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1 Business Cycles

1.1 What Are Business Cycles?

The first example of business cycle analysis stems from the work of Burns and Mitchell in the 1940's. They define business cycles as expansions occurring at the same time in many economic sectors, followed by similarly general recessions, contractions and revivals. An important property of these phases is that they are recurrent but not periodic, and last from two to eight years. Lucas (1981) subsequently refines this definition. Business cycles are now defined as deviations of aggregate output from a trend, when there is comovement in these deviations for different aggregate time series.

This view of business cycles lead to the definition adopted by the NBER when it "declares" the start and the end of a recession in the US:

"...a recession is a persistent period of decline in total output, income, employment, and trade, usually lasting from six months to a year, and marked by widespread contractions in many sectors of the economy."

Initial attempts at modelling business cycles were based on the notion of deterministic oscillations. These attempts failed to deliver the persistent and recurring nature of fluctuations in the data. Frisch (in various articles written between 1933-1965) was the first to propose models of the business cycle where the impulse takes the form of a random shock and a propagation mechanism gives rise to damped fluctuations in response to the impulse. Slutzky (1937) first noted that AR(1) variables can display cyclical behavior. The breakthrough was to recognize that an AR(1) stochastic process can generate recurrent cycles despite the fact that the deterministic version of the model converges monotonically to a point.

Initial and more recent attempts at modelling business cycles have focused on the persistence and recurrent nature of business cycle fluctuations. Less attention has been devoted to understanding the second defining property of business cycles, the synchronization across many sectors. Lucas (1981) conjectures that comovement can be explained by a common shock that hits many sectors. Many economists agree with this conjecture. However, there are two difficulties. First, even if it is possible to identify a common shock to all sectors, it is not granted that all sectors would respond in the same way to this shock. Second, the work of Long and Plosser (1983) has shown that disturbances to individual industries, even if they are uncorrelated across industries, could result in comovement. Despite these difficulties, a standard business cycle model emerged, based on the work of Kydland and Prescott (1982). This model is based on the notion that fluctuations in economic variables are driven by a unique, exogenous, random shock.

1.2 Business Cycles Analysis

A critical component of business cycle analysis is the *measurement* of business cycles. This requires isolating a particular component of the fluctuations in economic variables, those at the business cycle frequency, with a phase of eight years or less. Having extracted the business cycle component, the analysis is directed to two features of this component: the size of the fluctuations, measured by the standard deviation, and the correlation of each variable with output at various leads and lags. A variable is defined as procyclical if it is positively correlated with output, countercyclical otherwise. A variable is said to lead the cycle it its peak occurs before that of output, it is said to lag the cycle otherwise.

1.2.1 Extracting Business Cycle Fluctuations

Hodrick and Prescott propose a filter for identifying the trend component of a time series and extracting the business cycle component. Their measure of trend behavior incorporates the fact that trends from most variables are not constant and is consistent with the Lucas (1981) definition of trend and cycles. The trend is a curve that can be drawn through a time plot of a relevant series, it should be a linear transformation of the series, such that lengthening the time period should not have a large effect on the trend.

Given a time series $\{Y_t\}_{t=1}^T$, a measure of the trend that satisfies these properties is τ_t, defined as:

$$\tau_t = \arg \min_{\{\tau_t\}_{t=1}^T} \sum_{t=1}^T (y_t - \tau_t)^2 + \lambda \sum_{t=1}^T [(\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1})]^2 = 0,$$

where $y_t = \ln(Y_t)$. The parameter λ determines the smoothness of the resulting trend. For quarterly data, $\lambda = 1600$ (eliminates fluctuations lower than about 32 quarters). For $\lambda = 0$, the trend coincides with the series[1]. For $\lambda \to \infty$, the trend is linear. The business cycle component is simply given by $y_{ct} = y_t - \tau_t$, $t = 1, 2, ...T$.

