Precision of Communication in Coordination Games of Regime Change*

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Sergey Kolbin†

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Abstract

I study a model of regime change in which the government can communicate with different levels of precision as a function of the underlying fundamentals. In the model, higher precision of communication corresponds to a lower dispersion of private information among market participants. I compare a policy of an uncommitted government, which chooses the precision of communication after it learns the realization of fundamentals, to a policy of a committed government, which commits to a state-dependent policy before it learns the realization of fundamentals. I find that an uncommitted government communicates imprecisely for weak fundamentals and precisely for strong fundamentals. In contrast, a committed government communicates precisely for weak fundamentals and imprecisely for strong fundamentals. Consequently, a committed government saves its regime more often than an uncommitted one. An uncommitted government can benefit from a rule that enforces constant precision of communication.

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† Columbia University, Department of Economics, 1022 International Affairs Building 420 West 118th Street, New York, NY 10027. Email: sk3404@columbia.edu.

1 Introduction

Policymakers sometimes make statements that are precise and at other times make statements that are vague. Moreover, policymakers can choose the level of precision of their statements based on the information they are communicating, and they are often precise when they are in a strong position, but vague when they are in a weak position.

For example, the Mexican government was forthcoming about the size of its foreign currency reserves when they were growing between 1990 and early 1994. When the risk of a speculative currency attack arose in Mexico in March 1994, the government pronounced that it was ready to spend $30 billion of foreign currency reserves to defend the peso against this attack.[1] In contrast, in April 1994, after reserves had declined significantly, Mexican Finance Minister Pedro Aspe refused to disclose the level of reserves, saying instead that "there were huge capital inflows in January and February. That portion of Mexico's reserves has been drawn down."[2] A week later, Aspe was even vaguer when he characterized the level of reserves as "adequate."[3] Market participants had limited information about the reserves and other important economic data until a series of speculative attacks forced the government to float the peso in December 1994.[4]

Did the Mexican government use the best possible communication policy when it became vaguer about the economic situation in the difficult year 1994? In this paper, I explore this question in a theoretical model, and I ask the following questions. First, how should the communication policy of a government respond to changes in economic fundamentals? Second, to what extent does such a communication policy require commitment? Finally, can institutionalized transparency requirements that prevent the government from manipulating the quality of information improve welfare?

To answer these questions, I extend the global game model of Morris and Shin (2003),

[1] The risk of a currency attack emerged as a consequence of the killing of a presidential candidate. See "Political Trauma: Mexicans Struggle To Cope With Shock Of Candidate's Killing." *The Wall Street Journal* 25 March 1994.

[2] "Aspe says rates have topped, now declining - analyst." *Reuters News* 27 April 1994.

[3] "Finance Secretary on back-up credit and consultation agreements with USA, Canada." *BBC Monitoring Service: Latin America* 3 May 1994.

[4] For a detailed discussion of the government's management of information on foreign reserves, see Braun, Mukherji, and Runkle (1996). See Adler (1994), Edwards (1998), and Section 3 for additional evidence that the Mexican government decreased communication and data disclosure in 1994.

which is a model used to study currency attacks, bank runs, and political revolutions.[5] In this model, a continuum of agents (currency speculators) decides whether or not to attack a regime. The payoffs of these agents depend on whether or not the regime falls, which in turn depends on the fundamental strength (type) of the government (the level of liquid reserves or the determination of the government) relative to the size of the attack by all agents. These agents are informed by a private signal regarding the fundamental strength of the government. The government's interest is in preserving the regime. I extend the Morris and Shin (2003) framework by allowing the government, which observes fundamentals, to change the precision of this private signal in response to the realization of the fundamental.[6]

My modeling of the precision of communication as directly related to the dispersion of private signals is motivated by evidence from neuroscience. This evidence suggests that the precision of a stimulus can be quantified by the induced dispersion in its perception by human subjects.[7,8] For example, in the context of public announcements, the dispersion of perception will be lower if the government uses precise language such as "$30 billion" instead of vaguer language such as "adequate level of reserves."[9] This modeling choice is

[5]For example, Morris and Shin (1998) analyze currency attacks, Morris and Shin (2004) study debt runs, Goldstein and Pauzner (2005) analyze bank runs, and Edmond (2013) studies political revolutions.

[6]In practice, the government can also bias the mean of signals that it distributes. As I discuss in Section 2, such an action may sometimes be infeasible, as may have been the case in Mexico's 1994-1995 crisis. Moreover, as I show in Subsection 6.2, the main results related to precision of communication survive if the government can also bias the mean of its communication.

[7]Neuroscience provides evidence that human perception of a stimulus is probabilistic, with a random element. Moreover, in experiments, neuroscientists can vary the accuracy of perception by changing qualities of the stimulus. For example, in an experiment conducted by Stocker and Simoncelli (2006), human subjects observed a moving object. By increasing the visual contrast of the object, the researchers could predictably decrease the dispersion of the subjects' perception of the object's speeds. For additional evidence, see Yang and Stevenson (1997), Ernst and Banks (2002), or Körding and Wolpert (2004).

[8]Several papers before me modeled higher quality of information distributed by policy-makers as lower dispersion of private signals. For example, Heinemann and Illing (2002) model higher "transparency" and Myatt and Wallace (2014) model higher "clarity of communication" as a lower dispersion of private signals.

[9]Linguists have a clear understanding of the characteristics of vague language as opposed to precise language. For example, Channell (1994) classifies vague words and phrases into several groups that include approximations of quantities involving numbers (about 15, 3 or 4), approximations of quantities with round numbers (100 instead of 98.2), nonnumerical approximations

also supported by the Mexican experience. As Latin American Newsletters reported in May 1994, "Reserves declined significantly, although estimates of how much vary widely from US $ 6bn to US $ 12bn."[10]

In the model, the government observes the realization of fundamentals and then determines the precision of information that it distributes. If the information is very precise, then all agents receive similar signals regarding the strength of the government. In contrast, if the information is very vague, then agents receive very different signals. Agents know that the precision of the information they are receiving is driven by the realization of fundamentals, and they take this into account when deciding whether to attack the regime.

In my framework, I compare the policy of an uncommitted government with the policy of a committed government. An uncommitted government chooses the precision of its communication after it learns the realization of fundamentals to maximize the probability of saving its own regime. A committed government commits to a state-dependent precision of communication policy before it learns the realization of fundamentals to maximize the expected probability of regime survival over all types of the government.

Because the government varies the precision of information with the state of the world, the agents' posterior beliefs do not belong to the same class of distributions as agents' signals (and in fact do not belong to any standard class of distributions). I make the analysis tractable by considering threshold strategies for the agents. Threshold strategies are intuitive, as low signals correspond to weak governments and high signals correspond to strong governments. I provide conditions under which agents choose to use threshold strategies both under lack of commitment and under commitment. The agents' attack threshold serves as a sufficient statistic for agents' beliefs, which allows for a simple characterization of the policies of both an uncommitted and a committed government.

I first show that an uncommitted government communicates imprecisely when it is weak and precisely when it is strong.[11] In particular, a government uses a threshold pol-

of quantities (a lot of, many), and vague references to categories (stuff like that, something).

[10] "Surviving the first attack; Banco de Mexico beats off devaluation." *Latin American Newsletters* 12 May 1994.

[11] The analysis of an uncommitted government is consistent with Edmond (2013), as I discuss later in the introduction.

icy and switches from the lowest available precision below the threshold to the highest available precision above the threshold. An uncommitted government chooses precision to minimize the size of the attack conditional on the realization of fundamentals, as a smaller attack allows the regime to be saved with a higher probability. Therefore, a weak, uncommitted government communicates imprecisely to leave an incorrect impression among some agents that it is strong to avoid their attacks. A strong, uncommitted government communicates precisely to persuade most agents that an attack is not worthwhile.[12]

A government's lack of commitment can lead to too much aggressiveness on the part of agents, which in turn increases the likelihood of the regime falling. This is because the policy of an uncommitted government can change agents' beliefs in a way that increases the agents' attack threshold and leads agents to attack for a larger set of signals. Indeed, the agents near the attack threshold are certain that the government cannot be very strong, as a very strong government communicates precisely and hence distributes signals above the threshold. At the same time, the agents near the threshold know that the government can be very weak, as a very weak government communicates vaguely, and hence the signals of agents near the attack threshold are consistent with a government being very weak. Consequently, attacks against the regime become larger, which causes a higher probability of the regime falling.

The policy of a committed government is determined as a result of the interaction of two motives. First, a government wants to minimize the attack to save the regime, given its type. Second, a government seeks to manipulate the beliefs of agents to make them less aggressive and to minimize the attack against governments of other types. Whereas the first motive affects the policies of both an uncommitted and a committed government, only a committed government considers the effect of its policy on agents' beliefs and hence obtains a better outcome. The interaction of these two motives, which can either compete or agree depending on the government's type, results in a communication policy that is non-monotone in fundamentals. I analytically characterize the policy of a committed government and show that, for extreme fundamentals, the second

[12] In the model, the chosen dispersion of information is unobserved. My main results continue to hold if agents receive several signals that allow them to better estimate a government's action. See Subsection 6.1 for details.

motive dominates and prescribes a policy that is opposite to a policy of an uncommitted government.

Particularly, I show that a committed government communicates precisely when it is weak and imprecisely when it is strong to lower the agents' attack threshold and help governments of other types. If a government communicates precisely when it is very weak, it distributes only weak signals. Consequently, agents that receive signals near the attack threshold realize that these signals cannot come from a very weak government. If a government communicates imprecisely when it is very strong, it distributes a wide range of signals. Therefore, agents that receive signals near the threshold realize that these signals likely come from a very strong government. More intuitively, by revealing its weakness and concealing its strength, a committed government ensures that agents that receive signals near the threshold, and therefore are pivotal for the determination of whether the regime survives, realize that the regime is not very weak but can be very strong.

My final set of results is motivated by the fact that implementing a fully optimal policy of a committed government may be difficult for some countries. First, some governments may lack the willpower to commit to the optimal policy and may instead use the policy of an uncommitted government. Second, the optimal policy has a complex non-monotone structure, and hence may be difficult to implement. A simple rule of constant precision of communication can be easier to commit to and implement. I show that this simple rule can save the regime more often than a policy of an uncommitted government. Constant precision of communication should be chosen to approximate the policy of a committed government. Particularly, the rule prescribes constant precise communication if the government's type is weak on average, and constant imprecise communication if the government's type is strong on average. Under some conditions, a communication policy of any constant precision is better than a policy of an uncommitted government.

My theoretical results allow me to provide an interpretation of Mexico's experience as follows. Since the Mexican government communicated precisely in good times and vaguely in bad times, its policy was consistent with a policy of an uncommitted government in my model. This policy, however, was blamed for worsening the crisis. For example, Edwards (1998) writes, "In what in retrospect proved to be a serious mistake that greatly eroded credibility, the authorities decided against the general disclosure of information." According to the International Monetary Fund (IMF), "In the wake of the 1994 crisis in Mexico, the international financial community recognized the essential role of data transparency for meeting the challenges and risks of globalization and reducing the likelihood of financial crises."[13] This attribution of Mexico's crisis to the government's communication policy is consistent with my model's implication that a policy of an uncommitted government can make agents more aggressive, which leads to a greater chance of regime falling.

In 1996, the IMF responded to the Mexico's crisis by introducing the Special Data Dissemination Standard (SDDS). A country that subscribes to the SDDS must publish specific data with specific periodicity under control of the IMF, and hence the IMF views the SDDS as a mechanism for committing countries to transparency. Currently, 65 countries, including Mexico, have subscribed to the SDDS. This policy response by the international community is consistent with my theoretical result that constant precision can be more effective than a policy implemented by an uncommitted government.

My paper makes a contribution to the study of global games (Carlsson and Van Damme, 1993; Morris and Shin, 1998) by analyzing a situation in which agents do not know the precision of their private information because it is chosen strategically by the government as a function of its type. Most closely related are global game models with endogenous information structures,[14] and policy applications of global game models that study how to prevent currency attacks, debt runs, bank runs, and political revolutions.[15]

The most closely related global game paper is Edmond (2013), who studies how the government chooses dispersion of private signals conditional on government's type to save its own regime with the highest probability (is uncommitted in terms of my paper). My analysis of an uncommitted government is consistent with the analysis of Edmond (2013). The main contribution of my paper is the analysis of a committed government and of simple policies that can help an uncommitted government to obtain a better outcome. In particular, I show that a policy of a committed government is opposite to

[13] "The Special Data Dissemination Standard 2013. Guide for Subscribers and Users." IMF 2013.

[14] For example, Heinemann and Illing (2002), Metz (2002), Bannier and Heinemann (2005), Angeletos and Werning (2006), Hellwig, Mukherji, and Tsyvinsky (2006), Angeletos, Hellwig, and Pavan (2007), Edmond (2015), Szkup and Trevino (2015), and Yang (2015).

[15] For example, Angeletos, Hellwig, and Pavan (2006), Angeletos and Pavan (2013), Edmond (2013), and Szkup (2015).

the policy of an uncommitted government for extreme fundamentals, and that constant precision of communication can be more effective than a policy of an uncommitted government.

Heinemann and Illing (2002) and Bannier and Heinemann (2005) study government transparency in global games and model higher transparency as a lower dispersion of private signals. Their analysis of conditions under which constant precise communication is better than constant imprecise communication is consistent with my analysis of the policies of constant precision of communication. These papers, however, restrict their attention to policies of constant precision of communication. In contrast, I allow the government to vary dispersion of private signals with the state of the world and concentrate on comparing policies of uncommitted and committed governments. I show that both an uncommitted government and a committed government necessarily use policies of non-constant communication, and discuss commitment mechanisms to bring a policy of a government that cannot commit closer to a fully optimal policy of a committed government.

My paper is related to the literature on the disclosure of public information by policymakers (Morris and Shin, 2002; Angeletos and Pavan, 2007),[16] as it also demonstrates that proper choice of the precision of information, publicly distributed by a policymaker, can help avoid socially undesirable coordination. This literature studies whether disclosure of public information improves welfare, given that public information contains noise; and hence agents, which coordinate on public information, can over-coordinate on that noise. I model public communication differently—more precise communication as lower dispersion of private signals—that is, the government chooses how vaguely to communicate a correct message, and the aggregate noise does not play a role in my paper. Another difference is that I study the role of precision of communication in a model of regime change, as opposed to the very different "beauty contest" model.[17]

My analysis of a committed government shares intuitions with the literature on

[16]See also Woodford (2005), Amador and Weil (2010, 2012), Myatt and Wallace (2014), Chahrour (2014), Angeletos, Iovino, and La'O (2015).

[17]In a "beauty contest" model, the agent's payoff depends continuously on the state of the world and the average action of other agents, as opposed to inherently discontinuous global game models of regime change.

Bayesian persuasion, such as Kamenica and Gentzkow (2011).[18] In particular, the optimal communication policy of a committed government in my paper also manipulates posterior beliefs, to elicit a specific action from agents. Whereas papers in the Bayesian persuasion literature consider a general information structure, I restrict my attention to a specific class of information structures (unbiased communication with continuously distributed private signals), which enables me to study public communication by the government. I also consider a fairly general model of coordination (with a continuum of states of the world and agents), which is difficult to address using the approach of the Bayesian persuasion literature.

Finally, my paper contributes to the literature on the role of vagueness in economics,[19] due to its focus on how a policymaker can vary vagueness of its communication to avoid socially undesirable coordination in a model of regime change.

2 Model of State-Dependent Communication Policy

In this section, I set up a model of the state-dependent precision of government communication. For this purpose, I extend a standard global game of regime change as in Morris and Shin (2003) by allowing the government to choose the dispersion of private signals as a function of economic fundamentals. I use this model for policy analysis in the rest of the paper.

I consider a one-period model with a government and a continuum of agents.

Government

In the model, the government tries to defend its regime against an attack by many small agents. The regime survives if the attack size is smaller than the realization of economic

[18] See Bergemann and Morris (2015) and Taneva (2015) for an alternative approach to information design. See Goldstein and Leitner (2015), Faria-e-Castro, Martinez, and Philippon (2015), Bouvard, Chaigneau, and de Motta (2015) for an application of the Bayesian persuasion approach to the disclosure of banks' stress test results.

[19] For example, de Jaegher (2003), Lipman (2009), Serra–Garcia, van Damme, and Potters (2011), Agranov and Schotter (2012, 2013), and Blume and Board (2012, 2013).

fundamentals:

$$\text{Attack} \leq \theta + \xi. \tag{1}$$

The attack size is the share of agents that attack, and thus is always between 0 and 1. Higher realizations of fundamentals allow to defeat larger attacks, and thus correspond to stronger governments. For an example of a currency attack, the fundamentals can characterize the amount of foreign reserves, the government's desire to support its exchange rate, or the ability and willingness of foreign governments to help.

The fundamentals consist of θ, the part of the fundamentals that the government knows, and ξ, the part of the fundamentals that the government does not know. Fundamentals θ can be interpreted as the government's *type*. I assume that the prior distribution of θ is normal, $\mathcal{N}(\mu_{prior}, \sigma^2_{prior})$, with probability density function $p(\theta)$. I also assume that the prior distribution of ξ is normal, $\mathcal{N}(0, \sigma^2_\xi)$, with probability density function $\varphi(\xi)$. The expectation of ξ is 0, which implies that on average the government knows the realization of fundamentals. The only role of shock ξ is to select an equilibrium for an uncommitted government, as for any, even arbitrarily small $\sigma^2_\xi > 0$, there exists a finite number of equilibriums, whereas for $\sigma^2_\xi = 0$ there exists a continuum of equilibriums.[20]

The government gets utility 1 if the regime survives and 0 if it falls. I separately analyze a government that cannot commit to its policy and a government that can commit. An uncommitted government chooses the precision of its communication after it learns θ to save its regime with the highest probability. A committed government chooses which precision to use for each realization of θ before it learns θ. The goal of a committed government is to maximize the expected probability of regime survival.

Information structure

The government observes θ and communicates it to the agents. The government can use either more precise sentences or vaguer sentences. More precise sentences result in a

[20]The reason is that the action of a government do not affect the regime's survival for fundamentals below 0 or above 1 if $\sigma^2_\xi = 0$. At the same time, the government's action will be determined uniquely for any $\sigma^2_\xi > 0$. In the analysis of a committed government I will assume that $\sigma^2_\xi = 0$, as the policy of a committed government will be determined uniquely even if $\sigma^2_\xi = 0$.

lower dispersion of information obtained by the agents, whereas vaguer sentences result in higher dispersion. Each agent receives a private signal $x_i = \theta + \varepsilon_i$. Private noise ε_i is distributed normally with mean 0 and standard deviation $\sigma(\theta)$ chosen by the government based on the realization of the fundamentals.[21] That is $\varepsilon_i \sim \mathcal{N}(0, \sigma^2(\theta))$. I use $f_{\sigma(\theta)}(x)$ to denote the probability density function of normal distribution with standard deviation $\sigma(\theta)$ and mean 0 at x.

The government's policy is a measurable function $\sigma(\theta) : \mathbb{R} \to [P, I]$. I denote the set of available policies as $\mathcal{S} \equiv \{\sigma(\cdot) : \mathbb{R} \to [P, I]\}$. I use $\sigma(\cdot)$ to denote a government's policy, which is a function of fundamentals, and $\sigma(\theta)$ to denote a government's action, that is the value of the function at the realization of fundamentals θ. The lowest available standard deviation $P \in \mathbb{R}$ corresponds to precise communication and the highest available standard deviation $I \in \mathbb{R}$ corresponds to imprecise communication. I assume that $P > 0$, that is agents always receive at least slightly different impressions from the same press conference. I also assume that $I < +\infty$, that is agents always have at least some agreement about the government's message.

