Optimal Fiscal and Monetary Policy

Outline

(1) Background: Phelps-Friedman Debate

(2) Some Ideas from Public Finance - Ramsey Theory

– Policy

– Private Sector Equilibrium

– Private Sector Allocation Rule

– Ramsey Problem

– Ramsey Equilibrium

– Implementability Constraint

– Ramsey Allocation Problem

– Ramsey Allocations

(3) Evaluating Phelps-Friedman Debate Using Lucas-Stokey Cash-Credit Good Model

(a) General Remarks

(b) Model

(c) Ramsey Problem, Ramsey Allocation Problem

(d) Surprising Result:
Friedman is "Right" for Lots of Parameterizations (Used Homotheticity and Separability).

(4) Interpretation of Result

(a) Homotheticity and Separability Corresponds to Unit Consumption Elasticity of Money Demand

(b) Uniform Taxation Result in Public Finance for Non-Monetary Economies

(c) What Happens When You Don't Have Unit Elasticity?

(d) Who Is Right, Friedman or Phelps?

Friedman-Phelps Debate

- Money Demand:

$$\frac{M}{P} = \exp[-\alpha R]$$

- Friedman:
  (a) Efforts to Economize Cash Balances when R High is Socially Wasteful
  (b) Set R as Low As Possible - $R = 1$.
  (c) Since $R = r + \pi$, Frieman Recommends $\pi = -r$.
    (i) $r \sim$ exogenous real interest rate rate
    (ii) $\pi \sim$ inflation rate, $\pi = (P - P_{-1})/P_{-1}$

- Phelps:

(a) Inflation Acts Like a Tax on Cash Balances -

$$\begin{aligned} \text{Seignorage} &= \frac{M_t - M_{t-1}}{P_t} = \frac{M_t}{P_t} - \frac{P_{t-1}}{P_t}\frac{M_{t-1}}{P_{t-1}} \\ &\approx \frac{M}{P}\frac{\pi}{1+\pi} \end{aligned}$$

(b) Use of Inflation Tax Permits Reducing Some Other Tax Rate

(c) Extra Distortion in Economizing Cash Balances Compensated by Reduced Distortion Elsewhere.

(d) With Distortions a Convex Function of Tax Rates, Would Always Want to Tax All Goods (Including Money) At Least A Little.

(e) Inflation Tax Particularly Attractive if Interest Elasticity of Money Demand Low.

