Discretion Versus Commitment with Information Acquisition

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Abstract

This paper revisits the classic discretion versus commitment debate when a policymaker issues signals and agents choose to ignore them or not. Besides responding with discretion to agents' actions or committing to a plan ex ante, a policymaker can also choose an adaptation policy, where she commits after agents make learning decisions but before agents take actions. Two mechanisms are at work in the choice of monetary policy. First, the policymaker could give agents incentives to learn if she commits before agents make learning decisions. Second, a discretionary policy can react to the aggregate shock flexibly, track agents' actions closer, and partly control the effect of noisy information. However, this flexibility results in an inflation bias and an information bias. Overall, commitment is always the most efficient policy and adaptation may be worse than discretion under certain circumstances, especially when the policy is less important to agents than their idiosyncratic conditions.

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1 Introduction

Most central banks make two types of decisions: how to change policy actions; and how to communicate with agents. At the same time, agents react to central banks' decisions based on their information of the economy. In the classic literature, such as Barro and Gordon (1983), agents are assumed to be perfectly informed, therefore they are affected by central banks' policy

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actions, but not their communication strategies. In reality, however, agents are always imperfectly informed. Therefore, they should be affected by both types of decisions made by central banks. Moreover, central bank decisions on policy action and communication strategy can be incoherent, e.g. commitment in one and discretion in the other, hence their effects to agents need to be investigated separately. Specifically, in terms of communication, the central bank typically has two strategies. The first one is to commit to real information and have agents absorb it by having preannounced events such as FOMC meetings (commitment); the second one is to change policy reactions according to the agents' information set of the aggregate condition (discretion).

This paper investigates whether the central bank should commit to both policy action and information. In particular, it investigates which kind of policy is better with endogenous information acquisition in terms of time consistency. In the classical literature, such as Barro and Gordon (1983) and Kydland and Prescott (1977), commitment eliminates the inflation bias, while discretion allows flexible response to shocks. However, this conclusion relies on rational expectation with perfect information. This paper presents a novel model in which agents are allowed to have imperfect information, and they endogenously choose whether to acquire information or not. This model reveals that committing to both action and information is the most efficient strategy. However, committing to only action may be worse than committing to neither action nor information.

The presented model extends the classic Barro-Gordon problem to include endogenous information acquisition. The policymaker faces a trade-off between tracking agents' aggregate action more closely and tracking the aggregate shock more closely, also known as the trade-off between output and inflation. Moreover, it is assumed that she has perfect knowledge of the aggregate economic shock, but can not communicate perfectly with agents. At the same time, each agent tries to minimize the total distance from his action to aggregate agents' action, idiosyncratic shock, and monetary policy. However, following earlier work by Sims (2003), agents are rationally inattentive. Their inattentiveness gives rise to idiosyncratic information errors. Agents face a trade-off between better informed about the aggregate shock versus better informed about the idiosyncratic shock. Therefore, agents now have two decisions to make: how to allocate their limited attention between the aggregate shock and the idiosyncratic shock, and how to act given their information. The key objective of this model is to determine when the central bank should move with respect to these two decisions. There are three alternatives: 1) the commitment rule which sets policy before agents' two decisions; 2) the discretionary rule which chooses policy after agents' two decisions; 3) the adaptation rule which reacts after agents' learning decision but before agents' reaction, so it commits to its policy action but exercises discretion on information.

Based on the proposed model, the endogenous information can introduce two different effects, depending on the timing of the policy. One is the *learning incentive effect*. Policy is a reaction function of the real aggregate shock, while agents' action is a reaction function of the expected aggregate shock. As a result, inaccurate information differentiates agents' action from the policy, which results in output loss. This loss is small when agents put larger learning effort on the aggregate shock. Notably, the aggregate shock affects the agents' decision through its role in monetary policy. As a result, if the policy commits to reacting more on the aggregate shock, agents will have a larger incentive to learn that shock. Thus, committing monetary policy before agents' learning decisions can affect that learning decision. This effect is referred to in this paper as the learning incentive effect.

