Math Camp

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Unit 3

MSSM Program
Columbia University
Dr. Satyajit Bose

Unit 3 Outline

- Financial Return Assessment
  - Payback
  - NPV
  - IRR
- Capital Structure
  - Equity/Mezzanine/Debt

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Interlude

Financial Return Measures

Financial returns of investments vary in timing and risk. Return measures provide imperfect methods of ranking different investments based on timing and risk.

- Payback (Simple and modified)
- Discounted Cash Flow (NPV/IRR)

Simple Payback

is the period of time, in years, required for a return on an investment to repay the sum of the original investment.

- Conceptually simple
- Easy to calculate (except in Excel)
- Limitations:
  - Ignores cash flows after payback
  - Does not take into account time value of money

Payback examples - #1

- Compute payback for the three investments:

Cash Flow for:	Period							
	0	1	2	3	4	5	6	7
Investment A	$(100)	$ 20	$ 20	$ 20	$ 20	$ 20	$ 20	$ 20
Investment B	$(100)	$ 25	$ 25	$ 25	$ 25	$ 25	$ 25	$ 25
Investment C	$(100)	$ 30	$ 30	$ 30	$ 30	$ 30	$ 30	$ 30

Payback examples - #2

- Compute payback for the two investments:

Cash Flow for:	Period							
	0	1	2	3	4	5	6	7
Investment A	$(100)	$ 20	$ 20	$ 20	$ 20	$ 20	$ 20	$ 20
Investment B	$(100)	$ 25	$ 25	$ 25	$ 25	$ -	$ -	$ -

Payback examples - #3

- Compute payback for the four investments:

Cash Flow for:	Period							
	0	1	2	3	4	5	6	7
Investment A	$(100)	$ 5	$ 15	$ 25	$ 35	$ 45	$ 55	$ -
Investment B	$(100)	$ -	$ 30	$ 30	$ 30	$ 30	$ 30	$ 30
Investment C	$(100)	$ -	$ -	$ 45	$ 45	$ 45	$ 45	$ -
Investment D	$(100)	$ 60	$ -	$ -	$ -	$ -	$ -	$ 120