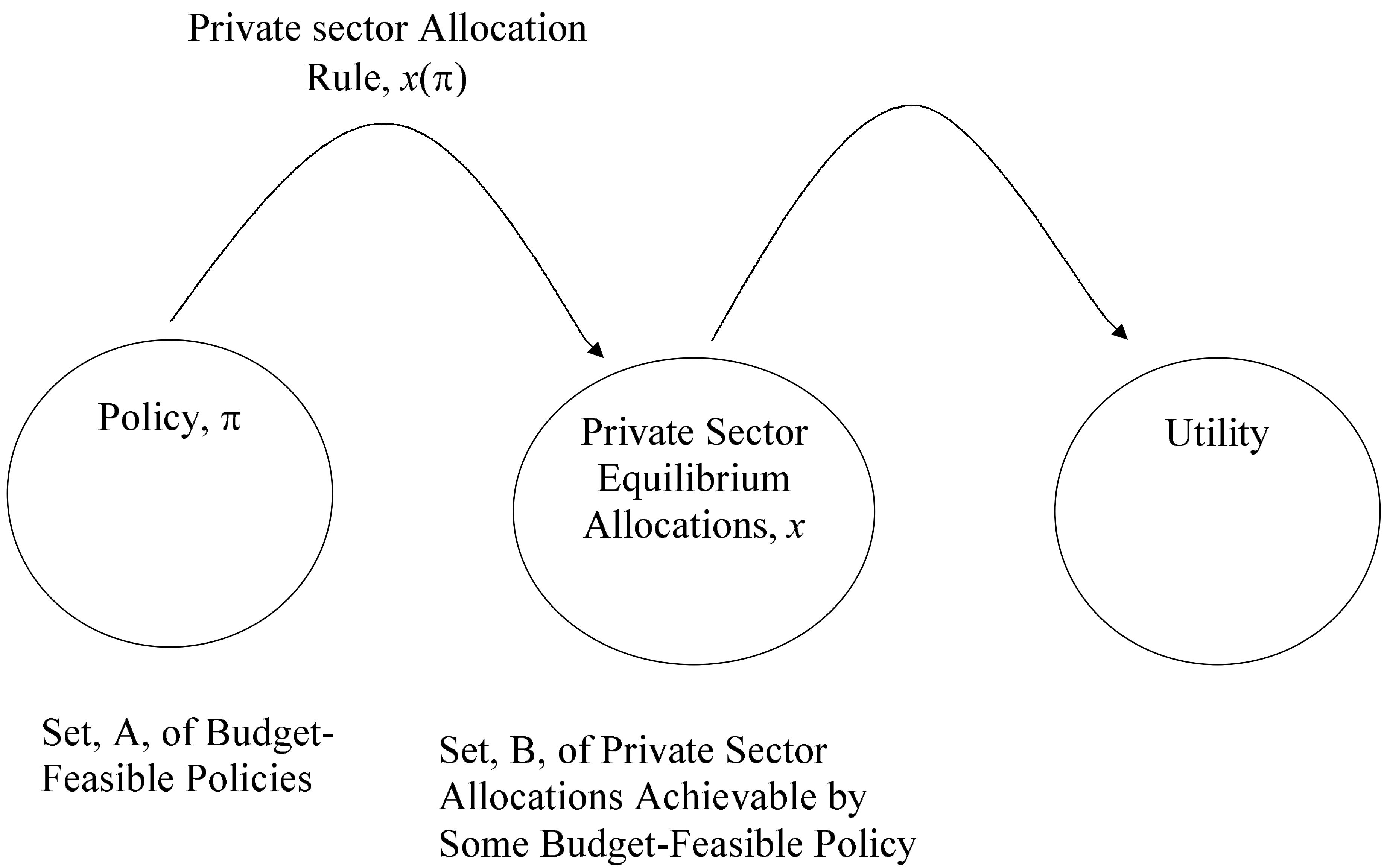
Question: Who is Right, Friedman or Phelps?

- Answer: Friedman Right Surprisingly Often
- Depends on Income Elasticity of Demand for Money
- Will Address the Issue From a Straight Public Finance Perspective, In the Spirit of Phelps.
- Easy to Develop an Answer, Exploiting a Basic Insight From Public Finance.

Some Basic Ideas from Ramsey Theory

- **Policy**, π, Belonging to the Set of 'Budget Feasible' Policies, A.
- **Private Sector Equilibrium Allocations**, Equilibrium Allocations, x, Associated with a Given π; $x \in B$.
- **Private Sector Allocation Rule**, mapping from π to x (i.e., $\pi : A \to B$).
- **Ramsey Problem**: Maximize, w.r.t. π, $U(x(\pi))$.
- **Ramsey Equilibrium**: $\pi^* \in A$ and x^*, such that π^* solves Ramsey Problem and $x^* = x(\pi^*)$. 'Best Private Sector Equilibrium'.

- **Ramsey Allocation Problem**: Solve, $\tilde{x} = \arg max\, U(x)$ for $x \in B$
- Alternative Strategy for Solving the Ramsey Problem:
  (a) Solve Ramsey Allocation Problem, to Find $\tilde{x}$.
  (b) Execute the Inverse Mapping, $\tilde{\pi} = x^{-1}(\tilde{x})$.
  (c) $\tilde{\pi}$ and $\tilde{x}$ Represent a Ramsey Equilibrium.
- **Implementability Constraint**: Equations that Summarize Restrictions on Achievable Allocations, B, Due to Distortionary Tax System.