1.2.2 Business Cycle Facts

The following are a list of facts about business cycles in the US, restricting attention to real variables. See Cooley and Prescott (1995), Chapter 1, Table 1.1 for the details.

I. Hours worked are highly procyclical, % deviation is similar to that of GDP.

II. The capital stock has no contemporaneous correlation with output. Business capital lags cycle by one year.

III. Employment lags cycle, hours very slightly lead cycle. Most of the volatility in total hours worked can be accounted for by volatility in employment[2].

IV. Average hourly compensation is uncorrelated with output.

V. There is a very small correlation between hours and productivity.

VI. Real wages vary less than productivity.

VII. Productivity is slightly procyclical but varies less than output and has small correlation with output.

VIII. All GDP components are highly procyclical.

IX. Consumption of non-durables and services, which amounts to 2/3 of output, is very smooth.

XI. Investment in consumer and producer durables fluctuates more than output (by a factor of 5).

XII. Labor income and capital income are highly procyclical. The share of capital income is procyclical.

[1] King and Rebelo (2000) display evidence that the HP filter may distort sample second moments of the data in small samples. An alternative procedure to extract business cycle components from time series is the band-pass filter. While the Hodrick-Prescott filter removes all fluctuations below a certain frequency, band-pass filters also remove fluctuations above a specified frequency. Removing fluctuations that occur at a higher frequency relative to business cycles improves the properties of sample second moments. See Baxter and King (1999) or Christiano and Fitzgerald (2001) for details.

[2] There is a measurement problem here. Since, we are not accounting for variations in the skill level over the cycle, hence, we are not correctly measuring labor input.

1.3 The Canonical Real Business Cycle Model

The canonical business cycle model is an enriched version of the one sector growth model. Two features are added: variable labor and random shocks to technology. The planning problem is given by:

$$\max_{\{c(z^t),n(z^t),k(z^t)\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \sum_{z^t} \mu\left(z^t\right) u\left(c\left(z^t\right), n\left(z^t\right)\right),$$

for k_0 given and:

$$0 \leq c\left(z^t\right) \leq f\left(k\left(z^{t-1}\right), n\left(z^t\right), z_t\right) - k\left(z^t\right), \tag{1}$$

$$f\left(k\left(z^{t-1}\right), 1\right) - k\left(z^t\right) \geq 0, \tag{2}$$

$$n\left(z^t\right) \in [0, 1], \tag{3}$$

where $f\left(k\left(z^{t-1}\right), n\left(z^t\right), z_t\right) = z_t k\left(z^t\right)^{\alpha} n\left(z^t\right)^{1-\alpha} + (1-\delta) k\left(z^{t-1}\right)$ for $\alpha \in (0,1)$. Here, $n\left(z^t\right)$ denotes per capita hours and the time endowment is 1. Leisure is given by $l\left(z^t\right) = 1 - n\left(z^t\right)$ and is assumed to be a normal good.

The random process z^t denotes the state of the economy at time t, given by the history of realizations of the random shock $z_t : \{z_0, z_1, ..., z_t\}$. The variable $\mu\left(z^t\right)$ denotes the probability of such a history. The variable z_t is the technology shock and is assumed to follow a stationary AR(1) process, with autocorrelation coefficient ρ. Typically, the following specification is assumed:

$$\ln\left(z_t\right) = (1-\rho)\ln\left(\bar{z}\right) + \rho \ln\left(z_{t-1}\right) + \varepsilon_{zt},$$

with $\rho \in (0,1)$ and $\varepsilon_z \sim N\left(0, \sigma_z^2\right)$.

The aggregate productivity shock z_t is the only source of fluctuations in the model. The variables in the planning problem are indexed by the information available when they are chosen, which corresponds to the whole history of realizations of z_t. For example, consumption at time t, is denoted with $c\left(z^t\right)$ to indicate that consumption is chosen after z_t is realized. Conversely, capital at time t is denoted with $k\left(z^{t-1}\right)$, since it is chosen at time $t-1$, before z_t is realized.

