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Wedges in Economies with Private Information

1 A Canonical Economy with Private Information

Consider an economy populated by a continuum of ex ante identical agents, that live for two periods. In each period they consume a vector of consumption goods $c_t = [c_{t1}, c_{t2}, ..., c_{tn}]$, $t = 0, 1$ $n \geq 1$. In the first period of their life they receive an endowment y_0. In the second period, the supply labor effort l and receive a skill shock θ. Their labor output is $y_1 = \theta l$. The skill shock is distributed iid across agents, with support $\Theta = \{\underline{\theta}, \bar{\theta}\}$, $\underline{\theta} < \bar{\theta}$, and probability distribution $\pi(\theta)$, with $\sum_\theta \pi(\theta) = 1$ and $\pi(\theta) > 0$, all $\theta \in \Theta$. Agents preferences are represented by the following utility function:

$$U(c_0, c_1, l) = u(c_0) + u(c_1) - v(l),$$

where $v : \mathbb{R}_+ \to \mathbb{R}$ and $u : \mathbb{R}^n_+ \to \mathbb{R}$ are strictly increasing in all arguments and twice continuously differentiable functions. We assume that u is strictly concave while v is strictly convex. For simplicity we also assume that u and v satisfy Inada conditions.

The informational structure is as follows: consumption, y_0 and y_1 are public information, while θ and l are private information.

We will assume that a benevolent planner chooses allocations to maximize the present discounted value of agents' lifetime utility. Here, agents are ex post different, hence, the can be characterized by their type θ. An allocation will be given by a triple $\{c_0(\theta), c_1(\theta), y(\theta)\}$. Since agents do not learn their skill until the beginning of period 1, $c_0(\theta) = c_0$. This amounts to the requirement that the time zero elements of the allocation be measurable with respect to the time 0 information. See Golosov, Kocherlakota and Tsyvinski (2003) for the generalized treatment of this requirement. Given the measurability restriction, we will denote allocations simply with $\{c_0, c_1(\theta), y(\theta)\}$. Note that the allocation is formulated in terms of observables.

The planner has access to a storage technology with rate of return $r \geq 0$. Hence, the resource constraints for the planner are:

$$\sum_{j=1}^{n} c_{0j} + K_1 \leq y_0, \tag{1}$$

$$\sum_{\theta} \pi(\theta) \sum_{j=1}^{n} c_{1j}(\theta) - \sum_{\theta} \pi(\theta) y(\theta) \leq K_1 (1+r). \tag{2}$$

Given the information structure, the planner also faces incentive compatibility constraints. These constraints require that the allocation designed for an agent of type θ is preferred by that agent to the allocation designed for an agent of type θ'. If incentive compatibility is satisfied, agents will self-select the allocation that is designed for their type. The incentive compatibility conditions for this economy can be written as:

$$U(c_0, c_1(\theta), y(\theta)/\theta) \geq U\left(c_0, c_1(\theta'), y(\theta')/\theta\right), \text{ for all } \theta, \theta' \in \Theta. \tag{3}$$

The planner's problem can then be formulated as follows:

$$V^* = \max_{K_1 \geq 0, \{c_0, c_1(\theta), y(\theta)\}_{\theta \in \Theta}} \sum_{\theta} \pi(\theta) U(c_0, c_1(\theta), y(\theta)/\theta)$$

subject to (1), (2), and (3).

We will refer to the allocation that solves the planning problem as constrained-efficient and denote with $\{c_0^*, c_1^*(\theta), y^*(\theta)\}$. Then, $V^* = U(c_0^*, c_1^*(\theta), y^*(\theta)/\theta)$.

The planning problem can be solved by using Lagrangian methods. Constraint (1) can be substituted into the problem. We denote with λ the multiplier on constraint (2), and with $\mu(\theta)$ the multipliers on the two incentive compatibility constraints in (3). The first order necessary conditions for the planner's problem are:

$$-U_{0j} + \lambda(1+r) = 0, \text{ for } j = 1, 2, ..n, \tag{1}$$

$$\pi(\theta) U_{1j}(\theta) - \lambda\pi(\theta) - \mu(\theta)[-U_{1j}(\theta)] - \mu(\theta')\left[U_{1j}(\theta;\theta')\right] = 0, \text{ for } j = 1, 2, ..n, \ \theta \in \Theta, \tag{2}$$

$$\pi(\theta) U_l(\theta)/\theta + \lambda\pi(\theta) - \mu(\theta)[-U_l(\theta)/\theta] - \mu(\theta')\left[U_l(\theta;\theta')/\theta'\right] = 0, \text{ for } \theta \in \Theta. \tag{3}$$

Here, U_{0j}, U_{1j} and U_y denote the derivative of U with respect c_{0j}, c_{1j} for $j = 1, 2, ...n$, and l, respectively. Moreover, $U(\theta)$ denotes the utility obtained from the allocation $\{c_0, c_1(\theta), y(\theta)\}$ by an agent of type θ, while $U(\theta;\theta') = U(c_0, c_1(\theta), y(\theta)/\theta')$ denotes the utility obtained from allocation $\{c_0, c_1(\theta), y(\theta)\}$ by an agent of type θ'.

