote the amount of time the household requires to purchase a particular level of consumption c_t. Then:

$$s_t = H\left(c_t, \frac{M_{t+1}}{P_t}\right), \tag{24}$$

where H, H_c, H_{cc}, $H_{m/p,m/p} \geq 0$, and $H_{m/p}$, $H_{c,m/p} \leq 0$.

The only constraint that households face in a sequence of markets equilibrium is a dynamic budget constraint:

$$c_t + Q_t B_{t+1} + \frac{M_{t+1}}{P_t} = \frac{W_t}{P_t}(1 - l_t) + B_t + X_t + \frac{M_t}{P_t},$$

where M_t is nominal cash balances accumulated in period $t-1$ and M_{t+1} are nominal cash balances usable for transactions at time t.

Note that the formulation in (24) allows households to adjust their holdings of currency in response to current period realization of the money growth process. This is the counterpart to a "Lucas timing" formulation for this class of models. An alternative is to replace M_t for M_{t+1} in (24), implying that households cannot adjust their holdings of currency in response to the realization of the money growth process in the current period. This is the counterpart to the "Svensson timing" formulation.

Consider the following parametric example for $H(\cdot)$:

$$H\left(c, \frac{M}{P}\right) = \frac{c}{M/P}\varepsilon.$$

This corresponds to a transactions costs that would arise in a Baumol-Tobin framework. The ratio $\frac{c}{M/P}$ represents the number of trips to the bank to finance a level of consumption purchases c with currency. Then, ε is the time it takes to make a trip to the bank.

For a very interesting formulation of an economy in this class, see Jovanovic (1982).

2.2.2 Money in the Utility Function Models

In this class of models, currency provides direct utility to the households, whose preferences are specified as:

$$U\left(c, l, \frac{M}{P}\right).$$

Therefore, the value of liquidity services (in terms of consumption goods) is exogenously given by the ratio of the marginal utility of consumption and the marginal utility of real currency holdings. Here, M can be either currency which is inherited from the previous period or a level of currency which is chosen after the realization of the money growth process in the current period has been observed.

These alternative timings give very different results in terms of the properties of equilibria, as illustrated in Matsuyama (1991).

Correia and Teles (1997) describe conditions under which shopping time models are equivalent to particular money in the utility function models. Cash-credit good models are equivalent to money in the utility function models in which consumption corresponds to the sum of cash and credit good consumption while money balances are equal to cash good consumption. A general procedure to map a monetary model into a money in the utility function model is to construct an indirect utility function for the household problem at a given level of real money balances.

CIA or shopping time models have been preferred to money in the utility function models for policy analysis. This is due to the fact that the properties of the equilibrium in these classes of models are easier to relate to the primitives of the economic problems faced by agents. In money in the utility function models, results on the welfare implications of different monetary policies depend on the details of the preference specification. These details are hard to justify unless a primitive trading structure giving rise to a behavior of cash balances analogous to the money-in-the-utility function in question is invoked.

3 Models with Limited Information

In this section, we introduce two workhorse monetary models, the sticky price model and the limited participation model. Versions of these models are widely used as building blocks of richer models designed to closely replicate the empirical response of various economic variables to monetary shocks. The definition of a monetary shock in the context of the model is an unanticipated change in the money growth rate. If the money growth rate follows a known stochastic process, the monetary shock is the innovation in this process. If the money growth rate is deterministic, the monetary shock can be seen as a one time unforeseen change in the money growth rate.

Sticky price and limited participation models are widely used for policy analysis. In both models, a subset of agents in the economy makes decisions under "limited information" with respect to the current realization of the monetary growth process. We analyze very simple versions of each of the models.

3.1 A Sticky Price Economy

Sticky price models have are based on two main assumptions. First, a subset of the firms in the economy are monopolistically competitive. This means that they set prices to maximize profits, taking as given the demand for their product and factor prices. These firms are competitive on factor markets but behave as monopolists in the product market. One way to think about this assumption is that these firms produce a special good but they are relatively small. Then, each of these firms will have market power but their factor demand will not affect the equilibrium on factor markets. The second crucial assumption in sticky price models is that a subset of the monopolistically competitive firms set their price before the current realization of monetary policy is realized. Here, we analyze a particular version of a sticky price model with a CIA structure, Lucas-Stokey timing and no physical capital.