Restrictions on preferences need to be imposed to ensure that k_{t+1} is increasing in k_t. See the handout on "Introducing variable hours in the canonical growth model".

The presumption of this model is that there is an aggregate shock, given by the technology shock z_t that hits all sectors of the economy and gives rise to business cycle fluctuations. Implicit is the assumption that all sectors respond in the same way to the shock, so that a unique production sector needs to be considered in the model.

Question: Can a model that is consistent with long run growth also account for business cycle fluctuations?

There are four basic steps in answering this question:

1. Restrict processes to a parametric class and restrict preferences and technology to be consistent with the long run growth facts.

2. Construct a set of measurements that are consistent with the parametric class.

3. Assign parameter values- calibration.

4. Simulate the model economy.

Calibration involves setting parameter values so that the behavior of the model economy matches features of the measured data in as many dimensions as there are unknown parameters. The model is matched along the dimensions required to match long run growth, based on the fact that certain ratios are constant.

The model simulation occurs as follows. a statistic for the technology shock z_t is constructed from the data, according to:

$$\hat{z}_t = \frac{GNP_t}{k_t^{\alpha} n_t^{1-\alpha}},$$

where α is matched to the capital share of income. An AR(1) process is fitted to the business cycle component of the statistic $\hat{z}_t$. This delivers a estimates for the values of ρ and σ_ε. The planning problem is solved and the relevant policy functions for consumption, labor and capital are obtained. A sequence of random realizations for the estimated process for the business cycle component of $\hat{z}_t$ are generated. Using the policy functions for the planning problem, the resulting behavior of the variables of interest as a function of the realized values of the simulated technology shock is traced out for a number of periods.

The simulated behavior of these variables can be used to construct statistics for the model economy and analyze business cycles in the model. See Cooley and Prescott, 1995, Chapter 1, Table 1.2.

1.4 Business Cycle Puzzles

The business cycle puzzles are the business cycle facts that cannot be accounted for with the standard business cycle model.

1. Output in the model does not fluctuate as much as output in the data. This is referred to as the canonical model lacking a *propagation* mechanism.

2. The standard deviation of total hours in the model is lower than the standard deviation of output, while in the data they have similar magnitude.

3. The fluctuation in hours in data can mostly be accounted for by fluctuations in employment, the *extensive* margin, as opposed to the fluctuations in hours per worker, the *intensive* margin, (Gary Hansen, (1985), and Rogerson, (1988)).

4. Average labor productivity is highly procyclical in the model, but is not procyclical in the data. The statistic y/n is slightly countercyclical using hours from the household survey and slightly procyclical in establishment survey.

5. The correlation between hours worked and productivity in the model is well above 0.90, while in the data it is very close to 0. See Christiano and Eichenbaum (1992) for extensive discussion.

6. Wages in the data fluctuate substantially less than hours and output, and earnings are not correlated with output. Wages are highly procyclical in the model.

7. The canonical model is unable to generate comovement, when interpreted as a two sector model with a consumption sector and an investment sector. (See Assignment 1. Christiano and Fitzgerald (1998) provide extensive discussion on this point.)

8. One of the maintained assumption of the canonical model is that the technology shock is exogenous. However, the Solow residual, which is the corresponding data statistic for the technology shock, is not

exogenous in the data (Hall, 1988). It is correlated with military expenditures, monetary aggregates and government consumption. Burnside, Eichenbaum and Rebelo (1993) show that the innovations in government spending and in the technology shock are also correlated. Hall advocates models with increasing returns and monopolistic competition to address this failure of the canonical model.

1.5 Confronting the Puzzles

The salient shortcomings of the canonical model are mostly concentrated in its labor market implications (points 2-6 in the previous section). A number of variations to the canonical model have been proposed to address these failures. Some of these are discussed below.

