W3600 HWK 4 Solution

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62. a.

$$P(X=2)=h(2;5,6,15)=\binom{6}{2}\binom{9}{3}\Big/\binom{15}{5}=\frac{1260}{3003}=0.42$$

b.

$$\begin{aligned}P(X &\le 2)=h(0;5,6,15)+h(1;5,6,15)+h(2;5,6,15)\\ &= 0.714\end{aligned}$$

c. $P(X\ge 1)=1-P(X=0)=1-h(0;5,6,15)=1-0.042=0.958$

d. $E(X)=(5*6)/15=2$, $Var(X)=\frac{(15-5)(5)(6)(15-6)}{15^2(15-1)}=0.857.$

70. The only possible values of X are 3, 4, and 5.

$P(3)=P(X=3)=P$(first 3 are B's or first 3 are G's)$=2(0.5)^3=0.250$.

$P(4)=P(X=4)=P$(two among the first three are B's and the fourth is a B)$+P$(two among the first three are G's and the 4th is a G)$=2\binom{3}{2}(0.5)^4=0.375$.

$P(5)=1-P(3)-P(4)=0.375$.

72. The interpretation of "roll" here is a pair of tosses of a single player's die (two tosses by A or two tosses by B). With S=doubles on a particular roll, $p=1/6$. Furthermore, A and B are really identical (each die is fair). Thus, we can equivalently imagine A rolling until 10 doubles appear. Then $P(X=x)$=P(9 doubles among the first $x-1$ rolls and a double among the xth roll), and

$$P(X=x)=\binom{x-1}{9}\left(\frac{5}{6}\right)^{x-10}\left(\frac{1}{6}\right)^{9}\left(\frac{1}{6}\right)=\binom{x-1}{9}\left(\frac{5}{6}\right)^{x-10}\left(\frac{1}{6}\right)^{10},$$

$x=10,11,...$

Let $Y=X-10$. Then

$$P(Y=y)=P(X=10+y)=\binom{y+10-1}{9}\left(\frac{5}{6}\right)^{y}\left(\frac{1}{6}\right)^{10},$$

and Y is a negative binomial random variable with $r=10$, $p=1/6$. Thus,

$$E(X)=10+E(Y)=10+\frac{10(1-1/6)}{1/6}=60,$$

$$Var(X) = Var(Y) = \frac{10(1-1/6)}{(1/6)^2} = 300.$$

76 a. $P(X=1) = e^{-0.2}(0.2)^1/1! = 0.164.$
b. $P(X \geq 2) = 1 - P(X=0) - P(X=1) = 1 - e^{-0.2} - 0.164 = 0.017.$
c. P(the first doesn't and the second doesn't)=P(the first doesn't)*P(the second doesn't)=$e^{-0.2}e^{-0.2} = 0.670$.

78 a. The experiment is bino(10,000, 0.005). So $\mu = np = 5$, and $\sigma = \sqrt{np(1-p)} = 2.2355$.
b. X can be approximated by a Poisson distribution with $\lambda = 5$. So $P(X > 10) = 1 - P(X \leq 5) = 1 - F(10; 5) = 1 - 0.986 = 0.014$, from the Poisson table.

86. a. $P(X = 10$ and no violations$)=P($no violations$|X = 10)P(X = 10)$

$$\begin{aligned} &= 0.5^{10}\{F(10;10) - F(9;10)\} \\ &= 9.7656 \times 10^{-4}(0.125) = 0.000122. \end{aligned}$$

b. P(y arrive and exactly 10 have no violations)=P(exactly 10 have no violations| y arrive)P(y arrive)

$$= \binom{y}{10} 0.5^{10} 0.5^{y-10} \cdot e^{-10}\frac{10^y}{y!} = \frac{e^{-10}5^y}{10!(y-10)!}.$$

c.

$$\begin{aligned} & P(\text{exactly 10 without a violation}) \\ = & \sum_{y=10}^{\infty} \frac{e^{-10}5^y}{10!(y-10)!} \\ = & \frac{e^{-10}5^{10}}{10!} \sum_{y=10}^{\infty} \frac{5^{y-10}}{(y-10)!} \\ = & \frac{e^{-10}5^{10}}{10!} \sum_{i=0}^{\infty} \frac{5^i}{i!} \\ = & \frac{e^{-10}5^{10}}{10!} e^5 = \frac{e^{-5}5^{10}}{10!}. \end{aligned}$$

105. The number sold is $\min(X, 5)$. Thus,

$$\begin{aligned}
& E(\min(X,5)) \\
= & \sum_{i=0}^{\infty} \min(i,5) P(X=i) \\
= & P(X=1) + 2P(X=2) + 3P(X=3) + 4P(X=4) + 5\sum_{i=5}^{\infty} P(X=i) \\
= & 1.735 + 5 * (1 - P(X \leq 4)) \\
= & 3.59
\end{aligned}$$