Optimal and Time Consistent Monetary and Fiscal Policy with Heterogeneous Agents

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Abstract

This paper studies the structure and time consistency of optimal monetary policy from a public finance perspective in an economy where agents differ in transaction patterns and asset holdings. I find that heterogeneity breaks the link between lack of government commitment and high inflation which characterizes representative agent models of optimal fiscal and monetary policy. Even under commitment, it may be optimal to depart from Friedman's rule for setting nominal interest rates. Moreover, optimal monetary and fiscal policy are time consistent. Time consistency does not require outstanding nominal claims on the government to be zero.

Keywords: Inflation, Heterogeneity, Distribution, Time Consistency

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1 Introduction

This paper explores the properties of optimal monetary policy from a public finance perspective in an economy where agents are heterogeneous in earning ability, transaction patterns and asset holdings. With a representative agent and no lump sum taxes, two main results hold. First, Friedman's rule for setting nominal interest rates is optimal in a large class of economies, if the government can commit to future policies. Second, as shown by Calvo (1978), if outstanding nominal claims on the government are positive, the incentive to reduce their present value via unanticipated inflation makes optimal fiscal and monetary policy time inconsistent. It follows that, in representative agent economies, high inflation is associated with lack of commitment in government policy. This paper shows that heterogeneity breaks the link between lack of commitment and high inflation. Even under commitment, it may be optimal to depart from the Friedman rule if labor income taxes are proportional or progressive and optimal monetary and fiscal policy can be made time consistent. Time consistency does not require that outstanding nominal claims on the government are zero. The sufficient conditions for time consistency impose restrictions on the distribution and maturity structure of both real and nominal government debt.

I analyze optimal policy and the issue of time consistency following the approach of Lucas and Stokey (1983). I consider a generalized version of their cash-credit good economy. Agents are infinitely lived, they value leisure and consumption and they differ in earning ability. Agents can make purchases with currency or pay a fixed cost and use an alternative transactions technology. In equilibrium, agents with higher earning ability hold less cash as a fraction of total purchases, consistent with empirical evidence on transaction patterns. The government must finance an exogenous stream of government consumption by proportional labor income taxes, seigniorage or by issuing real and nominal debt of different maturities.

I first solve the Ramsey problem, where policy is chosen to maximize a weighted average of agents' lifetime utility in the initial period. This amounts to assuming that the initial government has a commitment technology that binds the actions of future governments. In monetary economies with a representative agent, the Friedman rule is optimal when preferences are homothetic in consumption and separable between consumption and leisure as shown by Chari, Christiano and Kehoe (1996). Correia and Teles (1999) establish that the Friedman rule is a robust feature of Ramsey policies, in the sense that, even when these restrictions on preferences are not satisfied, optimal departures from the Friedman rule are small. Alvarez, Kehoe and Neumeyer (2002) argue that optimality of the Friedman rule is a very general result, since the restrictions on preferences required are the same as those for balanced growth.

I adopt a preference specification for which the Friedman rule is optimal in the corresponding representative agent economy and I show that, with heterogeneous agents, the Friedman rule is optimal only when agents with low earning ability have a sufficiently high weight in the government's objective function. Otherwise, distributional considerations determine significant departures from the Friedman rule. Since households with low earning ability hold more cash as a fraction of their total purchases, they are more exposed to anticipated inflation. Conversely, household with high earning ability bear a larger share of the labor income tax burden. Then, a departure from the Friedman rule shifts the tax burden from households with high earning ability to those with low earning ability. I show that this result also holds when the labor income tax is progressive.

These findings are related to Atkinson and Stiglitz (1976). They show that, with heterogeneous agents and weakly separable utility, uniform commodity taxation is optimal, only when the labor income tax schedule is sufficiently unconstrained. Hence, the introduction of heterogeneity extends and confirm the connection between optimality of the Friedman rule in monetary economies and optimality of uniform commodity taxation in real economies, established by Chari, Christiano and Kehoe (1996) in a representative agent setting.

I proceed to analyze time consistency of Ramsey policies. Lucas and Stokey (1983) show that a benevolent government may have an incentive to revise the intertemporal path of taxes to depreciate the present value of government debt. This *real time inconsistency* can be eliminated by appropriately restructuring outstanding claims on the government in a real economy. On the other hand, in a monetary economy optimal fiscal and monetary policy cannot be made time consistent in general. This is because to ensure time consistency, both real and nominal government debt held by the private sector must be non-zero. But a positive level of outstanding nominal government liabilities gives rise to *nominal time inconsistency* - the incentive to inflate away these nominal debt via changes in the price level. Lucas and Stokey conclude that commitment to a path for nominal prices is required for time consistency in a monetary economy. This ensures that payments on nominal debt represent a binding real commitment. Alvarez, Kehoe and Neumeyer (2002) establish that optimality of the Friedman rule is a necessary and sufficient condition for time consistency of optimal monetary and fiscal policies. They assume that agents can adjust their currency holdings in response to the current price level, as in Lucas and Stokey (1983). Their result is based on the observation that, under the Friedman rule, a monetary economy is equivalent to a real economy. Then, the restriction that the present value of nominal government liabilities must be zero is inconsequential and real debt restructuring is sufficient to make Ramsey policy time consistent. Persson, Persson and Svensson (2005) show that when agents cannot adjust their holdings of currency to the current price level, as in Svensson (1985), then Ramsey policies can be made time consistent even if the Friedman rule does not hold.

I show that, with heterogeneous agents, optimal monetary and fiscal policies can be made time consistent. Time consistency obtains irrespective of agents' ability to adjust their holdings of currency in response to the current price level and does not depend on whether the Friedman rule is optimal. The basis for this result is that nominal time inconsistency can be removed by implementing an appropriate *distribution* of nominal government liabilities. On the other hand, removing real time inconsistency imposes constraints on the *level* of both real and nominal claims on the government at all future dates. Since time consistency does not require that the present value of nominal claims on the government to be zero at any date, the sufficient conditions for time consistency determine the maturity structure of nominal, as well as real, government debt.

The findings on time consistency are in line with an observation put forth by Alexander Hamilton (1795). He argued in favor of the Federal assumption of the states' war debt as a way to reduce the risk of monetization. Debt assumption would provide powerful government creditors with a strong incentive to support Federal tax legislation, making the use of inflation to raise revenues less likely. The model is silent on how to implement a particular *distribution* of government debt as a competitive equilibrium outcome. Since agents are indifferent between different portfolio compositions in equilibrium, implementing a distribution of government debt requires breaking this indifference. For nominal government debt, only the relative positions across different agents matter. A possible strategy is to issue the appropriate quantity of nominal bonds and segment the market, so that only one type of agent holds nominal claims against the government. A natural way to achieve segmentation would be to issue large denomination nominal bonds. Then, only agents with sufficiently high earning ability would hold them in equilibrium. Calomiris (1991) documents that the US Federal government systematically pursued a debt policy tending to segment the government bond market between 1790 and 1880.

The plan of the paper is as follows. Section 2 describes the model. Section 3 studies optimal fiscal and monetary policy under commitment. Section 4 characterizes the sufficient conditions for time consistency. Section 5 concludes.

2 A Cash-Credit Good Economy with Heterogenous Households

In this section, I describe a version of Lucas and Stokey's cash-credit good economy. The economy is populated by households, firms and a government. Households consume, supply labor and trade in assets in each period. Households differ in labor productivity but have identical preferences. They make purchases with currency or with an alternative payment technology. A fixed cost associated with avoiding the use of cash implies that households with lower labor productivity and lower income hold more currency as a fraction of total purchases. This feature of the economy is consistent with cross–sectional evidence on transaction patterns and asset holdings[1]. In each period trade in

[1] Erosa and Ventura (2000) report that in the US low income households use cash for a greater fraction of their total purchases relative to high income households. Mulligan and Sala-i-Martin (2000) estimate the probability of adopting financial technologies that hedge against inflation and find that is positively related to the level of household wealth and inversely related to the level of education. Attanasio, Guiso and Jappelli (2001) find that the probability of using an interest bearing bank account increases with educational attainment, income and average consumption, based on cross-sectional household data for Italy.

goods and labor precedes trade in assets, as in Svensson (1985)[2]. Firms have access to a linear production technology that requires labor for the production of consumption goods. They are perfectly competitive. The government finances an exogenous stream of spending by issuing real and nominal debt of different maturities, printing money and taxing labor income at a uniform proportional rate. There is no uncertainty.

I now illustrate the problems faced by the agents in our economy in detail.

2.1 Firms

There are two types of competitive firms. All firms live for one period. Firms in the production sector hire labor to produce a continuum of differentiated consumption goods indexed on the interval $[0,1]$. The production technology is linear, with one efficiency unit of labor producing one unit of consumption. Different consumption goods are perfect substitutes in production. Hence, perfect competition implies:

$$P_t(j) = W_t, \tag{1}$$

for $j \in [0,1]$, where $P_t(j)$ is the price charged for good j and W_t the nominal wage for an efficiency unit of labor at time t. Hence, P_t will denote the price of consumption goods.

Firms in the financial sector produce transaction services, enabling households to purchase consumption goods without the use of cash, as in Prescott (1987). A financial firm's profit for providing transaction services for the purchase of good j is given by:

$$\pi_t(j) - W_t\theta(j), \tag{2}$$

where $\theta(\cdot)$ is measured in efficiency units of labor and satisfies $\theta' > 0$ on the interval $[\underline{z}, 1]$, with $\underline{z} \geq 0$. π_t is the dollar charge for arranging purchases of consumption good j without currency. Profit maximization implies: $\pi_t(j) = W_t\theta(j)$ for all t and all $j \in [0,1]$.

2.2 Households

There is a continuum of unit measure of households, divided into two types, where $0 < \nu_i < 1$ is the fraction of type i agents, with $i = 1,2$ and $\sum_i \nu_i = 1$. All households have identical preferences defined over a consumption aggregator c^i and over hours of work n^i, given by:

$$\sum_{t=0}^{\infty} \beta^t U\left(c_t^i, n_t^i\right),$$

$$c^i = \left[\int_0^1 c^i(j)^\rho \, dj\right]^{\frac{1}{\rho}}, \tag{3}$$

[2] An alertative timing is studied in Appendix D.

where $\rho \in (0,1)$ for a household of type $i = 1,2$. I will restrict attention to preferences of the class:

$$U\left(c^i, n^i\right) = h\left(c^i\right) + v\left(n^i\right),$$

where h is strictly increasing and strictly concave, while v is strictly decreasing and concave.

Households of the same type are identical. Households of different types differ in labor productivity, denoted with ξ_i, for $i = 1,2$. I will assume $\xi_2 > \xi_1$.

In each period, households choose transaction services, the level of consumption for each good, supply labor, accumulate currency and trade one-period nominal discount bonds. Given (3) and the assumption on transaction costs, households will optimally choose z_t^i, the fraction of consumption goods purchased without the use of cash, and $c_{1,t}^i$, $c_{2,t}^i$ is the level of consumption of goods purchased with and without currency, respectively, with:

$$c^i = \left[\left(1 - z^i\right)\left(c_1^i\right)^{\rho} + z^i\left(c_2^i\right)^{\rho}\right]^{\frac{1}{\rho}}.$$

They enter a period with M_t^i units of currency and are subject to a cash in advance constraint, given by:

$$P_t c_{1,t}^i\left(1 - z_t^i\right) - M_t^i \leq 0. \tag{4}$$

The asset market session follows trading in the goods and labor market. During the asset market session households receive labor income net of taxes, clear consumption liabilities and trade nominal and real bonds of different maturities issued by other households or by the government. Nominal (real) bonds purchased at time t entitle holders to one unit of currency (consumption) in the asset market section at $t+1$. I assume that the government and private agents are committed to debt repayments. This implies that agents are indifferent between holding privately or government issued bonds. The price in terms of currency of a nominal bond of maturity s at time t is $Q_{t,t+s}$. Analogously, the price in terms of currency of a real bond with maturity s at time t is $P_t q_{t,t+s}$. If the government does not issue debt, the bonds will be in zero net supply. Total holdings of nominal and real bonds by agent i at the end of time t are denoted with $B_{t,t+s}^i$ and $b_{t,t+s}^i$ for $i = 1,2$ and $s > 0$.

Households face the following constraint on the asset market:

$$\begin{aligned} & M_{t+1}^i + \sum_{s>0}\left(Q_{t,t+s}B_{t,t+s}^i + q_{t,t+s}P_t b_{t,t+s}^i\right) \qquad (5) \\ \leq \quad & M_t^i + \sum_{r=-1}^{t-1}\left(B_{r,t}^i + P_t b_{r,t}^i\right) - P_t c_{1,t}^i\left(1 - z_t^i\right) - P_t c_{2,t}^i z_t^i - \int_{\underline{z}}^{z_t^i}\pi_t(j)\,dj + W_t\xi_i\left(1 - \tau_t\right)n_t^i, \end{aligned}$$

where τ_t is the tax rate on labor income and $\int_{\underline{z}}^{z_t^i}\pi_t(j)\,dj$ the currency cost of arranging purchases of consumption goods with credit. In addition, the no-Ponzi game condition:

$$0 \leq \left[\sum_{r=-1}^{t} B^i_{r,t+1} \left(\prod_{s=0}^{r+1} Q_{s,s+1} \right)^{-1} + \sum_{r=-1}^{t} P_t b^i_{r,t+1} \left(\prod_{s=0}^{r+1} q_{s,s+1} \right)^{-1} \right] \Phi_{t+1} \tag{6}$$
$$+ Q^{-1}_{t,t+1} M^i_{t+1} \Phi_{t+1} + \sum_{s=1}^{\infty} \Phi_{t+s} W_{t+s} \left(1 - \tau^i_{t+s}\right) \xi_i,$$

is also required, with $\Phi_t = \prod_{t'=0}^{t-1} Q_{t',t'+1}$, $\Phi_0 = 1$.

2.3 Government

The government finances an exogenous stream of consumption $\bar{g}$ and is subject to the following dynamic budget constraint:

$$P_t \bar{g}_t + M_t + \sum_{\hat{t}=0}^{t-1} \left(B_{\hat{t},t} + P_t b_{\hat{t},t}\right) = \sum_{s>0} (Q_{t,t+s} B_{t,t+s} + q_{t,t+s} P_t b_{t,t+s}) + M_{t+1} + W_t T_t, \tag{7}$$

where M_t, B_t , b_t are the supply of currency, nominal and real bonds, respectively, and:

$$T_t = \sum_i \nu_i \tau_t \xi_i n^i_t.$$

2.4 Private Sector Equilibrium

The timing of events in each period is as follows:

1. Households come into the period with holdings of currency and debt given by M^i_t and $B^i_{t,t+s}$, $b^i_{t,t+s}$ for $t = -1, 0, 1, ...$ and $s > 0$. They choose z^i_t.