There are two incentive compatibility constraints, however, only one of them will be binding. We will establish that $\mu(\bar{\theta}) > 0$ and $\mu(\underline{\theta}) = 0$. Suppose instead that $\mu(\bar{\theta}) = 0$ and $\mu(\underline{\theta}) > 0$, so that:

$$v\left(y(\bar{\theta})/\underline{\theta}\right) - v(y(\underline{\theta})/\underline{\theta}) = u\left(c_1(\bar{\theta})\right) - u(c_1(\underline{\theta})) > v\left(y(\bar{\theta})/\bar{\theta}\right) - v\left(y(\underline{\theta})/\bar{\theta}\right).$$

Consider a perturbation to this candidate solution in which $y'(\bar{\theta}) = y(\bar{\theta}) + \Delta_H$ and $y'(\underline{\theta}) = y(\underline{\theta}) - \Delta_L$, with $\pi(\bar{\theta})\Delta_H = \pi(\underline{\theta})\Delta_L$. Such a perturbation is

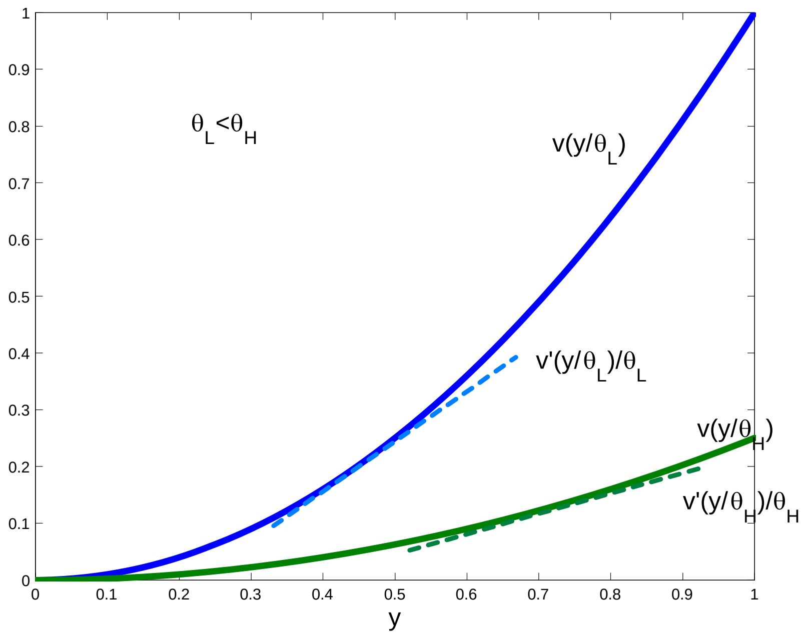

Figure 1: Convexity of $v(\cdot)$ and incentive compatibility.

feasible and incentive compatible, since the IC constraint on the high ability agent is slack. The effect on the planner's objective is given by:

$$\begin{aligned} & -\pi\left(\bar{\theta}\right)\Delta_H v'\left(y\left(\bar{\theta}\right)/\bar{\theta}\right)/\bar{\theta}+\pi\left(\underline{\theta}\right)\Delta_L v'\left(y\left(\underline{\theta}\right)/\underline{\theta}\right)/\underline{\theta} \\ = \; & \pi\left(\bar{\theta}\right)\Delta_H\left[v'\left(y\left(\underline{\theta}\right)/\underline{\theta}\right)/\underline{\theta}-v'\left(y\left(\bar{\theta}\right)/\bar{\theta}\right)/\bar{\theta}\right]>0, \end{aligned}$$

by the convexity of v. See figure. Since the perturbation is feasible, incentive compatible and it improves utility, the original candidate solution could not have been optimal. Hence, $\mu\left(\bar{\theta}\right)>0$ and $\mu\left(\underline{\theta}\right)=0$.

Equations 2 and 3 can then be rewritten as follows:

$$\pi\left(\bar{\theta}\right)U_{1j}\left(\bar{\theta}\right)-\lambda\pi\left(\bar{\theta}\right)-\mu\left(\bar{\theta}\right)\left[-U_{1j}\left(\bar{\theta}\right)\right]=0,\text{ for } j=1,2,..n, \tag{2H}$$

$$\pi\left(\underline{\theta}\right)U_{1j}\left(\underline{\theta}\right)-\lambda\pi\left(\underline{\theta}\right)-\mu\left(\bar{\theta}\right)\left[U_{1j}\left(\underline{\theta};\bar{\theta}\right)\right]=0,\text{ for } j=1,2,..n, \tag{2L}$$

$$\pi\left(\bar{\theta}\right)U_l\left(\bar{\theta}\right)/\bar{\theta}+\lambda\pi\left(\bar{\theta}\right)-\mu\left(\bar{\theta}\right)\left[-U_l\left(\bar{\theta}\right)/\bar{\theta}\right]=0, \tag{3H}$$

$$\pi\left(\underline{\theta}\right)U_l\left(\underline{\theta}\right)/\underline{\theta}+\lambda\pi\left(\underline{\theta}\right)-\mu\left(\bar{\theta}\right)\left[U_l\left(\underline{\theta};\bar{\theta}\right)/\bar{\theta}\right]=0. \tag{3L}$$

2 Partial Insurance, Labor and Intertemporal Wedge

Assume for simplicity that $n = 1$, so that in each period there is just one consumption good. We will now characterize three basic properties of the constrained-efficient allocation using equations 1, 2L, 2H, 3L, and 3H.