3.1.1 Model

The economy is inhabited by households, intermediate good producers and final good producers, as well as a monetary authority. The monetary authority prints money and transfer it to the households lump sum. The monetary transfer is the only exogenous driving variable in this economy. We describe the problems of each type of agents in this economy in turn.

Households There is a continuum of identical households. The household's expected utility function is:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t), \; u(c, l) = \log c + \gamma \log(l), \; l = 1 - n, \tag{25}$$

where c_t, and n_t denote consumption and employment, respectively, and l_t denotes leisure.

The sequence of events in the period is as follows. At the beginning of the period, the household trades in a securities market and allocates nominal assets between money and bonds. After trading in the securities market, the household supplies labor and consumes.

For securities market trading, the constraint is

$$A_t + X_t \geq M_t + Q_t B_t, \tag{26}$$

where A_t denotes beginning-of-period t nominal assets, X_t is the transfer from the monetary authority, M_t denotes the household's holdings of cash, B_t denotes the household's purchases of interest bearing bonds and Q_t is the bond price, and A_0 is given. Consumption goods must be paid for with currency from securities market trading. The cash in advance constraint is given by:

$$P_t c_t \leq M_t. \tag{27}$$

The household's sources of cash during securities market trading are cash left over from the previous period's goods market, $M_{t-1} - P_{1t-1} c_{t-1}$, earnings on bonds accumulated in the previous period, $R_{t-1} B_{t-1}$, labor income in the previous period, $W_{t-1} n_{t-1}$, and profits in the previous period, Π_{t-1}. Let P_{t-1} and R_{t-1} denote the period $t-1$ prices of consumption goods, and the gross nominal interest rate, respectively. The households' securities market constraint is:

$$A_t = W_{t-1} n_{t-1} + (M_{t-1} - P_{t-1} c_{t-1}) + B_{t-1} + \Pi_{t-1}. \tag{28}$$

We place the following restriction on the household's ability to borrow:

$$A_{t+1} \geq -\frac{1}{q_{t+1}} \sum_{j=1}^{\infty} q_{t+j+1} \left[W_{t+j} + T_{t+j-1} + \Pi_{t+j} \right], \text{ for } t = 0, 1, 2, ..., \tag{29}$$

where $q_t = \prod_{j=0}^{t-1} 1/R_j$, $q_0 \equiv 1$. Condition (29) says that the household can never borrow more than the maximum present value future income (it is the no Ponzi games condition).

The household's problem is to maximize (25) subject to (29)-(26) and the non-negativity constraints, n_t, c_t, $1 - n_t \geq 0$. We assume throughout that $R_t \geq 1$.

Monetary Authority At date t, the monetary authority transfers X_t units of cash to the representative household. It finances the transfers by printing money. Let x_t denote the growth rate of the money supply. Then, $X_t = x_t M_t$, where M_0 is given and $M_{t+1} = (1 + x_t) M_t$. A monetary policy is an infinite sequence, x_t, $t = 0, 1, 2, ...$.

Firms The firm sector is a variant of the production framework in Blanchard and Kiyotaki (1987). Since firms are static, we can delete time subscripts. In each period, there is a continuum of perfectly competitive, final goods firms that produce consumption goods using intermediate goods. Their production functions are:

$$y = \left[\int_0^1 y(\omega)^\lambda d\omega\right]^{\frac{1}{\lambda}}, \tag{30}$$

where y denotes output, and $y(\omega)$ is the quantity of intermediate good of type ω used to produce good i, and $0 < \lambda < 1$. These firms solve:

$$\max_{y,\{y(\omega)\}} Py - \int_0^1 P(\omega)y(\omega)d\omega.$$

The first order necessary condition for $y(\omega)$, after substituting (30) is:

$$Py(\omega)^{\lambda-1}\left[\int_0^1 y(\omega)^\lambda d\omega\right]^{\frac{1}{\lambda}-1} - P(\omega) = 0.$$