2. The government sets policy subject to (7) and (**??**).

3. Households, firms and the government trade on the goods and labor markets. The households' purchases of cash goods are subject to (4). Equilibrium on the goods market requires:

$$\sum_{i=1,2} \nu_i \left(c^i_{1,t} \left(1 - z^i_t\right) + c^i_{2,t} z^i_t + \int_0^{z^i_t} \theta(j)\, dj - \xi_i n^i_t \right) + g_t = 0. \tag{8}$$

4. Asset markets open. Households purchase bonds and acquire currency to take into the following period subject to the constraint (5). Equilibrium

in the asset market requires:

$$\begin{aligned}\sum_{i=1,2}\nu_i B^i_{t,t+s} &= B_{t,t+s},\ \text{for } s>0, \\ \sum_{i=1,2}\nu_i b^i_{t,t+s} &= b_{t,t+s},\ \text{for } s>0, \\ \sum_{i=1,2}\nu_i M^i_{t+1} &= M_{t+1}.\end{aligned} \tag{9}$$

Definition 1 *A private sector equilibrium is given by a government policy* $\{g_t,$ $\tau_t,\ M_{t+1},\ B_{t,t+s},\ b_{t,t+s}\}_{t\geq 0,s>0}$, *a price system* $\{P_t,\ W_t,\ Q_{t,t+s},\ q_{t,t+s},\ \pi_t\,(j)\}_{t\geq 0,s>0,j\in[0,1]}$ *and an allocation* $\{c^i_{1,t},\ c^i_{2,t},\ n^i_t,\ z^i_t,\ B^i_{t,t+s},\ b^i_{t,t+s}\}_{i=1,2,t\geq 0,s>0}$ *such that:*

1. *given the policy and the price system households and firm optimize;*
2. *government policy satisfies (7);*
3. *markets clear.*

The following proposition characterizes the competitive equilibrium.

Proposition 2 *An allocation* $\left\{c^i_{1,t}, c^i_{2,t}, n^i_t, z^i_t, B^i_{t,t+s}, b^i_{t,t+s}\right\}_{i=1,2,t\geq 0,s>0}$ *and a price system* $\{P_t, W_t, Q_{t,t+s}, q_{t,t+s}, \pi_t\,(j)\}_{t\geq 0,s>0,j\in[0,1]}$ *constitute a private sector equilibrium if and only if, for a given government policy* $\{g_t, \tau_t, M_{t+1}, B_{t,t+s}, b_{t,t+s}\}_{t\geq 0,s>0}$, *(8), (7) and the following conditions are verified:*

$$Q_{t,t+s} = \beta^s \frac{P_t}{P_{t+s}}\frac{\hat{u}^i_{2,t+s}}{\hat{u}^i_{2,t}},\ \textit{for } s>0, \tag{10}$$

$$q_{t,t+s} = \beta^s P_t \frac{\hat{u}^i_{2,t+s}}{\hat{u}^i_{2,t}},\ \textit{for } s>0, \tag{11}$$

$$\begin{aligned} 0 &< Q_{t,t+s} \leq 1, \\ 0 &< q_{t,t+s} \leq 1, \end{aligned}$$

$$\frac{-u^i_{n,t}}{\hat{u}^i_{2,t}} = \xi_i\,(1-\tau_t)\ \textit{for } t\geq 0, \tag{12}$$

$$W_t = P_t,$$

$$R_{t+1} \equiv \frac{\hat{u}^i_{1,t+1}}{\hat{u}^i_{2,t+1}} = Q^{-1}_{t,t+1}, \tag{13}$$

$$\begin{aligned} (R_t-1)\left(P_{t+1}c^i_{1,t+1}\left(1-z^i_{t+1}\right) - M^i_{t+1}\right) &= 0, \\ P_{t+1}c^i_{1,t+1}\left(1-z^i_{t+1}\right) &\leq M^i_{t+1}, \end{aligned}$$

$$\left[\left(\frac{1}{\rho}-1\right)\left(1-R_s^{\frac{\rho}{\rho-1}}\right)-\frac{\theta\left(z_s^i\right)}{c_{2,s}^i}\right]\left\{\begin{array}{l}\leq 0 \text{ for } z_s^i=\underline{z}, \\ =0 \text{ for } z_s^i\in\left(\underline{z},\bar{z}\right), \\ \geq 0 \text{ for } z_s^i=\bar{z}.\end{array}\right. \tag{14}$$

for $t\geq 0$, *and:*

$$P_0 c_{1,0}^i\left(1-z_0^i\right)\leq M_0^i, \tag{15}$$

$$\hat{u}_{1,0}^i\frac{M_0^i}{P_0}+\hat{u}_{2,0}^i\frac{B_{(-1),0}^i}{P_0}+\hat{u}_{2,0}^i\sum_{t=1}^{\infty}\frac{B_{(-1),t}^i}{P_0}\prod_{j=1}^{t}R_j+\sum_{t=0}^{\infty}\beta^t\hat{u}_{2,t}^i b_{(-1),t}^i \tag{16}$$
$$\leq \sum_{t=0}^{\infty}\beta^t\left[u_{1,t}^i c_{1,t}^i+u_{2,t}^i\hat{c}_{2,t}^i+u_{n,t}^i n_t^i\right].$$

for $i=1,2,$ *with* $C\left(z_t^i\right)=\int_{\underline{z}}^{z_t^i}\theta\left(j\right)dj.$

Here, $u_{j,t}^i=\partial U\left(c_t^i,n_t^i\right)/\partial c_{j,t}^i$, $u_{n,t}^i=U_2\left(c_t^i,n_t^i\right)$ and $\hat{c}_2^i=c_2^i+\frac{C\left(z^i\right)}{z^i}$, $\hat{u}_1^i=u_1^i/\left(1-z^i\right)$, $\hat{u}_2^i=u_2^i/z^i$ for $i,j=1,2$. Equation (16) is the households' intertemporal budget constraint and it incorporates the transversality condition. The proof of this proposition is in Appendix A.

3 Optimal Policy with Commitment

I define a Ramsey equilibrium as the private sector equilibrium which maximizes the government's objective function, which is given by the weighted sum of the households' lifetime utility. The Pareto weight on type i agents is η_i, with $\eta_1+\eta_2=1$. I assume that Pareto weights are time-invariant. The case $\eta_i=\nu_i$ corresponds to a utilitarian government.

A Ramsey equilibrium can be characterized by solving a *primal problem*, where the government chooses an allocation at time 0 subject to the constraint that it constitutes a private sector equilibrium. This problem's choice variables are $\left\{c_{1,t}^i,c_{2,t}^i,n_t^i,z_t^i,\right\}_{i=1,2,t\geq 0}$ and P_0. The level of P_0 determines the real value of nominal assets at time 0 and defines the boundary of the agents' intertemporal budget set. High values of P_0 amount to a tax on currency and outstanding nominal claims. The government is constrained to tax all nominal claims at the same rate. The extent to which each household is hit by this tax depends on the distribution of currency and nominal bonds at time 0.

Proposition 3 *An allocation* $\left\{c_{i1t},c_{i2t},n_{it},z_{it}\right\}_{i=1,2,t\geq 0}$ *and values of* $\left\{R_t\right\}_{t\geq 0}$ *and* P_0 *constitute a Ramsey equilibrium if and only if they solve the primal problem:*

$$\max_{P_0,\left\{c_{1,t}^i,c_{2,t}^i,n_t^i,z_t^i,\right\}_{i=1,2,t\geq 0}}\sum_{t=0}^{\infty}\beta^t\sum_{i=1,2}\eta_i U\left(c_t^i,n_t^i\right)$$

subject to:

$$\frac{\hat{u}_{1,t}^i}{\hat{u}_{2,t}^i}=R_t,\ \text{for } i=1,2, \tag{17}$$

$$R_t \geq 1, \tag{18}$$

$$\frac{-u^2_{n,t}}{\xi_2 \hat{u}^2_{2,t}} = \frac{-u^1_{n,t}}{\xi_1 \hat{u}^1_{2,t}}, \tag{19}$$

(14) and (8) for all t, as well as (16) and (15).

The linearity of the labor income tax implies that the wedge between the marginal rate of substitution between consumption and leisure and labor productivity are equated across households, which corresponds to constraint (19). The proof of proposition 3 parallels the one for a representative agents economy in Chari,Christiano and Kehoe (1996), and is omitted for brevity.

3.1 Properties of Optimal Policy for $t > 0$

I now illustrate the key properties of Ramsey equilibrium policy for $t > 0$.

Proposition 4 *Assume:*

(A1) $U(c,n) = h(c) + v(n)$, with $h(\cdot)$ strictly increasing and strictly concave and $v(\cdot)$ strictly decreasing and concave,

(A2) $b^i_{(-1),t} = 0$ and $B^i_{(-1),t} = 0$, $i = 1,2$, $t > 0$,

(A3) $\theta(j)$ is strictly increasing for $j \in [\underline{z}, 1]$, with $\underline{z} \geq 0$, and $\lim_{z \downarrow \underline{z}} \theta(z) > 0$.

Then, $R_t = 1$ for $t \geq 1$ in the Ramsey equilibrium, if and only if constraint (19) is not binding.

The proof of this proposition, which can be found in Appendix B, relies on the homotheticity of the consumption aggregator and separability of utility in consumption and leisure imposed in (A1). It is analogous to the proof of the optimality of the Friedman rule for a representative agent economy in Christiano, Chari and Kehoe (1996)

Assumption (A2) is imposed to make the analysis comparable with Chari, Christiano and Kehoe (1996) and Correia and Teles (1999)[3]. Assumption (A3) guarantees that $z^i_t > \underline{z}$ for $i = 1,2$ if $R_t > 1$, so that z^i_t is interior when the Friedman rule does not hold. Proposition 4 holds irrespective of the functional form of $v(\cdot)$ and initial conditions.

The intuition for the result lies in the effect of the inflation tax on the wedge between the marginal rate of substitution and the marginal rate of transformation between consumption and labor. Proportional labor income taxation forces this wedge to be equalized across agents in any private sector equilibrium, which results in constraint (19) in the Ramsey allocation problem. This constraint will be typically be binding, since for distributional or efficiency reasons a benevolent government would want to set agent specific labor income tax rates.

[3] Interest income accrues at the end of the period. Hence, without (A2) the Ramsey equilibrium nominal interest rate could be positive and non-stationary, since departing from the Friedman rule is a way to tax interest income from real and nominal bonds and a tax on interest income is not available. These consideration also arise with a representative agent, as pointed out in Lucas and Stokey (1983).

A departure from the Friedman rule can relax the constraint resulting from proportional labor income taxation, given that low productivity agents hold more cash as a fraction of total purchases. To illustrate how this affects the equilibrium wedge between the marginal rate of substitution between consumption and labor and the marginal rate of transformation, it is useful to define the type specific consumption price indexes, P_t^i, $\hat{P}_t^i$ for $i = 1, 2$ for $t > 0$:

$$P_t^i = \left[\left(1 - z_t^i\right)\left(R_t\right)^{\frac{\rho}{\rho-1}} + z_t^i\right]^{\frac{\rho-1}{\rho}},$$

$$\hat{P}_t^i = P_t^i + \frac{C\left(z_t^i\right)}{c_t^i}. \tag{20}$$

P_i^i measures the cost in efficiency units of labor of one unit of the consumption aggregator c_t^i for given z_t^i. $\hat{P}_i^i$ measures the cost in efficiency units of labor of one unit of c_t^i when z_t^i solves (14), including the cost of z_t^i [4]. At $R_t = 1$, $c_t^i = c_{1,t}^i = c_{2,t}^i$ holds, and, for $R_t > 1$ optimality implies $\hat{P}_t^i \leq R_t$ for $i = 1, 2$. For a given R_t, (14) implies $z_t^2 > z_t^1$ and $P_t^1 > P_t^2$ and $\hat{P}_t^1 \geq \hat{P}_t^2$.

Then, if $R_t > 1$, the wedge between the marginal utility of leisure and the marginal utility of consumption is higher for low productivity households:

$$\left(1 - \tau_t\right)\frac{1}{\hat{P}_t^2} > \left(1 - \tau_t\right)\frac{1}{\hat{P}_t^1}. \tag{21}$$

A departure from the Friedman rule is equivalent to a higher after tax real wage in efficiency units for high productivity households relative to low productivity households, for a given proportional labor income tax rate.

Proposition 4 easily extends to economies in which the tax rate on labor earnings can be agent specific by is constrained to be *progressive*[5]. Such a constraint can be captured as:

$$\kappa_1 \tau_t^1 + \kappa_0 \leq \tau_t^2,\ \kappa_0 \geq 0,\ \kappa_1 \geq 1, \tag{22}$$

[4] This price index is derived from the solution of the following static optimization problem:

$$\max_{c_{i1}, c_{i2}, z_i} \left[\left(1 - z_i\right) c_{i1}^{\rho} + z_i c_{i2}^{\rho}\right]^{1/\rho} \text{ subject to}$$
$$w = R c_{i1}\left(1 - z_i\right) + c_{i2} z_i + C\left(z_i\right),$$

where w is an exogeous endowment of real wealth. Let:

$$c_i = \left[\left(1 - z_i\right) c_{i1}^{\rho} + z_i c_{i2}^{\rho}\right]^{1/\rho},$$

and denote the expenditure function with $e(R;\theta)$ and the value function with $v\left(R; w, \theta\right)$. Then, the optimal value of c_i solves $c_i = v\left(R; w, \theta\right)$ and:

$$\hat{P}^i = \frac{e(R; w, \theta)}{c_i}.$$

[5] The requirement that labor income taxation is progressive, as capture in constraint (22), can be interpreted as a feature of the fiscal system that is specified at a constitutional stage and is taken as given by the government. For discussion, see Buchanan (1967).

where τ_t^i denotes the tax rate imposed on households of type i at time t. Constraint (22) imposes that the average labor income tax rate for high productivity households is higher than for low productivity households.

The following intuitive result can also be shown:

Proposition 5 *Assume (A1), (A2), (A3) and:*

(A4) $\frac{U_{cc}c}{U_c} + \frac{U_{nn}n}{-U_n} \geq 0.$

Then, if (22), imposed with $\kappa_0 = 0$*, is non-binding,* $\eta_1 \geq \bar{\eta}_1$*, with* $0 < \bar{\eta}_1$*. The Friedman rule is optimal for* $\eta_1 \geq \bar{\eta}_1$*.*

The proof is in appendix B. Proposition 5 illustrates that with progressive labor income taxation, provided (A4) holds, there exists a range of Pareto weights where the Friedman rule is not optimal. This range is characterized by a relatively high Pareto weight for high productivity households- a low value of η_1. This is consistent with the effect of departures from the Friedman rule on the wedge between the marginal utility of leisure and consumption. Under assumption (A4), the intertemporal rate of substitution of labor is greater than that of consumption. This assumption on preferences ensures that the optimal wedge between the marginal utility of labor and the marginal utility of consumption for each type of agent i is decreasing in η_i and in the value of the multiplier on the implementability constraint, which measures the cost of the distortions imposed by the fiscal system[6]. Hence, (A4) guarantees that is it is optimal for the government to set lower labor income taxes on agents with higher Pareto weights and on which the marginal cost of distortionary taxes is higher.