The second effect is the *flexible policy setting effect*, which has been discussed in the existing literature. Moving after agents' decisions allows the discretionary policymaker to flexibly respond to agents' aggregate action and the aggregate shock. However, flexibility results in not only the classical inflation bias but also a novel *information bias*, which induces high inflation and insufficient learning. Notably, when the learning decision is exogenous and a policy is less sensitive to the aggregate shock, the output loss is smaller, because the output gap is induced by the inaccurate information of agents. Consequently, a discretionary policymaker would like to react less on the aggregate shock in comparison with a commitment policymaker. Ex ante, agents further reduce their learning effort on the aggregate shock anticipating that the discretionary policymaker will accommodate their information imperfection ex post. As a result, agents have less accurate information on the aggregate shock. This effect is referred to in this paper as the information bias. Furthermore, an adaptation policymaker moves after the agents' learning decision and also generates an information bias.

Given these two effects, the model suggests that the optimal monetary policy in a Bayesian equilibrium is to commit both action and information before agents make any decisions. Commitment policy has a learning incentive effect, while the other two policies have flexibly policy setting effects but also face the information bias. The efficiency gap between commitment and discretion is even higher than it is in a perfect information framework. Also, under certain circumstances, adaptation, which only commits to action, is worse than discretion, because it has a larger information bias.

Literature Review. This paper follows the recent work on information frictions and endogenous information acquisition. The survey paper by Mankiw and Reis (2011) discusses the foundations for two classes of imperfect information models — partial information and delayed information. This paper contributes to the rational inattention literature with a study on commitment and discretion, which most previous work on endogenous information ignores. In terms of modeling the information acquisition, the model in this paper follows Reis (2011), which presents a dynamic model with rational inattention of the private sector and introduces the optimal timing of public announcements. However, unlike Reis (2011), the model proposed in this paper is static, because the question is about the effectiveness of commitment and discretionary monetary policy. In addition, this paper has a connection with the recent literature on transparency. Blinder et al. (2008) surveys the ever-growing literature on the communication of the monetary policy. Morris and Shin (2002) discusses the possibility that agents "over-coordinate" on public information. Chahrour (2013) presents the optimal quantity of communication of the authority to the public. This paper addresses the difference between commitment and discretion along with the efficiency study of communication transparency.

The novelty of this paper is the study on commitment and discretion in a rational inattention framework. Adam (2007, 2009) also studies commitment and discretion when firms pay limited attention to aggregate variables. These papers emphasize that ambitious attempts to stabilize the real economy via discretion could increase real and nominal aggregate volatility. The current paper discusses the welfare efficiency of different policies with the policymaker sending public signals and agents facing a trade-off between learning aggregate and idiosyncratic shocks. Also given that agents take two actions, there is a third policy studied in this paper, adaptation. Above all, the proposed model also provides some economic insight about the communication between the central bank and the private sector.

Outline. In Section 2, the Barro-Gordon economy with endogenous information is set up. Section 3 then presents the techniques of the commitment, discretion, and adaptation equilibria. After providing the equilibrium conditions and the policy biases of this model in Section 4, Section 5 studies policy efficiency. Section 6 discusses the role of the policy weight on the inflation gap, the aggregate shock volatility, and the agents learning capacities. Section 7 delves deeper on the objective function as well as further issues such as public signal property, output shocks, and public debt. Also, it discusses the key differences between Adam (2007) and this paper. Section 8 concludes.

2 Set up

This is a static game with a continuum of private agents indexed by i and uniformly distributed over the unit interval, and a central bank policymaker.

2.1 Central Bank

The central bank in this economy is concerned with both curbing inflation and stimulating the economy. The policymaker chooses the level of inflation, π, to minimize the following quadratic loss function:

$$Min \quad \mathbb{E}\{\underbrace{[(\pi - a) - b]^2}_{\mathcal{L}_O} + \underbrace{\lambda\left(\pi - \pi^* - \bar{\theta}\right)^2}_{\mathcal{L}_I}\}. \tag{1}$$

This is a reduced form objective function derived from a standard expectation augmented Philips curve. The first part of the loss, $\mathcal{L}_O$, is the output gap loss. In particular, the output gap is a function of the inflation deviation from the expected inflation, where the expected inflation in this model corresponds to the aggregate action of agents a. b is the output target of the central bank, which might reflect the distortion from monopolistic competition. The second part of the loss, $\mathcal{L}_I$, is the cost of inflation deviation from the inflation target, π^*. λ is the weight attached to price stability, a parameter capturing the degree of the "conservatism" of the central bank. The aggregate shock to this economy is:

$$\bar{\theta} \sim \mathcal{N}(0, \sigma_\theta^2).$$

It could be a shock to the inflation target or to the central bank's preference. According to this quadratic loss function, the central bank always sets inflation in order to balance the trade-off between the distance from the aggregate shock $\bar{\theta}$ and agents' action a.