This can be simplified to yield the following demand curves for each intermediate good:

$$y(\omega) = y\left[\frac{P}{P(\omega)}\right]^{\frac{1}{1-\lambda}}. \tag{31}$$

Intermediate good firms are monopolists in the product market and competitors in the market for labor. *They set prices for their goods and are then required to supply whatever final good producers demand at those prices.* We will consider two different cases. If prices are flexible, intermediate good firms set their prices after the current monetary transfer is realized. In this case, they solve the following problem:

$$\max_{y(\omega)} P(\omega)y(\omega) - Wn(\omega),$$

where W is the wage rate, subject to a production technology, $y(\omega) = n(\omega)$ and the demand curve in (31).

The first order necessary condition for $P(\omega)$, after substituting (31) and technology, is:

$$\frac{-\lambda}{1-\lambda}P(\omega)^{\frac{-1}{1-\lambda}}P^{\frac{1}{1-\lambda}} - W\frac{-1}{1-\lambda}P(\omega)^{\left(\frac{-1}{1-\lambda}-1\right)}P^{\frac{1}{1-\lambda}} = 0,$$

which simplifies to:

$$P(\omega) = \frac{W}{\lambda}. \tag{32}$$

Here, $1/\lambda$ is the mark-up of price over marginal cost, which in this version of the model with no capital is simply the nominal wage. The price set by intermediate good firms will be a function of the current value of x_t, which is the only exogenous state in this economy.

If intermediate good firms set their prices before the current monetary policy shock is realized, their problem is:

$$\max_{y(\omega)} P(\omega)y(\omega) - E_{-1}Wn(\omega),$$

where $E_{-1}W$ is the expected wage rate, which is a function of the state in the previous period, subject to a production technology, $y(\omega) = n(\omega)$ and the demand curve in (31). Profit maximization leads the intermediate good firms to set prices according to:

$$\bar{P}(\omega) = \frac{E_{-1}W}{\lambda}, \tag{33}$$

where we use a bar to denote the fact that the price is fixed. Note that firms set their prices and then satisfies whatever level of demand materializes at those prices. They hire labor after their demand is realized and they will pay the going wage rate.

3.1.2 Equilibrium

We begin by defining a private sector equilibrium (sequence of markets).

Definition 1 *A Private Sector Equilibrium with flexible prices is a set of sequences,* $\{P_t,\ W_t,\ R_t,\ c_t,\ n_t,\ B_t,\ M_t,\ x_t\}$ *with the properties:*

1. *Given the prices and the government policies the quantities solve the household problem.*
2. *The firm optimality conditions in (32) hold;*
3. $R_t \geq 1$;
4. *The various market clearing conditions hold:*

$$c_t = n_t,\ \ B_t = 0,\ \ M_{t+1} = M_t\left(1 + x_t\right). \tag{34}$$

The household problem can be solved by setting up the following Lagrangian:

$$\begin{aligned} L &= E_0 \sum_{t=0}^{\infty} \beta^t \left\{ u\left(c_t, l_t\right) - \mu_t\left(P_t c_t - M_t\right) \right. \\ &\quad \left. - \lambda_t\left[M_t + Q_t B_t - X_t - A_t\right] \right\}, \end{aligned}$$

where A_t can be substituted using (28) for $t > 0$. The households first order necessary conditions for optimality are, after simplification:

$$u_{ct} = P_t\lambda_t, \tag{35}$$

$$\frac{u_{ct}}{u_{lt}} = \frac{P_t\left(\mu_t + \beta E_t\lambda_{t+1}\right)}{W_t\beta E_t\lambda_{t+1}}, \tag{36}$$

$$R_t = 1/Q_t = \lambda_t/\beta E_t\lambda_{t+1}. \tag{37}$$

Flexible Prices Assuming prices are flexible, the additional equilibrium condition (32) can be used to solve for equilibrium consumption, jointly with (36). Assuming the cash in advance constraint holds with equality even when it is not binding, the cash in advance constraint determines equilibrium prices given the nominal money supply and consumption:

$$P_t = M_t^s / c_t.$$

Note that in equilibrium $A_t = M_{t-1}^s$.