As in a representative agent economy, a connection can be drawn between proposition 4 and the results on optimal uniform commodity taxation in public finance. Atkinson and Stiglitz (1976) show that, if the labor income tax schedule is sufficiently unconstrained and utility is weakly separable between consumption and leisure, then it is optimal to tax all commodities at the same rate, irrespective of the weighting of different agents in the social welfare function. Their result is based on the following logic. Since the wedge between leisure and consumption is only affected by the sub-utility derived from consumption, as long as resources are scarce and there are no constraints on distribution, a benevolent government seeks to deliver this sub-utility in the cost minimizing way. Constraints on the labor income tax schedule may give rise to a conflict between efficiency and distribution, which induces the government to abandon uniform commodity taxation. For the economy in this paper, the cost minimizing way of delivering a given sub-utility from consumption is to follow the

[6] If constraints (17)-(19) are not binding, this wedge is equal to:

$$-\frac{u_n^i}{u_c^i} = \frac{\eta_i + \lambda_i + \lambda_i \left(\frac{u_{cc}^i c^i}{u_c} \right)}{\eta_i + \lambda_i + \lambda_i \frac{u_{nn}^i}{u_n^i} n^i},$$

where $\lambda_i > 0$ is the multiplier on the implementability constraint.

Friedman rule. However, this policy is not optimal when the government must adopt proportional or progressive labor income taxes. These findings are also consistent with da Costa and Werning (2001), who examine the optimality of the Friedman rule with non-linear income taxation and idiosyncratic, unobservable shocks to the agents' labor productivity.

3.2 Numerical Illustration

To illustrate the findings in the previous section, I compute the Ramsey equilibrium as a function of the Pareto weights for a plausibly parametrized version of the economy. I focus on the utility specification:

$$U\left(c^i, n^i\right) = \frac{\left(c^i\right)^{1-\sigma} - 1}{1-\sigma} + v\left(n^i\right), \text{ for } i = 1, 2, \ \sigma > 0, \tag{23}$$

with:

$$v\left(n^i\right) = \gamma_0 \frac{\left(1 - n^i\right)^{1-\gamma_1}}{1-\gamma_1}, \ \gamma_0, \gamma_1 > 0. \tag{24}$$

The transactions technology is given by:

$$\begin{aligned} \theta(j) &= 0 \text{ for } j \leq \underline{z}, \\ &= \theta_0 \left(\frac{j - \underline{z}}{\bar{z} - j}\right)^{\theta_1} \text{ for } j \in (\underline{z}, \bar{z}) \\ &= \infty \text{ for } j \geq \bar{z}, \end{aligned}$$

where $0 \leq \underline{z} < \bar{z} \leq 1$, which is a generalization of the one used by Dotsey and Ireland (1996).

Parameters are set as follows. The fraction of low productivity households in the population is set to 0.6 and their productivity is set to $\xi_1 = 1$. The discount factor is set to 0.97. To evaluate to effect of different degrees of curvature on the utility from consumption I consider two values of σ, 0.8 and 1.2. Other parameter values are chosen so that in a steady state with $\tau_1 = \tau_2 = 0.30$ and $R = 1.05$ the model matches corresponding averages for the US economy, conditional on the value of σ. The parameters for $v(\cdot)$, γ_0, γ_1, are set to ensure that households work for approximately one third of their time endowment. Conditional on σ and the parameters for v, ξ_2 is set so that the Gini coefficient for consumption in the model is equal to 25.5% . This is equal to the average value of the consumption-Gini coefficient for the US in the post-war period based on CEX data, as reported by Krueger and Perri (2001). Government spending is set to equal 20% to GDP. Three parameters, ρ, θ_0 and θ_1, determine the behavior of money demand. I set ρ =0.5, which corresponds to an elasticity of substitution between consumption goods of 2. Then, I set θ_0 and θ_1 to approximate the interest elasticity and the average velocity of transactions accounts (currency plus checkable deposits, plus time and savings deposits) as a fraction of personal consumption expenditures[7]. These two statistics are equal to -5.11% and 1.37, respectively, in the post-war period, based on Flow of Funds data. This strategy gives rise to four parameterizations. The parameters that are kept constant across parameterizations are in Table 1, all other parameters values are in Table 2. Parameterization A.1 and A.2 correspond to (**??**). Parameterization B.1 and B.2 correspond to (24). Parameterization A.1 serves as a benchmark.

Table 1: Parameter Values

β	$\underline{z}$	$\overline{z}$	ν_1	ξ_1	ρ
0.97	0.10	0.75	0.6	1	0.5

Table 2: Alternative Parameterizations

A.1 (Benchmark)						B.1					
σ	γ_0	γ_1	ξ_2	θ_0	θ_1	σ	γ_0	γ_1	ξ_2	θ_0	θ_1
0.8	1.6	1	2.5	0.053	0.2	0.8	2.2	0.5	2.8	0.058	0.2
A.2						B.2					
1.2	2.6	1	3.3	0.056	0.2	0.8	2.2	1.5	2.8	0.058	0.2

I analyze the Ramsey equilibrium for these four parameterizations under different constraints on labor income taxation. Initial real and nominal debt holdings are set to 0 and the distribution of currency is symmetric.

The top panel of Figure 1 displays the properties of the Ramsey equilibrium as a function of $\eta_1$for Parameterization A.1 under: .

$$\tau^2 \geq \tau^1. \tag{25}$$

Under (25), labor income taxation is progressive. The top right panel exemplifies the results in propositions 4 and 5. There is a value of the Pareto weight, $\bar{\eta}_1$, for which the constraint on tax rates becomes non-binding, and, for $\eta_1 \geq \bar{\eta}_1$ the Friedman rule is optimal. The net nominal interest rate peeks at 16% at $\eta_1 = 0.25$ and is decreasing in η_1 as long as the Friedman rule is not optimal. The tax rate on type 2 households increases with η_1. As long as the constraint on distribution is binding, the tax rate on type 1 agents is equal to the tax rate on high skill agents. For $\eta_1 \geq \bar{\eta}_1$, it is decreasing with $\eta_{1,}$ since in this region of the Pareto space the government wishes to distribute to type 1 agents. The tax rate on type 2 systematically raises with η_1, from 0.19 to 0.65, while the tax rate on type 1 is increasing in η_1 as long as the constraint on distribution is binding and falls with η_1 otherwise. For η_1 greater than 0.5 the tax rate on low skill households hits the 0 lower bound. For $\eta_1 \geq 0.65$, the tax rates for low and high skill households are set so that the net of tax distribution of income is reversed and high skills households have lower consumption than low skill households in equilibrium. The vertical dotted line corresponds to that point.

[7] The interest elasticity in the model is computed as:

$$\frac{\partial \log (M/Pc)}{\partial \log(R)},$$

where M/P are aggregate real money balances and c is aggregate consumption.

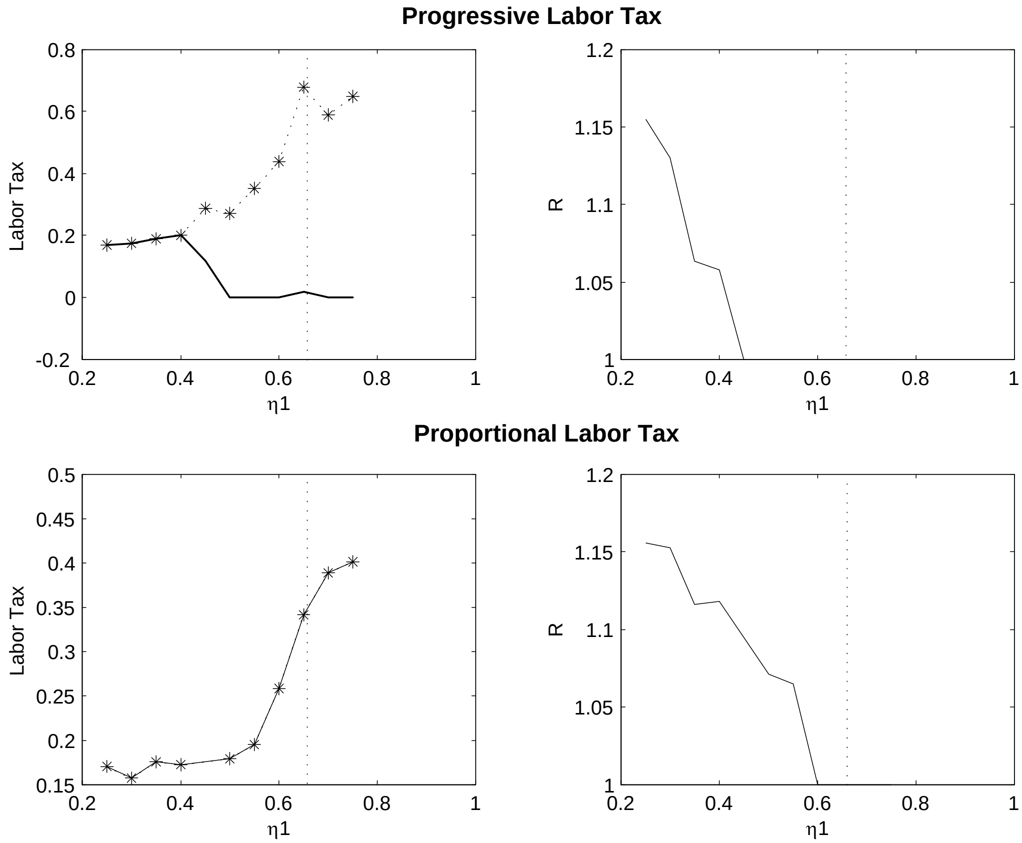

Figure 1: Benchmark parameterization

The bottom panel of Figure 1 displays the features of Ramsey equilibrium policy as a function of η_1 for $t > 0$, under Parameterization A.1, for proportional labor income taxation. As in the previous case, there is a value of the Pareto weight, $\bar{\eta}_1$, such that for $\eta_1 \geq \bar{\eta}_1$ the Friedman rule is optimal. The tax rate on labor is increasing in η_1, even for $\eta_1 > \bar{\eta}_1$. This is due to the fact that for higher η_1 the multiplier on the implementability constraint on type 2 falls (and the one for type 1 increases). This reduces the shadow cost of raising distortionary taxes from type 2 and induces a rise in the optimal tax rate. The tax rate on labor varies from 0.17 to 0.41, while the net nominal interest rate from 16% to 0.

Table 3 reports the main results under alternative parameterizations for progressive labor income taxes- under (25). The comparison between parameterizations A1 and A2 illustrate the effect on increasing the curvature on consumption in utility. The comparison between B_1 and B_2 illustrates the effects of labor supply elasticity on the properties of the Ramsey equilibrium.

Table 3: Comparison of Ramsey Equilibria

Parameterizations	A1	A2	B1	B2
max R	1.1549	1.1213	1.12615	1.1807
$\bar{\eta}_1$	0.5	0.45	0.6	0.5
max τ^1, τ^2	0.1895, 0.677	0.191, 0.86	0.177, 0.654	0.175, 0.675
min τ^1, τ^2	0, 0.0169	0, 0.162	0, 0.166	0, 0.164

Higher curvature on the utility from consumption corresponds to a lower value of R and a lower value of $\bar{\eta}_1$, implying that the range of Pareto weights for which the Friedman rule is not optimal is smaller for A2 relative to the benchmark parameterization, A1. This stems from the fact that a higher curvature on the utility from consumption intensifies the distortionary effects of departures from the Friedman rule. In parameterization B1 the Frisch labor supply elasticity is equal to 4 while it is approximately equal to 1.3 in B2. The distortionary effect of labor income taxation increases with the Frisch elasticity of labor supply. The nominal interest rate is higher for B1 and the value of $\bar{\eta}_1$ lower than in B2. The greater distortionary effect of labor income taxation on labor supply generates an incentive for the government to rely more heavily on the inflation tax.

4 Sufficient Conditions for Time Consistency

Asset holdings and transaction patterns do not respond to changes in current policy, thus giving rise to a potential for time inconsistency of Ramsey policies. As in Lucas and Stokey (1983), there is a *real* and a *nominal* time inconsistency problem. Real time inconsistency stems from the government's incentive to change the intertemporal path of labor tax rates and nominal interest rates, thus affecting equilibrium real interest rates and the real present value of claims on the government held by each type of agent. Nominal time inconsistency may arise given that outstanding nominal claims on the government are fixed and changes in the price level act as a lump-sum tax. The distributional consequences of possible deviations from the Ramsey plan that arise with heterogeneous agents may remove or exacerbate the time inconsistency problem.[8]

I derive sufficient conditions for time consistency of Ramsey policies following Lucas and Stokey (1983). For any $t \geq 0$, define the Ramsey problem at period t analogously to the Ramsey problem for period 0. The Ramsey problem at period

[8] This argument was first explored by Rogers (1986), who studies optimal wage and interest taxation in a two-period, multiple consumer economy. She finds that the incentive to raise interest taxation may be moderated if the resulting create utility distribution is unacceptable.

t is said to be time consistent for period $t+1$, if the continuation allocation of the solution to the Ramsey problem at period t solves the Ramsey problem at $t+1$. The Ramsey equilibrium is time consistent if the Ramsey problem at time t is time consistent for the Ramsey problem at $t+1$ for $t \geq 0$. In practice, it is sufficient to verify that initial conditions for the time 1 problem exist that would induce the government at time 1 to continue with the allocation that solves the Ramsey problem at time 0.

With this procedure, it is possible to establish the following result.

Proposition 6 *Assume (A1). The Ramsey equilibrium under (22) is time consistent.*

The proof is in Appendix C. Proposition 6 marks a stark contrast with the findings for a representative agent economy. As illustrated by Lucas and Stokey (1983), the Ramsey equilibrium policy cannot be made time consistent in a monetary economy in general. This is due to the fact that government policy determines two wedges, the one between the marginal utility of cash and credit goods and the one between leisure and consumption. This implies that two debt instruments are required to remove real time inconsistency and ensure that the government at future dates finds it optimal to continue with the policy chosen at a previous date. However, to remove the nominal time inconsistency problem, the present discounted value of nominal government debt must be equal to 0 at all dates. This restriction eliminates the possibility of solving the real time inconsistency problem.

To understand how nominal time inconsistency can be removed with heterogeneous agents, it is useful to analyze the following example.

The economy lasts for three periods and all goods are cash goods. The government issues indexed debt of all maturities and nominal debt of one period maturity. Firms are as in the general model.

The households' problem is:

$$\max_{\left\{c_t^i, n_t^i, M_{t+1}^i, {}_s b_t^i\right\}_{t=0}^2} \sum_{t=0}^{2} \beta^t u\left(c_t^i, n_t^i\right),$$

subject to

$$P_t c_t^i \leq M_t^i$$

$$P_t c_t^i + Q_t B_t^i + \sum_{t<s\leq 2} q_{t,s}\ b_{t,s}^i + M_{t+1}^i \leq M_t^i + W_t \xi_i n_t^i \left(1-\tau_t\right) + B_{t-1}^i + \sum_{-1\leq r<t} P_t\ b_{r,t}^i,$$

for $t = 0, 1, 2$, with M_0^i, $b_{-1,t}^i$, B_{-1}^i given.

