Inflation and Inequality

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February 2006

Appendix

1. Private Sector Equilibrium

Proof of proposition 3.2 Assume that an allocation $\{c_{i1t}, c_{i2t}, n_{it}, z_{it}, M_{it+1}\}_{i=1,2,t\geq 0}$, with $n_{it} > 0$ for $i = 1, 2$ and $t \geq 0$, and a price system $\{P_t, W_t, q_t(j)\}_{t\geq 0, j\in[0,1]}$ constitute a private sector equilibrium for a given policy $\{\bar{g}_t, \tau_t, M_{t+1}\}_{t\geq 0}$. Then, conditions (3.1) and (3.3) correspond to optimality in firm behavior, conditions (3.11) and (3.13) from clearing in the goods and assets markets. The other conditions follow from household optimization. The Lagrangian for the household problem is given by:

$$\begin{aligned} L &= \sum_{t=0}^{\infty} \beta^t \{u^i(c_{it}, n_{it}) - \mu_{it}(P_t c_{i1t}(1 - z_{it}) - M_{it}) - \lambda_{it}[M_{it+1} \\ &\quad - M_{it} - W_t(1 - \tau_t)\xi_i n_{it} + P_t c_{i1t}(1 - z_{it}) + P_t c_{i2t} z_{it} + \int_0^{z_{it}} q_t(j)\,dj\Big]\Big\}, \end{aligned}$$

where c_{it} is defined in (3.6) and μ_{it}, λ_{it} are the multipliers on the cash in advance constraint and the wealth evolution equation, respectively. Denote with u_{ijt} and u_{int} the marginal utility of good j and of labor for households $i = 1, 2$. The necessary conditions for household optimization are given by:

$$u_{i1t} = P_t(\mu_{it} + \lambda_{it})(1 - z_{it}), \tag{5.1}$$

$$\mu_{it}(P_t c_{it}(1 - z_{it}) - M_{it}) = 0, \; \mu_{it} \geq 0, \tag{5.2}$$

$$u_{i2t} = P_t \lambda_{it} z_{it}, \tag{5.3}$$

$$-u_{int} = W_t(1 - \tau_t)\xi_i \lambda_{it}, \tag{5.4}$$

$$P_t c_{i1t}(\mu_{it} + \lambda_{it}) - P_t c_{i2t}\lambda_{it} - q_t(z_{it})\lambda_{it} \begin{cases} < 0 \text{ for } z_{it} = \underline{z}, \\ = 0 \text{ for } z_{it} \in (\underline{z}, \bar{z}), \\ > 0 \text{ for } z_{it} = \bar{z}, \end{cases} \tag{5.5}$$

$$\lambda_{it} = \beta\left(\lambda_{it+1} + \mu_{it+1}\right), \tag{5.6}$$

$$\lambda_{it} R_{t+1}^{-1} = \beta \lambda_{it+1}, \tag{5.7}$$

$$\lim_{T \to \infty} \beta^T \lambda_{iT} M_{iT} = 0, \tag{5.8}$$

as well as (3.7) and (3.8). To see that (5.8) is a necessary condition for household optimization, suppose it does not hold and

$$\lim_{T \to \infty} \beta^T \lambda_{iT} M_{iT} > 0.$$

(The strictly smaller case is ruled out by (3.9).) Then, it is possible to construct a consumption sequence such that the budget constraint is satisfied in each period and utility for each type of household is greater, violating optimality. Combining (5.1)-(5.3) yields (3.14), while (5.3) and (5.4) determine (3.15). The expression in (5.7) follows from (5.4) and $u_{int} = \gamma$, and (**??**), while (3.18) follows from (5.1)-(5.3) at $t = 0$. To derive (3.19), multiply (3.8) by λ_{it} and apply (5.2) and (5.6). This yields:

$$\begin{aligned} 0 \;\; = \;\; & \left(\lambda_{it} + \mu_{it}\right) M_{it} + W_t \left(1 - \tau_t\right) \xi_i \lambda_{it} n_{it} - P_t c_{i1t} \left(\mu_{it} + \lambda_{it}\right) \left(1 - z_{it}\right) \\ & - P_t c_{i2t} z_{it} \lambda_{it} - \lambda_{it} \int_0^z q_t\left(j\right) dj - \beta \left(\lambda_{it+1} + \mu_{it+1}\right) M_{it+1}. \end{aligned}$$