Modified Payback

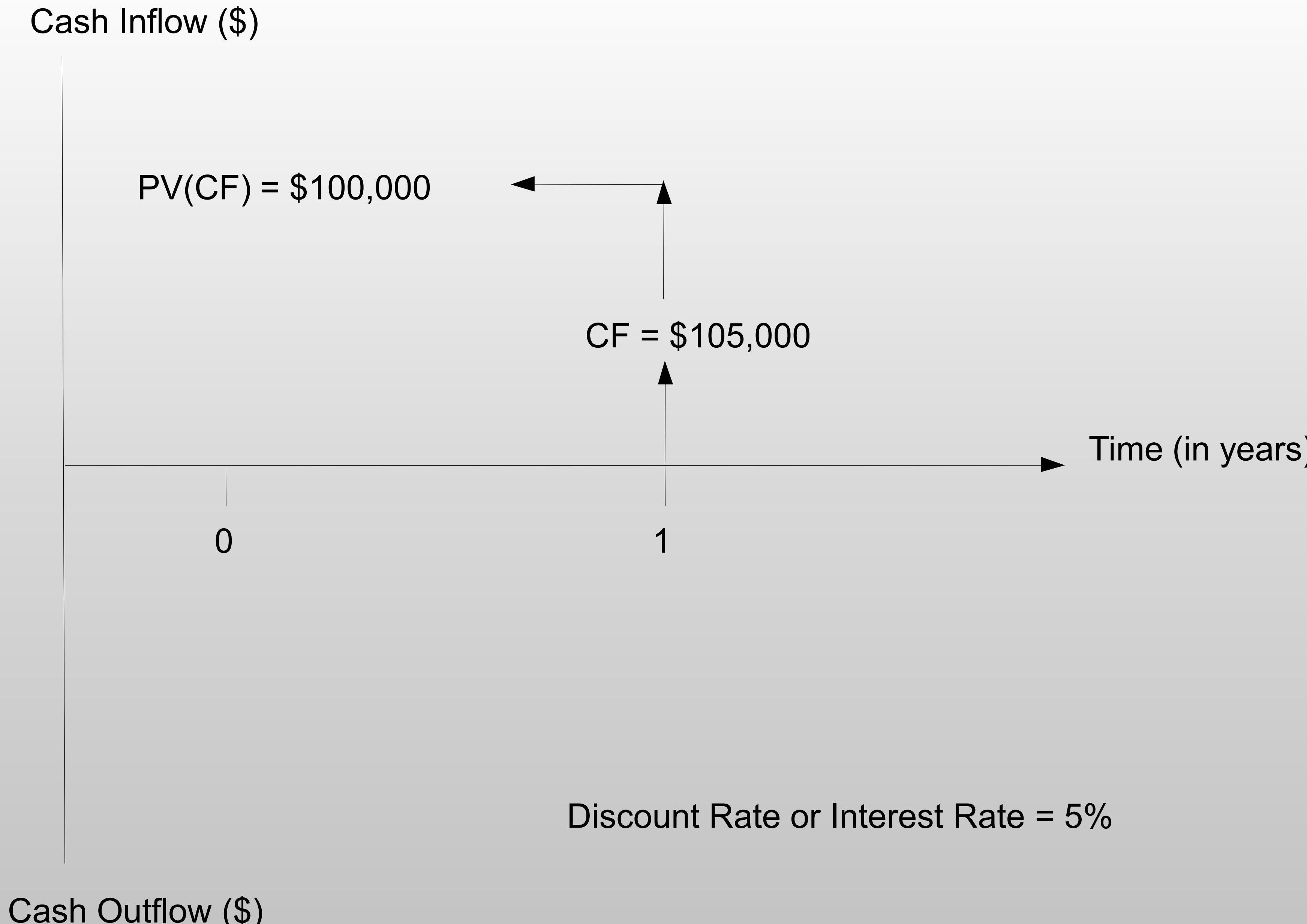
is the period of time, in years, required for the present discounted value of cash inflows to equal the sum of the original investment.

- Conceptually simple
- Easy to calculate (except in Excel)
- Limitations:
  - Ignores cash flows after payback

Present Value

The present value, *PV,* of a cash flow amounting to *CF* received *t* years from now, with interest compounded annually at rate *r* is:

$$PV = \frac{CF}{(1+r)^t}$$



Cash Inflow ($)
PV(CF) = $100,000
CF = $105,000
Time (in years)
0
1
Discount Rate or Interest Rate = 5%
Cash Outflow ($)

Discounted cash flow (DCF)

A DCF analysis converts all cash flows into present values, incorporating the time value of money

The "discount" refers to the fact that cash flows in the future (whether positive or negative) are valued less than cash in the present (i.e., are discounted)

- $100 today is generally worth more than $100 in five years
- Every cash flow is discounted using a formula: $CF_t / (1+r)^t$ where r is the discount rate (per period) and t is the number of periods into the future

The DCF analysis checks whether the positive discounted cash flows are greater than the negative discounted cash flows

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Net Present Value Analysis

NPV = PV(inflows) – PV(outflows)

A positive NPV means that the sum of the present values of all the cash inflows and outflows (including the initial investment) is positive.

A positive NPV project is "profitable" at a given discount rate (or cost of capital)

Choose the investment with the largest net present value at a given discount rate.