I assume that agents do not observe the government's action $\sigma(\theta)$. This assumption allows me to study the role of precision of communication in an analytically tractable framework easily comparable with the existing literature.[22] In reality, agents do not know the exact level of precision of communication used by the government, but may have some sense if the government is precise or vague. In Subsection 6.1, I show that my main results hold if agents receive several signals and thus can form precise estimates of government's action $\sigma(\theta)$.

I also assume that the government does not bias the information that it distributes. A government may choose not to bias its communication because of reputation costs, either of the government itself, or of the government officials who distribute the information.[23]

[21] The normality assumption streamlines the exposition, but my results hold under general assumptions on the noise distributions available to the government. See Subsection 6.1 for details.

[22] The assumption that agents perfectly observe $\sigma(\theta)$ is unreasonable in the current model, as it would allow the government to reveal the state of the world perfectly and with common knowledge by using different precision levels in different states of the world. In the model in the current section, agents can estimate the government's action $\sigma(\theta)$ by Bayes' rule as $\mathrm{E}[\sigma(\theta)|x_i, \sigma(\cdot)]$. Hence the information structure allows agents to know the precision of the government's communication with some noise.

[23] In the case of Mexico's crisis, the official statements regarding foreign reserves were usually

In the case of Mexico's crisis, for example, the government's officials did not exaggerate the level of reserves. At the same time, as I show in Subsection 6.1, my main results hold if the government biases the mean of the signals it distributes.

Agents

In the model, agents know the distributions of θ and ξ, and whether the government is committed or not. Agents do not know the realizations of θ and ξ, but use their private information x_i to estimate θ and make a binary decision whether to attack the regime. In the case of a currency attack, for example, an agent either sells short one unit of the currency or does nothing.[24] Agent i's strategy is $X_i \subset \mathbb{R}$, which is a set of signals for which the agent attacks. In what follows, we will often consider strategies of threshold form, which can be described with one number $x^* \in \mathbb{R}$ such that an agent attacks for signals $x_i \in (-\infty, x^*]$.

If an agent attacks and the regime falls, an agent receives payoff $\pi(\theta + \xi)$. The cost of attacking the regime is $c \in (0, 1)$ and does not depend on whether the attack is successful. Consequently, the agent's payoff is as follows:

$$U_{AGENT} = \begin{cases} \pi(\theta + \xi) - c & \text{if attacks and regime falls,} \\ -c & \text{if attacks and regime survives,} \\ 0 & \text{if does not attack.} \end{cases} \tag{2}$$

The agent's payoff $\pi(\theta + \xi)$ weakly decreases in the fundamentals $\theta + \xi$, that is the agent's payoff is lower if the government is stronger. In the case of a currency attack, for example, stronger fundamentals may correspond to lower subsequent devaluation and thus to a lower payoff. I assume that $c < \pi(\theta + \xi) < C$ for all values of $\theta + \xi$, where

made by Finance Minister Pedro Aspe and Central Bank Governor Miguel Mancera, who had a great reputation for their professionalism in Mexico and the international community. Since detailed data on Mexico's reserves during 1994 was disclosed in 1995, a lie about the volume of reserves during 1994 would have been revealed in 1995, and would have damaged the reputation of a lying official.

[24]The binary choice is a standard assumption in global games. See, for example, Morris and Shin (1998). In a currency attack model with risk-neutral agents and bounded trading positions, an agent either does nothing or sells the currency short up to the boundary of its position.

$C \in \mathbb{R}$ is a constant. The assumption that $c < \pi(\theta + \xi)$ reduces the number of cases I have to consider, but is not important for the results. The assumption that $\pi(\theta+\xi) < C$ guarantees that the behavior of the payoff function at $-\infty$ does not determine the behavior of agents and can be relaxed.

Timing

If the government can commit, the timing is as follows:

0. The government publicly announces and commits to a policy $\sigma(\cdot) \in \mathcal{S}$.
1. Nature chooses fundamentals $\theta \sim \mathcal{N}(\mu_{prior}, \sigma^2_{prior})$ and $\xi \sim \mathcal{N}(0, \sigma^2_{\xi})$.
2. The government learns θ and distributes private signals $x_i = \theta + \varepsilon_i$, $\varepsilon_i \sim \mathcal{N}\left(0, \sigma^2\left(\theta\right)\right)$.
3. Each agent i observes a private signal x_i and decides whether to attack.
4. An attack happens, the regime survival and payoffs are determined.

If the government cannot commit, then the timing is as above but without period 0. That is an uncommitted government chooses which precision of communication to use in period 2.

3 Equilibrium without Commitment

In this section I study the communication policy of an uncommitted government. I show that a weak uncommitted government communicates imprecisely and a strong uncommitted government communicates precisely. The policy of an uncommitted government may be a natural real world policy to consider, and I use it as a benchmark when I study the fully optimal policy of a committed government in Section 4.

I am looking for perfect Bayesian equilibriums in symmetric threshold strategies. In that equilibrium each type of the government chooses the precision of its communication to save its own regime with the highest probability. Each agent uses Bayes' rule to infer the distribution of fundamentals conditional on the agent's information and chooses whether to attack the government to maximize its expected payoff.

I focus on the threshold strategies for agents to simplify the exposition. Particularly, agents that receive signals $x_i \leq x^*$ attack, and agents that receive signals $x_i > x^*$ do not attack. Threshold strategies are intuitive as low signals correspond to weak fundamentals, and thus high expected payoff, whereas high signals correspond to strong fundamentals, and thus low expected payoff. I will show in Lemma 1 in Subsection 3.5 that an equilibrium in threshold strategies exists if the government uses levels of precision which are not too different.[25]

3.1 The probability of regime survival

To describe the government's problem, we have to derive the probability of regime survival conditional on the realization of fundamentals θ. The probability that the regime survives given θ, $\sigma(\theta)$, and x^* is as follows:

$$P^S(\theta, \sigma(\theta), x^*) = \Phi\left(\frac{\theta - \Phi\left(\frac{x^* - \theta}{\sigma(\theta)}\right)}{\sigma_\xi}\right), \tag{3}$$

where $\Phi(\cdot)$ is the cumulative distribution function of standard normal.

To derive expression (3), note that the attack size is $\Phi\left(\frac{x^*-\theta}{\sigma(\theta)}\right)$. Indeed, the government distributes signals centered at θ with dispersion $\sigma(\theta)$, and agents that receive signals below x^* attack. Thus, according to (1), the regime survives if:

$$\Phi\left(\frac{x^* - \theta}{\sigma(\theta)}\right) \leq \theta + \xi. \tag{4}$$

By computing the probaility that (4) holds, we obtain (3).

Expression (3) for the probability of regime survival is intuitive. The regime survives with higher probability if the fundamental θ is stronger. That happens because stronger governments survive larger attacks and because under stronger fundamentals agents receive higher signals and attack less. The regime survives with lower probability if the agents' attack threshold x^* is higher, that is if the agents attack more often. I discuss the impact of $\sigma(\theta)$ on the probability of regime survival in Subsection 3.4.

[25]That is for each $P > 0$ and $c \in (0, 1)$ we can find $r > 1$ such that as long as $I < rP$, there exists an equilibrium in which agents use threshold strategies.

3.2 Agent's behavior

An agent attacks the regime if the expected payoff from the attack is at least c. Indeed, this strategy maximizes the expected payoff in (2) because the attack cost is always equal to c. Thus an agent attacks if it receives a signal in the following set:

$$X^* = \{x_i : \text{ E[agent's payoff}|x_i, x^*, \sigma(\cdot)] \geq c\}. \tag{5}$$

Agents condition their expected payoff on their private information x_i, the strategy used by other agents x^*, and the government's communication policy $\sigma(\cdot)$.

Agents use Bayes' rule to infer the distribution of θ. According to the rule, the probability density function of θ conditional on the private signal x_i is as follows:

$$f(\theta|x_i) = \frac{p(\theta)f_{\sigma(\theta)}(x_i - \theta)}{\int\limits_{-\infty}^{+\infty} p(\theta)f_{\sigma(\theta)}(x_i - \theta)d\theta}. \tag{6}$$

The numerator of (6) is the product of the prior probability of θ and the probability of receiving signal x_i conditional on θ; the denominator is the normalization factor. Note that the conditional distribution of θ is neither normal nor continuous in θ.

The agents compute the expectation in (5) using the distribution of θ in (6) as follows:

$$\text{E[agent's payoff}|x_i, x^*, \sigma(\cdot)] = \iint\limits_{\Phi\left(\frac{x^*-\theta}{\sigma(\theta)}\right) > \theta+\xi} \pi(\theta+\xi)\varphi(\xi)f(\theta|x_i)d\xi d\theta. \tag{7}$$

The agents take the expected payoff over the range of fundamentals (θ, ξ) for which the regime falls. According to (4), this range of fundamentals is given by $\Phi\left(\frac{x^*-\theta}{\sigma(\theta)}\right) > \theta + \xi$.

3.3 Government's problem and equilibrium definition

An uncommitted government maximizes the probability that its regime survives separately for each realization of fundamentals θ:

$$P^S(\theta, \sigma(\theta), x^*) \longrightarrow \max_{\sigma(\theta)} \tag{8}$$

$$\text{s.t. E[agent's payoff}|x_i = x^*, x^*, \sigma(\cdot)] = c. \tag{9}$$

An uncommitted government ignores the affect of its policy on the determination of x^* in (9) because by changing $\sigma(\theta)$ for a single realization of θ the government cannot change x^*. Hence, even though each type of government understands how x^* is determined, it takes it as if it was exogenous when it chooses its action $\sigma(\theta)$, and hence ignores the effect of its action on strategy x^* used by agents.

A perfect Bayesian equilibrium in symmetric threshold strategies is defined below. In a threshold equilibrium, a set of signals X^* that lead agents to attack must have a threshold form $(-\infty, x^*]$.

Definition 1. ***Threshold equilibrium without commitment*** *is* $(\sigma^{NC}(\cdot), x^*)$ *such that:*

1. *The government chooses* $\sigma^{NC}(\theta)$ *to solve (8) for each* θ *and given* x^**.*

2. *Agents use (6) and (7) to compute their expected payoff given* x^* *and* $\sigma^{NC}(\cdot)$*.*

3. *Agents attack for signals* $x_i \leq x^*$*, where* x^* *is determined by (9).*

3.4 Communication policy of an uncommitted government

Proposition 1 characterizes the government's policy in a threshold equilibrium.

Proposition 1. *In any threshold equilibrium, an uncommitted government communicates imprecisely for weak fundamentals and precisely for strong fundamentals:*

$$\sigma^{NC}(\theta) = \begin{cases} I & \textit{if } \theta < x^*, \\ [P, I] & \textit{if } \theta = x^*, \\ P & \textit{if } \theta > x^*. \end{cases} \tag{10}$$

Proof. The proof follows immediately from (3) and (8). □

Proposition 1 is consistent with Proposition 7 in Edmond (2013). The main contribution of my paper, however, is the analysis of the policy of a committed government in Section 4 and of simple policies in Section 5.

To understand the result of Proposition 1, note that the government's problem (8) is equivalent to:

$$\Phi\left(\frac{x^* - \theta}{\sigma(\theta)}\right) \longrightarrow \min_{\sigma(\theta)}. \tag{11}$$

An uncommitted government minimizes the attack size for each realization of the fundamentals, as smaller attacks are less likely to overthrow the regime. It follows that for $\theta < x^*$, the government communicates imprecisely, and for $\theta > x^*$, the government communicates precisely. Intuitively, a weak government communicates imprecisely to leave an incorrect impression among some agents that it is strong and thereby avoid their attacks. A strong government communicates precisely to persuade most agents that it is strong and also experience a small attack.

The communication policy of the government of Mexico during the 1994 crisis is consistent with a policy of an uncommitted government described by Proposition 1. While the government officials were forthcoming about announcing the data on GDP growth and foreign reserves while the economy and reserves were growing, they started to avoid disclosing the data after a recession started at the second half of 1993 and reserves declined in March 1994. On March 16, 1994 *The New York Times* published an article titled "Mexico Slips Quietly into Recession," where quietly referred to that the Mexican government officials avoided acknowledging the unfavorable data on GDP growth, even though "The Treasury and the central bank have been known to make elaborate formal announcements when there is good news [. . .]." As Adler (1994) noted, "There appear to have been delays starting in January, 1994 in the release of customary government data (e.g. monetary statistics, production, employment, trade and capital flows figures)." The government's lack of transparency during 1994 was blamed for exacerbating the crisis, which is consistent with my model, as I discussed in the introduction and discuss in more detail in Section 5.

3.5 Existence of a threshold equilibrium without commitment

Lemma 1 provides a sufficient condition for the existence of a threshold equilibrium without commitment.

Lemma 1. *There exists $r > 1$ such that for any $I \leq rP$ there exists a threshold equilibrium without commitment $(\sigma^{NC}(\cdot), x^*)$. In the equilibrium, the agents' attack threshold x^* is a solution to (9) and the government's communication policy is given by (10).*

Proof. The proof is in the Appendix. □

According to Lemma 1, a threshold equilibrium exists if the government uses levels of precision that do not vary to a great extent. Note that as in any global game with an informative prior, multiple equilibriums are possible under some parameter values.[26] It is easy to see that a solution $x^* \in \mathbb{R}$ to (9) exists. Indeed, the expected payoff becomes larger than c for low enough signals x_i and close to 0 for high enough signals x_i.[27] Since the expected payoff changes continuously in private signals, the existence of x^* follows.

Lemma 1 assumes that I and P are not too different to guarantee that agents' strategies have a threshold form. To explain the intuition behind this result, consider $\sigma(\cdot) \equiv P$. Then the posterior distribution of θ conditional on x_i is $\mathcal{N}(\frac{P^2}{\sigma^2_{prior}+P^2}\mu_{prior} + \frac{\sigma^2_{prior}}{\sigma^2_{prior}+P^2}x_i, \frac{\sigma^2_{prior}P^2}{\sigma^2_{prior}+P^2})$. This distribution shifts to the right as x_i increases. Consequently, agents with higher signals expect their payoff to be lower because $\pi(\theta + \xi)$ decreases in θ. It follows from (5) that agents use threshold strategies.

4 Optimal Policy of a Committed Government

In this section, I characterize the outcome for a commitment and its optimal communication policy. The optimal policy allows the government to save its regime more often compared to the policy of an uncommitted government or a policy of constant precision of communication.

I examine the policy of a committed government for the following reasons. First, this policy allows a government to save its regime with the highest probability, and hence is the best possible policy. Second, the corresponding analysis demonstrates the government's incentives and helps to understand whether the best possible communication

[26]Global game models have multiple equilibriums if the precision of prior information is sufficiently higher than the precision of private information. See Hellwig (2002) or Morris and Shin (2003) for details.

[27]More formally, this is guaranteed by $I^2 - P^2 < \sigma^2_{prior}$, which follows from $I \leq rP$ for an appropriate choice of r.

policy can be implemented in practice. Finally, I use this analysis to propose simple communication policies in Section 5.

Subsection 4.1 formulates the government's problem, states the main result, and explains its intuition. Subsection 4.2 characterizes the threshold outcome under commitment. Subsection 4.3 describes properties of the optimal communication policy. Subsection 4.4 provides the conditions that guarantee the existence of the threshold outcome under commitment.

4.1 Government's problem, incentives and the main result

In this subsection, I describe the problem of a committed government. Then I show that a committed government communicates with a very different precision relative to an uncommitted government: precisely for very weak fundamentals and imprecisely for very strong fundamentals. This motivates a more detailed analysis of a policy enacted by a committed government in Subsections 4.2-4.4.

To present the main result first, we assume that the outcome under commitment exists and has a threshold form. That is agents with signals $x_i \leq x^*$ attack and agents with signals $x_i > x^*$ do not attack. I will provide conditions for the existence of the threshold outcome under commitment in Lemma 6.

4.1.1 Government's problem and incentives

A committed government maximizes the expected probability that the regime survives:

$$\int_{-\infty}^{+\infty} P^S(\theta, \sigma(\theta), x^*) p(\theta) d\theta \longrightarrow \max_{\sigma(\cdot)} \tag{12}$$

$$\text{s.t. E[agent's payoff}|x_i = x^*, x^*, \sigma(\cdot)] = c. \tag{13}$$

Note that we can express x^* from (13) as a function of $\sigma(\cdot)$, that is $x^* = x^*(\sigma(\cdot))$.[28] It follows, that a committed government considers how its policy affects the position of the threshold agent x^* when it chooses its policy $\sigma(\cdot)$.

A change in the government's action $\sigma(\widetilde{\theta})$ at fundamentals $\widetilde{\theta}$ has two effects. First,

[28] If there are several solutions x^* to (13), then the government chooses the smallest one.

it changes the survival probability of government $\widetilde{\theta}$. Second, it changes the beliefs of the agents and hence the position of the threshold agent x^*, thus affecting the probability of survival for governments of all other types. More formally, the derivative of the government's utility (12) with respect to a change in the government's action $\sigma(\widetilde{\theta})$ is proportional to:[29]

$$\underbrace{\frac{\partial P^S(\widetilde{\theta}, \sigma(\widetilde{\theta}), x^*)}{\partial \sigma(\widetilde{\theta})}}_{\substack{\text{change in the survival} \\ \text{probability for } \widetilde{\theta}}} + \underbrace{\int_{-\infty}^{+\infty} \frac{\partial P^S(\theta, \sigma(\theta), x^*)}{\partial x^*} p(\theta) d\theta}_{\substack{\text{change in the survival} \\ \text{probability for } \theta \neq \widetilde{\theta} \text{ w.r.t. } x^*}} \cdot \underbrace{\frac{\partial x^* (\sigma (\cdot))}{\partial \sigma(\widetilde{\theta})}}_{\text{change in } x^*} . \tag{14}$$

The first term in expression (14) shows that by changing its action at fundamentals $\widetilde{\theta}$, the government changes the probability of regime survival at those fundamentals. The first term affects the decision of both a committed and an uncommitted governments. The second term shows that a change in the government's action $\sigma(\widetilde{\theta})$, also changes x^*, and hence the probability of survival of all governments $\theta \neq \widetilde{\theta}$. Only a committed government considers the second term, and hence the impact of its action on agents' strategy. Consequently only a committed government can make a fully optimal decision regarding its communication policy.

Assumption 1. The government perfectly knows the realization of fundamental, that is $\sigma_\xi^2 = 0$.

For the rest of the paper I assume that Assumption 1 holds. Under Assumption 1, the regime survives for a realization of fundamentals θ if

$$\int_{X^*} f_{\sigma(\theta)}(x_i - \theta) dx_i \leq \theta, \tag{15}$$

where the left-hand side is the attack size and X^* is a set of signals that trigger an attack by the agents. Consequently, for any realization of the fundamentals θ, the regime either

[29] More formally, consider $\widetilde{\theta} \in \mathbb{R}$ such that $\sigma(\cdot)$ is continuous at $\widetilde{\theta}$. If the government increases $\sigma(\widetilde{\theta})$ by δ in a neighborhood of $\widetilde{\theta}$ of length ϵ, then the government's utility changes by the product of $\epsilon\delta$ and the amount in expression (14).

survives or falls; that is, $P^S(\theta, \sigma(\theta), x^*) \in \{0, 1\}$. This property allows me to simplify the analysis under commitment without loss of intuition.

4.1.2 Definition of the outcome under commitment

The threshold outcome under commitment consists of a government's policy $\sigma^C(\cdot) \in \mathcal{S}$, an agents' attack threshold $x^* \in \mathbb{R}$, and a regime survival threshold $\theta^* \in \mathbb{R}$. The regime survival threshold θ^* is such that the regime survives for fundamentals $\theta \geq \theta^*$ and falls for fundamentals $\theta < \theta^*$; that is, strong regimes survive and weak regimes fall.[30]

Definition 2. ***Threshold outcome under commitment*** *is* $(\sigma^C(\cdot), x^*, \theta^*)$ *such that:*

1. *The government chooses* $\sigma^C(\cdot)$ *to solve (12) and (13).*
2. *Agents use Bayes' rule to compute their expected payoff given* θ^* *and* $\sigma^C(\cdot)$.
3. *The set of signals* x_i *for which an agent chooses to attack is* $X^* = (-\infty, x^*]$.
4. *The set of fundamentals for which the regime survives is given by (15) and is equal to* $[\theta^*, +\infty)$.