We summarize these results in the form of a proposition.

Proposition 2 *(Characterization Result) A sequence, $\{P_t,\ W_t,\ R_t,\ c_t,\ n_t,\ B_t,\ M_t,\ x_t\}$, is an equilibrium if and only if (32)-(39) are satisfied. Furthermore, for any $R_t \geq 1$, there exists a private sector equilibrium with employment and consumption allocations uniquely determined by:*

$$n_t = c_t = \frac{\lambda}{\lambda + \gamma R_t},\ \textit{all } t, \tag{38}$$

$$1 = \beta R_t E_t \left(\frac{1}{1 + x_{t+1}} \right). \tag{39}$$

The proof can be obtained as in Section 1. Note that equilibrium consumption is decreasing in the mark-up $(1/\lambda)$ and in the nominal interest rate R_t. Monopolistic competition introduces an additional wedge or distortion in the economy, which reduces equilibrium output and consumption.

Sticky Prices We assume that the process for the variable x_t is first order Markov and we focus on stationary equilibria. A stationary equilibrium is one in which all equilibrium variables are time independent and only depend on the "state". Here, despite the fact that x_t is the only exogenous variable, the state is given by $\{x_{t-1}, x_t\}$, since intermediate good producers decide on prices based on time $t-1$ information. The state can be further simplified by noticing that the only variable that depends on x_{t-1} is P_t. So a stationary equilibrium can equivalently be defined as a function of the state $\left\{\bar{P}_t, x_t\right\}$. Lastly, since M_t^s does not influence real variables in equilibrium, it is useful to rescale all nominal variables by beginning of period money supply. We will denote rescaled variables with small case letter, e.g. $a = A/M^s$. Then we can define the state as follows: $s = \{\bar{p}, x\}$.

Definition 3 *A stationary equilibrium with sticky prices is a private sector equilibrium in which, in each period, the variables $\{c, w, R\}$ depend on $s = \{\bar{p}, x\}$ and $p(s) = \bar{p}$.*

The conditions that pin down equilibrium allocations and prices are as follows. Final goods firms optimization implies $p(s) = \bar{p}$. The cash in advance constraint determines consumption:

$$c(s) = \frac{1 + x}{\bar{p}}. \tag{40}$$

Condition (36) can be used to determine the wage rate:

$$\frac{1-c(s)}{\gamma c(s)} = R\frac{\bar{p}}{w(s)}. \tag{41}$$

The intertemporal Euler equation can be used to determine the nominal interest rate:

$$1 = \beta R E\left(\frac{1}{1+x'}\right).$$

Properties of Stationary Equilibria

- Consumption and the wage are increasing in x.
- Profits are decreasing in x. To see this note that:

$$\pi(s) = \bar{p}c(s) - w(s)c(s) = (\bar{p} - w(s))c(s),$$

and substitute using (40)-(41).

3.2 A Limited Participation Economy

This is a class of models in firms need to borrow to finance purchases of working capital. Working capital should be interpreted as short run working expenses, such as rental costs of various factors or inventories. This generates a demand for liquidity by firms. Prices are completely flexible. Households are subject to a CIA constraint, and hence must hold cash to finance their purchases of consumption goods. In addition, they can make deposits with financial intermediaries and earn a gross rate of return equal to the nominal interest rate. Hence, the nominal interest rate is the opportunity cost of holding cash for households and the price of liquidity for firms. Financial intermediaries are perfectly competitive and just serve as a clearing house between household and the firm sector. The crucial assumption is that households cannot adjust their deposits to the current realization of the monetary policy process. This portfolio friction implies that expansionary monetary policy shocks have a positive effect on output, real wages and profits. A useful reference for this type of models is Christiano, Eichenbaum and Evans (1997).