The information structure of this model is as follows. The policymaker has perfect knowledge of $\bar{\theta}$. However, she cannot communicate it perfectly with agents, but can only provide them a noisy signal θ, which is a draw from a normal distribution with mean $\bar{\theta}$,

$$\theta = \bar{\theta} + \epsilon, \quad \text{with} \quad \epsilon \sim \mathcal{N}(0, \sigma_\epsilon^2), \tag{2}$$

where $\sigma_\epsilon^2 \leq \sigma_\theta^2$. Agents do not have perfect information on $\bar{\theta}$. Instead, they get the noisy public signal θ and they can make an extra learning effort to increase their information accuracy. Without private learning, agents' expectations on the true aggregate shock $\bar{\theta}$ is equal to its prior θ.

One possible interpretation of this imperfect communication is that the private sector has limited ability to understand the complicated economic situation announced by the central bank. Furthermore, some monetary authorities make less transparent announcements on purpose.[1] Later in Section 7, a scenario where the policymaker does not send any signal will be discussed. In either case, agents will be able to learn about the aggregate shock with some cost. This paper focuses on the effect of imperfect communication on the optimal conduct of monetary policy under a rational inattention framework.

2.2 Private Sector

The setup of the private sector is similar to Reis (2011), but the focus in on commitment versus discretion. Agents in this economy face a trade-off dividing their limited attention between idiosyncratic shock and aggregate shock. Each agent chooses an action a_i based on a minimization problem,

$$\min_{a_i} \quad \mathbb{E}_i \left[a_i - (1-\alpha)a - \alpha\pi - \omega u_i \right]^2, \tag{3}$$

where $\mathbb{E}_i$ refers to the expectations operator conditional on the information set of agent i. Each agent tries to minimize the distance from the movements of three variables, the aggregate action (a), inflation or monetary policy (π), and idiosyncratic shock (u_i). Agents face a quadratic loss function if a_i deviates from the target. Thus, each agent's action depends on his information set,

$$a_i = \mathbb{E}_i[(1-\alpha)a + \alpha\pi + \omega u_i], \tag{4}$$

[1] For example, this is discussed in Blinder et al. (2008) and Chahrour (2013).

where

$$a = \int_0^1 a_i di. \tag{5}$$

The parameter α in Equation (4) varies between zero and one. It determines the strength of strategic complementarity in this economy. If $\alpha = 1$, actions are strategically independent. The actions of the other agents are not important and agents only care about the monetary policy π and u_i. On the other hand, if $\alpha = 0$, agents pay full attention to the actions of the others. Besides, the last factor u_i refers to the idiosyncratic shock to each agent. It is an exogenous stochastic process and pairwise independent across agents, i.e. $\int u_i di = 0$. u_i is a random i.i.d. drawn from a normal distribution with mean 0 and variance ϕ^2, i.e.

$$u_i \sim \mathcal{N}(0, \phi^2).$$

The parameter ω in Equation (4) represents the relative importance to the agents of the idiosyncratic condition versus the aggregate condition.

To minimize the objective function (Equation (3)), agents need to learn their idiosyncratic shock (u_i) and the monetary policy (π). The monetary policy is closely related to the aggregate shock ($\bar{\theta}$). As a result, agents need to pay attention to both u_i and $\bar{\theta}$. However, they have limited learning capacity and face the attention trade-off. Firstly, if agents pay attention to u_i, they could increase their information accuracy. The posterior variance of u_i will be $\phi^2 e^{-y_i}$, where $y_i \geq 0$ denotes the reduction in variance, thus represents the agents' learning effort on u_i.[2] At the same time, the agents' prior belief on the aggregate shock ($\bar{\theta}$) is the noisy public signal (θ). Also, with an additional learning effort on $\bar{\theta}$, agents will get a private signal θ_i.[3] Subsequently, the variance of agents' posterior beliefs ($\hat{\theta}_i$) becomes $\sigma_\epsilon^2 e^{-x_i}$, where $x_i \geq 0$ is the reduction in variance and represents the learning effort on $\bar{\theta}$. Updating Bayes rule, the posterior of agents' beliefs on $\bar{\theta}$ is a linear interpolation of the private and the public signals,

$$\hat{\theta}_i = (1 - e^{-x})\theta_i + e^{-x}\theta. \tag{6}$$

[2] If the signal has zero (infinite) precision, then $y_i = 0$ ($y_i = \infty$).