Investments with Different Horizons

- Two investments with different horizons are not comparable.
- Adjust using:
  - Roll-Over Method
  - Equivalent Annual Net Benefits (EANB) Method
  - (both these methods underestimate the value of shorter projects due to their quasi-option value)

Internal Rate of Return (IRR)

The "internal rate of return" is the discount rate such that the project's NPV is $0

An IRR is normally quoted on an annual basis; if the discount rate for monthly cash flows is determined, it would normally be converted to an annual IRR

Project Cash Flows (Dollars in Millions)

Year	Cash Inflow (Outflow)	Cash Inflow (Outflow) @8.1% Discount Rate
0	-$100.0	-$100.0
1	$15.0	$13.9
2	$15.0	$12.8
3	$15.0	$11.9
4	$15.0	$11.0
5	$15.0	$10.1
6	$15.0	$9.4
7	$15.0	$8.7
8	$15.0	$8.0
9	$15.0	$7.4
10	$15.0	$6.9
	IRR = 8.1%	sum (NPV) = $0.0

Internal Rate of Return (IRR) - cont'd

Alternatively, if you could borrow 100% of project cost ($100) at 8.1% and any cash flow after interest would repay the borrowing, you would have exactly $0 left

Project Cash Flows (Dollars in Millions)

Year	Beginning Principal	Interest Payment @ 8.1%	Principal Payment ($15 - interest)	Ending Principal
1	$100.0	$8.1	$6.9	$93.1
2	$93.1	$7.6	$7.4	$85.7
3	$85.7	$7.0	$8.0	$77.7
4	$77.7	$6.3	$8.7	$69.0
5	$69.0	$5.6	$9.4	$59.7
6	$59.7	$4.9	$10.1	$49.5
7	$49.5	$4.0	$11.0	$38.6
8	$38.6	$3.1	$11.9	$26.7
9	$26.7	$2.2	$12.8	$13.9
10	$13.9	$1.1	$13.9	$0.0

What should be the discount rate?

Government Contexts:

- Social rate of discount e.g. long term Treasury bonds or calibrated optimal growth rate

Corporate Contexts:

- Weighted average cost of capital (WACC): an appropriate average of the cost of borrowing money (debt), the opportunity cost for using equity for a particular investment rather than on returning capital to shareholders

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Example

A private company is considering investing in the construction of a wind farm, with a total capacity of 2 MW. 1 MW is expected to generate 2,190 MWh per year. The installation cost is equal to $1,800 per kW, while the operating cost is $75 per kW per year. The price of the electricity generated by the wind is known in advance and is equal to $90 per MWh. Assume that the majority of the electricity consumed comes primarily from the combustion of fossil fuels. When electricity is generated by wind power and not by fossil fuels, there is a reduction in CO_2 emissions of 1kg per kWh. The shadow price of carbon emissions used to compute externalities is $12 per 1,000kg of CO_2 emitted during electricity production. However, there is no regulation requiring the company to pay any costs or fees for the carbon emissions generated as a byproduct of its activities. The life span of the wind turbine is expected to be 25 years.

Assume that except for the installation cost, all other costs and benefits are incurred at the end of the year.

Example

1. Perform a private cost benefit analysis for the investment, using 5% as a discount rate.
2. Use the performance indicators of NPV, IRR and payback period to discuss the results.
3. Should the company make the investment?

Shortcomings of NPV Analysis

- Is appropriate only for now-or-never decisions
- Deals only with *expected* cash flows
- Systematically undervalues most other investment opportunities

Equity vs. Debt

Equity represents the ownership interests in an entity, and is entitled to the residual cash flows/assets of that entity (e.g., common stock)

- No specified promise of repayment; may receive periodic distributions (dividends) if/as/when declared and uncertain time to recoup investment
- Greatest risk to investor, so investors require greatest expected return
- Dividends are not tax deductible

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Equity vs. Debt

Debt represents a claim to be repaid principal and paid interest in accordance with certain specified terms (e.g., $1,000 bond, repayable in five years, with 4% annual interest payable semi-annually)

- Specified promise of repayment; if not paid in accordance with terms, investors have certain rights designed to reduce risk of failure to repay
- Lower risk to investor, so investors require lower expected return
- Interest on debt is generally tax deductible