1.5.1 Indivisible Labor

The indivisible labor model is based on the notion that the production process has important non-convexities or fixed costs that may make varying the number of employed workers more efficient than varying the number of hours. It assumes that workers can only work a fixed number of hours $\hat{h} \in (0, 1)$. In the equilibrium of this model, individuals are randomly assigned to employment or unemployment in each period. Unemployment insurance implies that their consumption is not sensitive to their labor market status. Thus, in this model fluctuations in aggregate hours stem from fluctuations in employment only. The indivisible labor model addresses and "solves" puzzles 3 and 6 in the list above. It improves on the canonical model for puzzle 1, and only marginally for puzzles 2, 4 and 5.

The canonical model gives rise to puzzle 3 by construction. The basis of puzzles 2, 4, and 6 for the canonical model is that the implied labor supply curve is not sufficiently elastic. In the model, workers are not sufficiently willing to substitute for leisure in the current period with leisure in future periods. Hence, the response of hours to changes in real wages and productivity is small (the aggregate labor supply curve is not very elastic). This implies that in response to a technology shock, current hours do not react as much as in the data, which determines a rise in the equilibrium real wage and a rise in average labor productivity in the model, see figures 1 and 2.

The data, on the other hand, seems to be consistent with the labor market equilibrium represented in figure 3. The indivisible labor model delivers labor market implications consistent with the situation in figure 3.

Let N_t the probability that an agent is employed in period t, so that $H_t = N_t\hat{h}$ is per-capita hours worked if we assume a large number of aggregate agents. Let c_{ut} denote the consumption of an unemployed agent and c_{et} the consumption of an employed agent. The variables c_{ut}, c_{et} and N_t will be chosen as part of a dynamic planning problem, to maximize:

$$N_t u\left(c_{et}, 1 - \hat{h}\right) + (1 - N_t)\, u\,(c_{ut}, 1)\,,$$

in each period subject to the constraint:

$$N_t c_{et} + (1 - N_t)\, c_{ut} = c_t,$$

where c_t is total per-capita consumption. Assuming $u\,(c, l) = \ln(c) + A\ln(l)$, where l denotes leisure, the

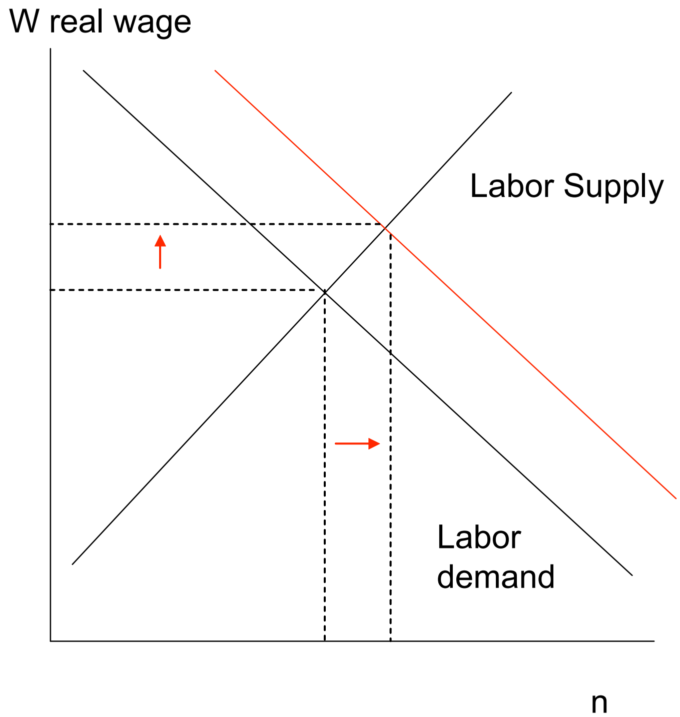

Figure 1: Labor market equilibrium in response to a positive technology shock in the canonical model

solution will imply $c_t = c_{ut} = c_{et}$. Expected utility can then be written as:

$$\ln(c_t) + N_t A \ln\left(1-\hat{h}\right) = \ln(c_t) - BH_t,$$

where $B = -A\ln\left(1-\hat{h}\right)/\hat{h}$, where H_t is per capita hours. Then, the indivisible labor model is equivalent to a divisible labor model, with preferences:

$$\tilde{U} = E\sum_{t=0}^{\infty}\beta^t \tilde{u}(c_t, H_t),$$

where $\tilde{u}(c_t, H_t) = \ln(c_t) - BH_t$.