[3] Through learning, agents get idiosyncratic private signals. The integral of this signal equals to the true shock, i.e. $\int_0^1 \theta_{it} = \bar{\theta}$.

The derivation of Equation (6) is presented in Section A.1 of the online Appendix.

Thus, through their choice on $\{x_i, y_i\}$, agents can control their information accuracy on the aggregate shock and the idiosyncratic shock. If there is no limit on their learning capacity, agents will choose both x_i and y_i equal to ∞ and the economy becomes a perfect information economy. To be more realistic, it is assumed that agents face the following constraint on learning capacity:

$$x_i + y_i \leq 2k. \tag{7}$$

This constraint follows Reis (2011) and is motivated by the rational inattention literature. It indicates that agents cannot learn everything. Instead, they face a trade-off between allocating their limited attention to aggregate shocks and idiosyncratic shocks.

2.3 Linear Monetary Policy

In regards to the central bank's decision, this paper focuses on linear Markovian strategies. In particular, the monetary policy is assumed to be an affine function of state variables $\bar{\theta}$ and θ, i.e.

$$\pi = p_0 + p_1\bar{\theta} + p_2\theta, \tag{8}$$

where p_0, p_1, and p_2 are the policy choice variables. Specifically, p_1 and p_2 are the weights the policymaker put on the aggregate shock and the public signal. It is notable that π also depends on the noisy signal θ. This is because that agents' action a depends on their posterior of the aggregate shock, which is a linear interpolation of $\bar{\theta}$ and θ (Equation (6)).[4] Therefore, with the objective function (1), the central bank would potentially put policy weight on θ, in order to reduce the output gap. Besides, the policymaker can choose $p_2 = 0$.

2.4 Agents' Information Acquisition Solution

Now that the problem is set up, this subsection presents the solution to the information acquisition problem, which is the first problem to solve the model. To recap, the central bank chooses $\{p_0, p_1, p_2\}$ to minimize their objective function (1), while agents choose $\{x_i, y_i, a_i\}$ to minimize

[4]Each agent's posterior is a linear interpolation of θ_i and θ, Equation (6). The aggregate posterior is a linear interpolation of $\bar{\theta}$ and θ, because $\int_0^1 \theta_i = \bar{\theta}$. Therefore, a depends on $\bar{\theta}$ and θ.

function (3).

To solve the optimal learning decision $\{x_i, y_i\}$, a guess about the aggregate action a is made. With complete information, the aggregate action would be perfectly consistent with monetary policy, i.e. $a = \pi$, which minimizes Equation (3). With incomplete information, the following starting guess is made:

$$a = p_0 + p_1[\gamma\bar{\theta} + (1-\gamma)\theta] + p_2\theta, \tag{9}$$

where γ is a variable governed by the learning effort. The value of γ is to be determined later. $\gamma\bar{\theta} + (1-\gamma)\theta$ identifies agents' posterior of the aggregate shock $\bar{\theta}$.[5] Notably, γ is a crucial parameter in this model, which determines agents' learning effort on the aggregate shock $\bar{\theta}$.

As stated in Section A.1 of the online Appendix, plugging Equation (9) into agents' objective function (3) gives the optimal choice of a_i in terms of γ. Moreover, the integral of a_i should be equal to a as proposed in Equation (9). Thus, there is an equilibrium condition between γ and x_i, which also confirms the guess of a in the form of Equation (9). Plugging Equation (9) into Equation (3) gives the optimal learning effort of agents on the aggregate shock $\bar{\theta}$,

$$x_i^* = max\{k + \ln(\frac{\sigma_\epsilon}{\phi}) + \ln(\frac{(1-\alpha)\gamma + \alpha}{\omega}) + \ln p_1, 0\}. \tag{10}$$

Introducing $z \equiv \frac{\omega\phi}{\alpha e^k}$, Equation (10) implies that if $z < \sigma_\epsilon p_1$, then $x_i^* > 0$. Subsequently, y_i^* equals $2k - x_i^*$ (see Equation (7)). z governs the relative importance of individual circumstances. On the one hand, the larger z is, the less important the aggregate condition is to agents. Parameters $\{\omega, \phi, \alpha, k\}$ determine the value of z. For example, the larger ω is, the more weight agents will put on their idiosyncratic shocks and the less they will care about the aggregate economics condition and the monetary policy. A larger ϕ implies higher loss due to imperfect learning of their individual condition; thus, they also pay less attention to the aggregate shock. Furthermore, the learning capacity, thus agents learning effort increases as k increases. On the other hand, if z decreases, agents would increase their attention to the monetary policy, thus the aggregate shock. For example, as α increases, the loss of deviation from π is larger; and agents will put larger learning effort on the aggregate shock. Essentially, Equation (10) indicates a bijection between γ and $\{x_i, y_i\}$. Henceforth,