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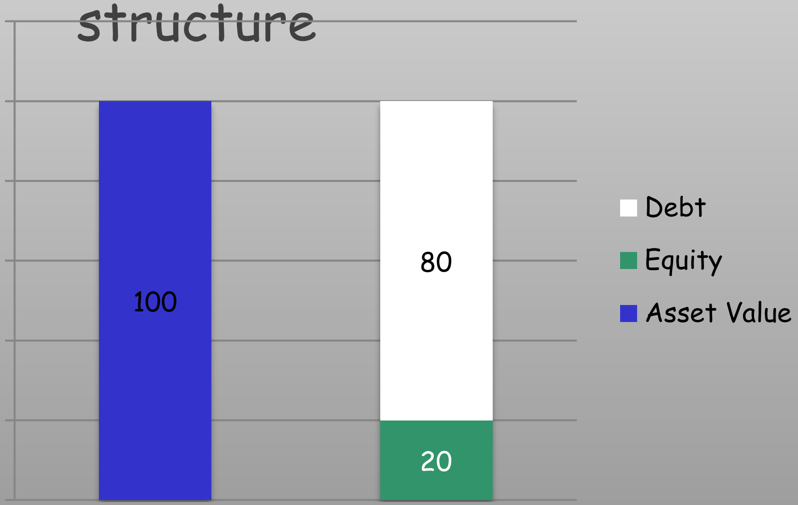
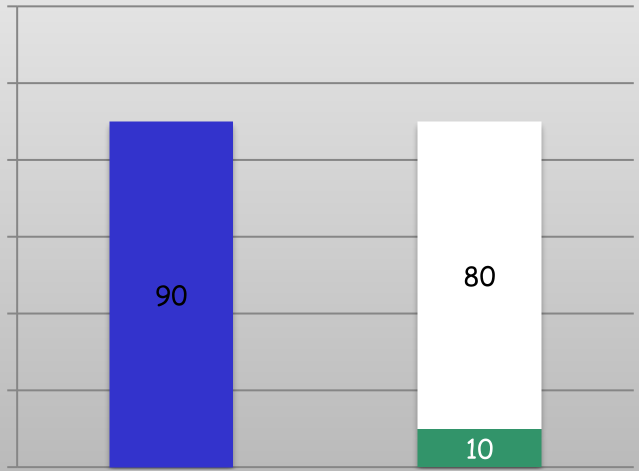
Capital Structure

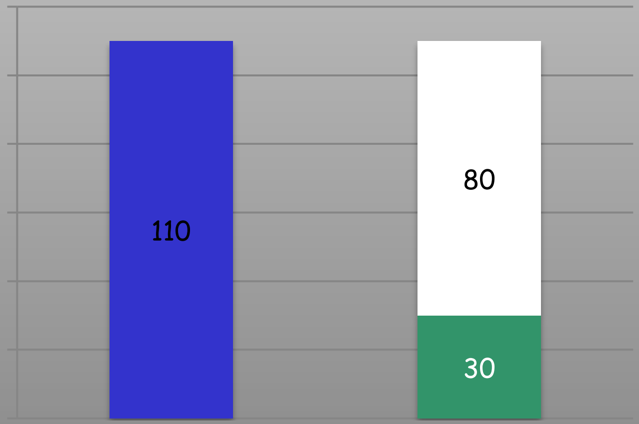
How a company or project or pool of assets is financed (or capitalized) is its capital structure



Asset value falls from 100 to 90, value of equity decreases from 20 to 10



Asset value rises from 100 to 110, value of equity increases from 20 to 30



Weighted Average Cost of Capital (WACC)

Is a weighted sum of the after tax cost of equity and debt. The weights are the proportions of the total investment financed by equity and debt, respectively.