Now use (5.1), (5.3)-(5.5), multiply by β^t and sum over t from 0 to T. Let T go to infinity and apply (5.8). Now assume that an allocation $\{c_{i1t}, c_{i2t}, n_{it}, z_{it}, M_{it+1}\}_{i=1,2,t\geq 0}$, with $n_{it} > 0$ for $i = 1, 2$ and $t \geq 0$, and a price system $\{P_t, W_t, R_t, q_t\left(j\right)\}_{t \geq 0, j \in [0,1]}$ satisfy (**??**)-(3.19) and (3.11) for a given policy $\{\bar{g}_t, \tau_t, M_{t+1}\}_{t \geq 0}$ for which (3.10) holds. Then, by (**??**) and (3.3) industrial and credit services firms optimize. To see that household optimization conditions are satisfied consider an alternative candidate plan $\{c'_{i1t}, c'_{i2t}, n'_{it}, z'_{it}\}_{i=1,2,t\geq 0}$ which satisfies the intertemporal budget constraint for the price system $\{P_t, W_t, R_t, q_t\left(j\right)\}_{t\geq 0, j\in[0,1]}$. This implies that:

$$\Delta \equiv \lim_{T \to \infty} \beta^t \left\{ u_{i1t} \left(c_{i1t} - c'_{i1t}\right) + u_{i2t} \left(c_{i2t} + \frac{C\left(z_{it}\right)}{z_{it}} - c'_{i2t} - \frac{C\left(z'_{it}\right)}{z'_{it}} \right) - \gamma \left(n_{it} - n'_{it}\right) \right\} \geq 0,$$

using (3.12) and the fact that $\{c_{i1t}, c_{i2t}, n_{it}, z_{it}\}_{i=1,2,t\geq 0}$ satisfies (3.14)-(3.19) and that the intertemporal budget constraint holds as a weak inequality using (3.9) and (3.8) for the price system $\{P_t, W_t, R_t, q_t\left(j\right)\}_{t\geq 0, j\in[0,1]}$. By concavity of u^i :

$$D \equiv \lim_{T \to \infty} \sum_{t=0}^{T} \beta^t \left(u^i\left(c_{it}, n_{it}\right) - u^i\left(c'_{it}, n'_{it}\right)\right) \geq \Delta,$$

where c'_{it} is defined by (3.6). This establishes the result since (3.13) and (3.11) guarantee market clearing.■

Proposition 5.1. *In a private sector equilibrium with continuation government policy $X_t = \{\tau_s, R_s\}_{s\geq t}$ and initial distribution of currency $M_{i,t}$ for $i = 1,2$ at $t \geq 0$, the allocation satisfies:*

$$\begin{aligned} c_{ij,t} &= \bar{\mathfrak{c}}_{ij}\left(M_{i,t}, P_t; \tau_t, R_t\right), & (5.9)\\ c_{ji,s} &= \mathfrak{c}_{ij}\left(\tau_s, R_s\right) \text{ for } s > t, & (5.10)\\ z_{i,s} &= \mathfrak{z}_i\left(\tau_s, R_s\right) \text{ for } s \geq t, & (5.11)\end{aligned}$$

$$\begin{aligned} n_{i,t} &= \mathfrak{n}_i\left(\tau_t, \left(\frac{c_{i1t}}{c_{i2t}}\right)^{\rho-1}; \tau_{t+1}, R_{t+1}\right), & (5.12)\\ n_{i,s} &= \mathfrak{n}_i\left(\tau, R; \tau', R'\right), \text{ for } s > t, & (5.13)\end{aligned}$$