I will show in Lemma 6 in Subsection 4.4 that the threshold outcome under commitment exists if the government uses levels of precision that are not too different.[31] To streamline the exposition of the main results, in Subsections 4.1–4.3 I assume that the conditions of Lemma 6 are satisfied, and hence the outcome under commitment exists and has a threshold form.

4.1.3 The main result

Proposition 2, the main result of the paper, establishes that for weak and strong fundamentals, a committed government communicates with the opposite precision relative to that of an uncommitted government.

[30] The role of θ^* is the same as the role of the probability of regime survival function $P^S(\theta, \sigma(\theta), x^*)$. That is, $P^S(\theta, \sigma(\theta), x^*) = 0$ for $\theta < \theta^*$ and $P^S(\theta, \sigma(\theta), x^*) = 1$ for $\theta \geq \theta^*$. It appears to be convenient to include θ^* in the definition of the outcome under commitment.

[31] That is for each $P > 0$ and $c \in (0, 1)$ we can find $r > 1$ such that as long as $I < rP$, the outcome under commitment has a threshold form.

Proposition 2. *A committed government communicates precisely when fundamentals are weak and imprecisely when fundamentals are strong. That is there exist two thresholds* $\underline{\theta}, \overline{\theta} \in \mathbb{R}$ *such that:*

$$\sigma^C(\theta) = \begin{cases} P & \text{if } \theta \leq \underline{\theta}, \\ I & \text{if } \theta \geq \overline{\theta}. \end{cases} \tag{16}$$

Proof. The proof is in the Appendix. □

To explain the intuition behind Proposition 2, let us examine (14) in more detail.[32] For weak and strong fundamentals, the government's policy does not affect the probability of regime survival, and hence the first term in (14) disappears. Indeed, if $\widetilde{\theta} < 0$ or $\widetilde{\theta} \geq 1$, the regime either falls or survives for any government's policy. Thus for weak and strong fundamentals, the government can affect only utility coming from the second term of (14). That is, a government of type $\widetilde{\theta}$ tries to increase the survival chances of governments $\theta \neq \widetilde{\theta}$ by minimizing x^*. A problem of a committed government for weak and strong fundamentals is as follows:[33]

$$x^* \longrightarrow \min_{\sigma(\widetilde{\theta})} \tag{17}$$

$$\text{s.t. } \mathrm{E}[\text{agent's payoff} | x_i = x^*, x^*, \sigma(\cdot)] = c. \tag{18}$$

Let us contrast the problem of an uncommitted government (11) with a problem of a committed government (17-18) when fundamentals are weak or strong. An uncommitted government minimizes the size of the attack against itself to maximize its own survival chances. A committed government minimizes x^* to help governments of other types to save the regime.

To lower the position of the threshold agent x^*, the government communicates precisely for very weak fundamentals and imprecisely for very strong fundamentals. If a government communicates precisely when it is very weak, it distributes only weak sig-

[32]Under Assumption 1 the derivative of $P^S(\cdot)$ with respect to $\sigma(\widetilde{\theta})$ is understood to belong to $\{-\infty, 0, +\infty\}$.

[33]I abuse notation here. The problem should be understood as with respect to changing the government's policy in a neighborhood of $\widetilde{\theta}$.

nals. Consequently, agents that receive signals near the threshold x^* realize that their signals cannot come from a very weak government, and thus must come from stronger governments, attacking which is unprofitable. If a government communicates imprecisely when it is very strong, it distributes a wide range of signals. Therefore, agents that receive signals near the threshold x^* realize that their signals likely come from a very strong government. More intuitively, by revealing its weakness and concealing its strength, a committed government ensures that agents that receive signals near the threshold x^*, and hence are pivotal for the determination of whether the regime survives, realize that the regime is not very weak but can be very strong.

4.2 Characterization of the threshold outcome under commitment

I characterize the outcome under commitment in three steps. First, I determine the position of the threshold agent x^* as a function of θ^* and $\sigma^C(\cdot)$. Second, I show that the government saves its regime if that is possible given x^* and determine θ^* as a function of x^* and $\sigma^C(\cdot)$. Third, I show how the government optimally manipulates agents' beliefs. This allows me to determine the government's communication policy $\sigma^C(\cdot)$ given x^*, θ^*. Together, these three relations determine the outcome under commitment.

4.2.1 Agents' beliefs and determination of x^*

In this subsection, I explain how agents compute their expected payoff. This allows to determine x^* as a function of θ^*.

Agents use Bayes' rule to compute the expected payoff given the government's policy $\sigma(\cdot) \in \mathcal{S}$ and the regime survival threshold θ^*. Under the assumption that $\sigma_\xi^2 = 0$, the expected payoff in a threshold outcome can be obtained from (6) and (7) as follows:

$$\mathrm{E}[\text{agent's payoff}|x_i, \theta^*, \sigma(\cdot)] = \frac{\int\limits_{-\infty}^{\theta^*} \pi(\theta) p(\theta) f_{\sigma(\theta)}(x_i - \theta) d\theta}{\int\limits_{-\infty}^{+\infty} p(\theta) f_{\sigma(\theta)}(x_i - \theta) d\theta}. \tag{19}$$

To concentrate on studying the threshold outcome, I must find sufficient conditions that guarantee that the agents use threshold strategies.

Lemma 2 establishes that the agents' attack region has a threshold form if I is sufficiently close to P and provides an equation that determines x^* as a function of θ^*.

Lemma 2. *Let $P > 0$. Then there exists $r > 1$ such that for any $I \leq rP$, any $\sigma(\theta)$: $\mathbb{R} \to [P, I]$, and any $\theta^* \in [0,1]$ the agent's attack region has a threshold form $(-\infty, x^*]$. The threshold x^* is the unique solution to*

$$\frac{\int\limits_{-\infty}^{\theta^*} \pi(\theta)p(\theta)f_{\sigma(\theta)}(x^* - \theta)d\theta}{\int\limits_{-\infty}^{+\infty} p(\theta)f_{\sigma(\theta)}(x^* - \theta)d\theta} = c. \tag{20}$$

Proof. The proof is in the Appendix. □

Assumptions of Lemma 2 will be enough to establish the existence of the threshold outcome under commitment in Lemma 6. As I mentioned before, in Subsections 4.1–4.3 I assume that conditions of Lemma 6 (or alternatively of Lemma 2) are satisfied, that is $I \leq rP$.

4.2.2 The government saves viable regimes, determination of θ^*

In this subsection, I show that, *given* x^*, the government chooses to save regimes that can be saved. This property allows me to determine the regime's survival threshold θ^* for any agents' attack threshold x^*.

I use an argument by contradiction to explain that the government saves regimes that can be saved given x^*. Pick a value of fundamentals θ such that the regime falls under the current communication policy, but the government can change $\sigma(\theta)$ to save the regime. This change in $\sigma(\theta)$ leads to a direct utility gain since now the regime survives for more fundamentals. Moreover, agents realize that their expected payoff becomes smaller and attack less. It follows that after the deviation, the regime survives under more fundamentals.

Given x^*, the regime survival region, that is the set of fundamentals for which the regime survives, has a threshold form. Indeed, the attack size $\Phi\left(\frac{x^*-\theta}{\sigma(\theta)}\right)$ decreases in θ for a fixed $\sigma(\theta)$. Hence the government that saves its regime for fundamentals θ', can save the regime for any stronger value of fundamentals θ by using $\sigma(\theta) = \sigma(\theta')$.

I then determine the regime survival threshold θ^*, which is the smallest value of fundamentals for which the regime survives. The attack size $\Phi\left(\frac{x^*-\theta^*}{\sigma(\theta^*)}\right)$ at the regime survival threshold must be equal to the regime survival threshold θ^*. Indeed, if the attack size was smaller than θ^*, the government would be able to save the regimes in a left neighborhood of θ^*. Also note that the government must minimize the attack size at θ^*. Indeed, if the government was able to decrease the attack size at θ^*, then the attack size would be lower than θ^* and the government would be able to save the regimes in a left neighborhood of θ^*. This argument allows me to characterize $\sigma^C(\theta^*)$ as a function of x^*:[34]

$$\sigma^C(\theta^*) = \begin{cases} P & \text{if } \theta^* < x^*, \\ \Sigma & \text{if } \theta^* = x^*, \\ I & \text{if } \theta^* > x^*. \end{cases} \tag{21}$$

Lemma 3 summarizes this discussion and expresses θ^* as a function of x^*.

Lemma 3. *Given x^*, the government saves its regime, if possible. The regime survival region is $[\theta^*, +\infty)$, where θ^* is the unique solution to*

$$\Phi\left(\frac{x^* - \theta^*}{\sigma^C(\theta^*)}\right) = \theta^*, \tag{22}$$

and $\sigma^C(\theta^*)$ is given by (21).

Proof. The proof is in the Appendix. □

4.2.3 Optimal belief manipulation, determination of $\sigma^C(\cdot)$

In this subsection, I show how the government manipulates agents' beliefs. This allows me to construct the optimal policy $\sigma^C(\cdot)$ given thresholds x^* and θ^*.

The government manipulates beliefs optimally if it cannot decrease the position of the threshold agent x^*. To prove this claim by contradiction, assume that by deviating on a subset of fundamentals the government could make the threshold agent x^* believe

[34] Note that the policy of a committed government at fundamentals θ^*, given by (21), is the same as the policy of an uncommitted government at any realization of θ, given by (10).

that the expected payoff is less than c. Then the position of the threshold agent would move to $x^{**} < x^*$. But then the government would be able to save the regime for a larger set of fundamentals according to (22).

The expected payoff of the threshold agent x^* can be written as follows:

$$\text{E}[\text{agent's payoff}|x^*, \theta^*, \sigma^*(\cdot)] = \frac{\int\limits_{-\infty}^{\theta^*} \pi(\theta)p(\theta)f_{\sigma(\theta)}(x^* - \theta)d\theta}{\int\limits_{-\infty}^{\theta^*} p(\theta)f_{\sigma(\theta)}(x^* - \theta)d\theta + \int\limits_{\theta^*}^{+\infty} p(\theta)f_{\sigma(\theta)}(x^* - \theta)d\theta}. \quad (23)$$

Signals that the threshold agent receives from $\theta < \theta^*$ correspond to fundamentals that produce payoff $\pi(\theta) > c$. When an agent receives more signals from $\theta < \theta^*$, it becomes more certain that the payoff from the attack will be higher than c. The government wants to minimize the amount of these pessimistic signals, given by

$$\int\limits_{-\infty}^{\theta^*} p(\theta)f_{\sigma(\theta)}(x^* - \theta)d\theta. \quad (24)$$

Signals that the threshold agent receives from $\theta \geq \theta^*$ correspond to regimes under which attacking brings no payoff. When an agent receives more signals from $\theta \geq \theta^*$, it becomes more certain that the payoff from the attack will be lower than c. The government wants to maximize the amount of these optimistic signals, given by

$$\int\limits_{\theta^*}^{+\infty} p(\theta)f_{\sigma(\theta)}(x^* - \theta)d\theta. \quad (25)$$

The government chooses $\sigma^C(\theta)$ to solve the corresponding minimization and maximization problems separately for each realization of fundamentals θ.

More intuitively, the government tries to lower the position of the threshold agent to make agents less aggressive. The government accomplishes this by redistributing the optimistic beliefs towards the threshold agent, to make the agent more optimistic about the survival chances of the regime, and thus move its position to the left. On the one hand, the redistribution requires insulating the threshold agent from signals that are more likely under weak fundamentals. The mass of pessimistic beliefs (24) held by the threshold agent must be minimized. On the other hand, this redistribution requires sending more signals to the threshold agent from fundamentals under which the regime survives. The mass of optimistic beliefs (25) held by the threshold agent must be maximized.

Proposition 3 demonstrates how the government constructs the optimal policy $\sigma^C(\cdot)$ given x^* and θ^*.

Proposition 3. *Given x^* and θ^*, the government's policy $\sigma^C(\cdot)$ satisfies the following conditions:*

1. *If $\theta < \theta^*$, then $\sigma^C(\theta)$ solves*

$$\min_{\sigma \in [P,I]} f_\sigma(x^* - \theta). \tag{26}$$

2. *If $\theta \geq \theta^*$, then $\sigma^C(\theta)$ solves*

$$\max_{\sigma \in [P,I]} f_\sigma(x^* - \theta) \tag{27}$$

$$s.t.\ \Phi\left(\frac{x^* - \theta}{\sigma}\right) \leq \theta. \tag{28}$$

Proof. The proof is in the Appendix. □

I next illustrate how the optimal policy $\sigma^C(\cdot)$ affects agents' beliefs and attack sizes. Let $c = 0.36$, $\pi(\theta) \equiv 1$, and $p(\theta) \equiv 1$.[35] When the government communicates with the constant precision of $P = 0.1$, the corresponding regime survival and the agents' attack thresholds are $\theta^*_C = 0.64$ and $x^*_C = 0.67$. When the government uses policy $\sigma^C(\cdot)$ with $[P, I] = [0.1, 0.5]$, the corresponding thresholds are $\theta^* = 0.1$ and $x^* = -0.03$.

The top panel of Figure 1 on page 45 shows agents' expected payoffs as a function of signal x_i in the case of the constant precision of communication $\sigma(\cdot) \equiv P$ and in the case of the optimal communication policy $\sigma^C(\cdot)$. Under policy $\sigma^C(\cdot)$, agents receiving intermediate signals are more confident that the regime will survive. At the same time, agents receiving strong signals are more confident that the regime will fall. Fortunately, the later feature does not induce larger attacks. Indeed, agents receiving higher signals

[35] $p(\theta) \equiv 1$ means that the prior distribution of θ is improper and is uniform on the real line.

still believe that the regime will fall with probability less than $c = 0.36$ (the horizontal dotted line), and thus do not attack.

The bottom panel of Figure 1 shows attack sizes as a function of fundamentals in the case of the constant precision of communication $\sigma(\cdot) \equiv P$ and in the case of the optimal communication policy $\sigma^C(\cdot)$. Under policy $\sigma^C(\cdot)$, the attacks are smaller exactly where it is important—for the intermediate values of fundamentals. At the same time, the government communicates imprecisely when fundamentals are strong, provoking larger attacks. Fortunately, the attacks are smaller than the regime strength (the 45-degree dotted line) and thus do not force the government to abandon its regime.

4.3 Properties of the optimal communication policy

I start this subsection by explaining how to construct the optimal policy $\sigma^C(\cdot)$ according to Proposition 3. Then I show that the optimal policy is non-monotone. Finally I provide an example of the optimal policy.

To construct the optimal policy $\sigma^C(\cdot)$, define $\sigma_L(\theta)$ as a solution to (26) and $\sigma_H(\theta)$ as a solution to (27):

$$\sigma_L(\theta) \equiv \underset{\sigma \in [P,I]}{\operatorname{argmin}}\, f_\sigma(x^* - \theta), \tag{29}$$

$$\sigma_H(\theta) \equiv \underset{\sigma \in [P,I]}{\operatorname{argmax}}\, f_\sigma(x^* - \theta). \tag{30}$$

Lemma 4 describes shapes of $\sigma_L(\theta)$ and $\sigma_H(\theta)$.[36]

Lemma 4. *1. Policy $\sigma_L(\theta)$ is $\bigcap$-shaped and symmetric around x^*.*

2. Policy $\sigma_H(\theta)$ is $\bigcup$-shaped and symmetric around x^*.

Proof. The proof is in the Appendix. □

Policy $\sigma_L(\theta)$ affects posterior beliefs of the threshold agent x^* in the weakest possible way and hence is $\bigcap$-shaped and symmetric around x^*. Indeed, for fundamentals far away from x^*, the government communicates precisely to isolate agent x^* from the signals. For fundamentals near x^*, the government communicates imprecisely to avoid persuading agent x^* that the fundamentals are in that range. Policy $\sigma_H(\theta)$ affects posterior

[36] Analytical expressions for $\sigma_L(\cdot)$ and $\sigma_H(\cdot)$ are in the Appendix.

beliefs of the threshold agent x^* in the strongest possible way and hence is $\bigcup$-shaped and symmetric around x^*. Indeed, for fundamentals far away from x^*, the government communicates imprecisely to send at least some signals to agent x^*. For fundamentals near x^*, the government communicates precisely to send as many signals to agent x^* as possible.

The optimal policy $\sigma^C(\cdot)$ consists of $\sigma_L(\theta)$ below θ^*, then is determined by constraint (28) in a right neighborhood of θ^*, and is given by $\sigma_H(\theta)$ for larger fundamentals. Communication policy $\sigma^C(\cdot)$ is non-monotone as a combination of a $\bigcap$-shaped policy below θ^* and a $\bigcup$-shaped policy above θ^*. This non-monotonicity result shows one more difference between policies of a committed and an uncommitted governments: the optimal policy $\sigma^C(\cdot)$ is not monotone, whereas according to Proposition 1 the policy of an uncommitted government $\sigma^{NC}(\cdot)$ is monotone.

Corollary 1. *The government's policy $\sigma^C(\cdot)$ is not monotone.*

Proof. The proof is in the Appendix. □

Figure 2 on page 46 illustrates how to construct the optimal policy $\sigma^C(\cdot)$ according to Proposition 3. The parameter values are the same as in the example of Figure 1. The top panel of Figure 2 shows policies $\sigma_H(\theta)$ and $\sigma_L(\theta)$ for $x^* = -0.03$ and $[P, I] = [0.1, 0.5]$. Consistently with Lemma 4, function $\sigma_H(\cdot)$ is $\bigcup$-shaped and $\sigma_L(\cdot)$ is $\bigcap$-shaped and both are centered at x^*.[37] To construct the optimal policy I combine $\sigma_L(\cdot)$ for $\theta < \theta^* = 0.1$, a policy determined by (28) in a right neighborhood of θ^*, and $\sigma_H(\cdot)$ after that. The combination of these policies is shown at the bottom panel of Figure 2.

The policy of Figure 2 satisfies the following properties. First, for weak fundamentals the government is precise and for strong fundamentals the government is imprecise, consistently with Proposition 2. Second, the government uses precision that minimizes the attack size in the right neighborhood of θ^*, that is constraint (28) is binding, consistently with Lemma 3. Finally, the optimal communication policy is non-monotone, consistently with Corollary 1.

[37] Moreover $\sigma_L(\cdot)$ takes only two values: P and I. The reason is that the probability density function $f_\sigma(t)$ is quasi-concave in σ and thus can be minimized only at the boundaries of $[P, I]$.

4.4 Existence of the outcome under commitment

This subsection establishes technical results which are necessary for a rigorous analysis of the outcome under commitment. I advise a reader who is not interested in these technical details to proceed to Section 5 without any loss of intuition.

Let $\Theta^* \subset \mathbb{R}$ be a regime survival region, that is, a set of fundamentals for which the regime survives. Agents infer the conditional distribution of θ using (6). The expression for expected payoff (7) must be modified to consider Assumption 1 and a general form of Θ^* and is as follows:

$$\text{E[agent's payoff}|x_i, \Theta^*, \sigma(\cdot)] = \frac{\int\limits_{\mathbb{R}\setminus\Theta^*} \pi(\theta)p(\theta)f_{\sigma(\theta)}(x_i - \theta)d\theta}{\int\limits_{\mathbb{R}} p(\theta)f_{\sigma(\theta)}(x_i - \theta)d\theta}. \tag{31}$$

The expected payoff in (31) is a continuous function of x_i for any $\sigma(\cdot) \in \mathcal{S}$.[38]

For each regime survival region Θ^* and communication policy $\sigma(\cdot)$, denote the set of signals that trigger an attack as follows:

$$X(\Theta^*, \sigma(\cdot)) = \{x_i : \text{E[agent's payoff}|x_i, \Theta^*, \sigma(\cdot)] \geq c\}. \tag{32}$$

Indeed, since the attack costs c, an agent with signal x_i attacks the regime if and only if the expected payoff is at least c. Function $X(\Theta^*, \sigma(\cdot))$ decreases; that is, if $\Theta_1^* \subset \Theta_2^*$, then $X(\Theta_1^*, \sigma(\cdot)) \supset X(\Theta_2^*, \sigma(\cdot))$. Intuitively, agents attack less if the regime survives more often.