[5] According to Equation (6) and the information structure presented in the online Appendix A.1, a conjecture of the aggregate agents' posterior belief on $\bar{\theta}$ is $\gamma\bar{\theta} + (1-\gamma)\theta$

γ is referred to as the *learning effort* on the aggregate shock $\bar{\theta}$, while $\{x_i, y_i\}$ will not be emphasized.

γ identifies the accuracy of agents' information on aggregate shock $\bar{\theta}$. According to Equation (9), if $\gamma = 0$, there is no learning on $\bar{\theta}$ and the posterior of $\bar{\theta}$ equals to agents' prior θ. If $\gamma = 1$, there is perfect learning and the posterior equals the true aggregate shock. As proved in the online Appendix, γ satisfies the following proposition:

Proposition 1. *In equilibrium,*

$$\gamma = \begin{cases} 1 - \frac{z}{\sigma_\epsilon p_1}, & \textit{if } z < \sigma_\epsilon p_1, \\ 0, & \textit{if } z \geq \sigma_\epsilon p_1, \end{cases} \tag{11}$$

where p_1 is the weight the policymaker puts on the aggregate shock. Notably, when there is learning, γ is an increasing function of p_1.

Note that Equation (11) identifies two regimes in this economy. The first one, characterized by $z < \sigma_\epsilon p_1$, is referred to as the *Learning Regime*, because there is private learning on the aggregate shock. The other one, characterized by $z \geq \sigma_\epsilon p_1$, is referred to as the *No-Learning Regime*, because in this regime agents put zero learning effort on the aggregate shock. The main property to focus on here is that as p_1 increases, the policymaker puts a higher weight on $\bar{\theta}$, giving agents higher learning incentives. Therefore, γ is an increasing function of p_1.

3 Equilibrium under Different Timing

After setting the problem of the policymaker and the private sector, this section defines the equilibrium condition of this model. In this economy, agents take two actions. First, they make learning decisions, i.e. the level of γ. Second, they take action a_i. The key of this model is the timing of the monetary policy with respect to the two actions of agents. Thus, this section discusses the central bank's commitment technologies on action and information, depending on the order of play. There are three kinds of rules — commitment, discretion, and adaptation. The equilibrium under each rule is a perfect Bayesian equilibrium.

3.1 Commitment Monetary Policy

Definition 1 (Commitment). *A commitment rule is set before agents make decisions on γ and a. After announcing the public signal θ, the policymaker chooses $\{p_0, p_1, p_2\}$ to minimize* (1) *expecting agents' optimal decision rule. Afterwards, each agent chooses his optimal level of $\{x_i, y_i, a_i\}$ to minimize* (3) *subject to* (7) *under perfect knowledge of $\{p_0, p_1, p_2\}$. They use Bayes rule to update their belief on the aggregate shock $\bar{\theta}$ according to* (6)*. The perfect Bayesian equilibrium under this commitment technology satisfies the equilibrium condition* (11).

The commitment policymaker commits both in action and in information. The policy is set before agents make learning decisions and take actions; its time-line is presented in the top row of Figure 1. In the real world, the central bank holds FOMC meetings every month and makes announcements afterward. Agents make learning decisions after receiving the public signal and the commitment from the central bank. Afterwards, they take actions based on their updated information set. For example, on December 12, 2012, the FOMC announced that the interest rate would remain at historic lows as long as the unemployment rate was above 6.5 percent, medium-term information forecasts stayed below 2.5 percent, and the long-run inflation expectations remained anchored. Under this commitment, agents would make learning effort especially on the key indicators stated in the public announcement, such as the unemployment rate before taking actions. And, the policymaker has committed to this policy so far.