$$M_{i,s+1} = \mathfrak{M}_i\left(\tau_{s+1}, R_{s+1}\right) \text{ for } s \geq t. \tag{5.14}$$

for $i, j = 1, 2$. Furthermore:

$$\begin{aligned} R_s &= \mathcal{R}(\tau_s; \{\tau_{s+1}, R_{s+1}\}, \bar{g}), & (5.15)\\ P_s &= \beta R_s P_{s-1}\frac{(1-\tau_{s-1})}{(1-\tau_s)}, & (5.16)\end{aligned}$$

for $s > t$, and

$$P_t = \mathfrak{P}(M_{1,t}, M_{2,t}; \{\tau_{t,}, R_t\}, \{\tau_{t+1}, R_{t+1}\}, \bar{g}). \tag{5.17}$$

Proof of Proposition 5.1 Equations (3.14), (3.15) and (3.17) determine (5.10)-(5.11). The function $\bar{\mathfrak{c}}_{ij}(\cdot)$ in (5.9) is implicitly defined by (3.18) and (3.15). The function $\mathfrak{n}_i(\cdot)$ is defined by:

$$\mathfrak{n}_i\left(\tau, R; \tau', R'\right) = \frac{\beta}{\gamma}u'_{i1}c'_{i1} + \frac{u_{i2}}{\gamma}\left(c_{i2} + \frac{C\left(z_i\right)}{z_i}\right). \tag{5.18}$$

Equations (3.12) and (3.16), together with (5.9)-(5.11) imply (5.14). A policy and a price sequence $\{P_s\}_{s\geq t}$ consistent with a private sector equilibrium must also satisfy the resource constraint and clearing on the money market. Then, if a policy X_t is part of a private sector equilibrium, $R_s = \mathcal{R}\left(\tau_s; \{\tau_{s+1}, R_{s+1}\}, \bar{g}\right)$ for all $s > t$, where $\mathcal{R}\left(\tau; \{\tau\prime, R\prime\}, \bar{g}\right)$ is implicitly defined by the resource constraint:

$$\begin{aligned} &\sum_{i=1,2} \nu_i \xi_i \mathfrak{n}_i\left(\tau, R; \tau', R'\right) & (5.19)\\ &= \bar{g} + \sum_{i=1,2} \nu_i[\mathfrak{c}_{i1}\left(\tau, R\right)\left(1 - \mathfrak{z}_i\left(\tau, R\right)\right) + \mathfrak{c}_{i2}\left(\tau, R\right)\mathfrak{z}_i\left(\tau, R\right) + \int_0^{\mathfrak{z}_i(\tau,R)} \theta\left(j\right) dj]. \end{aligned}$$

Given that (3.19) holds under the assumptions of proposition 5.1, (5.19) implies that the government's dynamic and intertemporal budget constraints are also satisfied. The equilibrium price sequence $\{P_s\}_{s\geq t}$ can be determined as a function of the policy X_t and of initial distribution of currency as follows. Equation (3.12) determines $\{P_s\}_{s>t}$,for given P_t, and guarantees money market claring for $s>t$. The equilibrium value of P_t satisfies $P_t=\mathfrak{P}\left(M_{1t},M_{2t};\{\tau_t,R_t\},\{\tau_{t+1},R_{t+1}\},\bar{g}\right)$, where $\mathfrak{P}\left(M_1,M_2;\{\tau,R\},\{\tau',R'\},\bar{g}\right)$ is implicitly defined by:

$$\begin{aligned}
&\sum_{i=1,2}\nu_i\xi_i\mathfrak{n}_i\left(\tau,\left(\frac{c_{i1}}{c_{i2}}\right)^{\rho-1};\tau',R'\right) \\
=\ &\bar{g}+\sum_{i=1,2}\nu_i[c_{i1}\left(1-\mathfrak{z}_i\left(\tau,R\right)\right)+c_{i2}\mathfrak{z}_i\left(\tau,R\right)+\int_0^{\mathfrak{z}_i(\tau,R)}\theta\left(j\right)dj],
\end{aligned} \tag{5.20}$$