For each agents' attack region X^* and communication policy $\sigma(\cdot)$, denote the set of fundamentals for which the regime survives as follows:

$$\Theta(X^*, \sigma(\cdot)) = \{\theta : \int\limits_{X^*} f_{\sigma(\theta)}(x_i - \theta)dx_i \leq \theta\}. \tag{33}$$

[38]A proof that the integral in the denominator of (31) is continuous in x_i is as follows. Function $f_{\sigma(\theta)}(x_i - \theta) : \mathbb{R}^2 \to \mathbb{R}$ is continuous in x_i and measurable in θ. Moreover $f_{\sigma(\theta)}(x_i - \theta)$ is locally uniformly integrably bounded because it is bounded and $p(\theta)$ is a finite measure. Consequently, the integral in the denominator of (31) is continuous. The proof that the integral in the numerator is continuous is the same. See Border (2002) for a definition of the local uniform integrability and sufficient conditions for functions defined with integrals to be continuous.

Indeed, the regime survives if and only if the attack size is smaller than the value of the fundamentals. Note that $\Theta(X^*, \sigma(\cdot))$ does not contain $\theta < 0$ and contains all $\theta \geq 1$. Function $\Theta(X^*, \sigma(\cdot))$ decreases; that is, if $X_1^* \subset X_2^*$, then $\Theta(X_1^*, \sigma(\cdot)) \supset \Theta(X_2^*, \sigma(\cdot))$. Intuitively, the government defends its regime more often if agents attack less.

I define the outcome conditional on communication policy $\sigma(\cdot) \in \mathcal{S}$. I consider only symmetric outcomes, so that all agents have the same strategies and beliefs.

Definition 3. ***Outcome conditional on*** $\sigma(\cdot)$ *is* $(\sigma(\cdot), X^*, \Theta^*)$ *such that* $X^* = X(\Theta^*, \sigma(\cdot))$ *and* $\Theta^* = \Theta(X^*, \sigma(\cdot))$.

Now I can define the outcome under commitment.

Definition 4. ***Outcome under commitment*** *is an outcome* $(\sigma(\cdot), X^*, \Theta^*)$ *conditional on* $\sigma(\cdot)$ *such that* $\sigma(\cdot)$ *is chosen by the government to solve:*

$$\int_{\Theta^*} p(\theta) d\theta \to \max_{\sigma(\cdot)} \tag{34}$$

$$s.t.\ \Theta^* = \Theta(X(\Theta^*, \sigma(\cdot)), \sigma(\cdot)) \tag{35}$$

Note that problem in (34-35) is similar to that in (12-13), but allows for a possibility of non-threshold outcomes.

Definition 5. ***Threshold outcome*** *is an outcome* $(\sigma(\cdot), X^*, \Theta^*)$ *with* $X^* = (-\infty, x^*]$ *and* $\Theta^* = [\theta^*, +\infty)$.

Lemma 5 guarantees that for any communication policy $\sigma(\cdot) \in \mathcal{S}$ there exists an outcome conditional on that policy, and the payoff of the government is well defined for all policies.[39]

Lemma 5. *For any* $\sigma(\cdot) \in \mathcal{S}$, *there exists an outcome* $(\sigma(\cdot), X^*, \Theta^*)$ *conditional on* $\sigma(\cdot)$. *Moreover, the outcome can be chosen such that for any* Θ_α: $\Theta_\alpha \subset \Theta\left(X\left(\Theta_\alpha, \sigma(\cdot)\right), \sigma(\cdot)\right)$, $\Theta_\alpha \subset \Theta^*$ *and* $X^* \subset X(\Theta_\alpha, \sigma(\cdot))$.

Proof. I sketch a proof under the assumption that fundamentals are distributed discretely on the real line. This assumption allows me to provide a simple proof in the

[39]That is, there exists a conditional outcome with a regime survival region Θ^*, such that for any other conditional outcome with a regime survival region $\Theta^{*\prime}$, $\Theta^* \supset \Theta^{*\prime}$.

main text. A detailed proof for the fundamentals distributed continuously on the real line is in the Appendix.

For each $\sigma(\cdot) \in \mathcal{S}$, function $\Theta X(\Theta^*) \equiv \Theta(X(\Theta^*, \sigma(\cdot)), \sigma(\cdot))$ increases in Θ^*. The set of subsets of fundamentals is a complete lattice.[40] By the Tarski fixed point theorem (see, e.g., Ok, 2010), there exists the greatest fixed point $\Theta^* = \Theta X(\Theta^*)$. The corresponding agents' attack region is $X^* \equiv X(\Theta^*, \sigma(\cdot))$. $\square$

Lemma 6 establishes that under additional assumptions, the outcome under commitment exists and has a threshold form.

Lemma 6. *Let $P > 0$. Then there exists $r > 1$ such that for any $I < rP$ there exists an outcome under commitment $(\sigma^*(\cdot), X^*, \Theta^*)$ and it has a threshold form. That is $X^* = (-\infty, x^*]$ and $\Theta^* = [\theta^*, +\infty)$.*

Proof. I sketch a proof under the assumption that fundamentals are distributed discretely on the real line. This assumption allows me to provide a simple proof in the main text. A detailed proof for the fundamentals distributed continuously on the real line is in the Appendix.

I start by constructing a candidate for the outcome under commitment. Let θ^* be the smallest regime that survives in at least one conditional outcome. Denote the corresponding outcome as $(\sigma_0(\cdot), X_0, \Theta_0)$. Note that $\min\{\Theta_0\} = \theta^*$. We can use $(\sigma_0(\cdot), X_0, \Theta_0)$ to construct a new outcome with $\Theta^* = [\theta^*, +\infty)$. Note that the government must abandon its regime for fundamentals outside $\Theta^* = [\theta^*, +\infty)$ because θ^* is the minimum of all regimes that can be saved. Define a candidate for the optimal policy as follows:

$$\sigma^*(\theta) = \begin{cases} \sigma_0(\theta^*) & \text{if } \theta \in [\theta^*, +\infty) \setminus \Theta_0, \\ \sigma_0(\theta) & \text{otherwise.} \end{cases} \tag{36}$$

I can establish that $(\sigma^*(\cdot), X(\Theta^*, \sigma^*(\cdot)), \Theta^*)$ is an outcome under commitment. First, note that for I sufficiently close to P, the agents' attack region has a threshold form: $X(\Theta^*, \sigma^*(\cdot)) = (-\infty, x^*]$ for some $x^* \in \mathbb{R}$. The intuition for that result was discussed in

[40] $\Theta X(\cdot)$ is defined on a set of measurable subsets of the fundamentals. When the fundamentals are distributed continuously on the real line, the set of all measurable subsets of the fundamentals is no longer a complete lattice. I modify the proof accordingly in the Appendix.

Subsection 3.5, and the result was stated in Lemma 2. Also note that agents become less aggressive if the government uses $\sigma^*(\cdot)$: $X(\Theta^*, \sigma^*(\cdot)) \subset X_0$. Indeed, the agents receive the same signals from $\theta \in (-\infty, \theta^*] \cup \Theta_0$, but signals $\theta \in (\theta^*, +\infty) \setminus \Theta_0$ now correspond to surviving regimes, and hence to 0 payoff.

Next, I verify that the regime survives for all fundamentals in $[\theta^*, +\infty)$. First, the government successfully defeats any attack that occurs for $\theta \in \Theta_0$, including $\theta = \theta^*$, because the agents are less aggressive. Second, the government defeats attacks for fundamentals $\theta \in [\theta^*, +\infty) \setminus \Theta_0$ because these fundamentals are stronger than θ^* and the government uses the same precision as in θ^* in them. More formally, for $\theta \in [\theta^*, +\infty) \setminus \Theta_0$: $\Phi\left(\frac{x^*-\theta}{\sigma^*(\theta)}\right) = \Phi\left(\frac{x^*-\theta}{\sigma_0(\theta^*)}\right) \leq \Phi\left(\frac{x^*-\theta^*}{\sigma_0(\theta^*)}\right) \leq \theta^* \leq \theta.$ □

5 Simple Policies in Theory and Practice

In this section, I show that an uncommitted government can benefit from a rule that prevents it from manipulating the quality of information it distributes. I show that a simple rule of constant precision of communication can bring a government's policy closer to a fully optimal policy of a committed government. The analysis of this section is consistent with a policy response of international community to the Mexico's 1994 crisis.

The analysis is motivated by the fact that it may be difficult to implement the optimal policy of a committed government. First, the optimal policy requires commitment, and hence may be infeasible for some impatient governments. Second, the optimal policy has a complex non-monotone structure, and hence its implementation may require the expertise not available to all governments. A government may be able implement a simple policy of constant precision of communication even if a fully optimal policy is infeasible.

5.1 A policy of constant precision can benefit an uncommitted government

Corollary 2 provides very narrow conditions under which an uncommitted government gets the same utility as a government that communicates with constant precision.

Corollary 2. *Let $\sigma_\xi^2 = 0$, $\pi(\cdot) \equiv 1$, and $p(\cdot) \equiv 1$. Then in the unique threshold equilibrium without commitment, the government uses policy as in (10) and $\theta^* = 1 - c$.*

Proof. The proof is in the Appendix. □

Under the restrictive conditions of Corollary 2, an uncommitted government gets the same outcome for any $P < I$, and also for any $P = I \in \mathbb{R}$. Corollary 2 assumes that the prior distribution of fundamentals is uniform on the real line and that agent's payoff $\pi(\theta)$ does not depend on θ. In reality, however, these two assumption likely fail, hence the conclusion of Corollary 2 does not hold.

Proposition 4 states that a government that communicates with constant precision can save its regime more often than an uncommitted government.

Proposition 4. *An uncommitted government can benefit from a rule that enforces constant precision of communication.*

Proof. A proof by example is provided below. □

I prove Proposition 4 by providing an example. In the example, I necessarily relax the assumptions of Corollary 2. I assume that θ is distributed as $\mathcal{N}(\mu, 1)$. I take $\sigma_\xi^2 = 0$, $[P, I] = [0.1, 0.5]$, $c = 0.5$. I choose $\pi(\theta) = \max\{1 - b \cdot \theta, 1\}$. Parameter μ determines the mean of the fundamentals and parameter b determines how important are weak fundamentals in affecting agents' payoffs. I consider $b \geq 0$, and hence an agent's payoff function weakly decreases in fundamentals.

For various values of μ and b, I compute the expected probabilities of regime survival under three policies: precise communication $\sigma(\cdot) \equiv P$, imprecise communication $\sigma(\cdot) \equiv I$, and a policy of an uncommitted government. Figure 3 on page 47 shows which of the three policies maximizes the government's utility for $\mu \in [0, 1.2]$ and $b \in [0, 4]$. First, the figure shows that when fundamentals are weak on average (μ is low), the best policy is $\sigma(\cdot) \equiv P$, and when fundamentals are strong on average (μ is high), the best policy is $\sigma(\cdot) \equiv I$. Intuitively, the simple rule of constant precision of communication approximates the policy used by a committed government, which communicates precisely for weak fundamentals and imprecisely for strong fundamentals.[41]

[41] This results is consistent with Bannier and Heinemann (2005), who show that if the prior distribution of fundamentals has low mean, then the regime falls less often if the dispersion of private signals is low whereas if the prior distribution of fundamentals has high mean, then the regime falls less often if the dispersion of private signals is high. The point of my analysis is to compare these constant dispersions of private signals with a policy of an uncommitted government, which varies dispersion with the state of the world.

Figure 3 also shows that precise communication $\sigma(\cdot) \equiv P$ is preferred when an agent's payoff is higher for lower values of fundamentals (that is b is larger).[42] The reason is that by being imprecise for weak fundamentals, a government increases the attack threshold of the agents, thus incentivizing the agents to be more aggressive. If a weak government communicates imprecisely, as an uncommitted government does, it sends both weak and intermediate signals to agents. Consequently, agents with signals near the attack threshold realize that there is a good chance that the government is very weak, and overthrowing its regime could bring a very high payoff (b is positive). Thus, agents become more aggressive, attack more, and thereby reduce the set of fundamentals for which the regime survives. The government can avoid this adverse effect of imprecise communication by communicating precisely in all states of the world.

A communication policy implemented by an uncommitted government can be worse than a policy of any constant precision. The reason is that the communication policy of an uncommitted government can change agents' beliefs in a way that increases their attack threshold and induces them to attack when signals are higher. The region of parameters for which this happens is indicated with plus signs on Figure 3. This happens when low fundamentals are important in determining agent's payoff (b is high) and fundamentals are high on average (μ is high). By communicating precisely when fundamentals are strong, the government also induces agents to be more aggressive, as the agents with signals near the attack threshold realize that the fundamentals cannot be very strong, as a very strong government distributes signals above the threshold. Consequently, for high b and μ, an uncommitted government uses almost the worst possible communication policy. Committing to any $\sigma(\cdot) \equiv \sigma \in [P, I]$ improves the government's outcome.

5.2 The SDDS of the IMF as a commitment mechanism

The lack of governmental transparency was considered a factor that contributed to the crisis in Mexico in 1994-1995 and the emerging market crises of 1997-1998. According to an IMF Staff Report, "Lack of transparency was a feature of the buildup to the Mexican

[42]This results is consistent with Heinemann and Illing (2002) and Iachan and Nenov (2015), who show that if agents' payoff decreases in fundamentals, then then the regime falls less often if the dispersion of private signals is low. The point of my analysis is to compare this constant dispersions of private signals with a policy of an uncommitted government, which varies dispersion with the state of the world.

crisis of 1994-95 and to the emerging market crises of 1997-98. . . . Inadequate economic data, hidden weaknesses in financial systems, and lack of clarity about government policies and policy formulation contributed to a loss of confidence that ultimately threatened to undermine global stability." This assessment by the IMF is consistent with a property of my model, that the communication policy of an uncommitted government can make agents more aggressive, leading to a regime falling more often.

The policy response of the IMF to the Mexico's crisis is consistent with Proposition 4, which states that a government can benefit from a rule that enforces a constant level of precision of communication. In 1996 the IMF introduced the Special Data Dissemination Standard (SDDS),[43] which prescribes the subscribed countries disclose specific data in a particular format with specific periodicity. Currently, 65 countries, including Mexico, are subscribed to the SDDS. The IMF views the SDDS as a mechanism for committing countries to transparency. For example, after China subscribed to the SDDS in 2015, First Deputy Managing Director of the IMF David Lipton said that "The subscription to the SDDS underscores China's strong commitment to transparency."[44]

Several features of the SDDS enforce a country's commitment to transparency. First, a country must publish specific data and metadata (the description of how the data is constructed), which must be accessible from the Dissemination Standard Bulletin Board, a website created by the IMF. Second, the SDDS prescribes specific periodicity with which the data have to be published, and restricts the amount of time, which a country has to collect and publish the data. Moreover, a country must publish the advance release calendar, that is announce in advance the dates on which it is publishing the data. Finally, the IMF controls that the countries follow the SDDS and issues an annual observance report for each country. These requirements make it costly for a country to be vaguer by publishing only favorable data or postponing the disclosure of unfavorable data, and hence bring a country closer to communicating with constant precision.

The SDDS prescribes a country to disclose detailed information on foreign currency reserves and debt, the data that agents use to decide whether to attack a country's currency or run on its debt. For example, a guide on reporting foreign reserves "In-

[43]The Mexico's crisis as a motivation behind the SDDS is mentioned in "The Special Data Dissemination Standard 2013. Guide for Subscribers and Users." IMF 2013.

[44]"The People's Republic of China Subscribes to the IMF's Special Data Dissemination Standard." Press Release No. 15/466. October 7, 2015.

ternational Reserves and Foreign Currency Liquidity. Guidelines for a Data Template" contains 108 pages of instructions including 4 table templates recommended for disclosing data. A guide on reporting debt, "Public Sector Debt Statistics. Guide for Compilers and Users" contains 231 pages including 13 table templates recommended for disclosing data.

6 Extensions

In this section, I extend the model of Section 2. In Subsection 6.1, agents can receive multiple signals and the distribution of noise can be non-normal. In Subsection 6.2, the government can communicate a biased message. The main results of the paper hold under these extensions.

6.1 Multidimensional signals and general noise distributions

I extend the model of Section 2 as described below. I keep the assumption that $\sigma_\xi^2 = 0$.
Assumption A1. Agents receive K private signals.
Under Assumption A1, each agent receives K signals, hence its signal x_i is a K-dimensional vector $x_i = (x_{i,1}, \ldots, x_{i,K})$. The signals are independent for each agent and between agents. The role of this extension is to relax the assumption that agents do not directly observe the precision of communication $\sigma(\theta)$ of the government. As K increases, the agents are able to better discriminate between various actions $\sigma(\theta)$, and thus there inference about θ becomes more related to the government's action $\sigma(\theta)$.
Assumption A2. The prior distribution of θ, $p(\theta)$, is strictly positive and continuous on $\mathbb{R}$ or is improper uniform on $\mathbb{R}$.
Assumption A3. An agent attacks the regime if all agent's signals are high and does not attack if all the signal are low for any government's policy and any actions of other agents.

Assumption A3 requires that low signals correspond to weak fundamentals and high signals correspond to strong fundamentals. This assumption is intuitive and I expect that any reasonably parametrized model should satisfy it.[45] More formally, the assump-

[45]The assumption is satisfied, for example, if the prior distribution of fundamentals is improper uniform on $\mathbb{R}$.

tion states that $\mathrm{E}\left[\text{agent's profit}|x_i + x \cdot \mathbf{1}, \Theta^* = [1, +\infty), \sigma(\cdot)\right]$ is strictly less than c for any government's policy if x is high enough (here $\mathbf{1}$ is the vector of 1s of length K). Also, $\mathrm{E}\left[\text{agent's profit}|x_i + x \cdot \mathbf{1}, \Theta^* = [0, +\infty), \sigma(\cdot)\right]$ is strictly greater than c for any government's policy for x low enough.

Assumption A4. The distribution of private noise has a probability density function $f_\sigma(\cdot)$ that satisfies the following properties:

1. Function $f_\sigma(\theta)$ is strictly positive. It is also continuous, unimodal and symmetric in θ.

2. Equation $f_s(t) = f_\sigma(t)$ has a unique positive solution $t(s, \sigma)$. Function $t(s, \sigma)$: $[P, I]^2 \to \mathbb{R}_+$ is strictly increasing and continuous.

Assumption A4 is satisfied by normal, Laplace, logistic, and Student's t-distribution with $\nu > 2$ degrees of freedom.[46]

Lemma 7 states that the main results of the paper hold under Assumptions A1-A4.

Lemma 7. *Consider the model in Section 2 that is extended by Assumptions A1-A4. Assume that there exists an equilibrium without commitment and an outcome under commitment.*[47] *There exist two thresholds $\underline{\theta}, \overline{\theta} \in \mathbb{R}$ such that:*

$$\sigma^{NC}(\theta) = \begin{cases} I & \text{if } \theta \leq \underline{\theta}, \\ P & \text{if } \theta \geq \overline{\theta}, \end{cases} \qquad \sigma^{C}(\theta) = \begin{cases} P & \text{if } \theta \leq \underline{\theta}, \\ I & \text{if } \theta \geq \overline{\theta}, \end{cases}$$

where $\sigma^{NC}(\cdot)$ is the government's communication policy in an equilibrium without commitment and $\sigma^{C}(\cdot)$ is the government's communication policy in an outcome under commitment.