3.2 Discretionary Monetary Policy

Definition 2 (Discretion). *A discretionary rule is set after agents make decisions on γ and a. After observing the public signal θ, each agent chooses his optimal level of $\{x_i, y_i, a_i\}$ to minimize* (3) *subject to* (7) *using expected $\{p_0, p_1, p_2\}$. They use Bayes rule to update their belief on the aggregate shock $\bar{\theta}$ according to* (6)*. Afterward, the policymaker chooses $\{p_0, p_1, p_2\}$ to minimize* (1) *under the perfect knowledge of the aggregate learning effort γ and action a. The perfect Bayesian equilibrium under this discretionary technology satisfies the equilibrium condition* (11).

The discretionary policymaker commits on neither information nor action. The timing of the economy under discretion is illustrated in the second row of Figure 1. In reality, the late 1960s and 1970s were a period where the central banks made many short-term fine-tuning without long-term thinking or commitment. In addition, in the real world, we have Calvo pricing model or menu-cost model, where some decisions of firms are made in advance. The timing of this game can be considered as a simple transform of these kinds of price setting models.

3.3 Adaptation Monetary Policy

Definition 3 (Adaptation)**.** *An adaptation rule is set after agents make learning decisions γ, but before agents take action a. After observing the public signal (θ), each agent chooses his optimal level of $\{x_i, y_i\}$ according to equation (10) using expected p_1. Afterward, the policymaker chooses $\{p_0, p_1, p_2\}$ to minimize* (1) *knowing aggregate learning effort γ. In the end, each agent takes action a_i to minimize* (3) *under perfect knowledge of $\{p_0, p_1, p_2\}$. The perfect Bayesian equilibrium under this adaptation technology satisfies the equilibrium condition* (11)*.*

The adaptation policymaker commits before agents take action but after they acquire information. The timing of the economy under adaptation rule is illustrated in the last row of Figure 1. Learning technology is probably fixed before firms or agents take actions. For instance, the fixed cost on technology investment may be very large. Agents revise their learning decision less frequently than monetary policy revisions. Thus, it is possible that γ is chosen before the monetary policy. Another example could be the unconventional monetary tool used during the recent financial crisis.

On November 25, 2008, the FOMC announced its QE1 decision,

> "This action is being taken to reduce the cost and increase the availability of credit for the purchase of houses, ... Further information regarding the operational detail of this program will be provided after consultation with market participants."

After this announcement, agents tried to understand this unconventional decision before making learning decisions and forming their expectations. Economists or correspondents discussed it through media. The central bank had not really done anything yet. As stated in the quote, the central bank would take further steps after consulting market's anticipation. This can be interpreted as a kind of adaptation rule.

4 Equilibrium

Before proceeding to solve this model, it is useful to revisit the classic model where information is perfect. With perfect information, the policymaker communicates the aggregate shock $\bar{\theta}$ perfectly to agents, i.e. $\epsilon = 0$ in Equation (2). In addition, agents know their idiosyncratic shock u_i precisely. This is the same economy as in Barro and Gordon (1983) and Kydland and Prescott (1977). In this case, the equilibrium conditions of commitment and discretion are

$$\begin{aligned} \text{[Inflation]}: \quad \pi^C &= \pi^A = \pi^* + \bar{\theta}, \\ \pi^D &= \pi^* + \bar{\theta} + \frac{b}{\lambda}, \end{aligned}$$

and

$$\begin{aligned} \text{[Total Loss]}: \quad \mathcal{L}^C &= \mathcal{L}^A = b^2, \\ \mathcal{L}^D &= b^2 + \frac{b^2}{\lambda}. \end{aligned}$$

With perfect information, commitment and adaptation are identical, while discretion has an inflation bias $\frac{b}{\lambda}$. Thus, commitment and adaptation are strictly better than discretion. However, with imperfect information and rational inattention, the properties of the optimal conduct of monetary policy become more complicated.

The key structure of this model is the wedge between the policymaker and agents. The policymaker faces a trade-off between tracking agents' actions a versus tracking the aggregate shock $\bar{\theta}$ (Equation (1)). Agents have a learning trade-off between better informed on u_i and better informed on $\bar{\theta}$ (Equation (3)). $\bar{\theta}$ does not affect agents' aggregate action a directly, but through policy π and agents' learning decision γ (Equation (9)). Accordingly, the output gap,

$$\pi - a - b = p_1(1-\gamma)(\bar{\theta} - \theta) - b, \tag{12}$$

is determined by γ and the policy rule p_1 (Equation (8) and Equation (9)). The inflation gap,

$$\pi - \pi^* - \bar{\theta} = p_0 - \pi^* + (p_1 - 1)\bar{\theta} + p_2\theta, \tag{13}$$

is determined by p_0, p_1, p_2. The key policy variable affecting the total loss is p_1, which has three effects. (1) γ is an increasing function of p_1 (the learning incentive effect, Equation (11)); (2) the output gap is an increasing function of p_1 but a decreasing function of γ (Equation (12)); (3) the inflation loss is a decreasing function of p_1 (Equation (13)).[6]