Private sector Allocation Rule, $x(\pi)$
Policy, π
Private Sector Equilibrium Allocations, x
Utility
Set, A, of Budget-Feasible Policies
Set, B, of Private Sector Allocations Achievable by Some Budget-Feasible Policy

Example

- Households:

$$\max_{c,l} u(c, l)$$

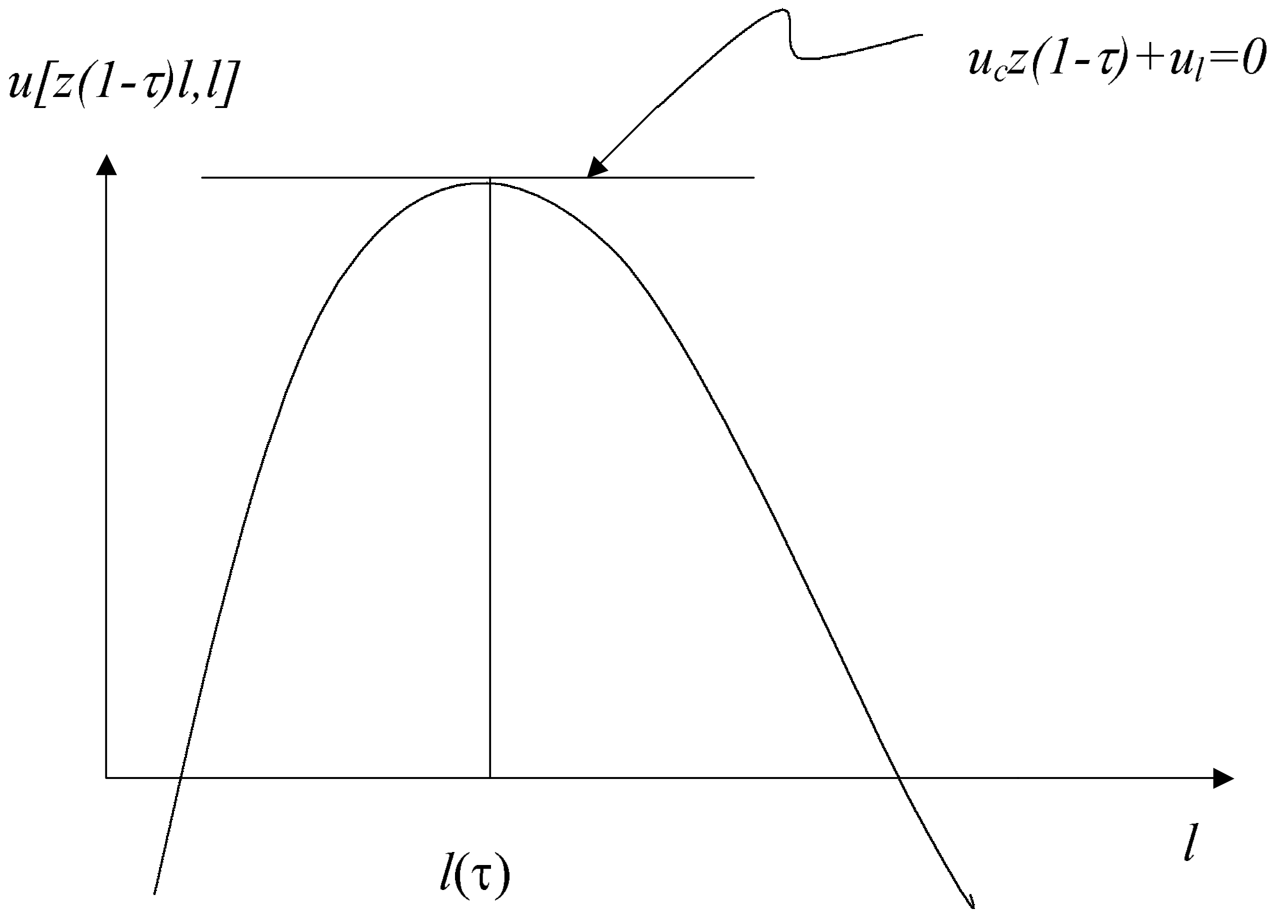
$$\begin{aligned} c &\leq z(1-\tau)l, \\ z &\sim \text{wage rate} \\ \tau &\sim \text{labor tax rate} \end{aligned}$$

