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G 6612 Adv. Macro Analysis
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You have two hours to complete the exam. The exam is closed book. Please answer each question on a separate blue book and write your name on each bluebook.

The maximum score is 100 points. The number of points assigned to each question can be used as an approximate guide for time allocation.

Question 1- Productivity Shocks in a Limited Participation Model (40 points)

Consider the following model of a monetary economy. A representative household maximizes the criterion function

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t)$$

where $0 < \beta < 1$, C_t is time t consumption and L_t is time t hours worked. The household begins the period with M_t units of money. The household faces the cash-in-advance constraint

$$P_t C_t \leq M_t - N_t + W_t L_t$$

where P_t is the time t dollar price of consumption, N_t is dollars deposited with a financial intermediary at time t, and W_t is the time t nominal wage rate. The gross interest rate on deposits is denoted by R_t. The household's stock of money evolves according to

$$M_{t+1} = M_t - N_t + W_t L_t - P_t C_t + R_t \left(N_t + X_t\right) + \pi_t^f + \pi_t^{fi}$$

Here π_t^f and π_t^{fi} denote time t lump sum profits received from firms and financial intermediaries.

A representative firm produces time t output using the technology

$$Y_t = \theta_t H_t$$

where θ_t is the time t level of technology, H_t denotes time t hours worked. θ_t evolves according to

$$\theta_t = \theta + \varepsilon_t$$

where θ is a positive constant and ε_t is a mean zero i.i.d. shock. The firm must borrow the wage bill $W_t H_t$ from a financial intermediary. The interest rate on these loans is given by R_t. All loans are repaid at the end of the period.

The firm maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{U_{C,t+1}}{P_{t+1}} \pi_t^f$$

where

$$\pi_t^f = P_t \theta_t H_t - W_t H_t R_t$$

and $U_{C,t+1}$ denotes the time $t+1$ marginal utility of the representative consumer.

A representative financial intermediary receives N_t from households at time t as well as a monetary injection X_t from the government. All funds received are lent out to firms at the rate R_t.

The aggregate stock of money evolves according to

$$M_{t+1}/M_t = (M_t + X_t)/M_t = 1 + x_t.$$

where x_t is an i.i.d. shock to the net growth rate of money. All markets and all agents in this economy (except for the government) are perfectly competitive. Suppose that N_t must be chosen by households before the current period realization of ε_t and x_t. All other time t variables under control of the household are chosen after the realization of ε_t and x_t.

(a) Formally state the problem of the representative household. Display a set of conditions that are jointly necessary and sufficient for optimization. *Hint*: You do not need to prove that they are. Make any additional assumptions that are necessary to ensure that the conditions you write are indeed jointly necessary and sufficient.

(b) Formally state the problem of the representative firm. Display the first order necessary conditions for the problem.

(c) Formally define a sequence-of-markets equilibrium for this economy.

For the rest of the problem, suppose that the household's utility function is given by $\ln(C_t) + \alpha \ln(1 - L_t)$.

(d) Prove that a positive shock to x_t reduces R_t and increases equilibrium time t employment.

(e) Show what happens to the equilibrium level of employment and R_t after a positive shock to ε_t. (Assume that the money supply doesn't change in response to the shock in ε_t).

Question 2- Monetary and Fiscal Policy (60 points)

Consider the following model economy. Final goods production is carried out in a perfectly competitive industry that utilizes intermediate goods. The technology for producing final goods is given by

$$Y_t = \left\{ \int_0^1 \left[y_t(i)^{(\eta-1)/\eta} \right] di \right\}^{\eta/(\eta-1)}$$

where Y_t denotes the final good, and $y_t(i)$ denotes intermediate good i, $i \in [0,1]$. The time t price of the final good is P_t.

Intermediate good firm i is a monopolist producer of intermediate good i. The firm is owned by the representative household and it distributes its profits to the household at the end of each period. The firm rents capital and labor from households in perfectly competitive factor markets and produces output according to the technology

$$y_t(i) = \theta_t K_t(i)^{\alpha} H_t(i)^{1-\alpha}.$$

Here θ_t denotes the state of technology at time t, $0 < \alpha < 1$, $K_t(i)$ denotes the amount of capital employed by firm i at time t, $H_t(i)$ is the amount of labor employed by firm i at time t. The time t real rental rate of capital and the real wage rate are r_t and w_t, respectively.