Proof. The proof is in the Appendix. □

[46]For normal distribution, for example, $t(\sigma, s) = \sqrt{\frac{2log(\sigma/s)}{\sigma^2 - s^2}}\sigma s$ with $t(s, s) = s$.

[47]If an equilibrium without commitment does not exist, then an uncommitted government still uses a strategy that satisfies Lemma 7 against any rationalizable strategy of agents. A committed government that uses a strategy that does not satisfy Lemma 7 on a set of positive measure can get a strictly higher utility by changing its strategy according to Lemma 7.

6.2 Biased communication

In the real world, a government may hide some information from the agents or provide agents with false information. In that case, agents' private signals are biased. We can model biased communication by introducing the bias function $b(\theta) : \mathbb{R} \to \mathbb{R}$, in which case agents receive signals $x_i = \theta + b(\theta) + \varepsilon_i$. The bias function $b(\cdot)$ can be a part of a fully optimal communication policy $(b(\cdot), \sigma(\cdot))$, in which a government chooses both the bias and the precision of communication.[48]

Even though in the real world a government can bias its communication, the magnitude of the bias is likely to be bounded. Indeed, by providing agents with information that is too distorted, the government may harm its reputation. Another justification for considering a finite bias is that the optimal bias choice is finite in a model with convex costs of biasing signals.

Lemma 8 states that the main results of the paper about the precision of communication hold if the government communicates with a finite bias.

Lemma 8. *Consider the model in Section 2 that contains bias $b(\cdot)$ bounded by $M > 0$. Assume that there exists an equilibrium without commitment and an outcome under commitment.*[49] *Then there exist two thresholds $\underline{\theta}, \overline{\theta} \in \mathbb{R}$ such that:*

$$\sigma^{NC}(\theta) = \begin{cases} I & \text{if } \theta \leq \underline{\theta}, \\ P & \text{if } \theta \geq \overline{\theta}. \end{cases} \qquad \sigma^{C}(\theta) = \begin{cases} P & \text{if } \theta \leq \underline{\theta}, \\ I & \text{if } \theta \geq \overline{\theta}, \end{cases}$$

where $\sigma^{NC}(\cdot)$ is the government's communication policy in an equilibrium without commitment and $\sigma^{C}(\cdot)$ is the government's communication policy in an outcome under commitment.

[48] Characterization of $b(\cdot)$ is beyond the scope of this paper. See Edmond (2013) for an analysis of the bias chosen by an uncommitted government.

[49] The statement of footnote 47 applies here.

7 Conclusion

In this paper, I examine the role of the precision of government communication in global coordination games of regime change. I characterize how the government chooses the precession of its communication based on the strength of economic fundamentals and the government's ability to commit to its policy. An uncommitted government chooses precision of communication discretionary and may have to abandon its regime more often relative to a government that uses constant precision of communication. A government that can commit to a communication policy uses a very different precision, relative to an uncommitted government, and saves its regime more often. I argue that a government may be able to commit to a simple policy of constant precision of communication, even if it cannot commit to a fully optimal policy, and thus can save its regime more often relative to a policy of an uncommitted government.

More generally, my results can be applied in other contexts, such as the business cycles management by a central bank. For example, Schaal and Taschereau-Dumouchel (2015) use a global game approach to study the business cycle dynamics in an economy where firms coordinate their production decisions. The intuition of my paper holds in such a framework, hence a central bank may be able to choose precision of communication to reduce the frequency and length of recessions.

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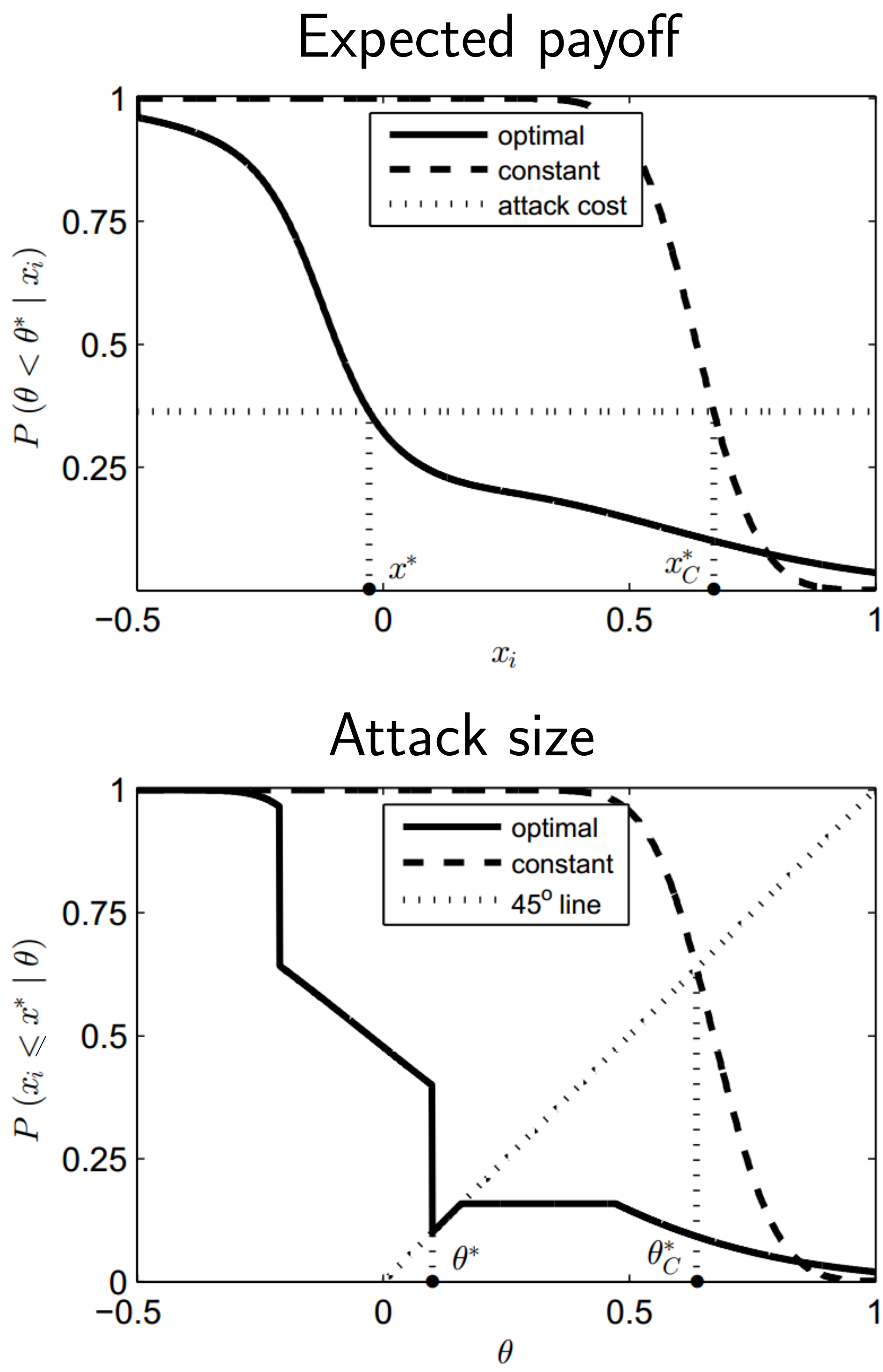

Figure 1: Expected payoff and attack sizes when the government uses the optimal communication policy $\sigma^C(\cdot)$ with $[P, I] = [0.1, 0.5]$ and communicates with constant precision $\sigma(\cdot) \equiv P$. Note that under the assumptions of the example, the expected payoff is equal to $P(\theta < \theta^*|x_i)$ and the attack size is equal to $P(x_i \leq x^*|\theta)$.

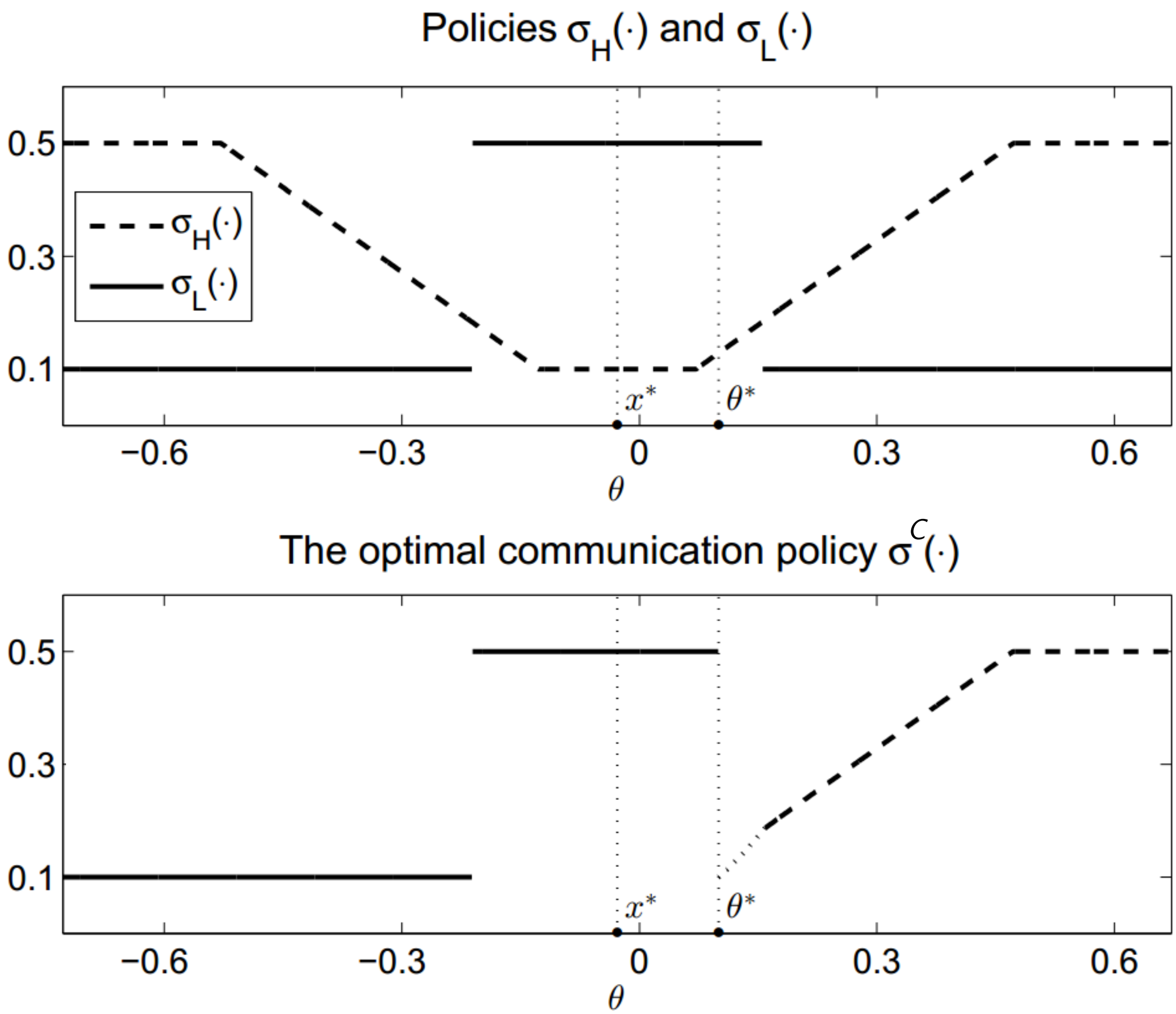

Figure 2: The top panel shows policy $\sigma_H(\cdot)$ that maximizes the impact of communication on posterior beliefs of the threshold agent and policy $\sigma_L(\cdot)$ that minimizes the impact of communication on posterior beliefs of the threshold agent. The bottom panel shows the optimal communication policy $\sigma^C(\cdot)$.

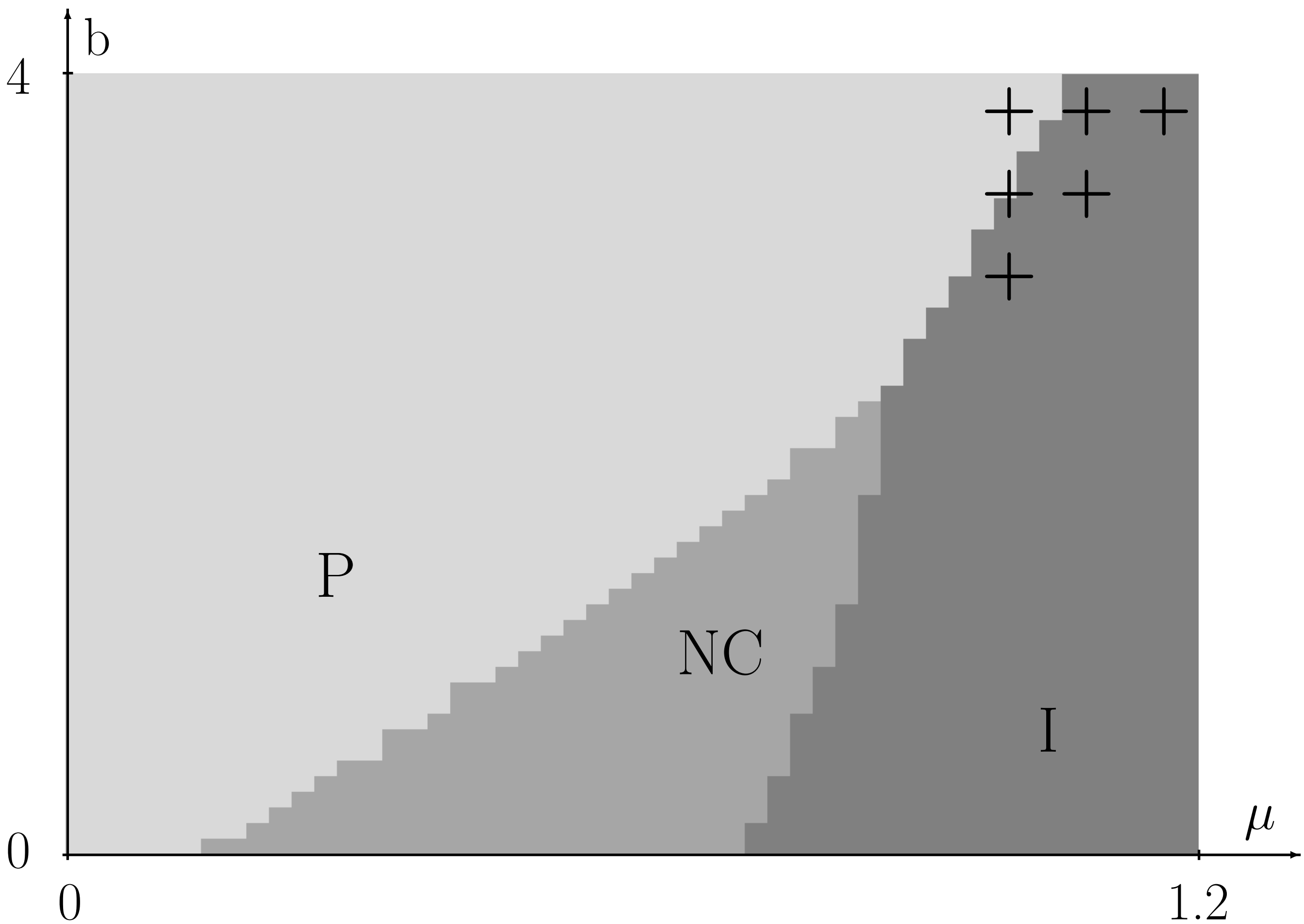

Figure 3: This figure shows the best policy out of three policies: $\sigma(\cdot) \equiv P$, $\sigma(\cdot) \equiv I$, and $\sigma^{NC}(\cdot)$ for various values of μ and b (the best policies correspond to regions marked P, I, and NC respectively). Parameter μ is the average value of fundamentals. Parameter b captures the importance of weak fundamentals. Plus signs show the region of parameters in which any policy of constant precision of communcition $\sigma(\cdot) \equiv \sigma \in [P, I]$ is better than $\sigma^{NC}(\cdot)$.

Appendix

Proof of Proposition 1

Proof. The proof is provided in the main text. □

Proof of Lemma 1

Proof. Let $P > 0$ and let $\widetilde{P} \in (P, \sqrt{P^2 + \sigma^2_{prior}})$.

Step 1. For any government's policy $\sigma(\cdot) : \mathbb{R} \to [P, \widetilde{P}]$ there exists x^* such that E[agent's payoff$|x_i = x^*, x^*, \sigma(\cdot)] = c$.

Proof. Define

$$\pi(x^*, \theta) \equiv \int\limits_{\Phi\left(\frac{x^*-\theta}{\sigma(\theta)}\right)-\theta>\xi} \pi(\theta + \xi)\varphi(\xi)d\xi. \tag{37}$$

Then, for the threshold agent x^*

$$\text{E[agent's payoff}|x_i = x^*, x^*, \sigma(\cdot)] = \int\limits_{-\infty}^{+\infty} \pi(x^*, \theta)f(\theta|x^*)d\theta. \tag{38}$$

Let us prove that (38) becomes less than c as x^* goes to $+\infty$. Pick $t \in \mathbb{R}$ such that $\pi(+\infty, \theta) < c/2$ for all $\theta > t$. Then

$$\frac{\int\limits_{-\infty}^{+\infty} \pi(x^*, \theta)p(\theta)f_{\sigma(\theta)}(x^* - \theta)d\theta}{\int\limits_{-\infty}^{+\infty} p(\theta)f_{\sigma(\theta)}(x^* - \theta)d\theta} \tag{39}$$

$$\leq \frac{\int\limits_{-\infty}^{t} \pi(+\infty, \theta)p(\theta)f_{\sigma(\theta)}(x^* - \theta)d\theta + \int\limits_{t}^{+\infty} \pi(+\infty, \theta)p(\theta)f_{\sigma(\theta)}(x^* - \theta)d\theta}{\int\limits_{-\infty}^{+\infty} p(\theta)f_{\sigma(\theta)}(x^* - \theta)d\theta} \tag{40}$$

$$\leq C \cdot \frac{\int_{-\infty}^{t} p(\theta) f_{\sigma(\theta)}(x^* - \theta) d\theta}{\int_{-\infty}^{+\infty} p(\theta) f_{\sigma(\theta)}(x^* - \theta) d\theta} + \frac{c}{2}. \tag{41}$$

Where C is a constant that bounds $\pi(\cdot)$ from above. On Step 1 in the proof of Lemma 2, I formally show that the ratio of integrals in (41) converges to 0 as x^* converges to $+\infty$ as long as $\widetilde{P} < \sqrt{P^2 + \sigma_{prior}^2}$. Hence (38) becomes less than c as x^* goes to $+\infty$

A similar argument establishes that (38) goes to $\pi(-\infty) > c$ as x^* goes to $-\infty$. Since both $\pi(x^*, \theta)$ and $f(\theta, x^*)$ are continuous in x^*, an argument similar to the one described in footnote 38 establishes that the expectation in (38) is continuous in x^*. Hence, there exists at least one x^* that solves E[agent's payoff$|x_i = x^*, x^*, \sigma(\cdot)] = c$. □

Step 2. E[agent's payoff$|x_i, x^*, \sigma(\cdot) \equiv P]$ is strictly decreasing in x_i.