- Household Problem Implies Private Sector Allocation Rules:

$$l(\tau),\ c(\tau)$$

- Ramsey Problem:

$$\max_{\tau} u(c(\tau), l(\tau))$$
$$\text{subject to } g \leq zl(\tau)\tau$$



Private Sector Allocation Rules:

$l(\tau),\ \ c(\tau) = z(1-\tau)l$

- Ramsey Equilibrium: τ^*, c^*, l^* such that

  (a) $c^* = c(\tau^*)$, $l^* = l(\tau^*)$
  ‘Private Sector Allocations are a Private Sector Equilibrium’

  (b) τ^* Solves Ramsey Problem
  ‘Best Private Sector Equilibrium’

OBSERVATIONS:

1) Ramsey equilibrium corresponds to the competitive equilibrium that maximizes social welfare. This implies that the resulting allocation is constrained by any restrictions that arise from the market structure.

The first best is the allocation that maximizes social welfare subject to only technological and resource constraints.

The first best allocation may not be attainable as a competitive equilibrium.

If this is the case, the Ramsey equilibrium is inferior to the first best.

2) The definition of competitive equilibrium takes as given the fiscal instruments of the government. It follows that the set of allocations that is attainable in a competitive equilibrium depends on those fiscal instruments. Therefore, the Ramsey equilibrium also depends on the fiscal instruments. though, often, there are multiple sets of fiscal instruments that can surport the same competitive equilibrium allocation.

The Lucas-Stokey Cash-Credit Good Model

Representative Household

$$\max \sum_{t=0}^{\infty} \beta^t u\left(c_{1t}, c_{2t}, l_t\right),$$

$$\begin{aligned} M_t^d + Q_t B_t^d \; &\leq \; M_{t-1}^d - P_{t-1}c_{1t-1} - P_{t-1}c_{2t-1} \quad (1) \\ &+ B_{t-1}^d + W_{t-1}\left(1-\tau_{t-1}\right) l_{t-1}, \quad (2) \end{aligned}$$

$$P_t c_{1t} \leq M_t^d$$

Euler Equations

$$\frac{u_{1t}}{u_{2t}} = R_t,$$

$$u_{1t} = \beta u_{1,t+1} R_t \frac{P_t}{P_{t+1}}, R_t = 1/Q_t$$

$$u_{lt} + \left(1-\tau_t\right)\frac{W_t}{P_t} u_{2t} = 0.$$

$$u_{lt} + (1 - \tau_t)\frac{W_t}{P_t}u_{2t} = 0.$$

Firms

$$c_{1t} + c_{2t} + g_t \leq z_t l_t^d,$$

$$\frac{W_t}{P_t} = z_t,$$

Government

$$\begin{aligned} & M_t^s - M_{t-1}^s + Q_t B_t^s + W_{t-1} z_{t-1} \tau_{t-1} l_{t-1} \\ \geq \ & B_{t-1}^s + P_{t-1} g_{t-1} \end{aligned}$$

Policy

$$\Pi = \{M_0^s, M_1^s, ..., B_0^s, B_1^s, ..., \tau_0, \tau_1, ...\}$$

For each Π, there is a private sector equilibrium:

$$x = [\{c_{1t}\}_t, \{c_{2t}\}_t, \{l_t\}_t, \left\{M_t^d\right\}_t, \left\{B_t^d\right\}_t],$$

$$p = (\{P_t\}_t, \{R_t\}_t),$$

$$M_t^d = M_t^s, allt,$$

$$B_t^d = B_t^s, allt,$$

$$R_t \geq 1, allt$$

Ramsey Policy

$$\max_{\Pi \in A} U(x(\Pi))$$

Ramsey Allocation Problem:

$$\max_{\{c_{1t},c_{2t},l_t\}\in D} \sum_{t=0}^{\infty} \beta^t u(c_{1t}, c_{2t}, l_t),$$

where D is the set of allocations, c_{1t}, c_{2t}, l_t, $t = 0, 1, 2, ...$, such that

$$\sum_{t=0}^{\infty} \beta^t [u_{1t}c_{1t} + u_{2t}c_{2t} + u_{3t}l_t] \;=\; u_{2,0}a_0,$$

$$c_{1t} + c_{2t} + g \;\leq\; zl_t,\; \frac{u_{1t}}{u_{2t}} \geq 1,$$

$a_0 = \dfrac{B_0}{P_0}$ $\sim$ real value of initial government debt.

Assumption:

$$B_0 = 0.$$

Lagrangian Representation of Problem:

There is a $\lambda \geq 0$, Such that the Solution to the RA Problem and the Following Problem Coincide:

$$\max_{\{c_{1t}, c_{2t}, l_t\}} \sum_{t=0}^{\infty} \beta^t W(c_{1t}, c_{2t}, l_t; \lambda)$$

subject to

$$c_{1t} + c_{2t} + g \leq z l_t, \ \frac{u_{1t}}{u_{2t}} \geq 1,$$

where

$$W(c_{1t}, c_{2t}, l_t; \lambda) = u(c_{1t}, c_{2t}, l_t) + \lambda [u_{1t} c_{1t} + u_{2t} c_{2t} + u_{3t} l_t].$$

Optimal Policy with No Uncertainty

Lagrangian of the Ramsey Allocation Problem:

$$\begin{aligned} \Lambda &= \sum_{t=0}^{\infty} \beta^t [W(c_{1t}, c_{2t}, l_t; \lambda) - \omega_t (c_{1t} + c_{2t} + g - z l_t) \\ &\quad - \psi_t (u_{2t} - u_{1t})] \end{aligned}$$

To solve assume that $\psi_t = 0$. If find that the solution to the unconstrained problem satisfies the constraint then we have the constrained solution.

Solution for a given P_0 at $t > 0$ satisfies:

$$W_{1t} - \omega_t = 0,$$

$$W_{2t} - \omega_t = 0,$$

$$W_{3t} + z\omega_t = 0$$

Then:

- Solution does not depend on t.
- $\frac{W_{1t}}{W_{2t}} = 1$
- $\frac{W_{2t}}{W_{3t}} = -1/z$

Prescription: Minimize inter- and intra-temporal distortions.

$\lambda \geq 0$ is the utility cost of raising revenues via distortionary taxes.

Recall from optimization theory, the solution of the Lagrangean problem satisfies a saddle condition: max Λ with respect to the controls and minimize Λ with respect to λ

Restricting the Utility Function

- Utility Function:

$$
\begin{aligned}
u(c_1, c_2, l) &= h(c_1, c_2)v(l), \\
h &\sim \text{homogeneous of degree } k \\
v &\sim \text{strictly decreasing.}
\end{aligned}
$$

- Then, $u_1 c_1 + u_2 c_2 + u_3 l = h\left[kv + v'\right]$, so

$$
\begin{aligned}
W(c_1, c_2, l; \lambda) &= hv + \lambda h\left[kv + v'\right] \\
&= h(c_1, c_2)Q(l, \lambda).
\end{aligned}
$$

- Conclude - Homogeneity and Separability Imply:

$$
\frac{W_1(c_1, c_2, l; \lambda)}{W_2(c_1, c_2, l; \lambda)} = \frac{u_1(c_1, c_2, l)}{u_2(c_1, c_2, l)}.
$$

Surprising Result: Friedman is Right More Often Than You Might Expect

- Equating 'Marginal Rate of Substitution' in W with Associated Marginal Rate of Technical Transformation:

$$\frac{W_1(c_1, c_2, l; \lambda)}{W_2(c_1, c_2, l; \lambda)} = 1.$$

- Under Homogeneity and Separability:

$$\frac{u_1(c_1, c_2, l)}{u_2(c_1, c_2, l)} = 1.$$

- Conclude

$$R = 1.$$

- Friedman Is Right!

