Solutions to In-Class Homework Problems 1, 2, 3.

1. Problem 30 in chapter 4.

Letting X_n be 0 if the nth vehicle is a car and letting it be 1 if the vehicle is a truck gives rise to a two-state Markov chain with transition probability matrix

$$\begin{bmatrix} 4/5 & 1/5 \\ 3/4 & 1/4 \end{bmatrix}.$$

The limiting probabilities are the solution of

$$\begin{aligned} \pi_0 &= 4/5\pi_0 + 3/4\pi_1 \\ \pi_1 &= 1/5\pi_1 + 1/4\pi_1 \\ \pi_0 + \pi_1 &= 1. \end{aligned}$$

Solving these yields

$$\pi_0 = 15/19,\ \pi_2 = 4/19.$$

That is, 4 out of every 19 vehicles are trucks.

2. Consider the gambler's ruin problem with $p = 0.3$, $N = 7$, and an initial wealth \$3. what is the probability that the gambler ever has a fortune of \$4?

Solution: We want to compute $f_{3,4}$. From the matrix given in the lecture notes, we have

$$f_{3,4} = \frac{s_{3,4} - \delta_{3,4}}{s_{4,4}} = \frac{0.91181 - 0}{2.2314} = 0.40863.$$

3. Problem 58 in Ch. 5
$\lambda = \frac{2}{3}3 = 2$. Thus,

$$P(X(t) = 0) = e^{-\lambda t}\frac{(\lambda t)^0}{0!} = e^{-\lambda t} = e^{-2t}$$

$$E[X(t)] = \lambda t = 2t.$$