$$WACC =$$

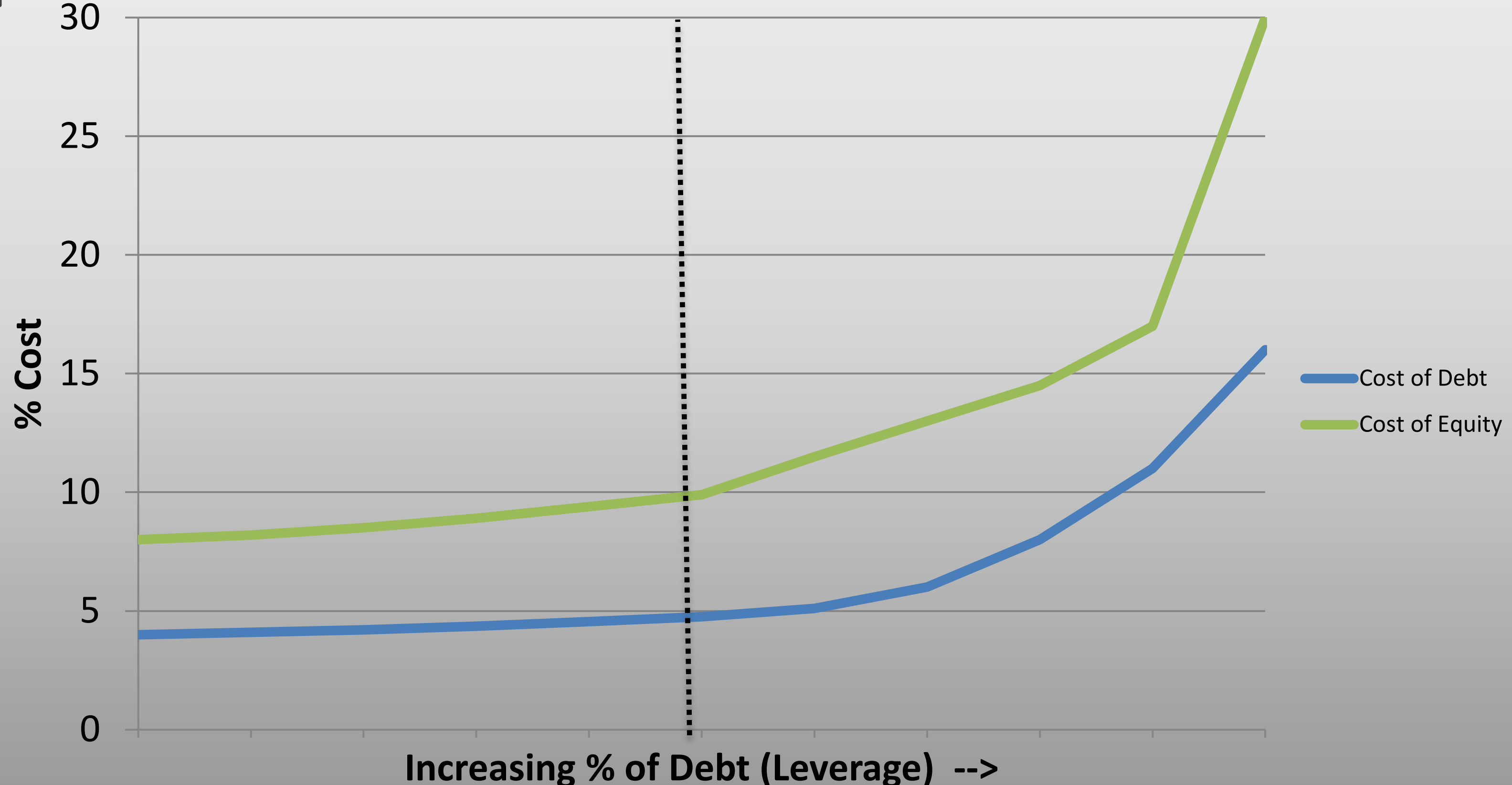
$$\frac{\mathrm{MV}_{equity}}{\mathrm{MV}_{debt} + \mathrm{MV}_{equity}} \mathrm{Return}_{equity}$$

$$+$$

$$\frac{\mathrm{MV}_{debt}}{\mathrm{MV}_{debt} + \mathrm{MV}_{equity}} \mathrm{Return}_{debt} (1 - \text{tax rate})$$