and c_{ij} are determined according to (5.9).■

The present discounted value of optimized household utility at time t under policy X_t, current price level P_t and initial distribution of currency $M_{i,t}$, $i=1,2$ can be written as:

$$U^i\left(M_{i,t};X_t,\{M_{1,t},M_{2,t}\}\right)=\bar{\mathcal{P}}^i\left(M_{i,t},P_t;\tau_t,R_t;\tau_{t+1},R_{t+1}\right)+\sum_{s=t+1}^{\infty}\beta^{s-t}\mathcal{P}^i\left(\tau_s,R_s;\tau_{s+1},R_{s+1}\right), \tag{5.21}$$

for all $t\geq 0$, where P_t satisfies (5.17). Here:

$$\bar{\mathcal{P}}^i\left(M_i,P;\tau,R;\tau',R'\right)=\left[\frac{(c_i)^{1-\sigma}-1}{1-\sigma}-\gamma\mathfrak{n}_i\left(\tau,\left(\frac{c_{i1}}{c_{i2}}\right)^{\rho-1};\tau',R'\right)\right],$$

with c_{ji} determined from (5.9), and

$$\mathcal{P}^i\left(\tau,R;\tau',R'\right)=\left[\frac{(c_i)^{1-\sigma}-1}{1-\sigma}-\gamma\mathfrak{n}_i\left(\tau,R;\tau',R'\right)\right], \tag{5.22}$$

with c_{ij} for $i,j=1,2$ given by (5.10)-(5.11). c_i is determined from (3.5) for $i=1,2$.

If $M_{i,t}=\mathfrak{M}_i\left(\tau_t,R_t\right)$, $c_{ij,t}$ is also satisfies (5.9) and the shadow price of cash goods relative to credit goods is exactly equal to R_t. Hence, in a one period deviation from a candidate equilibrium policy $\{\tau,R\}$, the initial distribution of currency will be given by $\{\mathfrak{M}_1\left(\tau,R\right),\mathfrak{M}_2\left(\tau,R\right)\}$, so that the present value of utility at the continuation X', can be expressed as:

$$\hat{U}^i\left(X'\right)\equiv U^i\left(\mathfrak{M}_i\left(\tau,R\right);X',\{\mathfrak{M}_1\left(\tau,R\right),\mathfrak{M}_2\left(\tau,R\right)\}\right), \tag{5.23}$$

for $i = 1, 2$.

2. The One-Period Economy

The following proposition characterizes the private sector equilibrium as a function of government policy $\{\tau, R\}$ in the one-period economy.

Proposition 5.2. *An allocation* $\{c_{i1,}c_{i2}, c_i, n_i, z_i\}_{i=1,2}$ *and a policy* $\{\tau, R\}$ *constitute a private sector equilibrium for the one-period economy for given* $\bar{g}$*, if and only if* z_i *solves (3.17) and* $c_{i1,}$ c_{i2}*,* c_i*,* n_i *are determined according to:*

$$c_i = w_i, \tag{5.24}$$

$$n_i = \frac{c_i}{w_i \gamma}\left[\left(RP^i\right)^{\frac{\rho}{\rho-1}} + \frac{\tilde{P}^i}{P^i}\right], \tag{5.25}$$

$$c_{i2} = c_i \left(P^i\right)^{\frac{1}{1-\rho}}, \tag{5.26}$$

$$\left(\frac{c_{i1}}{c_{i2}}\right)^{\rho-1} = R, \tag{5.27}$$

$$R = \mathcal{R}\left(\tau, \bar{g}\right),$$

where

$$P^i = \left[\left(1 - z_i\right) R^{\frac{\rho}{\rho-1}} + z_i\right]^{\frac{\rho-1}{\rho}}, \tag{5.28}$$

$$\tilde{P}^i = P^i + \frac{C\left(z_i\right)}{c_i}, \tag{5.29}$$

$$w_i = \frac{\xi_i\left(1-\tau\right)}{\gamma P^i}, \tag{5.30}$$

for $i = 1, 2$*, with* $\mathcal{R}\left(\cdot\right)$ *implicitly defined by (4.3).*