3.2.1 Model

The economy is populated by a continuum of identical households, final goods firms, intermediate good firms, financial intermediaries and a monetary authority. We describe each type of agent in turn.

Households The representative household orders sequences of consumption and leisure according to (3), with

$$u(c, l) = \log(c) + \gamma \log(l).$$

Households enter the period with currency holdings M_t and make deposits D_t with the financial intermediaries:

$$0 \leq D_t \leq M_t. \tag{42}$$

They face the following constraint on goods markets:

$$P_t c_t \leq W_t (1 - l_t) + M_t - D_t, \tag{43}$$

and their end of period wealth is given by:

$$M_{t+1} = W_t (1 - l_t) + M_t - D_t - P_t c_t + R_t (D_t + X_t) + \Pi_t, \tag{44}$$

where R_t is the return on deposits and π_t are profits.

The non-negativity constraints on M_t and D_t in effect rule out Ponzi schemes, so the intertemporal consumption set is well defined.

The Lagrangian for the household problem is given by:

$$\begin{aligned} \mathcal{L}_t = \; & E_t \sum_{s \geq 0} \beta^s \{[\log(c_{t+s}) + \gamma \log(1 - n_{t+s})] - \xi_s (D_{t+s} - M_{t+s}) \\ & -\mu_{t+s} [P_{t+s} c_{t+s} - (M_{t+s} - D_{t+s} + W_{t+s} n_{t+s})] \\ & -\eta_{t+s} [M_{t+s+1} - (M_{t+s} - D_{t+s} + W_{t+s} n_{t+s} - P_{t+s} c_{t+s}) - (D_{t+s} + X_{t+s}) R_{t+s} + \Pi_{t+s}]\}, \end{aligned}$$

where ξ, μ, λ are non-negative Lagrange multipliers. The necessary and sufficient conditions for an optimum for this problem are:

$$\frac{1}{c_t} - (\mu_t + \eta_t) P_t = 0, \tag{45}$$

$$\frac{-\gamma}{1 - n_t} + (\mu_t + \eta_t) W_t = 0, \tag{46}$$

$$\eta_t - \beta \xi_{t+1} - \beta E_t (\mu_{t+1} + \eta_{t+1}) = 0, \tag{47}$$

$$-\xi_t + E_{t-1} [-\mu_t + \eta_t (R_t - 1)] \begin{cases} \leq 0, \\ = 0, \text{ for } D_t > 0 \end{cases}, \tag{48}$$

$$\mu_t [P_t c_t - (M_t - D_t + W_t n_t)] = 0, \mu_t \geq 0, P_t c_t \leq M_t - Q_t + W_t n_t, \tag{49}$$

$$\xi_t (M_t - D_t) = 0, \xi_t \geq 0, \; Q_t \leq M_t \tag{50}$$

$$\lim_{T \to \infty} \beta^t E_{T-1} (\eta_T M_{T+1}) = 0. \tag{51}$$

Firms Final goods firms are as in section 3.1.1.

Intermediate good firms are as in section 3.1.1, except for the fact that they need to disburse their wage bill before they realize their revenues. In each period, they borrow an amount L_t from financial intermediaries and use it to finance the wage bill. They repay principal and interest, at rate R_t, on their loans at the end of the period. Intermediate goods firms solve the following problem:

$$\pi = \max_{P(\omega),L} P(\omega)\, y(\omega) - Wy(\omega) + L - RL,$$

subject to (31) and:

$$Wn \leq L. \tag{52}$$

The first order necessary condition for this problem is simply:

$$P(\omega) = \frac{WR}{\lambda}. \tag{53}$$

The need to borrow to finance wage disbursement increases the marginal cost of hiring workers.