Partial Insurance

Combining 2L and 2H:

$$u'\left(c_1^*\left(\underline{\theta}\right)\right) - u'\left(c_1^*\left(\bar{\theta}\right)\right) = \frac{\mu\left(\bar{\theta}\right)}{\pi\left(\underline{\theta}\right)} u'\left(c_1^*\left(\underline{\theta}\right)\right) + \frac{\mu\left(\bar{\theta}\right)}{\pi\left(\bar{\theta}\right)} u'\left(c_1^*\left(\bar{\theta}\right)\right) > 0. \quad \text{(PI)}$$

Hence, if the incentive compatibility constraint is binding, the c.-e. allocation features partial insurance, that is $c_1^*\left(\underline{\theta}\right) < c_1^*\left(\bar{\theta}\right)$.

Labor Wedge

Combining 2L and 3L and 2H and 3H, respectively obtains:

$$u'\left(c_1^*\left(\underline{\theta}\right)\right) - v'\left(y_1^*\left(\underline{\theta}\right)/\underline{\theta}\right)/\underline{\theta} = \frac{\mu\left(\bar{\theta}\right)}{\pi\left(\underline{\theta}\right)}\left[u'\left(c_1^*\left(\underline{\theta}\right)\right) - v'\left(y_1^*\left(\underline{\theta}\right)/\bar{\theta}\right)/\bar{\theta}\right] \neq 0, \quad \text{(LWL)}$$

$$u'\left(c_1^*\left(\bar{\theta}\right)\right) - v'\left(y_1^*\left(\bar{\theta}\right)/\bar{\theta}\right)/\bar{\theta} = 0. \quad \text{(LWH)}$$

Hence, when the incentive compatibility constraint is binding there is a wedge between the marginal rate of substitution between consumption and labor and the marginal rate of transformation, $\underline{\theta}$, for types with low ability. Instead, the labor wedge is not distorted for the high types.

Intertemporal Wedge

Combining equations 1, 2L and 2H:

$$Eu'\left(c_1^*\left(\theta\right)\right) - \frac{u'\left(c_0^*\right)}{1+r} = \mu\left(\bar{\theta}\right)\left[u'\left(c_1^*\left(\underline{\theta}\right)\right) - u'\left(c_1^*\left(\bar{\theta}\right)\right)\right] > 0, \quad \text{(IW)}$$

where the inequality follows from PI and $\mu\left(\bar{\theta}\right) > 0$. Then:

$$(1+r)\, Eu'\left(c_1^*\left(\theta\right)\right) - u'\left(c_0^*\right) > 0.$$

This inequality states that, at the optimal allocation, the cost of transfering expected utility to future period is greater than the cost in terms of marginal utility of current consumption. This additional cost stems from the adverse effect on incentives of increasing expected utility which occurs due to the concavity of u.

3 Consumption Wedges

If we allow $n > 1$, we can evaluate whether the c.-e. allocations displays any distortions between the marginal rate of substitution across different consumption goods and the marginal rate of transformation, which is equal to 1. Combining equation 2H for two goods i, j with $i \neq j$ obtains:

$$\left(\pi\left(\bar{\theta}\right) + \mu\left(\bar{\theta}\right)\right) U_{1j}\left(\bar{\theta}\right) - \left(\pi\left(\bar{\theta}\right) + \mu\left(\bar{\theta}\right)\right) U_{1i}\left(\bar{\theta}\right) = 0, \text{ for } i, j = 1, 2, ..n,$$

which implies $u_j\left(c_1^*\left(\bar{\theta}\right)\right) = u_i\left(c_1^*\left(\bar{\theta}\right)\right)$ for all i, j, where $u_i(\cdot) = \partial u(\cdot)/\partial c_{1i}$. Similarly, using equation 2L:

$$\left(\pi\left(\underline{\theta}\right) - \mu\left(\bar{\theta}\right)\right) U_{1j}\left(\underline{\theta}\right) = \left(\pi\left(\underline{\theta}\right) - \mu\left(\bar{\theta}\right)\right) U_{1i}\left(\underline{\theta}\right), \text{ for } j, i = 1, 2, ..n,$$

which implies $u_j\left(c_1^*\left(\underline{\theta}\right)\right) = u_i\left(c_1^*\left(\underline{\theta}\right)\right)$ for all i, j. Hence, there is no wedge between the marginal rate of substitution between any two consumption goods and the marginal rate of transformation, which is equal to one.

References

[1] Golosov, M., Kocherlakota, N., and Tsyvinski, A., "Optimal Indirect and Capital Taxation," *The Review of Economic Studies 70*, 569-88, 2003.