The first order necessary condition for the agents problem are:

$$\frac{-u_{n,t}^i}{\xi_i u_{c,t}^i} = \frac{W_t}{P_t}\frac{\left(1-\tau_t\right)}{R_t},$$

$$
\begin{aligned}
\frac{u^i_{c,t+1}}{P_{t+1}}\,Q_t-\beta\frac{u^i_{c,t+2}}{P_{t+2}} &= 0,\\
\frac{u^i_{c,t+1}}{P_{t+1}}q_{t,s}-\beta^{s-t}u^i_{c,t+s+1} &= 0.
\end{aligned}
$$

The government budget constraint is:

$$
P_tg_t+\sum_{-1\le r<t}\left(P_t\ b_{r,t}\right)+M_t\le W_t\tau_tn_t+Q_tB_t+\sum_{t<s\le 2}q_{t,s}b_{t,s}+M_{t+1},
$$

for $t=0,1,2$, with M_0 and ${}_tb_{-1}$ given.

The implementability constraints are:

$$
\sum_{t=0}^{2}\beta^t\left[u^i_{c,t}\left(c^i_t-b^i_{-1,t}\right)+u^i_{n,t}n^i_t\right]=u^i_{c,0}\left(\frac{M^i_0}{P_0}+\frac{B^i_{-1}}{P_0}\right). \tag{26}
$$

Let $W^i\left(c,n;\eta_i,\lambda_i\right)=\eta_iu\left(c,n\right)+\lambda_i\left(u_cc+u_nn\right)$. The Ramsey allocation problem is:

$$
\begin{aligned}
&\max\sum_{i=1,2}\left\{\sum_{t=0}^{2}\beta^t\left[W^i\left(c^i_t-b^i_{-1,t},n^i_t\right)-\zeta_t\left(-1\right)^i\frac{-u^i_{n,t}}{\xi_iu^i_{c,t}}\right]\right.\\
&\left.-\lambda_iu^i_{c,0}\left(\frac{M^i_0}{P_0}+\frac{B^i_{-1}}{P_0}\right)-\mu^i\left(c^i_0-M^i_0/P_0\right)\right\},
\end{aligned}
$$

by choice of $\left\{c^i_t,n^i_t\right\}_{t,i}$ and $P_0\ge 0$[9].

The first order necessary conditions for the Ramsey allocation problem are:

$$
\begin{aligned}
0 &= W^i_{c,t}-\zeta_t\left(-1\right)^i\frac{-u^i_{n,t}}{\xi_iu^i_{c,t}}\frac{-u^i_{cc,t}}{u^i_{c,t}}-\omega_t-\lambda_ib^i_{-1,t}u^i_{cc,t},\ t=1,2, && (27)\\
0 &= W^i_{c,0}-\zeta_t\left(-1\right)^i\frac{-u^i_{n,0}}{\xi_iu^i_{c,0}}\frac{-u^i_{cc,0}}{u^i_{c,0}}-\omega_0-\mu^i_0 && (28)\\
&\quad -\lambda_i\left(b^i_{-1,0}+\frac{B^i_{-1}}{P_0}+\frac{M^i_0}{P_0}\right)u^i_{cc,0},\\
0 &= W^i_{n,t}-\zeta_t\left(-1\right)^i\frac{-u^i_{nn,t}}{\xi_iu^i_{c,t}}+\xi_i\omega_t,\ t=0,1,2 && (29)
\end{aligned}
$$

$$
-\left[\sum_i\mu^i_0\frac{M^i_0}{P_0}-\sum_i\lambda_i\left(\frac{B^i_{-1}}{P_0}+\frac{M^i_0}{P_0}\right)u^i_{c,0}\right]\left\{\begin{array}{l}\le 0,\ P_0\ge 0\\ >0,\ P_0\to\infty,\end{array}\right. \tag{30}
$$

for $i=1,2$, jointly with the complementary slackness conditions on the proportional labor tax and on the cash in advance constraint at time 0.

[9] The time 0 cash in advance constraint needs to be imposed on the problem in addition to the implementability constraints, given the Svensson timing. This contributes the term $\delta_i\left(c^i_0-M^i_0/P_0\right)$ to the Lagrangian.

To verify whether the Ramsey equilibrium can be made time consistent, I evaluate the first order necessary condition for the time 1 Ramsey allocation problem at the allocations and prices that solve the time 0 Ramsey allocation problem:

$$0 = \eta_i W^i_{c,2} - \zeta'_2 (-1)^i \frac{-u^i_{n,2}}{\xi_i u^i_{c,2}} \frac{-u^i_{cc,2}}{u^i_{c,2}} - \omega'_2 - \lambda'_i b^i_{0,2} u^i_{cc,2}, \tag{31}$$

$$0 = W^i_{c,1} - \zeta'_1 (-1)^i \frac{-u^i_{n,1}}{\xi_i u^i_{c,1}} \frac{-u^i_{cc,1}}{u^i_{c,1}} - \omega'_1 - \mu^{i\prime}_1 \tag{32}$$
$$-\lambda'_i \left(b^i_{0,1} + \frac{B^i_0}{P_1} + \frac{M^i_1}{P_1} \right) u^i_{cc,1},$$

$$0 = W^i_{n,t} - \zeta'_t (-1)^i \frac{-u^i_{nn,t}}{\xi_i u^i_{c,t}} + \xi_i \omega'_t, \; t = 1,2 \tag{33}$$

$$-\left[\sum_i \mu^{i\prime}_1 \frac{M^i_1}{P_1} - \sum_i \lambda'_i \left(\frac{B^i_0}{P_1} + \frac{M^i_1}{P_1} \right) u^i_{c,1} \right] = 0, \tag{34}$$

$$c^i_1 = \frac{M^i_1}{P_1}, \tag{35}$$

for $i = 1, 2$. Here, a prime denotes variables that are part of the solution to the time 1 Ramsey allocation problem. With the implementability constraints for the time 1 Ramsey allocation problem, (**??**)-(35) comprise a system of thirteen equations in the 14 unknowns: M^i_1, B^i_1, $b^i_{0,t}$, ζ'_t for $t = 1, 2$, $\mu^{i\prime}_1$, λ'_i for $i = 1, 2$ and ω'_t for $t = 1, 2$. Hence, the system is under-determined and has more than one solution. It is possible to ensure time consistency.

The example makes clear that ensuring nominal time consistency pins down the distribution of nominal assets and not their level, as can be seen from (34). This property carries through in the general model, where the analogue of condition (34) is:

$$\sum_{i=1,2} \lambda'_i \left(\hat{u}^i_{2,1} \sum_{t=1}^{\infty} B^i_{0,t} \prod_{j=1}^{t} R_j \right) + \sum_{i=1,2} \left(\lambda'_i \hat{u}^i_{1,1} - \mu^{i\prime}_1 \right) M^i_1 = 0. \tag{36}$$

Hence, removing nominal time inconsistency requires setting the *shadow* present discounted value of nominal government liabilities to 0. This shadow value is a weighted average of the present discounted value of nominal claims on the government held by each type of agent. The weights are given by the multipliers on the implementability constraints, λ'_i, in the time 1 Ramsey equilibrium, which measure the marginal cost of raising distortionary taxes on agent of type i. The weight on currency holdings is adjusted for the liquidity value of outstanding currency in the time 1 Ramsey equilibrium, given by the multipliers on the cash in advance constraint at time 1, $\mu'_{i,1}$, This adjustment is due to the Svensson timing.

Condition (36) pins down a *distribution* of the present value of nominal government liabilities across agents of different types at time 1. This eliminates the incentive to change P_1 without imposing constraints on the *level* of nominal claims on the government at dates $t > 1$. Then, real time inconsistency can be removed by selecting an appropriate maturity structure of the shadow value of nominal government debt at all $t > 1$ and with an appropriate distribution and maturity structure of real government debt at all dates $t \geq 1$. Condition (36) has a simple interpretation. If the shadow present discounted value of nominal government debt is 0 at time 1, then a government selecting policy at time 1 cannot gain by changing P_1 relative to what is prescribed by the continuation of the time 0 Ramsey plan. This occurs because the gains from the standpoint of efficiency, which would result from an increase in P_1, are offset by distributional costs connected with the heterogeneity in nominal debt holdings in the population. Time consistency can be achieved without imposing that nominal government debt is 0.

Note that since M_1^i is determined by consumption needs at time 1, (36) can always be satisfied by $B_{0,t}^i = 0$ for $t > 1$ for $i = 1$ or $i = 2$. This implies that the distribution of nominal debt that guarantees time consistency can be implemented by segmenting the market for nominal government bonds so that only one type of agent holds all the nominal claims against the government. Since agents in equilibrium are indifferent to the composition of their portfolio between nominal and real government bonds, one way to achieve this outcome is to issue nominal bonds in large denominations. Then, only agents with high earning ability would hold them in equilibrium.

An important corollary of Proposition 6 is that the sufficient conditions for time consistency determine the maturity structure of both real and nominal government debt. The conditions on the maturity structure of real debt are analogous to those arising in a representative agent economy. The maturity structure of nominal debt that ensures time consistency removes that the government's incentive to change the path of the nominal interest rate at time 1 relative to what is prescribed by the continuation allocation of the time 0 Ramsey equilibrium. The conditions that determine the maturity structure of nominal debt constrain the shadow present discounted value of nominal bond payments outstanding at each date $t > 1$, which is an average of nominal bond payments due to each type of agent, weighted by the Lagrangian multipliers on the implementability constraints. Thus, they restrict the distribution of nominal debt at each maturity and, not the level of nominal debt.

Alvarez, Kehoe and Neumeyer (2002) show that for representative agent economies optimality of the Friedman rule is a necessary and sufficient condition for time consistency of Ramsey policies. The intuition for this finding is that a monetary economy under the Friedman rule is analogous to a real economy and only one debt instrument is sufficient to ensure time consistency. Therefore, the restriction that nominal debt is equal to 0 at all dates and all states, required to remove nominal time inconsistency, is inconsequential for real time consistency. By contrast, with heterogeneous agents, Ramsey policies are time consistent irrespective of whether the Friedman rule is optimal.

In Alvarez, Kehoe and Neumeyer (2002), as well as in Lucas and Stokey (1983), nominal time inconsistency is particularly severe since agents can adjust their currency holdings in response to changes in the price level and there are no costs associated with unanticipated inflation. Nicolini (1998) shows that nominal time inconsistency can be moderated if, as in Svensson (1985), agents cannot adjust their currency holdings in response to changes in the price level and unanticipated inflation is not a lump sum tax[10]. Persson, Persson and Svensson (2005) show that under the Svensson timing, it is possible to guarantee the time consistency of optimal fiscal and monetary policies in a representative agent economy, via an appropriate restructuring of real and indexed government debt, irrespective of whether the Friedman rule is optimal.

Appendix D derives sufficient conditions for time consistency under the timing convention adopted in Lucas and Stokey (1983). The analogue of Proposition 6 holds in this case. In a representative agent version of the economy, this would not hold, since under Lucas and Stokey timing there are no costs of unanticipated inflation. With heterogeneous agents, unanticipated changes in the price level do affect the consumption allocation since they redistribute wealth across agents with different outstanding levels of nominal claims on the government.

5 Concluding Remarks

I describe a monetary economy in which households have different earning ability and this implies that in equilibrium they are heterogeneous in transaction patterns and asset holdings. In this environment, monetary and fiscal policies have a distributional effect. I show that heterogeneity breaks the link between high inflation and lack of commitment in government policy. First, government commitment does not imply low inflation. Since agents with low earning ability are more vulnerable to inflation. surprisingly high rates of inflation are optimal under commitment, if the labor income tax is proportional or progressive and their weight in the government's objective function is sufficiently low. In addition, optimal fiscal and monetary policies are time consistent in the sense of Lucas and Stokey (1983). Time consistency does not require nominal government liabilities to be 0 at any date. These findings are in stark contrast with the results for a representative agent economy. The key for time consistency of Ramsey policies with heterogeneous agents is that the government's incentive to deviate from previously announced policies depends crucially on the distribution of nominal government debt. Then, this incentive can be eliminated by implementing an appropriate distribution of nominal government liabilities.

[10]Nicolini does not consider labor income taxation and stops short of analyzing the case with nominal government debt.

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6 Appendix

6.1 A: Characterization of Private Sector Equilibria

Assume that an allocation $\left\{c_{1,t}^i, c_{2,t}^i, n_t^i, z_t^i, B_{t,t+s}^i, b_{t,t+s}^i\right\}_{i=1,2,t\geq 0,s>0}$, with $n_t^i > 0$ for $i = 1, 2$ and $t \geq 0$, and a price system $\{P_t,\ W_t,\ Q_{t,t+s},\ q_{t,t+s},\ \pi_t(j)\}_{t\geq 0, j\in[0,1]}$ constitute a private sector equilibrium for a given policy $\left\{\bar{g}_t, \tau_t^i, M_{t+1}, B_{t+1}\right\}_{t\geq 0}$. Then, conditions (1) and (2) derive from optimality of firm behavior, conditions (8) and (9) from clearing in the goods and assets markets. The other conditions follow from household optimization.

The Lagrangian for the household problem is given by:

$$\begin{aligned} L \quad = \quad & \sum_{t=0}^{\infty} \beta^t \left\{ U\left(c_t^i, n_t^i\right) - \delta_t^i \left(P_t c_{1,t}^i \left(1 - z_t^i\right) - M_t^i\right) \right. \\ & -\phi_t^i \left[M_{t+1}^i + \sum_{s>0} \left(Q_{t,t+s} B_{t,t+s}^i + q_{t,t+s} P_t b_{t,t+s}^i\right) - M_t^i - \sum_{\hat{t}=0}^{t-1} \left(B_{\hat{t},t}^i + P_t b_{\hat{t},t}^i\right) \right. \\ & \left.\left. -W_t \left(1 - \tau_t^i\right) \xi_i n_t^i + P_t c_{1,t}^i \left(1 - z_t^i\right) + P_t c_{2,t}^i z_t^i + \int_0^{z_t^i} \pi_t(j)\, dj \right] \right\}, \end{aligned}$$

where c_t^i is defined in (3) and δ_t^i, ϕ_t^i are the multipliers on the cash in advance constraint and the wealth evolution equation, respectively.