Financial Intermediaries Financial intermediaries behave competitively. They receive D_t in deposits from households, and X_t on households' behalf from the central bank. The representative financial intermediary receives deposits from households and cash from the monetary authority which it lends out to firms. It maximizes its cash-flow given by:

$$(D_t + X_t - L_t)(1 - R_t),$$

subject to

$$D_t + X_t - L_t \geq 0.$$

Given perfect competition, it is optimal for financial intermediaries to loan all available funds when $R_t > 1$. Financial intermediaries are indifferent between making loans and just "sitting on" the available liquidity when $R_t = 1$ (this is a "liquidity trap"). We will make the assumption that financial intermediaries loan out the funds in case of indifference, so that $L_t = Q_t + X_t$.

The monetary authority controls the money supply. Its budget constraint is given by:

$$M^s_{t+1} - M^s_t = (x_t - 1)\, M^s_t = X_t,$$

where x_t is the gross rate of money growth and M_t the aggregate stock of money. We assume that the process for x_t is first order Markov.

3.2.2 Equilibrium

The timing of events is as follows:

1. Households choose D_t.

2. The monetary shock is realized.

3. The loan market clears.

4. The consumption and labor markets meet.

Definition 4 *A private sector equilibrium for this economy is a price system* $\{P_t, W_t, R_t\}_{t\geq 0}$ *, an allocation* $\{c_t, n_t, y_t, L_t, D_t, M_t\}_{t\geq 0}$ *and a monetary growth rate path* $\{X_t\}_{t\geq 0}$ *such that:*

1. *the allocation satisfies households', firms' and financial intermediaries' conditions for optimization given the price system and the money growth rate path;*

2. $R_t \geq 1$ *for all* $t = 0, 1, 2, ...$

3. *at the given allocation, price system and money growth rate path the following market clearing conditions hold:*

$$\begin{aligned} D_t + X_t &= L_t, \\ L_t &= W_t n_t, \\ c_t &= y_t = n_t. \end{aligned}$$

We will now focus attention on stationary equilibria. The exogenous state variable for this economy at time t is x_t. However, since households choose deposits before x_t is realized, they base their choices on x_{t-1}. Hence, the "economic" state for this model is $s_t = \{x_t, x_{t-1}\}$.

Let $d = d/M$, $p \equiv P/M, w \equiv W/M$, and $x \equiv X/M$.

Definition 5 *A stationary equilibrium is a private sector equilibrium in which the allocation* $\{c, n, d\}$*and the price system* $\{p,\ w,\ R\}$ *depend on the economic state,* $s = \{x_{-1}, x\}$ *, only.*

Note that d is a function of x_{-1} only, so that $d\,(x_{-1})$.

A stationary equilibrium can be characterized by solving for c, n, p, w , R for a given d and x first, and then solving for d as a function of of x_{-1}. The first step involves the following static optimization conditions and market clearing conditions.

Loan and money market clearing imply:

$$pc = 1 + x. \tag{54}$$

From the households' static Euler equations and firm optimality:

$$\frac{1-c}{\gamma c} = \frac{R}{\lambda}, \tag{55}$$

where $c = n = 1 - l$ from the resource constraint and clearing on the labor market.

The nominal interest rate can be determined as follows. Define:

$$\Gamma = \frac{Wn}{Pc},$$

the ratio of intermediated vs total liquidity in this economy. Then, from loan market clearing and (54):

$$\Gamma = \frac{d+x}{1+x} \leq 1,$$

since $d \in [0,1]$. This implies that Γ is increasing in x.

Using the resource constraint and optimality by firms, Γ can also be written as:

$$\Gamma = \frac{\lambda}{R}.$$

Therefore:

$$R(s) = \lambda \frac{1+x}{d+x}, \tag{56}$$

which implies that R is decreasing in x. From (55):

$$c(s) = \left[1 + \gamma \frac{1+x}{d+x}\right]^{-1},$$

so that output is increasing in x. Notice that these results hold independently from the properties of the stochastic process for x.