The indivisible labor model generates a large intertemporal substitution effect for the representative agent because instantaneous utility $\tilde{u}(c, H)$ is linear in H. Hence, the representative agent only cares about the present discounted utility cost of labor supply and is indifferent between different sequences of labor supply over her lifetime that deliver the same present discounted utility cost.

Simulating this model yields the following findings:

- Output in the indivisible labor (IL) model is considerably more volatile than the canonical model (σ_y in the model is approximately 1.70% for calibrated parameters).
- The ratio σ_h/σ_{alp} increases substantially in the IL model (alp denotes average labor productivity).

However, the correlation between hours and productivity, which is lower than in the canonical model, is still approximately 0.70, much higher than it is in the data. See Hansen and Wright (1992) for further details.

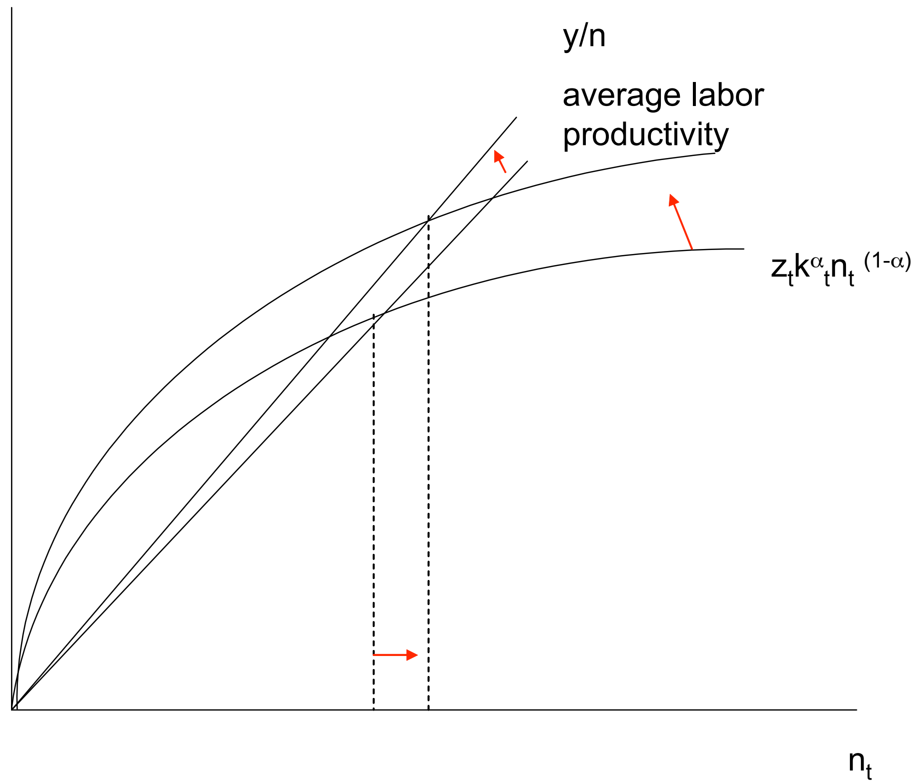

Figure 2: Response of y_t/n_t to a positive technology shock in the canonical model