The necessary conditions for household optimization are given by:

$$u_{1,t}^i = P_t \left(\delta_t^i + \phi_t^i\right)\left(1 - z_t^i\right), \tag{37}$$

$$\delta_t^i \left(P_t c_{1,t}^i \left(1 - z_t^i\right) - M_t^i\right) = 0,\ \delta_t^i \geq 0, \tag{38}$$

$$u_{2,t}^i = P_t \phi_t^i z_t^i, \tag{39}$$

$$-u_{n,t}^i = W_t \left(1 - \tau_t^i\right) \xi_i \phi_t^i, \tag{40}$$

$$P_t c_{1,t}^i \left(\delta_t^i + \phi_t^i\right) - P_t c_{2,t}^i \phi_t^i - \pi_t\left(z_t^i\right) \phi_t^i \begin{cases} < 0 \text{ for } z_t^i = \underline{z}, \\ = 0 \text{ for } z_t^i \in \left(\underline{z}, \bar{z}\right), \\ > 0 \text{ for } z_t^i = \bar{z}, \end{cases} \tag{41}$$

$$\phi_t^i = \beta\left(\phi_{t+1}^i + \delta_{t+1}^i\right), \tag{42}$$

$$\phi_t^i Q_{t,t+s} = \beta^s \phi_{t+s}^i,\ \text{for } s > 0, \tag{43}$$

$$\phi_t^i q_{t,t+s} = \beta^s \phi_{t+s}^i P_{t+s},\ \text{for } s > 0, \tag{44}$$

$$\lim_{T\to\infty} \beta^T \phi_T^i M_T^i = 0,\ \lim_{T\to\infty} \beta^T \phi_T^i B_{t,T}^i = 0,\ \lim_{T\to\infty} \beta^T \phi_T^i b_{t,T}^i P_T = 0, \tag{45}$$

as well as (4) and (5). To see that (45) is a necessary condition for household optimization, suppose it does not hold and

$$\lim_{T\to\infty} \beta^T \phi_T^i M_T^i > 0,\ \lim_{T\to\infty} \beta^T \phi_T^i B_{t,T}^i > 0,\ \lim_{T\to\infty} \beta^T \phi_T^i b_{t,T}^i P_T > 0.$$

(The strictly smaller case is rule out by (6).) Then, it is possible to construct a consumption sequence such that the budget constraint is satisfied in each period and utility for each type of household is greater, violating optimality.

Combining (37)-(39) yields (13), while (39) and (40) determine (12). The expression in (10) follows from (40), (43) and (1), while (15) follows from (37)-(39) at $t = 0$.

To derive (16), multiply (5) by ϕ_t^i and apply (38) and (42). Use (37), (39)-(41), multiply by β^t and sum over t from 0 to T. Let T go to infinity and apply (45). This yields:

$$\sum_{t=0}^{\infty} \beta^t \left(u_{1,t}^i c_{1,t}^i + u_{2,t}^i \left(c_{2,t}^i + \frac{C\left(z_t^i\right)}{z_t^i} - \frac{B_{(-1),t}^i}{P_t z_t^i} - \frac{b_{(-1),t}^i}{z_t^i} \right) + u_{n,t}^i n_t^i \right) = \frac{u_{1,0}^i}{1 - z_0^i} \frac{M_0^i}{P_0}. \tag{46}$$

From (42)-(43):

$$P_t = \beta^t \frac{\hat{u}_{2,t}^i}{\hat{u}_{2,0}^i} P_0 \prod_{j=1}^{t} R_j \text{ for } t > 1,$$

with $\prod_{j=1}^{1} R_j \equiv R_1$, $\prod_{j=1}^{0} R_j \equiv 1$, where $\hat{u}_{1,t}^i = u_{1,t}^i / \left(1 - z_t^i\right)$ and $\hat{u}_{2,t}^i = u_{2,t}^i / z_t^i$. Substitute into (46), to obtain (16).

Now assume that an allocation $\left\{c_{1,t}^i, c_{2,t}^i, n_t^i, z_t^i, B_{t,t+s}^i, b_{t,t+s}^i, M_{t+1}^i\right\}_{i=1,2,t\geq 0}$ and a price system $\left\{P_t, W_t, q_{t,t+s}, Q_{t,t+s}, q_t\left(j\right)\right\}_{s>0,t\geq 0,j\in[0,1]}$ satisfy (1)-(16) and (8) for a given policy $\left\{\bar{g}_t, \tau_t, M_{t+1}, B_{t,t+s}, b_{t,t+s}\right\}_{s>0,t\geq 0}$ for which (7) holds. Then, by (1) and (11) industrial and credit services firms optimize.

To see that household optimization conditions are satisfied consider an alternative candidate plan $\{c_{i1t}', c_{i2t}', n_{it}', z_{it}'\}_{i=1,2,t\geq 0}$ which satisfies the intertemporal budget constraint for the price system $\left\{P_t, W_t, q_{t,t+s}, Q_{t,t+s}, q_t\left(j\right)\right\}$. This implies that:

$$\Delta \equiv \lim_{T\to\infty} \beta^t \left\{ u_{1,t}^i \left(c_{1,t}^i - \left(c_{1,t}^i\right)' \right) + u_{2,t}^i \left(c_{2,t}^i + \frac{C\left(z_t^i\right)}{z_t^i} - \left(c_{2,t}^i\right)' - \frac{C\left(\left(z_t^i\right)'\right)}{\left(z_t^i\right)'} \right) - \gamma \left(n_t^i - \left(n_t^i\right)' \right) \right\} \geq 0,$$

using (10) and the fact that $\left\{c_{1,t}^i, c_{2,t}^i, n_t^i, z_t^i\right\}_{i=1,2,t\geq 0}$ satisfies (13)-(16) and that the intertemporal budget constraint holds as a weak inequality using (6) and (5) for the price system $\left\{P_t, W_t, Q_t, q_t\left(j\right)\right\}_{t\geq 0,j\in[0,1]}$. By concavity of u^i :

$$D \equiv \lim_{T\to\infty} \sum_{t=0}^{T} \beta^t \left(U\left(c_t^i, n_t^i\right) - U\left(\left(c_t^i\right)', \left(n_t^i\right)'\right) \right) \geq \Delta,$$

where c_{it}' is defined by (3). This establishes the result since (9) and (8) guarantee market clearing.

6.2 B: Solving the Ramsey problem

To solve the Ramsey allocation problem, it is useful to define the function $Z(R,c) = \max\{\underline{z}, \min\{z^*, 1\}\}$ where z^* solves:

$$c\left(\frac{1}{\rho}-1\right)\left(1-R^{\frac{\rho}{\rho-1}}\right)-\theta(z^*)=0.$$

By assumption A3, $Z_c > 0$ and $Z_R > 0$ for $c > 0$ and $R \geq 1$. The constraint $z_t^i = Z\left(R_t, c_{2,t}^i\right)$ needs to be imposed on the Ramsey allocation problem to ensure that the government chooses the same value of z_t^i that would be chosen by the agents in a private sector equilibrium. This constraint is substituted in the Ramsey allocation problem.

The Lagrangian for the Ramsey problem is:

$$\begin{aligned}
\Lambda \;=\; & \sum_{t=0}^{\infty}\beta^t \sum_i \Big\{ W^i\left(c_{1,t}^i, c_{2,t}^i, n_t^i, z_t^i; \eta_i, \lambda_i\right) \\
& -\omega_t\left(\bar{g}_t + \sum_i \nu_i\left(c_{1,t}^i\left(1-z_t^i\right) + z_t^i \hat{c}_{2,t}^i + C\left(z_t^i\right) - \xi_i n_t^i\right)\right)\Big\} \\
& -\sum_{t=1}^{\infty}\beta^t\left[\mu_t\left(1-R_t\right) + \sum_i \mu_t^i\left(\frac{\hat{u}_{1,t}^i}{\hat{u}_{2,t}^i} - R_t\right)\right] - \sum_{t=0}^{\infty}\beta^t\zeta_t\left(\frac{-u_{n,t}^2}{\xi_2\hat{u}_{2,t}^2} - \frac{-u_{n,t}^1}{\xi_1\hat{u}_{2,t}^1}\right) \\
& -\sum_i \lambda_i\left(\hat{u}_{1,0}^i\frac{M_0^i}{P_0} + \sum_{t=0}^{\infty}\beta^t\hat{u}_{2,t}^i b_{(-1),t}^i + \hat{u}_{2,0}^i\sum_{t=0}^{\infty}\frac{B_{(-1),t}^i}{P_0}\prod_{j=1}^{t}R_j\right) \\
& -\sum_i \mu_0^i\left(c_{1,0}^i\left(1-z_0^i\right) - \frac{M_0^i}{P_0}\right),
\end{aligned}$$

where $W^i\left(c_{1,t}^i, c_{2,t}^i, n_t^i, z_t^i; \eta_i, \lambda_i\right) = \eta_i U\left(c_t^i, n_t^i\right) + \lambda_i\left[u_{1,t}^i c_{1,t}^i + u_{2,t}^i\left(c_{2,t}^i + \frac{C(z_t^i)}{z_t^i}\right) + u_{n,t}^i n_t^i\right].$

The Ramsey allocation problem is to maximize Λ with respect to $c_{1,t}^i$, $c_{2,t}^i$, n_t^i, z_t^i, R_t, P_0, and minimize Λ with respect to μ_t^i, λ_i, ζ_t, μ_t, ω_t for $i = 1, 2$ and $t \geq 0$, subject to $z_t^i = Z\left(R_t, c_{2,t}^i\right)$. I will characterize the solution to the Ramsey allocation problem by deriving the first order necessary conditions for this problem. Since the second order necessary conditions for this problem involve third derivative of U, the task of verifying that they hold is intractable but for very specific assumptions on U. As Lucas and Stokey (1983), I simply assume that a solution of the system of equations resulting from the first order necessary conditions exists and constitutes an optimum for the Ramsey allocation problem.

It is convenient to introduce the following notation:

$$\hat{u}^i_{11} = \frac{u^i_{11}}{(1-z^i)^2},\ \hat{u}^i_{22} = \frac{u^i_{22}}{(z^i)^2},$$
$$\hat{u}^i_{12} = \frac{u^i_{12}}{(1-z^i)\, z^i} = \hat{u}^i_{21} = \frac{u^i_{21}}{(1-z^i)\, z^i},$$

$$\begin{aligned}
\hat{W}^i_{1,t} &= (\eta_i + \lambda_i)\, \hat{u}^i_{1,t} + \lambda_i \left(\hat{u}^i_{11,t} \left(1 - z^i_t\right) c^i_{1,t} + \hat{u}^i_{12,t} z^i_t \hat{c}^i_{2,t} \right), \\
\hat{W}^i_{2,t} &= (\eta_i + \lambda_i)\, \hat{u}^i_{2,t} + \lambda_i \left(\hat{u}^i_{12,t} \left(1 - z^i_t\right) c^i_{1,t} + \hat{u}^i_{22,t} z^i_t \hat{c}^i_{2,t} \right), \\
W^i_{n,t} &= (\eta_i + \lambda_i)\, u^i_{n,t} + \lambda_i u^i_{nn,t} n^i_t,
\end{aligned}$$

$$\begin{aligned}
W^i_{z,t} &= \eta_i u^i_{c,t} c^i_{z,t} + \lambda_i \frac{1}{\rho} c^i_{2,t} \left(1 - R_t^{\frac{\rho}{\rho-1}}\right) \hat{u}^i_{2,t} \\
&\quad + \lambda_i \left[R_t \left(1 - z^i_t\right) c^i_{1,t} + z^i_t c^i_{2,t} + C\left(z^i_t\right) \right] c^i_{z,t} \hat{u}^i_{2,t} \left(\frac{u^i_{cc,t}}{u^i_{c,t}} + \frac{(1-\rho)}{c^i_t} \right),
\end{aligned}$$

where

$$c^i_{z,t} = \left[\left(c^i_{2,t}\right)^\rho - \left(c^i_{1,t}\right)^\rho \right] \left(c^i_t\right)^{1-\rho}.$$

The first order conditions for the Lagrangian $i = 1, 2$ are:

$$\begin{aligned}
\nu_i \omega_t &= \hat{W}^i_{1,t} - \lambda_i \hat{u}^i_{12,t} b^i_{(-1),t} \\
&\quad - \mu^i_t \left(\frac{\hat{u}^i_{11,t}}{\hat{u}^i_{2,t}} - \frac{\hat{u}^i_{21,t}}{\hat{u}^i_{2,t}} \frac{\hat{u}^i_{1,t}}{\hat{u}^i_{2,t}} \right) - \zeta_t (-1)^i \frac{-u^i_{n,t}}{\xi_i \hat{u}^i_{2,t}} \frac{-\hat{u}^i_{21,t}}{\hat{u}^i_{2,t}},\ \text{for } t > 0,
\end{aligned} \tag{47}$$

$$\begin{aligned}
\nu_i \omega_t &= \hat{W}^i_{2,t} - \lambda_i \hat{u}^i_{22,t} b^i_{(-1),t} \\
&\quad + \left[W^i_{z,t} - \omega_t \nu_i \left(c^i_{2,t} - c^i_{1,t} + \theta\left(z^i_t\right) \right) \right] Z_c\left(R_t, c^i_{2,t}\right) \\
&\quad - \mu^i_t \left(\frac{\hat{u}^i_{12,t}}{\hat{u}^i_{2,t}} - \frac{\hat{u}^i_{22,t}}{\hat{u}^i_{2,t}} \frac{\hat{u}^i_{1,t}}{\hat{u}^i_{2,t}} \right) - \zeta_t (-1)^i \frac{-u^i_{n,t}}{\xi_i \hat{u}^i_{2,t}} \frac{-\hat{u}^i_{22,t}}{\hat{u}^i_{2,t}},\ \text{for } t > 0,
\end{aligned} \tag{48}$$

$$0 = W^i_{n,t} - \zeta_t (-1)^i \frac{-u^i_{nn,t}}{\xi_i \hat{u}^i_{2,t}} + \xi_i \nu_i \omega_t,\ \text{for } t \geq 0, \tag{49}$$

$$\begin{aligned}
&\sum_i \left[W^i_{z,t} - \omega_t \nu_i \left(c^i_{2,t} - c^i_{1,t} + \theta\left(z^i_t\right) \right) \right] Z_R\left(R_t, c^i_{2,t}\right) + \mu_t + \sum_i \mu^i_t \\
&= \sum_i \frac{\lambda_i \hat{u}^i_{2,0}}{R_t} \sum_{s=t+1}^{\infty} \frac{B^i_{(-1),s}}{P_0} \prod_{j=1}^{s} R_j,\ \text{for } t > 0,
\end{aligned} \tag{50}$$

$$\mu_t (1 - R_t) = 0,\ \mu_t \geq 0,\ R_t \geq 1,\ \text{for } t > 0, \tag{51}$$

$$\mu_t^i \left(\frac{\hat{u}_{1,t}^i}{\hat{u}_{2,t}^i} - R_t \right) = 0, \text{ for } t > 0,$$

$$\begin{aligned} \zeta_t \left(\frac{-u_{n,t}^2}{\xi_2 \hat{u}_{2,t}^2} - \frac{-u_{n,t}^1}{\xi_1 \hat{u}_{2,t}^1} \right) &= 0, \\ \frac{-u_{n,t}^2}{\xi_2 \hat{u}_{2,t}^2} - \frac{-u_{n,t}^1}{\xi_1 \hat{u}_{2,t}^1} &\leq 0, \ \zeta_t \geq 0, \text{ for } t \geq 0, \end{aligned} \tag{52}$$

$$\begin{aligned} 0 &= \hat{W}_{1,0}^i - \mu_0^i - \zeta_0 (-1)^i \frac{-u_{n,0}^i}{\xi_i \hat{u}_{2,0}^i} \frac{-\hat{u}_{21,0}^i}{\hat{u}_{2,0}^i} \\ &\quad -\lambda_i \hat{u}_{11,0}^i \frac{M_0^i}{P_0} - \nu_i \omega_0, \end{aligned} \tag{53}$$

$$\begin{aligned} 0 &= \hat{W}_{2,0}^i - \nu_i \omega_0 - \zeta_0 (-1)^i \frac{-u_{n,0}^i}{\xi_i \hat{u}_{2,0}^i} \frac{-\hat{u}_{22,0}^i}{\hat{u}_{2,0}^i} \\ &\quad -\lambda_i \hat{u}_{22,0}^i \left(b_{(-1),0}^i + \sum_{t=0}^{\infty} \frac{B_{(-1),t}^i}{P_0} \prod_{j=1}^{t} R_j \right), \end{aligned} \tag{54}$$

$$\sum_{i=1,2} \left(-\lambda_i \hat{u}_{1,0}^i M_0^i - \lambda_i \hat{u}_{2,0}^i \sum_{t=0}^{\infty} B_{(-1),t}^i \prod_{j=1}^{t} R_j + \mu_0^i M_0^i \right) \left(\frac{-1}{P_0^2} \right) = 0, \tag{55}$$

$$\mu_0^i \left(c_{1,0}^i \left(1 - z_0^i\right) - \frac{M_0^i}{P_0} \right) = 0, \text{ with } \mu_0^i \geq 0, \ c_{1,0}^i \left(1 - z_0^i\right) \leq \frac{M_0^i}{P_0}.$$

