SIEO W3600 Solution to Homework 11

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Note: During this week, there are also two in-class hwk problems

In class hwk on Nov. 24, Problem 9.8

Part a. $H_0 : \mu_1 - \mu_2 \geq -10$. $H_a : \mu_1 - \mu_2 \leq -10$.

We shall apply two sample z test.

$$\begin{aligned} Z &= \frac{\bar{X} - \bar{Y} - (-10)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \\ &= \frac{107.6 - 123.6 - (-10)}{\sqrt{\frac{1.3^2}{129} + \frac{2.0^2}{129}}} \\ &= \frac{-6}{0.21} = -28.57. \end{aligned}$$

Since $\Phi(-28.57) \approx 0$, we can reject H_0.

Part b. The 95% c.i. (using the two sample z interval) is given by

$$\begin{aligned} \bar{X} - \bar{Y} \pm 1.96\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} &= -6 \pm 1.96(0.21) \\ &= (-6.412, -5.588). \end{aligned}$$

In class hwk on Nov. 26, Problem 14.6

A $9:3:4$ ratio implies that $p_{10} = \frac{9}{16} = .5625, p_{20} = \frac{3}{16} = .1875$, and $p_{30} = \frac{4}{16} = .2500$.

With $n = 195 + 73 + 100 = 368$, the expected counts are $207.000, 69.000$, and 92.000, so

$$\chi^2 = [\frac{(195-207)^2}{207} + \frac{(73-69)^2}{69} + \frac{(100-92)^2}{92}] = 1.623.$$

With $d.f = 3-1 = 2$, our χ^2 value of 1.623 is less than $\chi^2_{.10,2} = 4.605$. Thus, the p-value $> .10$, which is not $< .05$, and we cannot reject H_0. The data does confirm the $9:3:4$ theory.

Regular homework

9.24 First of all we shall note that

$$S_1^2/S_2^2 = \frac{0.66^2}{0.39^2} = 2.8639 > 2.$$

Therefore, we cannot assume that $\sigma_1 = \sigma_2$. Hence, we shall use the formula on p. 367, instead of the pooled t procedure.

A 95% confidence interval for the difference between the true firmness of zero-day apples and the true firmness of 20-day apples is

$$(8.74 - 4.96) \pm t_{0.025,v}\sqrt{\frac{0.66^2}{20} + \frac{0.39^2}{20}}.$$

We calculate the degrees of freedom $v = \frac{(\frac{0.66^2}{20} + \frac{0.39^2}{20})^2}{\frac{(\frac{0.66^2}{20})^2}{19} + \frac{(\frac{0.39^2}{20})^2}{19}} = 30.83$ so we use d.f.=30, and $t_{0.025,30} = 2.042$, so the interval is $3.78 \pm 2.042(0.17142) = (3.43, 4.13)$. Thus, with 95% confidence, we can say that the true average firmness for zero-day apples exceeds that of 20-day apples by between 3.43 and 4.13 N.

9.33 Let μ_1= the true average weight gain for steroid treatment and let μ_2= the true average weight gain for the population not treated with steroids. The exercise asks if we can conclude that μ_2 exceeds μ_1 by mo re than 5g., which we can restate in the equivalent form:$\mu_1 - \mu_2 < -5$. Therefore , we conduct a lower tailed test of $H_0 : \mu_1 - \mu_2 = -5 vs. H_a : \mu_1 - \mu_2 < -5$. The test statistic is $t = \frac{(\overline{x}-\overline{y})-(\delta)}{\sqrt{\frac{s_1^2}{m}+\frac{s_2^2}{n}}} = \frac{(32.8-40.5)-(-5)}{\sqrt{\frac{2.6^2}{8}+\frac{2.5^2}{10}}} = \frac{-2.7}{1.2124} = -2.23 \approx 2.2$. The approximation d.f. is $v = \frac{(\frac{2.6^2}{8}+\frac{2.5^2}{10})}{\frac{\frac{2.6^2}{8}^2}{7}+\frac{\frac{2.5^2}{10}^2}{9}} = \frac{2.1609}{0.1454} = 14.876$, which we round down to 14. The p-value for a lower tailed test is $P(t < -2.2) = P(t > 2.2) = 0.22$. Since this p-value is larger than the specified significance level 0.01, we cannot reject H_0. Therefore, this data does not support the belief that average weight gain for the control group exceeds that of the steroid group by more than 5g.