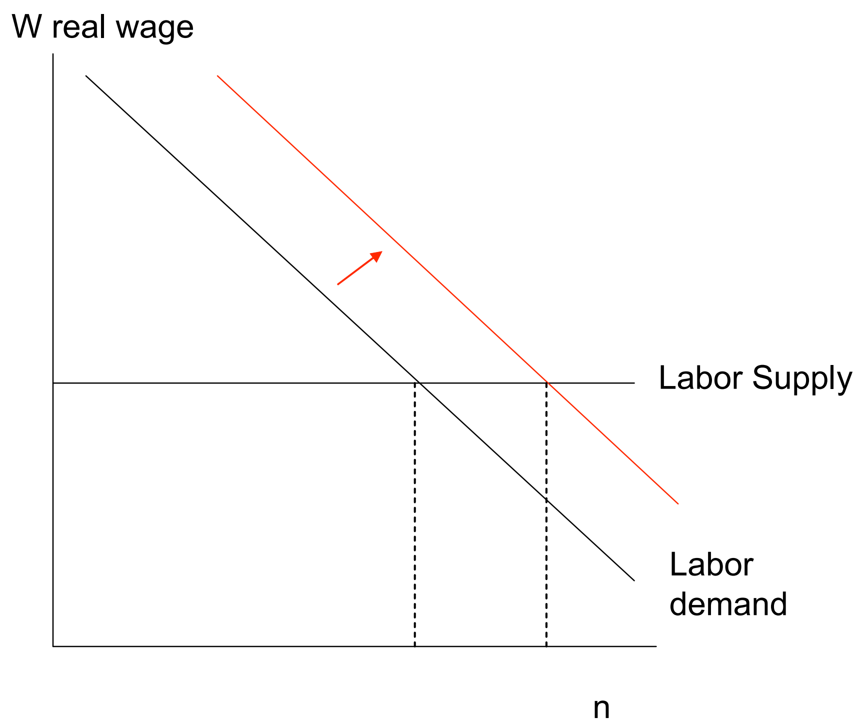

FIGURE 3: A labor market equilibrium consistent with data on hours and real wages over the business cycle

1.5.2 Government Spending Shocks

Christiano and Eichenbaum (1992) introduce stochastic government spending into the canonical model to address puzzle 5. They assume that government spending follows the following stationary AR(1) process:

$$\ln(g_{t+1}) = (1-\lambda)\ln(\bar{g}) + \lambda \ln(g_t) + \mu_t,$$

where $\lambda \in (0,1)$ and μ_t is iid with mean 0 and standard deviation σ_μ. A crucial assumption about μ_t is that it is independent from the technology shock z_t.

Government spending is financed with lump-sum (non-distortionary) taxes levied on household. The government's budget is balanced in each period. Government spending does not generate any utility for households, and it cannot be used in production. The resource constraint for this economy is:

$$c_t + i_t + g_t = y_t,$$

where y_t denotes output at time t and i_t is investment.

Given these assumptions, g_t is a pure drain on the economy. A rise in g_t generates a negative wealth effect on households and induces them to increase labor supply. Therefore, fluctuations in g_t shift the labor supply curve. Fluctuations in z_t shift the labor demand curve.

A calibrated version of this model behaves very much like the canonical model except for the correlation of hours and productivity which is approximately 0.50. While this is still larger than in the data, it is significantly smaller than in the canonical model. The scatter plot of the behavior of hours and productivity