Proof of Proposition 4

The first order conditions for the Ramsey allocation problem for $t > 0$ under assumption A2 simplify to:

$$\hat{W}_{1,t}^i - \omega_t \nu_i - \mu_t^i \left[\frac{\hat{u}_{11,t}^i}{\hat{u}_{2,t}^i} - R_t \frac{\hat{u}_{21,t}^i}{\hat{u}_{2,t}^i} \right] - \zeta_t (-1)^i \frac{-u_{n,t}^i}{\xi_i \hat{u}_{2,t}^i} \left(\frac{-\hat{u}_{21,t}^i}{\hat{u}_{2,t}^i} \right) = 0,$$

$$\begin{aligned} 0 &= \hat{W}_{2,t}^i - \omega_t \nu_i + \left[W_{z,t}^i - \omega_t \nu_i \left(c_{2,t}^i - c_{1,t}^i + \theta\left(z_t^i\right)\right)\right] Z_c \left(R_t, c_{2,t}^i\right) \\ &\quad -\mu_t^i \left[\frac{\hat{u}_{12,t}^i}{\hat{u}_{2,t}^i} - R_t \frac{\hat{u}_{22,t}^i}{\hat{u}_{2,t}^i} \right] - \zeta_t (-1)^i \frac{-u_{n,t}^i}{\xi_i \hat{u}_{2,t}^i} \left(\frac{-\hat{u}_{22,t}^i}{\hat{u}_{2,t}^i} \right), \end{aligned}$$

$$W_{n,t}^i + \omega_t \nu_i \xi_i - \zeta_t (-1)^i \frac{-u_{nn,t}^i}{\xi_i \hat{u}_{2,t}^i} = 0,$$

$$\sum_i \left[W^i_{z,t} - \omega_t \nu_i \left(c^i_{2,t} - c^i_{1,t} + \theta\left(z^i_t\right)\right)\right] Z_R\left(R_t, c^i_{2,t}\right) + \sum_i \mu_{i,t} + \mu_t = 0.$$

Further, using the definition of Z :

$$W^i_{z,t} - \omega_t \nu_i \left(c^i_{2,t} - c^i_{1,t} + \theta\left(z^i_t\right)\right) = W^i_{z,t} - \omega_t \nu_i c^i_{2,t} \left[R_t^{\frac{\rho}{\rho-1}} - R_t^{\frac{1}{\rho-1}} + \frac{1}{\rho}\left(1 - R_t^{\frac{\rho}{\rho-1}}\right)\right]$$

The term in square brackets is positive by $R_t \geq 1$ and $\rho \in (0,1)$. A sufficient condition for $W^i_z > 0$ is $\left(\frac{u^i_{cc,t}}{u^i_{c,t}} + \frac{(1-\rho)}{c^i_t}\right) > 0.$

To evaluate when the Friedman rule is optimal, note that:

$$\begin{aligned} \frac{\hat{W}^i_{1,t}}{\hat{u}^i_{1,t}} &= \eta_i + \lambda_i + \lambda_i \left(\frac{\hat{u}^i_{11,t} c^i_{1,t} + \hat{u}^i_{21,t} c^i_{2,t}}{\hat{u}^i_{1,t}}\right), \\ \frac{\hat{W}^i_{2,t}}{\hat{u}^i_{2,t}} &= \eta_i + \lambda_i + \lambda_i \left(\frac{\hat{u}^i_{12,t} c^i_{1,t} + \hat{u}^i_{22,t} c^i_{2,t}}{\hat{u}^i_{2,t}}\right), \end{aligned}$$

where by homotheticity of h^i :

$$\frac{\hat{u}^i_{12,t}\left(1 - z^i_t\right) c^i_{1,t} + \hat{u}^i_{22,t} z^i_t c^i_{2,t}}{\hat{u}^i_{2,t}} = \frac{\hat{u}^i_{11,t}\left(1 - z^i_t\right) c^i_{1,t} + \hat{u}^i_{12,t} z^i_t c^i_{2,t}}{\hat{u}^i_{1,t}}, \tag{56}$$

for any z^i_t, and by definition of $\hat{u}^i_j$ and $\hat{u}^i_{jk}$:

$$\hat{u}^i_{21} = \hat{u}^i_{12}. \tag{57}$$

Hence, $\frac{\hat{u}^i_{1,t}}{\hat{u}^i_{2,t}} = 1$ if and only if $\frac{\hat{W}^i_{1,t}}{\hat{W}^i_{2,t}} = 1.$

Consider the wedge:

$$\begin{aligned} \hat{W}^i_{1,t} - \hat{W}^i_{2,t} &= \mu_{i,t} R_t \left[\frac{\hat{u}^i_{11,t}}{\hat{u}^i_{1,t}} - \frac{\hat{u}^i_{21,t}}{\hat{u}^i_{2,t}} - \left(\frac{\hat{u}^i_{12,t}}{\hat{u}^i_{1,t}} - \frac{\hat{u}^i_{22,t}}{\hat{u}^i_{2,t}}\right)\right] \\ &\quad + \zeta_t (-1)^i \frac{-u^i_{n,t}}{\xi_i \hat{u}^i_{2,t}} \left(\frac{-\hat{u}^i_{21,t}}{\hat{u}^i_{2,t}} - \frac{-\hat{u}^i_{22,t}}{\hat{u}^i_{2,t}}\right) \\ &\quad + \left[W^i_{z,t} - \omega_t \nu_i \left(c^i_{2,t} - c^i_{1,t} + \theta\left(z^i_t\right)\right)\right] \frac{Z_c\left(R_t, c^i_{2,t}\right)}{z^i_t}. \end{aligned}$$

Note that:

$$\left[\frac{\hat{u}^i_{11,t}}{\hat{u}^i_{2,t}} - R_t \frac{\hat{u}^i_{21,t}}{\hat{u}^i_{2,t}}\right] = R_t \left[\frac{\hat{u}^i_{11,t}}{\hat{u}^i_{1,t}} - \frac{\hat{u}^i_{21,t}}{\hat{u}^i_{2,t}}\right] = \frac{\rho - 1}{\left(1 - z^i_t\right) c^i_{1,t}}$$

$$\left[\frac{\hat{u}^i_{12,t}}{\hat{u}^i_{2,t}} - R_t \frac{\hat{u}^i_{22,t}}{\hat{u}^i_{2,t}}\right] = R_t \left[\frac{\hat{u}^i_{12,t}}{\hat{u}^i_{1,t}} - \frac{\hat{u}^i_{22,t}}{\hat{u}^i_{2,t}}\right] = \frac{1 - \rho}{z^i_t c^i_{2,t}},$$

$$\left(\frac{-\hat{u}^i_{21,t}}{\hat{u}^i_{2,t}} - \frac{-\hat{u}^i_{22,t}}{\hat{u}^i_{2,t}}\right) = -\frac{1-\rho}{z^i_t c^i_{2,t}} + (1-R_t)\frac{\hat{u}^i_{22,t}}{\hat{u}^i_{2,t}}.$$

Then:

$$\begin{aligned} \hat{W}^i_{1,t} - \hat{W}^i_{2,t} &= -\mu^i_t R_t (1-\rho)\left[\frac{1}{\left(1-z^i_t\right)c^i_{1,t}} + \frac{1}{z^i_t c^i_{2,t}}\right] \\ &\quad -\zeta_t (-1)^i \frac{-u^i_{n,t}}{\xi_i \hat{u}^i_{2,t}}\left[-\frac{1-\rho}{z^i_t c^i_{2,t}} + (1-R_t)\frac{\hat{u}^i_{22,t}}{\hat{u}^i_{2,t}}\right] \\ &\quad + \left[W^i_{z,t} - \omega_t \nu_i \left(c^i_{2,t} - c^i_{1,t} + \theta\left(z^i_t\right)\right)\right]\frac{Z_c\left(R_t, c^i_{2,t}\right)}{z^i_t}. \end{aligned} \tag{58}$$

First, I show that $R_t = 1$ implies that $\zeta_t = 0$. Suppose not, that is suppose $R_t = 1$ and $\zeta_t > 0$. $R_t = 1$ implies $\hat{W}^i_{1,t} = \hat{W}^i_{2,t}$ and $\left[W^i_{z,t} - \omega_t \nu_i \left(c^i_{2,t} - c^i_{1,t} + \theta\left(z^i_t\right)\right)\right] = 0$. Then, by the first order necessary condition for R_t :

$$\sum_i \mu^i_t + \mu_t = 0,$$

and by (58):

$$0 = -\mu^i_t R_t (1-\rho)\left[\frac{1}{\left(1-z^i_t\right)c^i_{1,t}} + \frac{1}{z^i_t c^i_{2,t}}\right] - \zeta_t (-1)^i \frac{-u^i_{n,t}}{\xi_i \hat{u}^i_{2,t}}\left(-\frac{1-\rho}{z^i_t c^i_{2,t}}\right).$$

Dividing by the term $\left(\frac{1-\rho}{z^i_t c^i_{2,t}}\right)$ and using $z^i_t =\underline{z}$ at $R_1 = 1$:

$$0 = -\mu^i_t\left[\frac{\underline{z}}{(1-\underline{z})} + 1\right] + \zeta_t (-1)^i \frac{-u^i_{n,t}}{\xi_i \hat{u}^i_{2,t}}.$$

Summing over i :

$$0 = -\sum_i \mu^i_t\left[\frac{1}{(1-\underline{z})}\right] + \sum_i \zeta_t (-1)^i \frac{-u^i_{n,t}}{\xi_i \hat{u}^i_{2,t}}.$$

Then, $\sum_i \mu^i_t = 0$ by $\sum_i \zeta_t (-1)^i \frac{-u^i_{n,t}}{\xi_i \hat{u}^i_{2,t}} = 0$ which implies $\mu_t = 0$ and $R_t > 1$, from (50). Contradiction.

I now show that $\zeta_t = 0$ implies $R_t = 1$. Suppose not, that is assume that $\zeta_t = 0$ and $R_t > 1$. Then:

$$\begin{aligned} \hat{W}^i_{1,t} - \hat{W}^i_{2,t} &= -\mu^i_t R_t (1-\rho)\left[\frac{1}{\left(1-z^i_t\right)c^i_{1,t}} + \frac{1}{z^i_t c^i_{2,t}}\right] \\ &\quad + \left[W^i_{z,t} - \omega_t \nu_i \left(c^i_{2,t} - c^i_{1,t} + \theta\left(z^i_t\right)\right)\right]\frac{Z_c\left(R_t, c^i_{2,t}\right)}{z^i_t} > 0, \text{ for } i = 1, 2, \end{aligned}$$

and summing over i :

$$\sum_i \left(\hat{W}^i_{1,t} - \hat{W}^i_{2,t}\right) = \sum_i \left[W^i_{z,t} - \omega_t \nu_i \left(c^i_{2,t} - c^i_{1,t} + \theta\left(z^i_t\right)\right)\right] Z_R\left(R_t, c^i_{2,t}\right) + \sum_i \mu^i_t > 0.$$

But, by the first order necessary condition for R_t :

$$\sum_i \left[W^i_{z,t} - \omega_t \nu_i \left(c^i_{2,t} - c^i_{1,t} + \theta\left(z^i_t\right)\right)\right] Z_R\left(R_t, c^i_{2,t}\right) + \sum_i \mu^i_t = 0,$$

since $\mu_t = 0$ at $R_t > 1$. Contradiction.■

Proof of Proposition 5

Assume (22) is non-binding at time t, then $\zeta_t = 0$ and:

$$1 - \tau^2_t - \kappa_1\left(1 - \tau^1_t\right) - \widetilde{\kappa}_0 < 0,$$

where $\widetilde{\kappa}_0 = (1 - \kappa_1 - \kappa_0) \leq 0$, $\kappa_1 \geq 1$. Since, the Friedman rule holds, $z^i_t = \underline{z}$, $c^i = c^i_j$ for $j = 1, 2$, and by (58) $\mu^i_t = 0$. Then,. (49) and (12) imply:

$$\frac{-W^i_{n,t}}{\xi_i \hat{W}^i_{2,t}} = 1,$$

for $i = 1, 2$. Note that:

$$\tau^i = 1 - \frac{-u^i_n}{\xi_i \hat{u}^i_2} = \frac{-\left[\frac{u^i_{nn} n^i}{-u^i_n} + \frac{\hat{u}^2_{12}(1-\underline{z}) + \hat{u}^2_{22}\underline{z}}{\hat{u}^2_2} c^2_2\right]}{\frac{\eta_i}{\lambda_i} + 1 + \frac{u^i_{nn} n^i}{u^i_n}}. \tag{59}$$

where the time subscript has been dropped for convenience.