9.34

a. Following the usual format for most confidence intervals:

statistics $\pm$ (critical value)(standard error),

a pooled variance confidence interval for the difference between two means is

$(\overline{x} - \overline{y}) \pm t_{\frac{\alpha}{2},m+n-2} . s_p \sqrt{\frac{1}{m} + \frac{1}{n}}$.

b. The sample means and standard deviations of the two samples are $\overline{x} = 13.90, s_1 = 1.225, \overline{y} = 12.20, s_2 = 1.010$. The pooled variance estimate is $s_p^2 = (\frac{m-1}{m+n-2})s_1^2 + (\frac{n-1}{m+n-2})s_2^2 = (\frac{4-1}{4+4-2})1.225^2 + (\frac{4-1}{4+4-2})1.010^2 = 1.260$, so $s_p = 1.1227$. With $df = m + n - 1 = 6$ for this interval, $t_{0.025,6} = 2.447$ and the desired interval is $(13.90 - 12.20) \pm (2.447)(1.1227)\sqrt{\frac{1}{4} + \frac{1}{4}} = 1.7 \pm 1.943 = (-.24, 3.64)$. This interval contains 0, so it does not support the conclusion that the two population means are different.

c. The confidence interval is given by equation (9.2) on p. 366:

$$(\overline{x} - \overline{y}) \pm t_{\frac{\alpha}{2},\nu} . \sqrt{\frac{s_1^2}{m} + \frac{s_2^2}{n}},$$

where

$$v = \frac{(\frac{1.225^2}{4} + \frac{1.010^2}{4})^2}{\frac{(\frac{1.225^2}{4})^2}{3} + \frac{(\frac{1.010^2}{4})^2}{3}} = 5.7896 \approx 5.$$

Hence, the answer is

$$\begin{aligned} & 13.90 - 12.20 \pm t_{0.025,5} . \sqrt{\frac{1.225^2}{4} + \frac{1.010^2}{4}} \\ = \; & 1.7 \pm 2.571 * 0.79384 = 1.7 \pm 2.041 \\ = \; & (-0.341, 3.741). \end{aligned}$$

9.35 There are two changes that must be made to the procedure we currently use. First, the equation used to compute the value of the t test statistic is: $t = \frac{(\overline{x}-\overline{y})-(\delta)}{s_p\sqrt{\frac{1}{m}+\frac{1}{n}}}$ where s_p is defined as in Exercise 34 above. Second, the degrees of freedom = m+n-2. Assuming equal variances in the situation from Exercise 33, we calculate s_p as follows: $s_p = \sqrt{(\frac{7}{16})(2.6)^2 + (\frac{9}{16})(2.5)^2} = 2.544$. The value of the test statistic is, then, $t = \frac{(32.8-40.5)-(-5)}{2.544\sqrt{\frac{1}{8}+\frac{1}{10}}} = -2.24 \approx -2.2$. The degrees of freedom=16, and the p-value is $P(t < -2.2) = 0.021$. Since, $0.21 > 0.01$, we fail to reject H_0. This is the same conclusion reached in Exercise 33.

9.40

a. H_0 will be rejected in favor of H_a if either $t \geq_{0.005,15} = 2.947$ or $t \leq -2.947$. The summary quantities are $\overline{d} = -5.44$, and $s_d = 0.714$, so $t = \frac{-.544}{.1785} = -3.05$. Because $-3.05 \leq -2.947, H_0$ is rejected in favor of H_a.

b. $s_p^2 = 7.31, s_p = 2.70$, and $t = \frac{-.544}{.96} = -.57$, which is clearly insignificant; the incorrect analysis yields an inappropriate conclusion.

9.50 Let $\alpha = 0.05$. A95% confidence interval is

$$(\hat{p}_1 - \hat{p}_2) \pm z_{\frac{\alpha}{2}}\sqrt{(\frac{\hat{p}_1\hat{q}_1}{m} + \frac{\hat{p}_2\hat{q}_2}{n})} = (\frac{224}{395} - \frac{126}{266}) \pm 1.96\sqrt{(\frac{(\frac{224}{295})(\frac{171}{395})}{395} + \frac{(\frac{126}{266})(\frac{140}{266})}{266})}$$

$$= 0.0934 \pm 0.0774 = (0.0160, 0.1708).$$

9.52 Let p_1= true proportion of irradiated bulbs that are marketable; P_2=true proportion of untreated bulbs that are marketable; The hypotheses are $H_0 : p_1 - p_2 = 0 vs. H_0 : p_1 - p_2 > 0$. The test statistic is $z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}_1\hat{q}_2(\frac{1}{m}+\frac{1}{n})}}$. With $\hat{p}_1 = \frac{153}{180} = 0.850$, and $\hat{p}_2 = \frac{119}{180} = 0.661, \hat{p} = \frac{272}{360} = 0.756, z = \frac{.850-.661}{\sqrt{(.756)(.244)(\frac{1}{180}+\frac{1}{180})}} = \frac{.189}{.045} = 4.2$ The p-value $=1 - \Phi(4.2) \approx 0$, so reject H_0 at any reasonable level. Radiation appears to be beneficial.