Generality of the Result

- Result is True for the Following More General Class of Utility Functions:

$$u(c_1, c_2, l) = V(h(c_1, c_2), l),$$

  where h is homothetic.

- Analogous Result Holds in ‘Money in Utility Function’ Models and ‘Transactions Cost’ Models (Chari-Christiano-Kehoe, *Journal of Monetary Economics*, 1996.)

- Actually, strict homotheticity and separability are not necessary.

Consumption Elasticity of Demand

- Homotheticity and Separability Correspond to Unit Consumption Elasticity of Money Demand.
- Money Demand:

$$R = \frac{u_1}{u_2} = \frac{h_1}{h_2} = f\left(\frac{c_2}{c_1}\right) = f\left(\frac{c - \frac{M}{P}}{\frac{M}{P}}\right) = \tilde{f}\left(\frac{c}{M/P}\right).$$

- Note: Holding R Fixed, Doubling c Implies Doubling M/P

Uniform Taxation Result from Public Finance For Non-Monetary Economies

- Households:

$$\max_{c_1,c_2,l} u(c_1, c_2, l)$$
$$\text{s.t. } zl \geq c_1(1+\tau_1) + c_2(1+\tau_2)$$
$$\Rightarrow c_1 = c_1(\tau_1, \tau_2),\ c_2 = c_2(\tau_1, \tau_2),\ l = l(\tau_1, \tau_2).$$

- Ramsey Problem:

$$\max_{\tau_1,\tau_2} u(c_1(\tau_1,\tau_2), c_2(\tau_1,\tau_2), l(\tau_1,\tau_2))$$
$$\text{s.t. } g \geq c_1(\tau_1,\tau_2)\tau_1 + c_2(\tau_1,\tau_2)\tau_2$$

- *Uniform Taxation Result*:

$$\text{if } u = V(h(c_1, c_2), l),\ h \sim \text{homothetic}$$
$$\text{then } \tau_1 = \tau_2.$$

Proof : trivial! (just study Ramsey Allocation Problem)

Similarities to Monetary Economy

- Rewrite Budget Constraint:

$$\frac{zl}{1+\tau_2} \geq c_1 \frac{1+\tau_1}{1+\tau_2} + c_2.$$

- Similarities:

$$\frac{1}{1+\tau_2} \sim 1-\tau,\ \frac{1+\tau_1}{1+\tau_2} \sim R.$$

- Positive Interest Rate 'Looks' Like a Differential Tax Rate on Cash and Credit Goods.

- **Have the Same Ramsey Allocation Problem**, Except Monetary Economy Also Has:

$$\frac{u_1}{u_2} \geq 1.$$

What Happens if You Don't Have Unit Elasticity?

- Utility Function:

$$u(c_1, c_2, l) = \frac{c_1^{1-\sigma}}{1-\sigma} + \frac{c_2^{1-\delta}}{1-\delta} + v(l)$$

- Money Demand:

$$R = \frac{u_1}{u_2} = \frac{c_1^{-\sigma}}{c_2^{-\delta}} = \frac{\left(\frac{M}{P}\right)^{-\sigma}}{\left(c - \frac{M}{P}\right)^{-\delta}},$$

$$\varepsilon_M = \frac{d\log\left(\frac{M}{P}\right)}{d\log(c)}$$

- Can Verify:

Utility Function Parameters	ε_M	Non-Monetary Economy	Monetary Economy
$\delta > \sigma$	$\varepsilon_M > 1$	$\tau_2 \geq \tau_1$	$R = 1$
$\delta < \sigma$	$\varepsilon_M < 1$	$\tau_2 < \tau_1$	$R > 1$
$\delta = \sigma$	$\varepsilon_M = 1$	$\tau_1 = \tau_2$	$R = 1$

Who is Right, Friedman or Phelps?