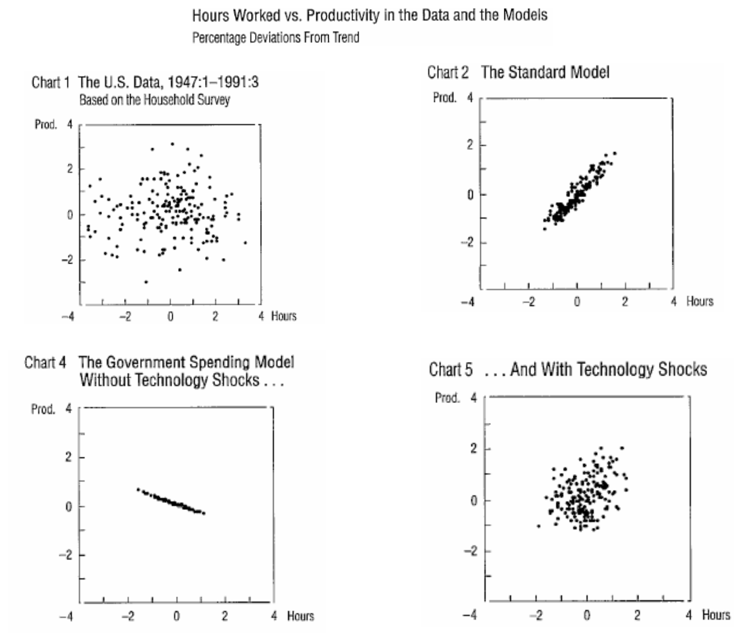

FIGURE 4: Aggregate hours and labor productivity. Source Hansen and Wright (1992).

in the canonical model and in the model with government spending shocks is displayed in figure 4. It is much closer to the scatter plot in the data.

An additional component that can be introduced in the canonical model to increase the intertemporal labor supply elasticity and reduce the correlation between hours and productivity is home production, with shocks to home production productivity. See Hansen and Wright (1992) and Assignment 1 for a discussion.

1.5.3 Factor Hoarding

The factor hoarding model was initially proposed by Burnside, Eichenbaum and Rebelo (1993) to deal with puzzles 1 and 8. Their point of departure is Hall's observation that the Solow residual is not exogenous, but it is instead correlated with military expenditures, various monetary aggregates and government consumption. In addition, the innovations in government consumption and in the technology shock are also correlated.

BER take as given the identifying restriction that innovations to technology should be uncorrelated with innovations in government spending. They then ask whether conventional empirical measures of the technology shock are sensitive to this restriction, and whether there are any quantitatively convincing mechanisms that can account for the correlation between the Solow residual and government consumption.

They find that imposing an orthogonality restriction on empirical measures of the technology shock reduces the variance of the technology shock by 60% and considerably worsens the performance of the model. They implement a labor hoarding model and they estimate the fraction of the variation in the innovation of the Solow residual that is due to true technology shocks. The resulting model can account for the empirical correlations of the Solow residual with government consumption and does as well as the canonical model, even if the underlying technology shock is a lot smaller.

Indivisible Labor Model with Factor Hoarding The model is a simple extension of the indivisible labor model with technology shocks. Individual preferences are assumed to take the form:

$$\ln(c_t) + A\ln\left(T - \xi - E_t\hat{h}\right),$$

where $0 \leq E_t \leq (T-\xi)/\hat{h}$ is effort, $\hat{h}$ is shift length, and $\xi > 0$ is a fixed cost of working. T is the time endowment. The production function is:

$$y_t = z_t k_t^\alpha \left[\hat{h} N_t E_t\right]^{1-\alpha}, \tag{4}$$

where N_t is the fraction of individuals who are employed. There are two shocks in this economy, a technology shock z_t and a government consumption shock, g_t. Both shocks are assumed to follow an AR(1) stochastic process:

$$\ln(z_t) = (1-\rho)\ln(\bar{z}) + \rho\ln(z_{t-1}) + \varepsilon_{zt},$$

$$\ln(g_t) = (1-\lambda)\ln(\bar{g}) + \lambda\ln(g_{t-1}) + \varepsilon_g,$$

with $\rho, \lambda \in (0,1)$. The innovations $\varepsilon_z, \varepsilon_g$ are iid, with mean 0 and variance σ_z^2, σ_g^2, respectively. In addition, ε_g and ε_z are uncorrelated.

The resource constraint is:

$$c_t + k_{t+1} - (1-\delta)k_t + g_t \leq y_t.$$

As in the indivisible labor model, individuals face a lottery of employment, with probability $N_t \in [0,1]$ of becoming employed. However, they are insured against unemployment, so their consumption is independent from their employment status. The planning problem with unemployment insurance is:

$$\max_{\{c_t N_t, k_{t+1}, E_t\}_{t\geq 0}} E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln(c_t) + AN_t \ln\left(T - \xi - E_t\hat{h}\right) + A(1-N_t)\ln(T)\right],$$

subject to the resource constraint, $0 \leq E_t \leq (T-\xi)/\hat{h}$ and $N_t \in [0,1]$, with k_0 given.