Proof. For $\sigma(\cdot) \equiv P$, the conditional posterior of θ is:

$$f(\theta|x_i) = \frac{p(\theta) f_{\sigma(\theta)}(\theta - x_i)}{\int_{-\infty}^{+\infty} p(\theta) f_{\sigma(\theta)}(\theta - x_i) d\theta} \sim \mathcal{N}\left(\frac{P^2}{\sigma_{prior}^2 + P^2}\mu_{prior} + \frac{\sigma_{prior}^2}{\sigma_{prior}^2 + P^2}x_i, \frac{\sigma_{prior}^2 P^2}{\sigma_{prior}^2 + P^2}\right). \tag{42}$$

Hence as x_i increases, the conditional distribution of θ shifts to the right. Also for $\sigma(\cdot) \equiv P$ function $\pi(x^*, \theta)$ strictly decreases in θ. It follows that

$$\text{E[agent's payoff}|x_i, x^*, \sigma(\cdot) \equiv P] = \int_{-\infty}^{+\infty} \pi(x^*, \theta) f(\theta|x_i) d\theta \tag{43}$$

strictly decreases in x_i. □

Step 3. We can choose $\widetilde{P} \in (P, \sqrt{P^2 + \sigma_{prior}^2})$ such that the following statement is correct. Let x^* be a solution to E[agent's payoff$|x_i = x^*, x^*, \sigma^{NC}(\cdot)] = c$ for P and $I \leq \widetilde{P}$. Then $\{x_i :$ E[agent's payoff$|x_i, x^*, \sigma^{NC}(\cdot)] \geq c\} = (-\infty, x^*]$.

Proof. By the argument similar to the argument of Step 1, there exists $\hat{x} \in \mathbb{R}$ such that for $x_i < -\hat{x}$ an agent attacks for any government's policy $\sigma(\cdot) : \mathbb{R} \to [P, \widetilde{P}]$ even if no other agent attacks. Moreover, we can choose $\hat{x}$ so that for $x_i > \hat{x}$ an agent does not attack for any government's policy $\sigma(\cdot) : \mathbb{R} \to [P, \widetilde{P}]$ even if all other agents attack. It follows that x^* necessarily satisfies $|x^*| < \hat{x}$.

To finish the proof of Lemma 1, it is enough to prove, that for $\widetilde{P}$ close enough to P the derivative of E[agent's payoff$|x_i, x^*, \sigma^{NC}(\cdot)$] with respect to x_i is negative for any x^* and x_i such that $|x_i| < \hat{x}$ and $|x^*| < \hat{x}$.

$$\begin{aligned}
&\mathrm{E}[\text{agent's payoff}|x_i, x^*, \sigma^{NC}(\cdot)] = \qquad (44)\\
&= \frac{\int\limits_{-\infty}^{x^*} \int\limits_{-\infty}^{\Phi\left(\frac{x^*-\theta}{I}\right)-\theta} \pi(\theta+\xi)\varphi(\xi)p(\theta)f_I(\theta-x_i)d\xi d\theta + \int\limits_{x^*}^{+\infty} \int\limits_{-\infty}^{\Phi\left(\frac{x^*-\theta}{P}\right)-\theta} \pi(\theta+\xi)\varphi(\xi)p(\theta)f_P(\theta-x_i)d\xi d\theta}{\int\limits_{-\infty}^{x^*} p(\theta)f_I(\theta-x_i)d\theta + \int\limits_{x^*}^{+\infty} p(\theta)f_P(\theta-x_i)d\theta}\\
&= \frac{\Delta\pi+\pi}{\Delta f+f},
\end{aligned}$$

where:

$$f = \int\limits_{-\infty}^{+\infty} p(\theta)f_P(\theta-x_i)d\theta, \tag{45}$$

$$\pi = \int\limits_{-\infty}^{+\infty} \int\limits_{-\infty}^{\Phi\left(\frac{x^*-\theta}{P}\right)-\theta} \pi(\theta+\xi)\varphi(\xi)p(\theta)f_P(\theta-x_i)d\xi d\theta, \tag{46}$$

$$\Delta f = \int\limits_{-\infty}^{x^*} p(\theta)f_I(\theta-x_i)d\theta - \int\limits_{-\infty}^{x^*} p(\theta)f_P(\theta-x_i)d\theta, \tag{47}$$

$$\Delta\pi = \int\limits_{-\infty}^{x^*} \int\limits_{-\infty}^{\Phi\left(\frac{x^*-\theta}{I}\right)-\theta} \pi(\theta+\xi)\varphi(\xi)p(\theta)f_I(\theta-x_i)d\xi d\theta - \int\limits_{-\infty}^{x^*} \int\limits_{-\infty}^{\Phi\left(\frac{x^*-\theta}{P}\right)-\theta} \pi(\theta+\xi)\varphi(\xi)p(\theta)f_P(\theta-x_i)d\xi d\theta. \tag{48}$$

We denote as $\Delta\pi'$, π', $\Delta f'$, f' the derivatives of $\Delta\pi$, π, Δf, f with respect to x_i. It follows, that the derivative of (44) is equal to:

$$\frac{(\Delta\pi'+\pi')(\Delta f+f)-(\Delta\pi+\pi)(\Delta f'+f')}{(\Delta f+f)^2}. \tag{49}$$

Straightforward derivations demonstrate that

$$|\Delta f| \leq \text{const}_1 \cdot \int_{-\infty}^{+\infty} |f_P(\theta) - f_I(\theta)| d\theta, \tag{50}$$

$$|\Delta f'| \leq \text{const}_2 \cdot \int_{-\infty}^{+\infty} |f'_P(\theta) - f'_I(\theta)| d\theta, \tag{51}$$

$$|\Delta \pi| \leq \text{const}_3 \cdot \int_{-\infty}^{+\infty} |f_P(\theta) - f_I(\theta)| d\theta, \tag{52}$$

$$|\Delta \pi'| \leq \text{const}_4 \cdot \int_{-\infty}^{+\infty} |f'_P(\theta) - f'_I(\theta)| d\theta. \tag{53}$$

Here const_1–const_4 are positive constants that do not depend on x_i or x^*. Note that π, π', f, f' are bounded from above by constants that do not depend on x_i or x^*.

By Step 2, $\pi' f - \pi f'$ is negative. Moreover, it is continuous in both x_i and x^*. Consequently, there exists $\epsilon > 0$ such that the derivative of (43), $\pi' f - \pi f'$, is less than $-\epsilon$ for all x^* and x_i such that $|x_i| < \hat{x}$ and $|x^*| < \hat{x}$. This argument together with (49-53) implies that the derivative of (44) is negative for I close enough to P. □

□

Proof of Proposition 2

Proof. Proposition 2 follows from Lemma 7. Assumption A3, required by Lemma 7, holds under assumptions of Proposition 2 by the result established on Step 1 in the proof of Lemma 2. □

Proof of Lemma 2

Proof. Let $P > 0$ and choose any $\widetilde{P} \in (P, \sqrt{\sigma^2_{prior} + P^2})$.

Step 1. There exists $\hat{x} \in \mathbb{R}$ such that for $x_i > \hat{x}$ an agent does not attack for any government's policy $\sigma(\cdot) : \mathbb{R} \to [P, \widetilde{P}]$ even if all other agents attack. Moreover, we can choose $\hat{x}$ so that for $x_i < -\hat{x}$ an agent attacks for any government's policy $\sigma(\cdot) : \mathbb{R} \to [P, \widetilde{P}]$ even if no other agents attack.

Proof. To save on notation denote $\mu \equiv \mu_{prior}$ and $s \equiv \sigma_{prior}$. Recall that the error function is defined as follows $\mathrm{erf}(x) \equiv \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$. Pick a positive constant $M > \widetilde{P}$.[50] Then for x_i *high enough* we can bound the expectation of agent's profit as follows:

$$
\begin{aligned}
&\frac{\int\limits_{-\infty}^{1} \pi(\theta)p(\theta)f_{\sigma(\theta)}(x_i-\theta)d\theta}{\int\limits_{\mathbb{R}} p(\theta)f_{\sigma(\theta)}(x_i-\theta)d\theta} \le C\frac{\int\limits_{-\infty}^{1} p(\theta)f_{\sigma(\theta)}(x_i-\theta)d\theta}{\int\limits_{\mathbb{R}} p(\theta)f_{\sigma(\theta)}(x_i-\theta)d\theta} \\
\le& C\frac{\int\limits_{-\infty}^{1} p(\theta)f_{\widetilde{P}}(x_i-(\theta+M))d\theta}{\int\limits_{-\infty}^{x_i} p(\theta)f_{P}(x_i-(\theta-M))d\theta + \int\limits_{x_i}^{+\infty} p(\theta)f_{P}(x_i-(\theta+M))d\theta} \\
=& C\sqrt{\frac{P^2+s^2}{\widetilde{P}^2+s^2}} \frac{e^{-\frac{(M+\mu-x_i)^2}{2(\widetilde{P}^2+s^2)}}\left(1-\mathrm{erf}\left(\frac{\widetilde{P}^2(\mu-1)+s^2(-M+x_i-1)}{\sqrt{2}\widetilde{P}s\sqrt{\widetilde{P}^2+s^2}}\right)\right)}{e^{-\frac{(M-\mu+x_i)^2}{2(P^2+s^2)}}\left(1+\mathrm{erf}\left(\frac{-Ms^2-\mu P^2+P^2x_i}{\sqrt{2}Ps\sqrt{P^2+s^2}}\right)\right)+e^{-\frac{(M+\mu-x_i)^2}{2(P^2+s^2)}}\left(1-\mathrm{erf}\left(\frac{Ms^2-\mu P^2+P^2x_i}{\sqrt{2}Ps\sqrt{P^2+s^2}}\right)\right)}
\end{aligned}
\tag{54}
$$

We can bound $1-\mathrm{erf}(\cdot)$ in the numerator of last expression in (54) by the following expression:

$$
\frac{\sqrt{2}\widetilde{P}s\sqrt{\widetilde{P}^2+s^2}}{\widetilde{P}^2(\mu-1)+s^2(-M+x_i-1)} \cdot e^{-\left(\frac{\widetilde{P}^2(\mu-1)+s^2(-M+x_i-1)}{\sqrt{2}\widetilde{P}s\sqrt{\widetilde{P}^2+s^2}}\right)^2}, \tag{55}
$$

which behaves asymptotically as $e^{-\frac{s^2x_i^2}{2\widetilde{P}^2(\widetilde{P}^2+s^2)}}$. The asymptotic properties of the denominator of (54) are the same as the asymptotic properties of $e^{-\frac{x_i^2}{2(P^2+s^2)}}$. Thus it follows that the expected profit of an agent is bounded from above by a function that asymptotically behaves as:

$$
e^{-\frac{x_i^2}{2(\widetilde{P}^2+s^2)}} \cdot e^{-\frac{s^2x_i^2}{2\widetilde{P}^2(\widetilde{P}^2+s^2)}} \cdot e^{\frac{x_i^2}{2(P^2+s^2)}}. \tag{56}
$$

The function in (56) converges to 0 when x_i converges to $+\infty$ if $\widetilde{P}^2 < s^2 + P^2$.

It follows that we can choose $\hat{x}$ high enough so that an agent that receives a signal $x_i > \hat{x}$ does not attack for any $\sigma(\cdot) \in \mathcal{S}$ and $\Theta^* = [1, +\infty)$. A similar argument establishes

[50] We will be a little more general in the proof than necessary to make the proof applicable under more general assumptions we use later in the paper.

that we can also choose $\hat{x}$ so that agents that receive signals $x_i < -\hat{x}$, attack for any $\sigma(\cdot) \in \mathcal{S}$ and $\Theta^* = [0, +\infty)$. $\square$

Step 2. Suppose that the government uses $\sigma(\cdot) \equiv P$. Then the derivative of agent's expected payoff with respect to x_i is less than $-\epsilon$ for $\epsilon > 0$ for all $x_i : |x_i| < \hat{x}$ and all $\theta^* \in [0, 1]$.

Proof. If the government uses the constant precision of P, then the expected payoff of an agent with signal x_i is:

$$\mathrm{E}[\text{agent's payoff}|x_i, \theta^*, \sigma(\cdot) \equiv P] = \frac{\int\limits_{-\infty}^{\theta^*} \pi(\theta)p(\theta)f_P(\theta - x_i)d\theta}{\int\limits_{-\infty}^{+\infty} p(\theta)f_P(\theta - x_i)d\theta}.$$

Note that the expectation is continuously differentiable in x_i and its derivative is continuous in θ^*. Moreover the expectation is strictly decreasing in x_i, which can be established with an argument similar to an argument on Step 2 in the proof of Lemma 1. It follows that there exists $\epsilon > 0$ such that the derivative of agent's payoff with respect to x_i is less than $-\epsilon$ for any x_i such that $|x_i| < \hat{x}$ and any $\theta^* \in [0, 1]$. $\square$

Step 3. For I close enough to P the derivative of $\mathrm{E}[\text{agent's payoff}|x_i, \theta^*, \sigma(\cdot)]$ with respect to x_i is negative for any $\sigma(\cdot) : \mathbb{R} \to [P, I]$, any $\theta^* \in [0, 1]$ and any x_i such that $|x_i| < \hat{x}$.

Proof.

$$\begin{aligned}
&\mathrm{E}[\text{agent's payoff}|x_i, \theta^*, \sigma(\cdot)] = \\
&= \frac{\int\limits_{-\infty}^{\theta^*} \pi(\theta)p(\theta)f_{\sigma(\theta)}(\theta - x_i)d\theta}{\int\limits_{-\infty}^{+\infty} p(\theta)f_{\sigma(\theta)}(\theta - x_i)d\theta} \\
&= \frac{\Delta\pi + \pi}{\Delta f + f},
\end{aligned} \tag{57}$$

where:

$$f = \int_{-\infty}^{+\infty} p(\theta) f_P(\theta - x_i) d\theta, \tag{58}$$

$$\pi = \int_{-\infty}^{\theta^*} \pi(\theta) p(\theta) f_P(\theta - x_i) d\theta, \tag{59}$$

$$\Delta f = \int_{-\infty}^{+\infty} p(\theta) f_{\sigma(\theta)}(\theta - x_i) d\theta - \int_{-\infty}^{+\infty} p(\theta) f_P(\theta - x_i) d\theta, \tag{60}$$

$$\Delta \pi = \int_{-\infty}^{\theta^*} \pi(\theta) p(\theta) f_{\sigma(\theta)}(\theta - x_i) d\theta - \int_{-\infty}^{\theta^*} \pi(\theta) p(\theta) f_P(\theta - x_i) d\theta. \tag{61}$$

We denote as $\Delta\pi'$, π', $\Delta f'$, f' derivatives of $\Delta\pi$, π, Δf, f with respect to x_i. It follows, that the derivative of (57) is equal to:

$$\frac{(\Delta\pi' + \pi')(\Delta f + f) - (\Delta\pi + \pi)(\Delta f' + f')}{(\Delta f + f)^2}. \tag{62}$$

Straightforward derivations demonstrate that

$$|\Delta f| \le \text{const}_1 \cdot \int_{-\infty}^{+\infty} \max_{\sigma \in [P,I]} |f_P(\theta) - f_\sigma(\theta)| d\theta, \tag{63}$$

$$|\Delta f'| \le \text{const}_2 \cdot \int_{-\infty}^{+\infty} \max_{\sigma \in [P,I]} |f'_P(\theta) - f'_\sigma(\theta)| d\theta, \tag{64}$$

$$|\Delta \pi| \le \text{const}_3 \cdot \int_{-\infty}^{+\infty} \max_{\sigma \in [P,I]} |f_P(\theta) - f_\sigma(\theta)| d\theta, \tag{65}$$

$$|\Delta \pi'| \le \text{const}_4 \cdot \int_{-\infty}^{+\infty} \max_{\sigma \in [P,I]} |f'_P(\theta) - f'_\sigma(\theta)| d\theta. \tag{66}$$

Here const_1–const_4 are positive constants that do not depend on x_i or θ^*. Note that π, π', f, f' are bounded from above by constants that do not depend on x_i or θ^*.

As we argued before, $\pi' f - \pi f'$ is negative and less than $-\epsilon$ for all θ^* and x_i such that

$|x_i| < \hat{x}$ and $\theta^* \in [0,1]$. This argument together with (62-66) implies that the derivative of (57) is negative for I close enough to P. $\square$

$\square$

Proof of Lemma 3

Proof. Step 6 in the proof of Lemma 6 shows that the governments saves all the regimes that can be saved given the agents' strategy X^*.

Let us find the weakest regime θ^* that can be saved. Given x^*, the government can save its regime for the following fundamentals:

$$\{\theta : \Phi\left(\frac{x^* - \theta}{\sigma}\right) \le \theta \text{ for some } \sigma \in [P, I]\}. \tag{67}$$

Note that $\Phi\left(\frac{x^*-\theta^*}{\sigma}\right) = \theta^*$ for some $\sigma \in [P, I]$ since $\Phi\left(\frac{x^*-\theta^*}{\sigma}\right) < \theta^*$ would imply that regimes slightly weaker than θ^* can be saved. Moreover, σ must minimize $\Phi\left(\frac{x^*-\theta^*}{\sigma}\right)$ because that expression is equal to θ^*. Particularly, for $x^* - \theta^* < 0$ precision $\sigma = P$ minimizes the attack size and thus θ^*. For $x^* - \theta^* > 0$ precision $\sigma = I$ minimizes the attack size and thus θ^*. It follows that the precision policy for $\theta = \theta^*$ is as follows:

$$\sigma^*(\theta^*) = \begin{cases} P & \text{if } x^* < 0.5, \\ \Sigma & \text{if } x^* = 0.5, \\ I & \text{if } x^* > 0.5. \end{cases} \tag{68}$$

We can combine (67) and (68) to find θ^* as a function of x^*:

$$\begin{cases} \Phi\left(\frac{x^*-\theta^*}{P}\right) = \theta^* & \text{if } x^* \le 0.5, \\ \Phi\left(\frac{x^*-\theta^*}{I}\right) = \theta^* & \text{if } x^* > 0.5. \end{cases} \tag{69}$$

$\square$

Proof of Proposition 3

Proof. Consider a threshold outcome $(\sigma_0(\cdot), x_0, \theta_0)$ conditional on $\sigma_0(\cdot)$ and let $\Theta_0 \equiv [\theta_0, +\infty)$. If $\sigma_0(\cdot)$ does not satisfy (26-28) on a set of positive measure, then there exist an outcome with a larger regime survival region.

Let $\sigma_d(\cdot) \in \mathcal{S}$ be the policy constructed according to (26-28) given thresholds x_0 and θ_0. Let us show that agent x_0 does not attack given Θ_0 and $\sigma_d(\cdot)$:

$$\begin{aligned} c &= \mathrm{E}[\text{agent's payoff} | x_i = x_0, \theta_0, \sigma_0(\cdot)] \\ &= \frac{\int_{-\infty}^{\theta_0} \pi(\theta) p(\theta) f_{\sigma_0(\theta)}(x_0 - \theta) d\theta}{\int_{-\infty}^{\theta_0} p(\theta) f_{\sigma_0(\theta)}(x_0 - \theta) d\theta + \int_{\theta_0}^{+\infty} p(\theta) f_{\sigma_0(\theta)}(x_0 - \theta) d\theta} \\ &> \frac{\int_{-\infty}^{\theta_0} \pi(\theta) p(\theta) f_{\sigma_d(\theta)}(x_0 - \theta) d\theta}{\int_{-\infty}^{\theta_0} p(\theta) f_{\sigma_d(\theta)}(x_0 - \theta) d\theta + \int_{\theta_0}^{+\infty} p(\theta) f_{\sigma_d(\theta)}(x_0 - \theta) d\theta} \\ &= \mathrm{E}[\text{agent's payoff} | x_i = x_0, \theta_0, \sigma_d(\cdot)] \end{aligned} \tag{70}$$

By Lemma 2, $X(\Theta_0, \sigma_d) = (-\infty, x_d]$ for some $x_d \in \mathbb{R}$. The argument in (70) implies that $x_d < x_0$. It follows that $\Theta_0 \subset \Theta\left(X_0, \sigma_d(\cdot)\right) \subset \Theta\left(X\left(\Theta_0, \sigma_d(\cdot)\right), \sigma_d(\cdot)\right)$. By Lemma 5 there exists an outcome conditional on $\sigma_d(\cdot)$ such that the corresponding regime survival region includes Θ_0.