Note further that at the Friedman rule:

$$\frac{\hat{u}^2_{12}\left(1 - \underline{z}\right) + \hat{u}^2_{22}\underline{z}}{\hat{u}^2_2} c^2_2 = \frac{U_{cc}\left(c^i, n^i\right) c^i}{U_c\left(c^i, n^i\right)}.$$

Assumption A4 guarantees that the numerator of (59) is negative. Then, τ^i is decreasing in η^i and the result follows if $\lambda_i > 0$ for $i = 1, 2$. Algebraic manipulation (see Lucas and Stokey (1983)) of the first order necessary conditions of the Ramsey allocation problem obtains the following restriction on λ_i :

$$\lambda_i Q^i + \omega_0 M^i_0 + \sum_{t=0}^{\infty} \beta^t \omega_t \left(b^i_{-1,t} + \frac{B^i_{-1,t}}{P_t}\right) + \sum_{t=0}^{\infty} \beta^t \omega_t g_t = 0,$$

for $i = 1, 2$, where Q^i is a negative constant. Hence, $g_t > 0$ in at least one t and assumption A2 guarantee $\lambda_i > 0$ for $i = 1, 2$. ■

6.3 C: Sufficient Conditions for Time Consistency

Proof of proposition 6

The first order necessary conditions for the time 1 Ramsey problem are:

$$\begin{aligned}\nu_i \omega'_t &= \hat{W}^i_{1,t} - \lambda'_i \hat{u}^i_{12,t} b^i_{(0),t} \\ &\quad -\mu'^i_t \left(\frac{\hat{u}^i_{11,t}}{\hat{u}^i_{2,t}} - \frac{\hat{u}^i_{21,t}}{\hat{u}^i_{2,t}} \frac{\hat{u}^i_{1,t}}{\hat{u}^i_{2,t}} \right) - \zeta'_t (-1)^i \frac{-u^i_{n,t}}{\xi_i \hat{u}^i_{2,t}} \frac{-\hat{u}^i_{21,t}}{\hat{u}^i_{2,t}}, \text{ for } t > 1, \end{aligned} \tag{60}$$

$$\begin{aligned}\nu_i \omega'_t &= \hat{W}^i_{2,t} - \lambda'_i \hat{u}^i_{22,t} b^i_{(0),t} \\ &\quad + \left[W^i_{z,t} - \omega'_t \nu_i \left(c^i_{2,t} - c^i_{1,t} + \theta\left(z^i_t\right)\right)\right] Z_c \left(R_t, c^i_{2,t}\right) \\ &\quad -\mu'^i_t \left(\frac{\hat{u}^i_{12,t}}{\hat{u}^i_{2,t}} - \frac{\hat{u}^i_{22,t}}{\hat{u}^i_{2,t}} \frac{\hat{u}^i_{1,t}}{\hat{u}^i_{2,t}} \right) - \zeta'_t (-1)^i \frac{-u^i_{n,t}}{\xi_i \hat{u}^i_{2,t}} \frac{-\hat{u}^i_{22,t}}{\hat{u}^i_{2,t}}, \text{ for } t > 1, \end{aligned} \tag{61}$$

$$0 = W^i_{n,t} - \zeta'_t (-1)^i \frac{-u^i_{nn,t}}{\xi_i \hat{u}^i_{2,t}} + \xi_i \nu_i \omega'_t, \text{ for } t \geq 1, \tag{62}$$

$$\begin{aligned} & \sum_i \left[W^i_{z,t} - \omega'_t \nu_i \left(c^i_{2,t} - c^i_{1,t} + \theta\left(z^i_t\right)\right)\right] Z_R \left(R_t, c^i_{2,t}\right) + \mu'_t + \sum_i \mu'^i_t \\ = & \sum_i \frac{\lambda'_i \hat{u}^i_{2,1}}{R_t} \sum_{s=t+1}^{\infty} \frac{B^i_{(0),s}}{P_1} \prod_{j=1}^{s} R_j, \text{ for } t > 1, \end{aligned} \tag{63}$$

$$\begin{aligned} 0 &= \hat{W}^i_{1,1} - \mu'^i_1 - \zeta'_1 (-1)^i \frac{-u^i_{n,1}}{\xi_i \hat{u}^i_{2,1}} \frac{-\hat{u}^i_{21,1}}{\hat{u}^i_{2,1}} \\ &\quad -\lambda'_i \hat{u}^i_{11,1} \frac{M^i_1}{P_1} - \nu_i \omega'_1 - \lambda'_i \hat{u}^i_{21,1} \left(b^i_{(0),1} + \sum_{t=1}^{\infty} \frac{B^i_{(0),t}}{P_1} \prod_{j=1}^{t} R_j \right), \end{aligned} \tag{64}$$

$$\begin{aligned} 0 &= \hat{W}^i_{2,1} - \nu_i \omega'_1 - \zeta'_1 (-1)^i \frac{-u^i_{n,1}}{\xi_i \hat{u}^i_{2,1}} \frac{-\hat{u}^i_{22,1}}{\hat{u}^i_{2,1}} \\ &\quad -\lambda'_i \hat{u}^i_{22,1} \left(b^i_{(0),1} + \sum_{t=1}^{\infty} \frac{B^i_{(0),t}}{P_1} \prod_{j=1}^{t} R_j \right), \end{aligned} \tag{65}$$

$$\sum_{i=1,2} \left(-\lambda'_i \hat{u}^i_{1,1} \frac{M^i_1}{P_1} - \lambda'_i \hat{u}^i_{2,1} \sum_{t=1}^{\infty} \frac{B^i_{(0),t}}{P_1} \prod_{j=1}^{t} R_j + \mu'^i_1 \frac{M^i_1}{P_1} \right) = 0, \tag{66}$$

$$\mu'_t (1 - R_t) = 0, \ \mu_t \geq 0, \ R_t \geq 1, \text{ for } t > 1,$$

$$\mu_t'^i \left(\frac{\hat{u}_{1,t}^i}{\hat{u}_{2,t}^i} - R_t \right) = 0, \text{ for } t > 1,$$

$$\begin{aligned} \zeta_t' \left(\frac{-u_{n,t}^2}{\xi_2 \hat{u}_{2,t}^2} - \frac{-u_{n,t}^1}{\xi_1 \hat{u}_{2,t}^1} \right) &= 0, \\ \frac{-u_{n,t}^2}{\xi_2 \hat{u}_{2,t}^2} - \frac{-u_{n,t}^1}{\xi_1 \hat{u}_{2,t}^1} &\leq 0, \ \zeta_t' \geq 0, \text{ for } t \geq 1, \end{aligned}$$

$$\mu_1'^i \left(c_{1,1}^i \left(1 - z_1^i\right) - \frac{M_1^i}{P_1} \right) = 0, \text{ with } \mu_1'^i \geq 0, \ c_{1,1}^i \left(1 - z_1^i\right) \leq \frac{M_1^i}{P_1}.$$

Here, primes denote values of multipliers for the time 1 Ramsey allocation problem, where the function W_t^i is also evaluated at λ_i'. When evaluated at the allocation the solves the time 0 Ramsey allocation problem, equations (60)-(66) plus the time 1 implementability constraints for each type of agent constitute a system of equations in the unknowns $b_{0,t}^i$, $B_{0,t}^i$, M_1^i, λ_i', ω_t', $\mu_t'^i$, ζ_t' for $t \geq 1$ and $i = 1, 2$ and μ_t' for $t > 1$.

To prove the result, I construct a solution to this system by finding $b_{0,t}^i$, $B_{0,t}^i$, M_1^i, ω_t', $\mu_t'^i$, ζ_t' and μ_t as a function of λ_1' and λ_2', and then using the implementability constraints for the time 1 Ramsey allocation problem to determine λ_i'.

Equations (60), (61) and (62) at each $t > 1$, comprise a system of six equations in the six unknowns $\omega_t', \zeta_t', \mu_t'^i$, $b_{0,t}^i$ for $i = 1, 2$ for given λ_i'. Equation (63) determines $\sum_i \frac{\lambda_i' \hat{u}_{2,1}^i}{R_t} \sum_{s=t+1}^{\infty} \frac{B_{(0),s}^i}{P_1} \prod_{j=1}^{s} R_j$ for each $t > 1$ as a function of λ_i' for $i = 1, 2$. The only restriction on μ_t' is that $\mu_t' = 0$ if $\mu_t = 0$. Hence, for Ramsey equilibria in which $R_t = 1$, I set $\mu_t' = \mu_t$, given the stationarity of the Ramsey allocation problem at time 0 and at time 1 for $t > 1$. Let:

$$\chi_t \equiv \sum_i \frac{\lambda_i' \hat{u}_{2,1}^i}{R_t} \sum_{s=t+1}^{\infty} \frac{B_{(0),s}^i}{P_1} \prod_{j=1}^{s} R_j, \tag{67}$$

denote the implied value of this term. Note that (63) imposes a linear restriction on the maturity structure of nominal debt held by each type of household, but does not separately pin down nominal bond holdings for each type.

I now construct the time 1 currency and real and nominal bond positions. Recall that z_1^i is predetermined from the standpoint of the government at time 1. Set $M_1^i = P_1 c_1^i \left(1 - z_1^i\right)$ for $i = 1, 2$. Then, equations (64), (65) and (62) at $t = 1$ for $i = 1, 2$ constitute a system of six equations in the six unknowns ω_1', ζ_1', $\mu_1'^i$, and $\left(b_{(0),1}^i + \sum_{t=1}^{\infty} \frac{B_{(0),t}^i}{P_1} \prod_{j=1}^{t} R_j \right)$. Equation (66) then imposes a linear restriction on $\sum_{t=1}^{\infty} \frac{B_{(0),t}^i}{P_1}$, for $i = 1, 2$, for given λ_i', without separately pinning down the value of this quantity for each i. The term

$\left(b^i_{(0),1} + \sum_{t=1}^{\infty} \frac{B^i_{(0),t}}{P_1} \prod_{j=1}^{t} R_j\right)$ for $i = 1, 2$ is determined from the system of equations given by (64), (65) and (62). Denote:

$$\varrho_i \equiv \left(b^i_{(0),1} + \sum_{t=1}^{\infty} \frac{B^i_{(0),t}}{P_1} \prod_{j=1}^{t} R_j\right), \; i = 1, 2, \tag{68}$$

from the solution to this system of equations. Using (67), (66) implies:

$$\chi_1 \equiv \sum_{i=1,2} \left(\lambda_i' \hat{u}^i_{2,1} \frac{B^i_{(0),1}}{P_1}\right) = \sum_{i=1,2} \left[\left(\mu_i' - \lambda_i' \hat{u}^i_{1,1}\right) \frac{M^i_1}{P_1}\right] - \sum_{t=2}^{\infty} \chi_t. \tag{69}$$

Then, (68) and (69) is a system of three equations in the four unknowns $b^i_{(0),1}$, $\frac{B^i_{(0),1}}{P_1}$, which is underdetermined and has more than one solution.

Hence, we have used (60)-(66) to determine $b^i_{0,t}$, $B^i_{0,t}$, M^i_1, ω_t', $\mu_t'^i$, ζ_t' and μ_t for all t and $i = 1, 2$ as a function of λ_i', $i = 1, 2$. The last step is to use the implementability constraints from the time 1 Ramsey allocation problem to pin down λ_i'. This will ensure that the household and government's intertemporal budget constraints are satisfied at the nominal and real bond positions constructed in the previous steps. The time 1 implementability constraint, after substituting for M^i_1, is:

$$\sum_{t=1}^{\infty} \beta^{t-1} \left[u^i_{1,t} c^i_{1,t} + u^i_{2,t} \hat{c}^i_{2,t} + u^i_{n,t} n^i_t\right] - \hat{u}^i_{1,1} c^i_{1,1} \left(1 - z^i_1\right) = \hat{u}^i_{2,1} \left(b^i_{(0),1} + \sum_{t=1}^{\infty} \frac{B^i_{(0),t}}{P_1} \prod_{j=1}^{t} R_j\right) + \sum_{t=1}^{\infty} \beta^{t-1} \hat{u}^i_{2,t} b^i_{0,t}.$$

Only the right hand side of this equation depends, implicitly, on λ_i'. This determines λ_i' for $i = 1, 2$.

This construction also ensures that all the complementary slackness conditions are satisfied.■

6.4 D: The Model with Lucas and Stokey Timing

Under Lucas and Stokey timing, the households maximize lifetime utility under an asset market constraint:

$$M^i_t + \sum_{s>0} \left(Q_{t,t+s} B^i_{t,t+s} + q_{t,t+s} P_t b^i_{t,t+s}\right) \leq A^i_t + \sum_{\hat{t}=0}^{t-1} \left(B^i_{\hat{t},t-1} + P_{t-1} b^i_{\hat{t},t-1}\right),$$

where A^i_t denotes financial wealth and a cash-in-advance constraint:

$$P_t c^i_{1,t} \left(1 - z^i_t\right) \leq M^i_t.$$

Their financial wealth evolves according to:

$$A_t = W_{t-1} \left(1 - \tau^i_{t-1}\right) \xi_i n^i_{t-1} + M^i_{t-1} - P_{t-1} c^i_{1,t-1} \left(1 - z^i_{t-1}\right) - P_{t-1} c^i_{2,t-1} z^i_{t-1} - \int_0^{z^i_{t-1}} \pi_{t-1}(j)\, dj,$$

where A_0^i is exogenously given. In all period, including time 0, household choose both M_t^i and z_t^i after the government selects policy.

The Lagrangian for the household problem is:

$$\begin{aligned} L &= \sum_{t=0}^{\infty} \beta^t \left\{ u^i\left(c_{it}, n_{it}\right) - \delta_t^i \left(P_t c_{1,t}^i \left(1 - z_t^i\right) - M_t^i\right) \right. \\ &\quad -\phi_t^i \left[M_t^i + \sum_{s>0} \left(Q_{t,t+s} B_{t,t+s}^i + q_{t,t+s} P_t b_{t,t+s}^i\right) - M_{t-1}^i - \sum_{\hat{t}=0}^{t-1} \left(B_{\hat{t},t-1}^i + P_{t-1} b_{\hat{t},t-1}^i\right) \right. \\ &\quad \left.\left. -W_{t-1}\left(1-\tau_{t-1}^i\right)\xi_i n_{t-1}^i + P_{t-1} c_{1,t-1}^i \left(1 - z_{t-1}^i\right) + P_{t-1} c_{2,t-1}^i z_{t-1}^i + \int_0^{z_{t-1}^i} \pi_{t-1}(j)\, dj \right] \right\}, \end{aligned}$$

The solution to this optimization problem, implies the following implementability constraint for $i = 1, 2$:

$$\sum_{t=0}^{\infty} \beta^t \left(u_{1t}^i c_{1,t}^i + u_{2,t}^i \hat{c}_{2,t}^i + u_{n,t}^i n_t^i\right) = \hat{u}_{2,0}^i \frac{A_0^i}{P_0} + \sum_{t=0}^{\infty} \beta^t \hat{u}_{2,t}^i b_{(-1),t}^i + \hat{u}_{2,0}^i \sum_{t=0}^{\infty} \frac{B_{(-1),t}^i}{P_0} \prod_{j=1}^{t} R_j.$$

In equilibrium, $M_t = \sum_{i=1,2} A_t^i$.