We denote with Ω_t the state of the economy at time t, with $\Omega_t = \{k_t, z^t, g^t\}$. The planner's information set at time t is denoted with, Ω_t^*. By assumption, N_t must be chosen before g_t and z_t are known, so that $\Omega_t^* = \Omega_t / \{z_t, g_t\}$. This assumption corresponds to firms making employment decisions conditional on their expectations about the future realization of government consumption and technology. Employment is treated symmetrically with capital. This captures the fact that there are often fixed costs associated with hiring workers and that it takes time for a newly hired worker to become fully productive within a firm, due to the need to acquire firm specific skills. Then, when shocks to government consumption or technology hit, firms adjust along the intensive margin, by changing effort, rather then on the extensive margin, by changing employment.

Implications for the Solow Residual The canonical model calls for identifying technology shocks from the statistic:

$$\hat{S}_t = \frac{GNP_t}{{k_t}^\alpha \left(\hat{H}_t\right)^{1-\alpha}},$$

with $\hat{H}_t = N_t\hat{h}$, where $\hat{H}_t$ are total hours measured from the data and α is the empirical share of capital.

Based on the canonical model, the statistic $\hat{S}_t$ is equal to the Solow residual,

$$S_t = \frac{y_t}{k_t^\alpha \left(N_t\hat{h}\right)^{1-\alpha}},$$

which is given by the technology shock, $\exp(z_t)$, and is exogenous. The labor hoarding model provides a theory of the Solow residual. To see this, note that if output is produced according to (4):

$$S_t = \frac{z_t k_t^\alpha \left(E_t N_t \hat{h}\right)^{1-\alpha}}{k_t^\alpha \left(N_t\hat{h}\right)^{1-\alpha}} = z_t E_t^{1-\alpha}.$$

Then:

$$\begin{aligned} \ln(S_t) &= \ln(z_t) + (1-\alpha)\ln(E_t), \\ \ln(y_t/H_t) &= \ln(z_t) + (1-\alpha)\ln(E_t) + \alpha\left[\ln(k_t) - \ln\left(\hat{h}N_t\right)\right], \end{aligned}$$

where y_t/H_t is labor productivity. Note that N_t and k_t do not respond to current values of z_t and g_t.

Observations:

1. Any variable which is correlated with effort will be correlated with the Solow residual, even if they are not correlated with $\ln(z_t)$. In the model, E_t responds positively to a rise in the government spending shock, due to the presence of a negative wealth effect on the supply of effort in the preferences of the representative agent. Then, both the Solow residual and average labor productivity will rise in response to a government consumption shock. Standard business cycle accounting falsely attributes this rise to a positive technology shock. E_t is correlated with innovations in g_t and z_t. Then, S_t will be correlated with innovations in g_t.

2. If a positive technology shock occurs effort will rise. The rise in effort implies that the resulting increase in the Solow residual and in average labor productivity is greater than the technology shock. Then, the endogenous response of effort provides a propagation mechanism for the technology shock. The empirical measure of the Solow residual, $\hat{S}_t$, when used as a measure of technology, overestimates technology shocks in booms and underestimates technology shocks in recessions.

3. Since average labor productivity depends on E_t, the correlation between productivity and hours will be lower in this model than in the canonical model. Moreover, this framework delivers the prediction that ALP slightly leads the cycle (is positively correlated with future hours worked) and is negatively correlated with lagged hours worked.

Finally, the empirical estimation of the model requires identifying a series for effort in the data. This can be done by using the first order necessary condition for effort:

$$A\left(T - \xi - E_t\hat{h}\right)^{-1} = \frac{(1-\alpha)\,y_t}{c_t E_t H_t}.$$

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