The more debt in a project…

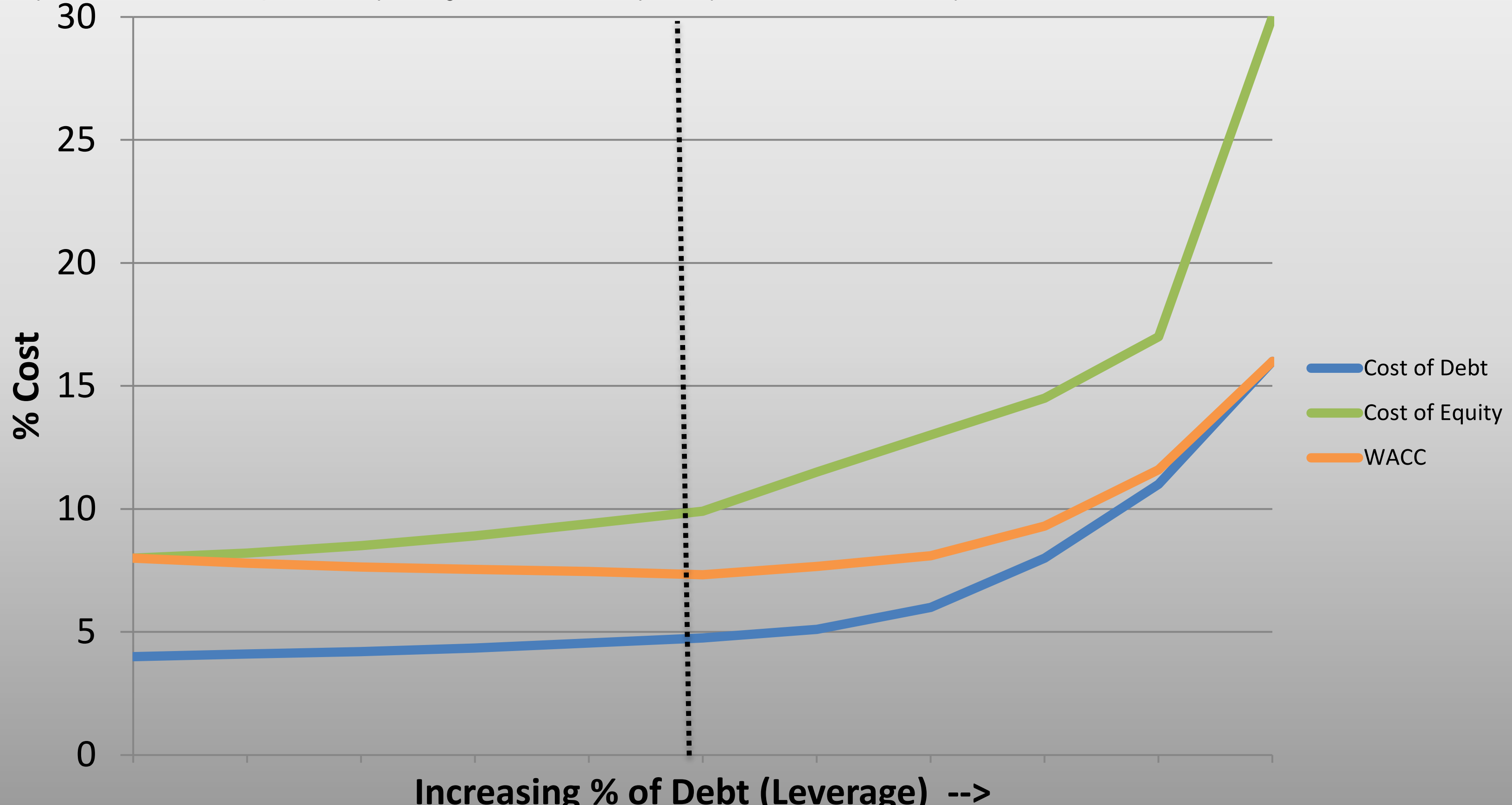
the more expensive the equity and the more expensive the debt, although the weighted average cost of capital does not necessarily get more expensive…



Source: Brealey, Richard A. and Myers, Stewart C., Principles of Corporate Finance.

Capital structure drives WACC

Theoretically, there may be an optimal amount of leverage and cost of capital for a given project, company or industry



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