We assume that θ_t evolves according to

$$\begin{aligned} \theta_t &= \exp(\varphi_t) \\ \varphi_t &= \rho_\theta \varphi_{t-1} + \varepsilon_{\theta,t} \end{aligned}$$

where $|\rho_\theta| < 1$ and $\varepsilon_{\theta,t}$ is an i.i.d. random variable. Firm i maximizes the objective function

$$E_s \left\{ \beta \frac{U_{c,t+1}}{P_{t+1}} \left[P_t(i) y_t(i) - P_t w_t H_t(i) - P_t r_t K_t(i) \right] \right\}$$

where $U_{c,t+1}$ is the time $t+1$ marginal utility of the representative consumer, and E_s is the expectations operator conditional on time s information. Below we will consider two scenarios: (i) contemporaneous pricing, $s = t$, and (ii) prices set one period in advance, $s = t-1$.

The representative household is infinitely lived with preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, 1 - L_t)$$

where E_0 denotes the expectations operator conditional on time 0 information, $\beta \in (0,1)$, c_t is time t consumption, L_t is time t labor and leisure endowment is normalized to one. The function $U(.,.)$ is twice continuously differentiable, strictly increasing in both its arguments and strictly concave. Households face the cash in advance constraint

$$P_t c_t \leq M_t + M_t^s (G_t - 1) - B_t$$

where M_t denotes the household's beginning-of-period cash balances, M_t^s denotes the per household money supply and G_t denotes the gross money growth rate, $G_t = M_{t+1}^s / M_t^s$. The household views M_t^s and G_t as exogenous random variables beyond its ability to influence. So monetary injections are carried out as lump sum transfers at the beginning of each period. B_t denotes the household's choice of one period nominal bonds, each promising to pay R_t dollars at the end of time period t. The household's stock of money evolves according to

$$\begin{aligned} M_{t+1} &= M_t + M_t^s (G_t - 1) + P_t (w_t L_t + r_t K_t) \\ & -P_t c_t - P_t T_t - P_t [K_{t+1} - (1-\delta) K_t] + \Pi_t + B_t (R_t - 1). \end{aligned}$$

Here T_t denotes time t real lump sum tax payments to the fiscal authority. Note that we are assuming that capital accumulation occurs at the household level and that investment is a credit good.

Fiscal policy is modeled as a stochastic process for government purchases, g_t that evolves according to

$$\begin{aligned} g_t &= \exp(\psi_t) \\ \psi_t &= \rho_g \psi_{t-1} + \varepsilon_{g,t} \end{aligned}$$

where $\left|\rho_g\right| < 1$ and $\varepsilon_{g,t}$ is an i.i.d. random variable. Assume that

$$g_t = T_t \text{ for all } t.$$

Monetary policy is given by a process for either R_t or G_t. When monetary policy is specified as a rule for R_t, then G_t is endogenous. When monetary policy is specified as a rule for G_t, then R_t is endogenous.

(a) What is final good producers' demand for $y_t(i)$ as a function of Y_t and $P_t(i)/P_t$? Find an expression that specifies equilibrium P_t solely as a function of intermediate good prices.

(b) What are the first order conditions to intermediate good firm i's problem? There are two cases to consider. In case (i) $s = t$, so that intermediate goods producers set their prices after seeing the time t values of g_t and θ_t. In case (ii), $s = t-1$, and intermediate good producers set their prices before seeing the time t values of g_t and θ_t. Firms understand that they must satisfy the demand for the good given the realized values of g_t and θ_t.

(c) What are the first order conditions to the household's problem?

(d) Define a sequence of markets equilibrium for this economy.

(e) Consider the version of the model in which prices are not set in advance, i.e. $s = t$.

- **(e1)** Show that a monetary policy which sets $R_t = R$ for all t will eliminate monetary distortions on capital accumulation (i.e. the household's Euler equation for investment looks the same as in the standard one sector growth model).
- **(e2)** Show that the policy of setting $R_t = R$ for all t places no restrictions on G_0. Show that this implies that nominal prices are not determinate.

(e3) Show that if preferences are separable and logarithmic in consumption, a monetary policy which sets the growth rate of money according to

$$\frac{1}{G_{t+1}} = \frac{1}{R\beta} + v_{t+1}$$

induces the equilibrium relationship $R_t = R$ for all t. Here v_{t+1} is any mean zero i.i.d. random variable. Provide intuition for this result.