Proof of Proposition 5.2 The first order conditions for the household problem are given by:

$$u_{i1} - R\left(1 - z_i\right)\lambda_i = 0, \tag{5.31}$$

$$u_{i2} - z_i \lambda_i = 0, \tag{5.32}$$

$$\gamma - \left(1-\tau\right)\xi_i \lambda_i = 0, \tag{5.33}$$

plus the analogous of (3.17) and (4.2), where λ_i is the Lagrange multiplier on (4.2). (5.24) follows from (5.32), (5.28) and (5.30), (5.27) follows from (5.31)-(5.32). (5.25) follows from (4.2) using (5.28), (5.29) and (5.30).■

Proposition 4.2 characterizes sufficient conditions for increased inequality to correspond to higher equilibirum nominal interest rate in the bargaining equilibrium.

Proof of Proposition 4.2 The necessary condition for the bargaining problem is:

$$p\left[\frac{\mathcal{V}_2}{\mathcal{V}_1}\right]\frac{dU^1\left(\{\tau,R\}\right)}{d\tau}+\frac{dU^2\left(\{\tau,R\}\right)}{d\tau}=0. \tag{5.34}$$

Assume that it is satisfied at $\{\tau,R\}$ for a given value of ξ_1 and ξ_2. The proof of this Proposition requires establishing that the expression on the LHS of (5.34) is negative at $\xi_2'>\xi_2$, since U^i is quasiconvex with respect to $(1-\tau)$, which implies that U^i is quasiconcave with respect to τ. Given (4.6), it is sufficient to show that $\mathcal{V}_2$ is decreasing in ξ_2 and that $\frac{dU^i(\{\tau,R\})}{d\tau}$ is non-increasing in ξ_i.

By proposition 5.2:

$$U^i\left(\{\tau,R\}\right)=1-\gamma\frac{\tilde{P}^i}{P^i\gamma}.$$

This simplifies to:

$$U^i\left(\{\tau,R\}\right)=-\frac{\gamma}{\xi_i}\frac{C\left(z\left(\tau,R;\xi_i\right)\right)}{1-\tau}, \tag{5.35}$$

where $z\left(\tau,R;\xi_i\right)$ is implicitly defined by:

$$\left(\frac{1}{\rho}-1\right)\left(1-R^{\frac{\rho}{\rho-1}}\right)\left(\frac{\xi_i\left(1-\tau\right)}{\gamma}\right)\left(P^i\right)^{\frac{\rho}{1-\rho}}-\theta=0, \tag{5.36}$$

for z_i interior. (5.36) is the first order condition for z_i in the household problem. Differentiating (5.36) with respect to ξ_i obtains:

$$\frac{\partial z\left(\tau,R;\xi_i\right)}{\partial\xi_i}=\frac{1}{\xi_i}\frac{\left(1-R^{\frac{\rho}{\rho-1}}\right)z_i+1}{\left(1-R^{\frac{\rho}{\rho-1}}\right)}\geq 0. \tag{5.37}$$

From (5.35):

$$\mathcal{V}_i=\max\left\{0,-\frac{\gamma C\left(z\left(\tau,R;\xi_i\right)\right)}{\left(1-\tau\right)\xi_i}+\frac{\gamma C\left(z\left(\tau^T,R^T;\xi_i\right)\right)}{\left(1-\tau^T\right)\xi_i}\right\}.$$