The households' intertemporal Euler equation can be used to solve for d as a function of x_{-1}. This is obtained by combining (47) and (48) at an equilibrium in which $d \in (0,1)$, so that $\xi = \xi' = 0$.

$$E_{-1}\left(\frac{1}{pc}\right) = \beta E_{-1}\left(\frac{R}{1+x} E\left(\frac{1}{p'c'}\right)\right), \tag{57}$$

where a prime denotes one period ahead variables. This simplifies to:

$$E_{-1}\left(\frac{1}{1+x}\right) = \beta E_{-1}\left[\left(\lambda \frac{1+x}{d+x}\right) \frac{1}{1+x} E\left(\frac{1}{1+x'}\right)\right], \tag{58}$$

using (54) and (56). This is a functional equation in $d(x_{-1})$.

Example: x is i.i.d., with support $\{x_l, x_h\}$ and probability distribution $\{\pi_l, \pi_h\}$, $\pi_l, \pi_h > 0$ and $\pi_l + \pi_h = 1$. Then, $d(x_{-1}) = d$ and (58) implies:

$$1 = \lambda \beta \sum_{j=l,h} \pi_j \frac{1}{d + x_j}.$$

This equation can be used to solve for d.

Properties of Stationary Equilibria

- Consumption is decreasing in the nominal interest rate.
- A monetary expansion determines a decline in R, from (56). This is known as the "liquidity effect".
- Consumption prices move less than one for one with money growth:

$$p(s) = \frac{1+x}{c(s)} = (1+x)\left[1+\gamma\frac{1+x}{d+x}\right],$$

  since the second term falls with x.
- Equilibrium profits for intermediate good firms are given by:

$$\pi(s) = (1-\lambda R(s))\, p(s)\, c(s),$$

  and they are increasing in x.

4 Appendix: Monetary Shocks in Deterministic Models

We present a simple way to analyze the effects of monetary policy shocks in models in which the money growth process is deterministic. This is exercise is useful for isolating the effects of innovations in the monetary policy variable for models in which it is stochastic.

We assume that agents expect monetary policy to be a certain sequence $\{x_t\}_{t=0}^{\infty}$, but at time τ, the monetary policy variable is actually $\hat{x}_\tau \neq x_\tau$. This is a one-time, unanticipated change in monetary policy. We assume that $R_t > 1$ for all t, so that the cash in advance constraint is binding for the expected monetary policy. We will denote with the symbol ˆ equilibrium values of variables for the monetary process corresponding to the unanticipated change in policy at time τ, and we will compare them with the equilibrium values of the same variables absent the change.

We denote with $\bar{W}_\tau$ the wage that would prevail at time τ under the predicted monetary policy. The the price level will be fixed at $P_\tau = \bar{P} = \bar{W}_\tau/\lambda$, irrespective of the value of x_τ. The cash in advance constraint determines the demand for consumption:

$$\hat{c}_\tau = (1+\hat{x}_\tau)\, M_\tau^s/\bar{P}. \quad (59)$$

Condition (36) determines the equilibrium nominal wage rate W_τ :

$$\frac{1-\hat{c}_\tau}{\gamma\hat{c}_\tau} = \frac{\bar{P}}{W_\tau} R_\tau. \quad (60)$$

The nominal interest rate at time τ, is pinned down by the intertemporal Euler equation for bonds and is independent of x_τ. R_τ depends on $x_{\tau+1}$ only.

Note that if $\hat{x}_\tau > x_\tau$, $\hat{c}_\tau > c_\tau$, from (59).[1] Then, (60) implies that $\hat{x}_\tau > x_\tau$ induces a rise in the real wage in equilibrium. Therefore, it reduces the mark-up, which is equal to the inverse of the real wage. Hence, if monetary policy is more expansionary than expected in a given period, it increases consumption and real wages in that period and reduces the mark-up, thereby reducing the degree of distortion in equilibrium for the economy.

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[1] Note that if $\hat{x}_\tau = x_\tau$, the equilibrium with sticky prices has the same prices and allocations that the equilibrium with flexible prices. Sticky prices only matter when there is an unanticipated component in monetary policy.