The Lagrangian for the Ramsey problem is:

$$\begin{aligned} \Lambda &= \sum_{t=0}^{\infty} \beta^t \sum_i W^i\left(c_{1,t}^i, c_{2,t}^i, n_t^i, z_t^i; \eta_i, \lambda_i\right) \\ &\quad - \sum_{t=0}^{\infty} \beta^t \left[\mu_t \left(1 - R_t\right) + \sum_i \mu_t^i \left(\frac{\hat{u}_{1,t}^i}{\hat{u}_{2,t}^i} - R_t\right)\right] - \sum_{t=0}^{\infty} \beta^t \zeta_t \left(\frac{-u_{n,t}^2}{\xi_2 \hat{u}_{2,t}^2} - \frac{-u_{n,t}^1}{\xi_1 \hat{u}_{2,t}^1}\right) \\ &\quad - \sum_i \lambda_i \left(\hat{u}_{i2,0} \frac{M_0^i}{P_0} + \sum_{t=0}^{\infty} \beta^t \hat{u}_{2,t}^i b_{(-1),t}^i + \hat{u}_{2,0}^i \sum_{t=0}^{\infty} \frac{B_{(-1),t}^i}{P_0} \prod_{j=1}^{t} R_j\right), \end{aligned}$$

where $W^i\left(c_{1,t}^i, c_{2,t}^i, n_t^i, z_t^i; \eta_i, \lambda_i\right) = \eta_i U\left(c_t^i, n_t^i\right) + \lambda_i \left[u_{1t}^i c_{1,t}^i + u_{2,t}^i \left(c_{2,t}^i + C\left(z_t^i\right)\right) + u_{n,t}^i n_t^i\right].$

The first order conditions for the Ramsey problem at $t > 0$ are the same as for the Svensson timing. They differ as follows for time 0:

$$0 = \hat{W}_{1,t}^i - \mu_0^i \left(\frac{\hat{u}_{11,0}^i}{\hat{u}_{2,0}^i} - \frac{\hat{u}_{21,0}^i}{\hat{u}_{2,0}^i} R_0\right) - \nu_i \omega_0, \tag{70}$$

$$\begin{aligned} 0 &= \hat{W}_{2,t}^i - \nu_i \omega_0 \\ &\quad + \zeta_0^i \frac{u_{n,0}^i}{\hat{u}_{2,0}^i} \frac{\hat{u}_{22,0}^i}{\hat{u}_{2,0}^i} - \mu_0^i \left(\frac{\hat{u}_{12,0}^i}{\hat{u}_{2,0}^i} - \frac{\hat{u}_{22,0}^i}{\hat{u}_{2,0}^i} R_0\right) \\ &\quad - \lambda_i \hat{u}_{22,0}^i \left(\frac{M_0^i}{P_0} + b_{(-1),0}^i + \sum_{t=0}^{\infty} \frac{B_{(-1),t}^i}{P_0} \prod_{j=1}^{t} R_j\right), \end{aligned} \tag{71}$$

$$-\sum_i \lambda_i \left(\hat{u}_{i2,0} \frac{M_0^i}{P_0} + \hat{u}_{2,0}^i \sum_{t=0}^{\infty} \frac{B_{(-1),t}^i}{P_0} \prod_{j=1}^{t} R_j \right) \left(\frac{-1}{P_0^2} \right) = 0,$$

$$\mu_0^i \left(\frac{\hat{u}_{1,0}^i}{\hat{u}_{2,0}^i} - R_0 \right) = 0, \tag{72}$$

$$\mu_0 \left(1 - R_0\right) = 0, \ \mu_0 \geq 0, \ R_0 \geq 1, \tag{73}$$

and (50), which holds for $t \geq 0$.

Proposition 7 *Under (A1), the Ramsey equilibrium is time consistent.*

Proof. I need to show that it is possible to find λ_i', $\mu_t'^i$, μ_t', ζ_t', ω_t', $B_{0,t}^i$, $b_{0,t}^i$ for $t \geq 1$, and M_1^i for $i = 1, 2$ such that the continuation allocation for the time 0 Ramsey problem solves the time 1 Ramsey problem. Here, a prime denotes the multipliers on the corresponding constraints in the Lagrangian for the time 1 Ramsey problem. ■

The first order necessary conditions for the time 1 Ramsey problem are:

$$\begin{aligned} \nu_i \omega_t' &= \hat{W}_{1,t}^i - \lambda_i' \hat{u}_{12,t}^i b_{(0),t}^i \\ &\quad - \mu_t'^i \left(\frac{\hat{u}_{11,t}^i}{\hat{u}_{2,t}^i} - \frac{\hat{u}_{21,t}^i}{\hat{u}_{2,t}^i} \frac{\hat{u}_{1,t}^i}{\hat{u}_{2,t}^i} \right) - \zeta_t' \left(-1\right)^i \frac{-u_{n,t}^i}{\xi_i \hat{u}_{2,t}^i} \frac{-\hat{u}_{21,t}^i}{\hat{u}_{2,t}^i}, \text{ for } t > 1, \end{aligned} \tag{74}$$

$$\begin{aligned} \nu_i \omega_t' &= \hat{W}_{2,t}^i + \left[W_{z,t}^i - \omega_t' \nu_i \left(c_{2,t}^i - c_{1,t}^i + \theta\left(z_t^i\right) \right) \right] Z_c\left(R_t, c_{2,t}^i\right) - \lambda_i' \hat{u}_{22,t}^i b_{(0),t}^i \\ &\quad - \mu_t'^i \left(\frac{\hat{u}_{12,t}^i}{\hat{u}_{2,t}^i} - \frac{\hat{u}_{22,t}^i}{\hat{u}_{2,t}^i} \frac{\hat{u}_{1,t}^i}{\hat{u}_{2,t}^i} \right) - \zeta_t' \left(-1\right)^i \frac{-u_{n,t}^i}{\xi_i \hat{u}_{2,t}^i} \frac{-\hat{u}_{22,t}^i}{\hat{u}_{2,t}^i}, \text{ for } t > 1, \end{aligned} \tag{75}$$

$$0 = W_{n,t}^i - \zeta_t' \left(-1\right)^i \frac{-u_{nn,t}^i}{\xi_i \hat{u}_{2,t}^i} + \xi_i \nu_i \omega_t', \text{ for } t \geq 1, \tag{76}$$

$$\begin{aligned} &\sum_i \left[W_{z,t}^i - \omega_t' \nu_i \left(c_{2,t}^i - c_{1,t}^i + \theta\left(z_t^i\right) \right) \right] Z_R\left(R_t, c_{2,t}^i\right) + \mu_t' + \sum_i \mu_t'^i \\ = &\sum_i \frac{\lambda_i' \hat{u}_{2,1}^i}{R_t} \sum_{s=t+1}^{\infty} \frac{B_{(0),s}^i}{P_1} \prod_{j=1}^{s} R_j, \text{ for } t \geq 1, \end{aligned} \tag{77}$$

$$\begin{aligned} 0 &= \hat{W}_{1,1}^i - \mu_1'^i \left(\frac{\hat{u}_{11,1}^i}{\hat{u}_{2,1}^i} - \frac{\hat{u}_{21,1}^i}{\hat{u}_{2,1}^i} \frac{\hat{u}_{1,1}^i}{\hat{u}_{2,1}^i} \right) - \zeta_1' \left(-1\right)^i \frac{-u_{n,1}^i}{\xi_i \hat{u}_{2,1}^i} \frac{-\hat{u}_{21,1}^i}{\hat{u}_{2,1}^i} \\ &\quad - \lambda_i' \hat{u}_{21,1}^i \left(\frac{A_1^i}{P_1} + b_{(0),1}^i + \sum_{t=1}^{\infty} \frac{B_{(0),t}^i}{P_1} \prod_{j=1}^{t} R_j \right) - \nu_i \omega_1', \end{aligned} \tag{78}$$

$$0 = \hat{W}^i_{2,1} - \mu'^i_1 \left(\frac{\hat{u}^i_{12,1}}{\hat{u}^i_{2,1}} - \frac{\hat{u}^i_{22,1}}{\hat{u}^i_{2,1}} \frac{\hat{u}^i_{1,1}}{\hat{u}^i_{2,1}} \right) - \zeta'_1 (-1)^i \frac{-u^i_{n,1}}{\xi_i \hat{u}^i_{2,1}} \frac{-\hat{u}^i_{22,1}}{\hat{u}^i_{2,1}} \tag{79}$$

$$-\lambda'_i \hat{u}^i_{22,1} \left(\frac{A^i_1}{P_1} + b^i_{(0),1} + \sum_{t=1}^{\infty} \frac{B^i_{(0),t}}{P_1} \prod_{j=1}^{t} R_j \right) - \nu_i \omega'_1,$$

$$- \sum_{i=1,2} \lambda'_i \hat{u}^i_{2,1} \left(\frac{A^i_1}{P_1} + \sum_{t=1}^{\infty} \frac{B^i_{(0),t}}{P_1} \prod_{j=1}^{t} R_j \right) = 0, \tag{80}$$

$$\mu'_t (1 - R_t) = 0,\ \mu_t \geq 0,\ R_t \geq 1,\ \text{for } t \geq 1,$$

$$\mu'^i_t \left(\frac{\hat{u}^i_{1,t}}{\hat{u}^i_{2,t}} - R_t \right) = 0,\ \text{for } t \geq 1,$$

$$\zeta'_t \left(\frac{-u^2_{n,t}}{\xi_2 \hat{u}^2_{2,t}} - \frac{-u^1_{n,t}}{\xi_1 \hat{u}^1_{2,t}} \right) = 0,$$

$$\frac{-u^2_{n,t}}{\xi_2 \hat{u}^2_{2,t}} - \frac{-u^1_{n,t}}{\xi_1 \hat{u}^1_{2,t}} \leq 0,\ \zeta'_t \geq 0,\ \text{for } t \geq 1.$$

Here, primes denote values of multipliers for the time 1 Ramsey allocation problem, where the function W^i_t is also evaluated at λ'_i.. When evaluated at the allocation the solves the time 0 Ramsey allocation problem, equations (74)-(80) plus the time 1 implementability constraints for each type of agent constitute a system of equations in the unknowns $b^i_{0,t}$, $B^i_{0,t}$, A^i_1, λ'_i, ω'_t, μ'^i_t, ζ'_t for $t \geq 1$ and $i = 1, 2$ and μ'_t for $t > 1$.

To prove the result, I construct a solution to this system by finding $b^i_{0,t}$, $B^i_{0,t}$, A^i_1, ω'_t, μ'^i_t, ζ'_t and μ_t as a function of λ'_1 and λ'_2, and then using the implementability constraints for the time 1 Ramsey allocation problem to determine λ'_i.

Equations (74), (75) and (76) at each $t > 1$, comprise a system of six equations in the six unknowns $\omega'_t, \zeta'_t, \mu'^i_t$, $b^i_{0,t}$ for $i = 1, 2$ for given λ'_i. Equation (77) determines $\sum_i \frac{\lambda'_i \hat{u}^i_{2,1}}{R_t} \sum_{s=t+1}^{\infty} \frac{B^i_{(0),s}}{P_1} \prod_{j=1}^{s} R_j$ for each $t > 1$ as a function of λ'_i for $i = 1, 2$. The only restriction on μ'_t is that $\mu'_t = 0$ if $\mu_t = 0$. Hence, for Ramsey equilibria in which $R_t = 1$, I set $\mu'_t = \mu_t$, given the stationarity of the Ramsey allocation problem at time 0 and at time 1 for $t > 1$. Let:

$$\chi_t \equiv \sum_i \frac{\lambda'_i \hat{u}^i_{2,1}}{R_t} \sum_{s=t+1}^{\infty} \frac{B^i_{(0),s}}{P_1} \prod_{j=1}^{s} R_j, \tag{81}$$

denote the implied value of this term.

I now construct the time 1 financial wealth and real and nominal bond positions. Equations (78), (79) and (76) at $t = 1$ for $i = 1, 2$ constitute a system of six equations in the six unknowns ω_1', ζ_1', $\mu_1'^i$, and $\left(\frac{A_1^i}{P_1} + b_{(0),1}^i + \sum_{t=1}^{\infty} \frac{B_{(0),t}^i}{P_1} \prod_{j=1}^{t} R_j\right)$.

Equation (80) then imposes a linear restriction on the terms $\frac{A_1^i}{P_1} + \sum_{t=1}^{\infty} \frac{B_{(0),t}^i}{P_1}$, for $i = 1, 2$, for given λ_i', without separately pinning down the value of this quantity for each i. Let:

$$\varrho_i \equiv \left(\frac{A_1^i}{P_1} + b_{(0),1}^i + \sum_{t=1}^{\infty} \frac{B_{(0),t}^i}{P_1} \prod_{j=1}^{t} R_j\right), \; i = 1, 2, \tag{82}$$

be the solution for the term $\left(\frac{A_1^i}{P_1} + b_{(0),1}^i + \sum_{t=1}^{\infty} \frac{B_{(0),t}^i}{P_1} \prod_{j=1}^{t} R_j\right)$ for $i = 1, 2$ from the system of equations given by (78), (79) and (76). Using (81), (80) implies:

$$\chi_1 \equiv \sum_{i=1,2} \lambda_i' \hat{u}_{2,1}^i \left(\frac{B_{(0),1}^i}{P_1}\right) = -\sum_{i=1,2} \lambda_i' \hat{u}_{2,1}^i \frac{A_1^i}{P_1} - \sum_{t=2}^{\infty} \chi_t. \tag{83}$$

Using:

$$A_1^i = M_0^i - P_0 c_{1,0}^i \left(1 - z_0^i\right) - P_0 c_{2,0}^i z_0^i - \int_0^{z_0^i} \pi(i)\, di + W_0 \xi_i n_0^i (1 - \tau_0),$$

equations (82) and (83) define a system of three equations in the four unknowns $b_{(0),1}^i$, $\frac{B_{(0),1}^i}{P_1}$. The system is underdetermined and has more than one solution.

Hence, we have used (74)-(80) to determine $b_{0,t}^i$, $B_{0,t}^i$, A_1^i, ω_t', $\mu_t'^i$, ζ_t' and μ_t for all t and $i = 1, 2$ as a function of λ_i', $i = 1, 2$. The last step is to use the implementability constraints from the time 1 Ramsey allocation problem to pin down λ_i'. This will ensure that the household and government's intertemporal budget constraints are satisfied at the nominal and real bond positions constructed in the previous steps. The time 1 implementability constraint is:

$$\sum_{t=1}^{\infty} \beta^{t-1} \left[u_{1,t}^i c_{1,t}^i + u_{2,t}^i \hat{c}_{2,t}^i + u_{n,t}^i n_t^i\right] = \hat{u}_{2,1}^i \left(\frac{A_1^i}{P_1} + b_{(0),1}^i + \sum_{t=1}^{\infty} \frac{B_{(0),t}^i}{P_1} \prod_{j=1}^{t} R_j\right) + \sum_{t=1}^{\infty} \beta^{t-1} \hat{u}_{2,t}^i b_{0,t}^i.$$

Only the right hand side of this equation depends, implicitly, on λ_i'. This determines λ_i' for $i = 1, 2$.

This construction also ensures that all the complementary slackness conditions are satisfied.■