We are done if the regime survival region in the outcome conditional on $\sigma_d(\cdot)$ is strictly larger than Θ_0. Otherwise, we can construct an outcome with a strictly larger regime survival region. Since $F_{\sigma(\theta_0)}(x_0 - \theta_0) = \theta_0$, we have that $F_{\sigma(\theta_0)}(x_d - \theta_0) < \theta_0$. Thus the government can save some regimes in a left neighborhood of θ_0 given $X(\Theta_0, \sigma_d(\cdot))$. For that purpose it is enough for the government to use strategy $\sigma(\theta_0)$ in a left neighborhood of θ_0. Then Step 6 in the proof of Proposition 6 implies that there exists an outcome with a regime survival region that is strictly larger than Θ_0. □

Proof of Lemma 4

Proof. Let $t(\sigma, s)$ be the unique positive solution to $f_\sigma(t) = f_s(t)$. Direct computation shows that $t(\sigma, s) = \sqrt{\frac{2log(\sigma/s)}{\sigma^2 - s^2}}\sigma s$. Then direct computations demonstrate that:

$$\sigma_L(\theta) = \begin{cases} P & \theta \leq x^* - t(P, I), \\ I & x^* - t(P, I) < \theta < x^* + t(P, I), \\ P & \theta \geq x^* + t(P, I). \end{cases} \tag{71}$$

$$\sigma_H(\theta) = \begin{cases} I & \theta \leq x^* - I, \\ x^* - \theta & x^* - I \leq \theta \leq x^* - P, \\ P & x^* - P \leq \theta \leq x^* + P, \\ \theta - x^* & x^* + P \leq \theta \leq x^* + I, \\ I & \theta \geq x^* + I. \end{cases} \tag{72}$$

It is easy to see that the result of Lemma 4 follows.

□

Proof of Corollary 1

Proof. Recall from Proposition 2 that $\sigma^*(\theta) = P$ for $\theta \leq \underline{\theta}$, and $\sigma^*(\theta) = I$ for $\theta \geq \overline{\theta}$. We will consider three cases.

Case 1. $x^* < \theta^*$. By (22) $\sigma^*(\theta^*) = P$. By (26) $\sigma^*(x^*) = I$.

Case 2. $\theta^* < x^*$. By (22) $\sigma^*(\theta^*) = I$. By (27-28) $\sigma^*(x^*) = P$.

Case 3. $\theta^* = x^*$. By (26) $\sigma^*(\theta) = I$ in a left neighborhood of θ^*. By (27-28) $\sigma^*(x^*) = P$ in a right neighborhood of θ^*. □

Proof of Lemma 5

Proof. The proof essentially repeats a standard proof of the Tarski fixed point theorem (see, e.g., Chapter 5 in Ok, 2010) with modifications necessary for the problem that we have[51].

[51]Unfortunately, we cannot directly apply the Tarski fixed point theorem. The reason is as follows. We want to prove the existence of the largest fixed point for a function between

Define $\Theta X(\Theta_\alpha) \equiv \Theta\left(X\left(\Theta_\alpha, \sigma(\cdot)\right), \sigma(\cdot)\right)$. While $\Theta X(\cdot)$ depends on $\sigma(\cdot)$, we omit it from the arguments to lighten notation. Importantly, operator $\Theta X(\cdot)$ is monotone, that is for any measurable subsets of the real line A and B such that $A \subset B$, it follows that $\Theta X(A) \subset \Theta X(B)$. Let $D = \{\Theta_\alpha : \Theta_\alpha \subset \Theta X(\Theta_\alpha)\}$. Let $\Theta_1 \equiv [1, +\infty)$ and note that $\Theta_1 \in D$.

Let us now construct a sequence $\{\Theta_n\}$ as follows. Given Θ_n, define $d_n = \sup_{\Theta_\alpha \in D} p(\Theta_\alpha \setminus \Theta_n)$, where $\setminus$ is the difference operator. Then pick any $\Theta_n^A \in D$ such that $\mu(\Theta_n^A \setminus \Theta_n) > d_n/2$ and let $\Theta_{n+1} \equiv \Theta_n \cup \Theta_n^A$. Note that the sequence constructed above converges to a measurable set $\Theta_0^* \equiv \cup_{n\in\mathbb{N}}\Theta_n$. Moreover, $p(\Theta_\alpha \setminus \Theta_0^*) = 0$ for all $\Theta_\alpha \in D$[52].

Let $\Theta^* = \Theta X(\Theta_0^*)$. Our goal is to show that Θ^* is the largest fixed point of $\Theta X(\cdot)$, that is Θ^* is a fixed point and it contains all fixed points of $\Theta X(\cdot)$. First, let us show that $\Theta^* \subset \Theta X(\Theta^*)$. Indeed, we have that

$$\Theta_0^* = \cup_{n\in\mathbb{N}}\Theta_n \subset \cup_{n\in\mathbb{N}}\Theta X(\Theta_n) \subset \Theta X\left(\cup_{n\in\mathbb{N}}\Theta_n\right) = \Theta X\left(\Theta_0^*\right). \tag{73}$$

Here the first and the last equalities follow from the definition of Θ_0^*. The first inclusion follows from the fact that Θ_n is a finite union of elements of D, and thus $\Theta_n \in D$[53]. The second inclusion follows from monotonicity of $\Theta X(\cdot)$. By applying $\Theta X(\cdot)$ to (73), we get that $\Theta X(\Theta_0^*) \subset \Theta X(\Theta X(\Theta_0^*))$, thus $\Theta^* \subset \Theta X(\Theta^*)$.

Second, let us show that $\Theta^* \supset \Theta X(\Theta^*)$. It is enough to prove that $\Theta X(\Theta^*) \in D$ and Θ^* contains all elements of D. Monotonicity of $\Theta X(\cdot)$ and the result that $\Theta^* \subset \Theta X(\Theta^*)$ together imply that $\Theta X\left(\Theta^*\right) \subset \Theta X\left(\Theta X(\Theta^*)\right)$, and thus $\Theta X(\Theta^*) \in D$. To prove that Θ^* contains all elements of D, let us take any $\Theta_\alpha \in D$. Then

$$\Theta^* = \Theta X\left(\Theta_0^*\right) = \Theta X\left(\Theta_0^* \cup \Theta_\alpha\right) \supset \Theta X\left(\Theta_\alpha\right) \supset \Theta_\alpha. \tag{74}$$

measurable subsets of the real line, but the set of all measurable subsets of the real line is not a complete lattice. Fortunately, an argument very similar to the Tarski theorem still establishes the existence of the largest fixed point in our case.

[52]Indeed, note that for any $n \in \mathbb{N}$, $[1, +\infty) \subset \Theta_n \subset [0, +\infty)$ and $\mu(\Theta_{n+1} \setminus \Theta_n) \geq d_n/2$. Thus $d_n \to 0$. Now assume $\mu(\Theta_\alpha \setminus \Theta_0^*) = \varepsilon > 0$ for some $\Theta_\alpha \in D$, and let $m \in \mathbb{N}$ be such that $d_n/2 < \varepsilon$ for all $n > m$. This implies, that Θ_α has been added to Θ_n for some $n < m$, and thus $\Theta_\alpha \subset \Theta_0^*$ and $\mu(\Theta_\alpha \setminus \Theta_0^*) = 0$ which contradicts $\mu(\Theta_\alpha \setminus \Theta_0^*) = \varepsilon > 0$.

[53]Indeed, If $A \subset \Theta X(A)$ and $B \subset \Theta X(B)$, then $A \subset \Theta X(A \cup B)$ and $B \subset \Theta X(A \cup B)$ by monotonicity of $\Theta X(\cdot)$. It follows that $A \cup B \subset \Theta X(A \cup B)$.

The first equality follows from the definition of Θ^*. The second equality follows from the fact that by construction of Θ_0^*, $p(\Theta_0^* \setminus \Theta_\alpha) = 0$. The first inclusion follows from the monotonicity of $\Theta X(\cdot)$ and the second inclusion follows from the definition of D. We have established that $\Theta^* \supset \Theta X(\Theta^*)$.

The results of the previous two paragraphs imply that $\Theta^* = \Theta X(\Theta^*)$, and thus Θ^* is a fixed point of $\Theta X(\cdot)$. Moreover, since any fixed point of $\Theta X(\cdot)$ is an element of D, and Θ^* contains all elements of D, it must be the largest fixed point of $\Theta X(\cdot)$.

To finish the proof let us define $X^* \equiv X(\Theta^*, \sigma(\cdot))$. Then $\Theta(X^*, \sigma(\cdot)) = \Theta(X(\Theta^*, \sigma(\cdot)), \sigma(\cdot)) = \Theta^*$. Thus (Θ^*, X^*) is a fixed point of the pair of operators $\Theta(\cdot, \sigma(\cdot))$ and $X(\cdot, \sigma(\cdot))$. Finally note that $X^* = X(\Theta^*, \sigma(\cdot)) \subset X(\Theta_\alpha, \sigma(\cdot))$ for each $\Theta_\alpha \in D$ by monotonicity of $X(\cdot, \sigma(\cdot))$.

□

Proof of Lemma 6

Proof. To simplify notation I assume that the government can commit to abandon a regime even if it can be saved. I will show the existence of the outcome under this assumption. To finish the proof of the result formulated in the main text I will show that the government never chooses to voluntarily abandon a regime that can be saved.

The proof proceeds in 6 steps. On Steps 1-2 I establish that we can choose a sequence of outcomes such that the lower bounds of the corresponding regime survival regions converge to the infimum of all the regimes that can potentially be saved. Moreover, the corresponding regime survival regions can be chosen to be threshold. On Steps 3 I establish that the corresponding agents' attack regions also converge. On Step 4 this allows me to construct a converging sequence of government policies, which preserves all the properties of the regime survival and agents' attack regions stated above. On Step 5 I show that the policy constructed on Step 4 is the policy that gives the government the largest possible regime survival region. On Step 6 I argue that the government never chooses to voluntarily abandon regimes that can be saved, which finishes the proof.

Step 1. Consider an outcome $(\sigma_0(\cdot), X_0, \Theta_0)$ conditional on $\sigma_0(\cdot)$. If Θ_0 does not have a threshold form, then there exists $\sigma_d(\cdot) \in \mathcal{S}$ and a corresponding outcome $(\sigma_d(\cdot), X_d, \Theta_d)$ such that $(\inf\{\Theta_0\}, +\infty) \subset \Theta_d$, $X_d \subset X_0$. Moreover Θ_d has a threshold form.

Proof. Let $\underline{\theta} \equiv \inf\{\Theta_0\}$. If $\underline{\theta} \in \Theta_0$, then define $\underline{\sigma} \equiv \sigma_0(\underline{\theta})$ and $\Theta_d \equiv [\underline{\theta}, +\infty)$. If $\underline{\theta} \notin \Theta_0$, define $\underline{\sigma} \equiv \liminf_{\theta \to \underline{\theta}} \sigma_C(\theta)$, where $\sigma_C : \Theta_0 \to \mathbb{R}$ and $\sigma_C(\Theta_0) = \sigma_0(\Theta_0)$. Define $\Theta_d \equiv (\underline{\theta}, +\infty)$. Note that in both cases $\Theta_d \supset \Theta_0$. Define $\sigma_d(\cdot)$:

$$\sigma_d(\theta) = \begin{cases} \underline{\sigma} & \text{if } \theta \in \Theta_d \setminus \Theta_0, \\ \sigma_0(\theta) & \text{otherwise.} \end{cases} \tag{75}$$

Let us show that $X_d \equiv X(\Theta_d, \sigma_d(\cdot)) \subset X_0$. The idea behind this result is simple—relative to the initial outcome, signals from $\Theta_d \setminus \Theta_0$ now correspond to the regime survival (and signal from outside of $\Theta_d \setminus \Theta_0$ are the same). Thus agents become less inclined to attack. The formal proof is as follows. Take $x_i \in X_d$, then:

$$\begin{aligned} c &\leq \mathrm{E}[\text{agent's payoff}|x_i, \Theta_d, \sigma_d(\cdot)] \qquad (76)\\ &= \frac{\int\limits_{\mathbb{R}\setminus\Theta_d} \pi(\theta)p(\theta)f_{\sigma_d(\theta)}(x_i-\theta)d\theta}{\int\limits_{\mathbb{R}\setminus(\Theta_d\setminus\Theta_0)} p(\theta)f_{\sigma_d(\theta)}(x_i-\theta)d\theta + \int\limits_{\Theta_d\setminus\Theta_0} p(\theta)f_{\sigma_d(\theta)}(x_i-\theta)d\theta} \\ &\leq \frac{\int\limits_{\mathbb{R}\setminus\Theta_d} \pi(\theta)p(\theta)f_{\sigma_d(\theta)}(x_i-\theta)d\theta}{\int\limits_{\mathbb{R}\setminus(\Theta_d\setminus\Theta_0)} p(\theta)f_{\sigma_d(\theta)}(x_i-\theta)d\theta} \\ &= \frac{\int\limits_{\mathbb{R}\setminus\Theta_d} \pi(\theta)p(\theta)f_{\sigma_0(\theta)}(x_i-\theta)d\theta}{\int\limits_{\mathbb{R}\setminus(\Theta_d\setminus\Theta_0)} p(\theta)f_{\sigma_0(\theta)}(x_i-\theta)d\theta} \end{aligned}$$

Then:

$$\begin{aligned} &\mathrm{E}[\text{agent's payoff}|x_i, \Theta_0, \sigma_0(\cdot)] \qquad (77)\\ =&\frac{\int\limits_{\mathbb{R}\setminus\Theta_d} \pi(\theta)p(\theta)f_{\sigma_0(\theta)}(x_i-\theta)d\theta + \int\limits_{\Theta_d\setminus\Theta_0} \pi(\theta)p(\theta)f_{\sigma_0(\theta)}(x_i-\theta)d\theta}{\int\limits_{\mathbb{R}\setminus(\Theta_d\setminus\Theta_0)} p(\theta)f_{\sigma_0(\theta)}(x_i-\theta)d\theta + \int\limits_{\Theta_d\setminus\Theta_0} p(\theta)f_{\sigma_0(\theta)}(x_i-\theta)d\theta} \\ \geq& c, \qquad (78) \end{aligned}$$

because $\pi(\theta) \geq c$ for $\theta \in \Theta_d \setminus \Theta_0$. Hence $x_i \in X_0$.

Let us prove, that $\Theta_d \subset \Theta(X_d, \sigma_d(\cdot))$. Obviously $\Theta_0 = \Theta(X_0, \sigma_0(\cdot)) \subset \Theta(X_d, \sigma_d(\cdot))$

because $X_d \subset X_0$ and $\sigma_0(\Theta_0) = \sigma_d(\Theta_0)$. It is left to prove, that $\Theta_d \setminus \Theta_0 \subset \Theta(X_d, \sigma_d(\cdot))$. Note that Lemma 2 implies $X_d = (-\infty, x_d]$ for some $x_d \in \mathbb{R}$. Now pick any $\theta \in \Theta_d \setminus \Theta_0$, recall that $\sigma_d(\theta) = \underline{\sigma}$. By construction there is a point at Θ_0 strictly to the left of θ such that the government uses precision arbitrarily close to $\underline{\sigma}$ and the regime survives. Thus the regime survives at θ if the government uses precision $\underline{\sigma}$. Let the government commit to abandon all the regimes outside Θ_d. In that case $(\sigma_d(\cdot), X_d, \Theta_d)$ is an outcome conditional on $\sigma_d(\cdot)$ with Θ_d having a threshold form and $(\inf\{\Theta_0\}, +\infty) \subset \Theta_d$. $\square$

Step 2. Let θ^* be the infimum of all the regimes that can be saved in all outcomes. Then there exists a sequence $(\sigma_n^1(\cdot))$ such that the corresponding regime survival regions have threshold form $(\theta_n^1, +\infty)$ and their infimumfs converge to θ^*.

Proof. Let $\widetilde{\Theta} \equiv \{\inf\{\Theta\} : \Theta \text{ is a regime survival region given } \sigma(\cdot), \sigma(\cdot) \in \mathcal{S}\}$. Let $\theta^* \equiv \inf\{\widetilde{\Theta}\}$. We can pick a sequence $\sigma_n^0(\cdot) \in \mathcal{S}$ such that $\inf\{\Theta_n^0\} \to \theta^*$, where Θ_n^0 is a regime survival region that corresponds to $\sigma_n^0(\cdot) \in \mathcal{S}$.

By the result of Step 1, for each n we can use outcome $(\sigma_n^0(\cdot), X_n^0, \Theta_n^0)$ to construct $\sigma_n^1(\cdot) \in \mathcal{S}$ such that in the corresponding conditional outcome $(\sigma_n^1(\cdot), X_n^1, \Theta_n^1)$ the regime survival region Θ_n^1 has a threshold form for threshold $\theta_n^1 \equiv \inf\{\Theta_n^0\}$. Moreover, by Lemma 2 the corresponding agents' attack regions are also threshold: $X_n^1 = (-\infty, x_n^1]$ for some $x_n^1 \in \mathbb{R}$. $\square$

Step 3. $\theta_n^1 \to \theta^*$ and there exists $x^* \in \mathbb{R}$ such that $x_n^1 \to x^*$.

Proof. The first results follows from the construction of θ_n^1, so it is left to verify that for some x^*, $x_n^1 \to x^*$. Consider the case of $\theta^* > 0.5$ (the case of $\theta^* \leq 0.5$ can be analyzed similarly). Then $\theta_n^1 > 0.5$. Note that the corresponding $x_n^1 > 0.5$, because $x_n^1 \leq 0.5$ would have implied that all regimes $\theta \geq 0.5$ survive in the corresponding outcome. For each $x_n^1 > 0.5$ infimum of regimes that can be saved is $\theta_n' : \Phi\left(\frac{x_n^1 - \theta_n'}{I}\right) = \theta_n'$[54]. Note that $\theta^* \leq \theta_n' \leq \theta_n^1$, and thus $\theta_n' \to \theta^*$. It follows, that $x_n^1 \to x^* \equiv \theta^* + I\Phi^{-1}(\theta^*)$.

$\square$

[54]Indeed, conditional on x_n^1 regime θ can be saved as long as there exists $\sigma \in \Sigma$ such that $\Phi\left(\frac{x_n^1 - \theta}{\sigma}\right) \leq \theta$. For the smallest regime that can be saved $\Phi\left(\frac{x_n^1 - \theta}{\sigma}\right) = \theta$, since a strict inequality would imply that weaker regimes can be saved. In the last equality σ must minimize $\Phi\left(\frac{x_n^1 - \theta}{\sigma}\right)$ because it is equal to θ. In the case of $x_n^1 > 0.5$, we necessarily have that the solution to this equation must have $x_n^1 > \theta > 0.5$. Thus σ that allows to get the smallest surviving regime is I.

Step 4. Given $\{\sigma_n^1(\cdot), X_n^1, \Theta_n^1\}$ we can construct $\{\sigma_n^2(\cdot), X_n^2, \Theta_n^2\}$ such that $\Theta_n^2 = \Theta_n^1$, $X_n^2 = (-\infty, x_n^2] \subset X_n^1$ for some x_n^2, and σ_n^2 pointwise converges to some $\sigma^*(\cdot) \in \mathcal{S}$.