To see that $\mathcal{V}_2/\mathcal{V}_1$ is decreasing in ξ_2, it is sufficient so analyze the derivative of $\mathcal{V}_i$ with respect to ξ_i equal to:

$$\frac{\partial\mathcal{V}_i}{\partial\xi_i}=\gamma\left[-\frac{\partial z\left(\tau,R;\xi_i\right)}{\partial\xi_i}\frac{\theta\left(z\left(\tau,R;\xi_i\right)\right)}{\left(1-\tau\right)\xi_i}-\frac{1}{\xi_i}\mathcal{V}_i\right]\leq 0,$$

by (5.37) and (4.5). (4.6) implies that $\frac{dU^2(\{\tau,R\})}{d\tau} \leq 0$. To show that $\frac{dU^2(\{\tau,R\})}{d\tau}$ is non-increasing in ξ_2 note that:

$$\frac{dU^i}{d\tau} = \frac{\partial U^i}{\partial \tau} + \frac{\partial U^i}{\partial R}\frac{\partial R}{\partial \tau},$$

and

$$\begin{aligned}
\frac{\partial U^i}{\partial \tau} &= \frac{-\gamma\theta}{\xi_i(1-\tau)}[\frac{\partial z(\tau,R;\xi_i)}{\partial \tau} + \frac{z_i}{1-\tau}], \\
\frac{\partial U^i}{\partial R} &= \frac{-\gamma}{\xi_i(1-\tau)}\theta\frac{\partial z(\tau,R;\xi_i)}{\partial R}.
\end{aligned}$$

From:

$$\begin{aligned}
\frac{\partial z(\tau,R;\xi_i)}{\partial R} &= \frac{\rho}{1-\rho}\frac{R^{\frac{1}{\rho-1}}z_i}{\left(1-R^{\frac{\rho}{\rho-1}}\right)}, \\
\frac{\partial z(\tau,R;\xi_i)}{\partial \tau} &= -\frac{\left(1-R^{\frac{\rho}{\rho-1}}\right)z_i+1}{\left(1-R^{\frac{\rho}{\rho-1}}\right)(1-\tau)},
\end{aligned}$$

one can show that:

$$\begin{aligned}
\frac{d\left(\frac{\partial U^i}{\partial \tau}\right)}{d\xi_i} &= \frac{-\gamma}{\xi_i^2(1-\tau)^2} \leq 0, \\
\frac{d\left(\frac{\partial U^i}{\partial R}\right)}{d\xi_i} &= \frac{\gamma\theta}{\xi_i(1-\tau)}\frac{\partial z}{\partial R}\left(\frac{1}{\xi_i(1-\tau)} + \frac{\partial z_i}{\partial \xi_i}\frac{1}{z_i}\right) \geq 0.
\end{aligned}$$

Then, $\frac{d\left(\frac{dU^i}{d\tau}\right)}{d\xi_i} \leq 0$ follows from $\partial R/\partial\tau \leq 0$.■

3. Data

The data on inflation from Easterly, Rodriguez and Schmidt-Hebbel (1994) and the data on income inequality is from the Deinenger and Squire (1996) source file. The sample of countries included in the analysis is smaller than the "high quality" data sample. The criteria for inclusion in the sample are availability of before tax income inequality measures and quintile income distribution measures at least every five years[23].

[23] This significantly restricts the sample number of countries included relative to previous studies, such as Romer and Romer (1998). In addition, it addresses the concerns raised in Atkinson and Anderlini (2001) on using secondary datasets on inequality.

Inequality measures for before tax income for Norway, Sweden, Finland and the UK are drawn from data from the Luxemburg Income Study. For Argentina no comparable data with national coverage is available. The measures provided are based on household surveys conducted in urban centers and the greater Buenos Aires area.

Political instability is measured as the frequency of power transfers in the period 1971-1982, from Edwards and Tabellini (1992). A transfer of power is defined as a situation where there is a break in the governing political party control of executive power. This measure varies between 0 and 1, where 0 represents perfect stability. Data on central bank independence is from Cukierman (1992). Legal central bank independence is measured based on a number of indicators, including the power of the central bank governor, the independence in policy formulations and in the definitions of objectives and on the presence of limitations on lending to the treasury. The included index measures overall independence for the 1980's. The values of this variable range from 0 (minimal independence) to 1 (maximum independence). The turnover rate for central bank governors is the average number of changes per annum in the period 1950-1989. The IMF International Financial Statistics are used for data on GDP per capita.

A list of countries and variables included in the sample is provided in Albanesi (2002a).