- Friedman is Right ($R = 1$) When Consumption Elasticity of Money Demand is Unity or Greater
- Close Connection to Uniform Taxation in Public Finance
  (But, $R = 1$ Holds More Generally Because of $R \geq 1$ Constraint in Monetary Economies)
- Basic Idea:
  Implicitly, High Interest Rates Tax Some Goods More Heavily that Others. Under Certain Conditions, Don't Want to Do That.
- What is Consumption Elasticity in the Data?

OBSERVATION:
The common view for monetary economies without nominal frictions (such as sticky prices or portfolio frictions) is that the Friedman rule is optimal.
This result itself is non generic. However, most models make assumptions on preferences that imply a unitary elasticity of money demand. Under those assumptions, the Friedman rule is optimal.

An economy with uncertainty

s_t realization of the state at t, drawn from a discrete distribution

$\hat{s} \in [s^1, s^2, ..s^N]$

$s^t = (s_0, s_1, ..., s_t)$

Variables chosen after s^t realized:

$M^d\left(s^t\right)$, $B^d\left(s^t\right)$, $c_i\left(s^t\right), i = 1, 2,$ $l\left(s^t\right)$

Asset market constraint: $M\left(s^t\right) + Q\left(s^t\right)B\left(s^t\right) \leq A\left(s^{t-1}\right) + B\left(s^{t-1}\right)$

Nominal bond returns are *not* state contingent.

Household preferences:

$$\sum_t \sum_{s^t} \beta^t \mu\left(s^t\right) u\left(c_1\left(s^t\right), c_2\left(s^t\right), l\left(s^t\right)\right)$$

Analogous set up of the Ramsey Allocation Problem

First order necessary conditions for the Ramsey allocation problem:

- $\frac{W_{1t}}{W_{2t}} = 1$

- $\frac{W_{2t}}{W_{3t}} = -1/z\left(s^t\right)$

Stationarity property of Ramsey allocation preserved as in real economy.

Predictions for bond repayments:

$$\left[\frac{M\left(s^{t-1}\right)}{P\left(s^{t}\right)}+\frac{B\left(s^{t-1}\right)}{P\left(s^{t}\right)}\right]u_2\left(s^t\right)$$
$$=\sum_{j=0}^{\infty}\sum_{s^{t+j}}\mu\left(s^{t+j}|s^t\right)\left[\sum_{i=1,2}u_i\left(s^t\right)c_i\left(s^t\right)+u_l\left(s^t\right)l\left(s^t\right)\right]$$

Currency and bonds are nominal liabilities of the government.

They are not state contigent in nominal terms but are state contingent in real terms.

→Government can minimize inter-temporal distortions by taxing lump sum by changes in price level.

Implications:

- Low average inflation
- High volatility of realized inflation.

TABLE 3

PROPERTIES OF THE MONETARY MODELS

Rates	Models: Baseline	High Risk Aversion	I.I.D.
Labor Tax			
Mean	20.05	20.18	20.05
Standard Deviation	.11	.06	.11
Autocorrelation	.89	.89	.00
Correlation with			
Government Consumption	.93	−.93	.93
Technology Shock	−.36	.35	−.36
Output	.03	−.06	.02
Inflation			
Mean	−.44	4.78	−2.39
Standard Deviation	19.93	60.37	9.83
Autocorrelation	.02	.06	−.41
Correlation with			
Government Consumption	.37	.26	.43
Technology Shock	−.21	−.21	−.70
Output	−.05	−.08	−.48
Money Growth			
Mean	−.70	4.03	−2.78
Standard Deviation	18.00	54.43	3.74
Autocorrelation	.04	.07	.00
Correlation with			
Government Consumption	.40	.28	.92
Technology Shock	−.17	−.20	−.36
Output	.00	−.07	.02