Proof. Given (x_n^1, θ_n^1) let us construct $\sigma_n^2(\cdot)$ as follows:

1. If $\theta < \theta_n^1$, then $\sigma_n^2(\theta)$ solves

$$\min_{\sigma \in \Sigma} f_\sigma(x_n^1 - \theta). \tag{79}$$

2. If $\theta \geq \theta_n^1$, then $\sigma_n^2(\theta)$ solves

$$\max_{\sigma \in \Sigma} f_\sigma(x_n^1 - \theta) \tag{80}$$

$$s.t.\ \Phi\left(\frac{x_n^1 - \theta}{\sigma}\right) \leq \theta. \tag{81}$$

By construction

$$\text{E[agent's payoff}|x_i = x_n^1, \theta_n^1, \sigma_n^2(\cdot))] = \frac{\int\limits_{-\infty}^{\theta_n^1} \pi(\theta)p(\theta)f_{\sigma_n^2(\theta)}(x_n^1 - \theta)d\theta}{\int\limits_{-\infty}^{\theta_n^1} p(\theta)f_{\sigma_n^2(\theta)}(x_n^1 - \theta)d\theta + \int\limits_{\theta_n^1}^{+\infty} p(\theta)f_{\sigma_n^2(\theta)}(x_n^1 - \theta)d\theta} \tag{82}$$

$$\leq \frac{\int\limits_{-\infty}^{\theta_n^1} \pi(\theta)p(\theta)f_{\sigma_n^1(\theta)}(x_n^1 - \theta)d\theta}{\int\limits_{-\infty}^{\theta_n^1} p(\theta)f_{\sigma_n^1(\theta)}(x_n^1 - \theta)d\theta + \int\limits_{\theta_n^1}^{+\infty} p(\theta)f_{\sigma_n^1(\theta)}(x_n^1 - \theta)d\theta} = \text{E[agent's payoff}|x_i = x_n^1, \theta_n^1, \sigma_n^1(\cdot))] = c.$$

By Lemma 2, $X_n^2 \equiv X(\Theta_n^1, \sigma_n^2(\cdot)) = (-\infty, x_n^2]$ for some $x_n^2 \in \mathbb{R}$. Hence (82) implies $x_n^2 \leq x_n^1$ and $X_n^2 \subset X_n^1$. It follows that $\Theta(X_n^2, \sigma_n^2(\cdot)) \supset \Theta_n^1$. Let $\Theta_n^2 = \Theta_n^1$ and let the government commit to voluntarily abandon the regime for fundamentals outside of Θ_n^2. Then $(\sigma_n^2(\cdot), X_n^2, \Theta_n^2)$ is an outcome conditional on $\sigma_n^2(\cdot)$.

Given (x^*, θ^*), define $\sigma^*(\cdot)$ according to (79-81). Since $x_n^1 \to x^*$ and $\theta_n^1 \to \theta^*$, it follows that $\sigma_n^2(\theta) \to \sigma^*(\theta)$ almost everywhere. See Lemma 4 and its proof for the analytical expressions for a strategy defined by (79-81). □

Step 5. Let $\sigma^*(\cdot)$ be defined by (79-81) given (x^*, θ^*). Let $X^* \equiv (-\infty, x^*]$, $\Theta^* \equiv [\theta^*, +\infty)$. Then $\{\sigma^*(\cdot), x^*, \theta^*\}$ is the outcome under commitment.

Proof. It is enough to show that $X(\Theta^*, \sigma^*(\cdot)) = X^*$ and $\Theta(X^*, \sigma^*(\cdot)) = \Theta^*$. Note that by construction $\Theta^* = \bigcup\limits_{n\in\mathbb{N}} \Theta_n^2 \cup \theta^*$, $X^* = \bigcap\limits_{n\in\mathbb{N}} X_n^2$.

Let us first prove that $X(\Theta^*, \sigma^*(\cdot)) = X^*$. By the construction of X^* and the definition of $X(\cdot,\cdot)$:

$$x_i \in X^* \iff \forall n \in \mathbb{N} : \frac{\int\limits_{-\infty}^{\theta_n^2} \pi(\theta)p(\theta)f_{\sigma_n^2(\theta)}(x_i - \theta)d\theta}{\int\limits_{\mathbb{R}} p(\theta)f_{\sigma_n^2(\theta)}(x_i - \theta)d\theta} \geq c, \tag{83}$$

$$x_i \in X(\Theta^*, \sigma^*(\cdot)) \iff \frac{\int\limits_{-\infty}^{\theta^*} \pi(\theta)p(\theta)f_{\sigma^*(\theta)}(x_i - \theta)d\theta}{\int\limits_{\mathbb{R}} p(\theta)f_{\sigma^*(\theta)}(x_i - \theta)d\theta} \geq c. \tag{84}$$

We can use Lebesgue's Dominated Convergence Theorem to show that

$$\frac{\int\limits_{-\infty}^{\theta_n^2} \pi(\theta)p(\theta)f_{\sigma_n^2(\theta)}(x_i - \theta)d\theta}{\int\limits_{\mathbb{R}} p(\theta)f_{\sigma_n^2(\theta)}(x_i - \theta)d\theta} \longrightarrow \frac{\int\limits_{-\infty}^{\theta^*} \pi(\theta)p(\theta)f_{\sigma^*(\theta)}(x_i - \theta)d\theta}{\int\limits_{\mathbb{R}} p(\theta)f_{\sigma^*(\theta)}(x_i - \theta)d\theta}. \tag{85}$$

Therefore $X^* \subset X(\Theta^*, \sigma^*(\cdot))$.

Note that $X(\Theta^*, \sigma^*(\cdot)) = (-\infty, x^{**}]$ for some $x^{**} \in \mathbb{R}$. For any $x_i < x^{**}$ inequality in (84) is strict (by Lemma 2), thus $x_i \in X^*$. Moreover $x^{**} \in X^*$ because X^* is a closed set.

Now let us establish, that $\Theta(X^*, \sigma^*(\cdot)) = \Theta^*$. By the definitions of Θ^* and $\Theta(\cdot,\cdot)$

$$\theta \in \Theta^* \iff \exists N \in \mathbb{N} \text{ such that } \forall n > N : \int\limits_{-\infty}^{x_n^2} f_{\sigma_n^2(\theta)}(x_i - \theta)dx_i \leq \theta, \tag{86}$$

$$\theta \in \Theta(X^*, \sigma(\cdot)) \iff \int\limits_{-\infty}^{x^*} f_{\sigma^*(\theta)}(x_i - \theta)dx_i \leq \theta. \tag{87}$$

We can use Lebesgue's Dominated Convergence Theorem to show that

$$\int_{-\infty}^{x_n^2} f_{\sigma_n^2(\theta)}(x_i-\theta)dx_i \longrightarrow \int_{-\infty}^{x^*} f_{\sigma^*(\theta)}(x_i-\theta)dx_i. \tag{88}$$

Therefore $\Theta^* \subset \Theta(X^*, \sigma^*(\cdot))$. Moreover, it follows that $\Theta^* \subset \Theta(X(\Theta^*, \sigma^*(\cdot)), \sigma^*(\cdot))$. Thus, by Lemma 5 there exist an outcome conditional on $\sigma^*(\cdot)$ such that the corresponding regime survival region is larger than Θ^*. However, Θ^* is the largest possible regime survival region by its construction. □

Step 6. The government gets strictly lower utility if it abandons its regime voluntarily for some realization of fundamentals. It follows that $\{\sigma^*(\cdot), X^*, \Theta^*\}$ from Step 5 is the outcome under commitment even if the government cannot abandon regimes voluntarily.

Proof. Consider any outcome $(\sigma^*(\cdot), X^*, \Theta^*)$. Assume that the government abandons regimes even though they can be saved given agents' policy X^*. That is

$$A \equiv \{\theta \in \mathbb{R} \setminus \Theta^* : \int_{X^*} f_{\sigma_d}(x_i-\theta)dx_i \leq \theta \text{ for some } \sigma_d \in \Sigma\} \tag{89}$$

is not empty. Consider a policy $\sigma_d(\cdot)$ that is similar to $\sigma^*(\cdot)$ except that it saves the regime for fundamentals inside A. That is

$$\sigma_d(\theta) = \begin{cases} \sigma^*(\theta) & \text{if } \theta \notin A, \\ \sigma_\theta \in \Sigma : \sigma_\theta \text{ saves the regime given } X^* & \text{if } \theta \in A. \end{cases} \tag{90}$$

After this deviation fever agents attack the regime. For any $x_i \in X(\Theta^* \cup A, \sigma_d(\cdot))$:

$$\begin{aligned} c \leq & \mathrm{E}[\text{agent's payoff} | x_i, \Theta^* \cup A, \sigma_d(\cdot)] \qquad (91) \\ = & \frac{\int_{(\mathbb{R}\setminus\Theta^*)\setminus A} \pi(\theta)p(\theta)f_{\sigma_d(\theta)}(x_i-\theta)d\theta}{\int_{\mathbb{R}} p(\theta)f_{\sigma_d(\theta)}(x_i-\theta)d\theta} \end{aligned}$$

$$
\begin{aligned}
&= \frac{\int\limits_{(\mathbb{R}\backslash\Theta^*)\backslash A} \pi(\theta)p(\theta)f_{\sigma^*(\theta)}(x_i-\theta)d\theta}{\int\limits_{\mathbb{R}\backslash A} p(\theta)f_{\sigma^*(\theta)}(x_i-\theta)d\theta + \int\limits_{A} p(\theta)f_{\sigma_d(\theta)}(x_i-\theta)d\theta} \\
&\leq \frac{\int\limits_{(\mathbb{R}\backslash\Theta^*)\backslash A} \pi(\theta)p(\theta)f_{\sigma^*(\theta)}(x_i-\theta)d\theta}{\int\limits_{\mathbb{R}\backslash A} p(\theta)f_{\sigma^*(\theta)}(x_i-\theta)d\theta}.
\end{aligned}
$$

It follows that

$$
\begin{aligned}
&\text{E}[\text{agent's payoff}|x_i, \Theta^*, \sigma^*(\cdot)] \qquad (92)\\
&= \frac{\int\limits_{(\mathbb{R}\backslash\Theta^*)\backslash A} \pi(\theta)p(\theta)f_{\sigma^*(\theta)}(x_i-\theta)d\theta + \int\limits_{A} \pi(\theta)p(\theta)f_{\sigma^*(\theta)}(x_i-\theta)d\theta}{\int\limits_{\mathbb{R}\backslash A} p(\theta)f_{\sigma^*(\theta)}(x_i-\theta)d\theta + \int\limits_{A} p(\theta)f_{\sigma^*(\theta)}(x_i-\theta)d\theta} \\
&\geq c, \qquad (93)
\end{aligned}
$$

and thus $x_i \in X^*$. Thus we have proved that $X(\Theta^* \cup A, \sigma_d(\cdot)) \subset X^*$.

Notice that $\Theta^* \cup A = \Theta(X^*, \sigma_d(\cdot)) \subset \Theta\left(X\left(\Theta^* \cup A, \sigma_d(\cdot)\right), \sigma_d(\cdot)\right)$. It follows from Lemma 5 that there exists an outcome conditional on $\sigma_d(\cdot)$ such that the corresponding regime survival region includes $\Theta^* \cup A$. It follows, that the government never chooses to abandon a regime if it can be saved given agents' strategy. □

□

Proof of Corollary 2

Proof. Given x^* and $\sigma^{NC}(\cdot)$, the regime survival threshold θ^* is the value of fundamentals which is equal to the attack size at that value of fundamentals:

$$
\Phi\left(\frac{x^* - \theta^*}{\sigma^{NC}(\theta^*)}\right) = \theta^*. \qquad (94)
$$

Given θ^* and $\sigma^{NC}(\cdot)$, we can find the position of agent x^* that is indifferent whether to attack the regime as follows:

$$\frac{\int\limits_{-\infty}^{\theta^*} f_{\sigma(\theta)}(x^* - \theta)d\theta}{\int\limits_{\mathbb{R}} f_{\sigma(\theta)}(x^* - \theta)d\theta} = c. \tag{95}$$

The government's policy $\sigma^{NC}(\cdot)$ conditional on x^* is described by (10). Combining (94), (95), and (10) we get the result of Corollary 2. To check that agents use threshold strategies, it is enough to differentiate the expected payoff of an agent with respect to x_i and verify that the derivative is negative. □

Proof of Proposition 4

Proof. Provided in the main text by means of a numerical example. □

Proof of Lemma 7

Let me introduce some notation. Recall that $x_i \in \mathbb{R}^K$ and $x_i = (x_{i,1}, x_{i,2}, \ldots, x_{i,K})$. $\mathbf{1}$ is a vector of ones of length K. To simplify notation I also define $f_\sigma(x_i - \theta) \equiv \prod_{k=1}^K f_\sigma(x_{i,k} - \theta)$. Define $\tau \equiv t(I, I)$.

Proof. **Proof of the result for a committed government.**

Proof. Let us consider an outcome $(\sigma^*(\cdot), \Theta^*, X^*)$ and assume that $\sigma^*(\cdot)$ does not satisfy conditions of Lemma 7. We will construct policy $\sigma_d(\cdot)$ such that the corresponding regime survival region is larger than Θ^*.

Given $\overline{\theta} \in \mathbb{R}$ define a set of signals $X_{NA}(\overline{\theta})$ such that all $x_i \in X_{NA}(\overline{\theta})$ are in a τ neighborhood of $[\overline{\theta}, +\infty) \cdot \mathbf{1}$. That is

$$X_{NA}(\overline{\theta}) \equiv \{x_i \in \mathbb{R}^K : d_2(x_i, \theta \cdot \mathbf{1}) \leq \tau \text{ for some } \theta \geq \overline{\theta}\}. \tag{96}$$

Pick $\overline{\theta}$ high enough so that it satisfies two conditions. First, for any policy $\sigma(\cdot) \in \mathcal{S}$, agents with $x_i \in X_{NA}(\overline{\theta})$ do not attack in any corresponding conditional outcome. Second, regime survives in any outcome for any $\theta \geq \overline{\theta}$. The existence of such $\overline{\theta}$ is guaranteed by Assumption A3.

For $\underline{\theta} \in \mathbb{R}$ define $X_A(\underline{\theta})$ as a set of signals which are in a τ neighborhood of $(-\infty, \underline{\theta}]$. We can pick $\underline{\theta} \in \mathbb{R}$ low enough so that it satisfies two conditions. First, for any policy $\sigma(\cdot) \in \mathcal{S}$, agents with $x_i \in X_A(\underline{\theta})$ attack in any corresponding conditional outcome. Second, regime falls in any outcome for any $\theta \leq \underline{\theta}$. The existence of such $\underline{\theta}$ is guaranteed by Assumption A3.

Assume that $\sigma^*((-\infty, \underline{\theta})) \neq P$ and $\sigma^*((\overline{\theta}, +\infty) \neq I$ on a set of positive measure. Then define

$$\sigma_d(\theta) = \begin{cases} P & \theta < \underline{\theta}, \\ \sigma^*(\theta) & \underline{\theta} < \theta < \overline{\theta}, \\ I & \theta > \overline{\theta}. \end{cases} \tag{97}$$

If the government uses policy $\sigma_d(\cdot)$, agents becomes less aggressive and attack less. Indeed, for $x_i \in X(\Theta^*, \sigma_d(\cdot)) \setminus X_A(\underline{\theta})$:

$$\begin{aligned} c \leq & \mathrm{E}[\text{agent's payoff}|x_i, \Theta^*, \sigma_d(\cdot)] = \frac{\int\limits_{\mathbb{R}\setminus\Theta^*} \pi(\theta)p(\theta)f_{\sigma_d(\theta)}(x_i - \theta)d\theta}{\int\limits_{\mathbb{R}} p(\theta)f_{\sigma_d(\theta)}(x_i - \theta)d\theta} \qquad (98) \\ = & \frac{\int\limits_{(-\infty,\underline{\theta})} \pi(\theta)p(\theta)f_{\sigma_d(\theta)}(x_i - \theta)d\theta + \int\limits_{[\underline{\theta},\overline{\theta}]\setminus\Theta^*} \pi(\theta)p(\theta)f_{\sigma_d(\theta)}(x_i - \theta)d\theta}{\int\limits_{(-\infty,\underline{\theta})} p(\theta)f_{\sigma_d(\theta)}(x_i - \theta)d\theta + \int\limits_{[\underline{\theta},\overline{\theta}]} p(\theta)f_{\sigma_d(\theta)}(x_i - \theta)d\theta + \int\limits_{(\overline{\theta},+\infty)} p(\theta)f_{\sigma_d(\theta)}(x_i - \theta)d\theta}. \end{aligned}$$

Then

$$\begin{aligned} & \mathrm{E}[\text{agent's payoff}|x_i, \Theta^*, \sigma^*(\cdot)] = \frac{\int\limits_{\mathbb{R}\setminus\Theta^*} \pi(\theta)p(\theta)f_{\sigma^*(\theta)}(x_i - \theta)d\theta}{\int\limits_{\mathbb{R}} p(\theta)f_{\sigma^*(\theta)}(x - \theta)d\theta} \qquad (99) \\ = & \frac{\int\limits_{(-\infty,\underline{\theta})} \pi(\theta)p(\theta)f_{\sigma^*(\theta)}(x_i - \theta)d\theta + \int\limits_{[\underline{\theta},\overline{\theta}]\setminus\Theta^*} \pi(\theta)p(\theta)f_{\sigma^*(\theta)}(x_i - \theta)d\theta}{\int\limits_{(-\infty,\underline{\theta})} p(\theta)f_{\sigma^*(\theta)}(x_i - \theta)d\theta + \int\limits_{[\underline{\theta},\overline{\theta}]} p(\theta)f_{\sigma^*(\theta)}(x_i - \theta)d\theta + \int\limits_{(\overline{\theta},+\infty)} p(\theta)f_{\sigma^*(\theta)}(x_i - \theta)d\theta} \\ \geq & c. \end{aligned}$$

It follows that after the government starts to use strategy $\sigma_d(\cdot)$ agents attack for a smaller set of signals: $X(\Theta^*, \sigma_d(\cdot)) \subset X^*$. Moreover, the agents attack for a strictly smaller set of signals because equality in (99) implies strict inequality in (98). Thus the government can save its regime for a strictly larger set of fundamentals. We can use an argument which is essentially identical to the one of Lemma 5 to show that there exists a conditional outcome $(\sigma_d(\cdot), X_d, \Theta_d)$ such that Θ_d is strictly larger than Θ^*. $\square$

Proof of the result for an uncommitted government.

Proof. For any $\theta > \overline{\theta}$ the attack size given action $\sigma \in [P, I]$ is equal to

$$\begin{aligned}
&\int\limits_{X^*} f_\sigma(x_i - \theta) dx_i \\
=&\int\limits_{X^*} \prod_{k=1}^{K} f_\sigma(x_{i,k} - \theta) dx_{i,1} dx_{i,2} \ldots dx_{i,K} \\
=&\int\limits_{X^* \setminus X_{NA(\overline{\theta})}} \prod_{k=1}^{K} f_\sigma(x_{i,k} - \theta) dx_{i,1} dx_{i,2} \ldots dx_{i,K}. \qquad (100)
\end{aligned}$$

The goal of an uncommitted government is to choose $\sigma \in [P, I]$ to minimize (100). Note that for $\theta \geq \overline{\theta}$ and $x_i \in X^* \setminus X_{NA(\overline{\theta})}$ we have $d_2(\theta \cdot \mathbf{1}, x_i) > \tau$. Hence we have $f_P(x_i - \theta) < f_\sigma(x_i - \theta)$ for $\sigma > P$. It follows that an uncommitted government chooses $\sigma([\overline{\theta}, +\infty)) = P$. A similar argument establishes that an uncommitted government chooses $\sigma((-\infty, \underline{\theta}]) = I$. $\square$

$\square$

Proof of Lemma 8

Proof. Note that the result of Lemma 5 holds even with the bias. The boundedness of bias $b(\cdot)$ allows us to apply the proof of Step 1 in the proof of Lemma 2 and thus establish that Assumption A3 holds (constant M in the proof of Lemma 2 should be chosen to bound the bias function chosen by the government). This, in turn, allows us to define $X_{NA}(\overline{\theta})$ and $X_A(\underline{\theta})$ in the same way as in the proof of Lemma 7. The rest of the proof is similar to the proof of Lemma 7. $\square$