4.1 Commitment Rule

The key difference of commitment from the other rules is that under this condition, agents make a learning decision γ based on their perfect knowledge of $\{p_0, p_1, p_2\}$. Essentially, committing before the agents' learning decision brings the *learning incentive effect.* As p_1 increases, π becomes more sensitive with $\bar{\theta}$. Consequently, aggregate shock $\bar{\theta}$ becomes more important to agents (Equation (3)). Hence, γ is an increasing function of p_1 (Equation (11)). The learning incentive effect is the first effect introduced by the information structure of this model. The commitment policymaker will take learning incentives into consideration when choosing $\{p_0, p_1, p_2\}$. As derived in Section B.1 of the online Appendix, the equilibrium proposition of the commitment rule is:

Proposition 2. *Equilibrium under Commitment Rule*

(C1) There is private learning on aggregate shock ($\bar{\theta}$) if and only if $\left(\dfrac{z}{\sigma_\epsilon}\right)^2 < \dfrac{\lambda}{\lambda + 1 + \sigma_\epsilon^2/\sigma_\theta^2}$.

(C2) In the no-learning regime ($\gamma = 0$),

$$\pi^C(\bar{\theta}, \theta) = \underbrace{\pi^*}_{p_0^C} + \underbrace{\frac{\lambda}{\lambda + 1 + \sigma_\epsilon^2/\sigma_\theta^2}}_{p_1^C}\bar{\theta} + \underbrace{\frac{1}{\lambda + 1 + \sigma_\epsilon^2/\sigma_\theta^2}}_{p_2^C}\theta. \tag{14}$$

(C3) In the learning regime ($\gamma \neq 0$),

$$\text{[Inflation]} \quad : \quad \pi^C(\bar{\theta}, \theta) = \pi^* + \bar{\theta}, \tag{15}$$

$$\text{[Learning]} \quad : \quad \gamma^C = 1 - \frac{z}{\sigma_\epsilon}. \tag{16}$$

The commitment policymaker takes into account the three effects of p_1 when making policy decisions. In the no-learning regime ($\gamma = 0$), the output gap is an increasing function of p_1 while

[6] In this set up, the equilibrium p_1 and p_2 are between 0 and 1. Thus, $|p_1 - 1|$ is a decreasing function of p_1, which controls the inflation gap $\pi - \pi^* - \bar{\theta}$.

the inflation gap is a decreasing function of p_1, according to the second and the third effect. Consequently, the optimal commitment policy satisfies (C2) with $p_1^C \in (0,1)$. In the learning regime, the policymaker notices that the output loss, $\mathcal{L}_O = (\pi - a - b)^2 = z^2 + b^2$, is independent of p_0, p_1 and p_2 (combine the first and the second effect, see the online Appendix B.1). The policymaker takes only the third effect into consideration and chooses π to minimize the inflation loss $\mathcal{L}_I$. Consequently, the optimal commitment policy satisfies (C3) with $p_1^C = 1$ and $p_2^C = 0$.

Moreover, according to (11), the two learning regimes are determined by the policy variable p_1 and parameters z, σ_ϵ. With a learning incentive effect, the commitment policymaker can control the learning regime ex ante. Obviously, if there are two possible equilibria (one in the learning regime, the other one in the no-learning regime), the policymaker chooses the one with a smaller total loss.

4.2 Discretionary Rule

The advantage of discretion lies in its flexibility in tracking aggregate shocks and agents' actions. π is chosen after $\bar{\theta}$ and a are realized. The *flexible policy setting effect* is the second effect introduced in this model. The equilibrium conditions are:

Proposition 3. *Equilibrium under Discretionary Rule*

(D1) There is private learning on aggregate shock if and only if $\dfrac{z}{\sigma_\epsilon} < \dfrac{\lambda}{1+\lambda}$.

(D2) In the no-learning regime ($\gamma = 0$)*,*

$$\pi^D(\bar{\theta}, \theta) = \underbrace{\pi^* + \frac{b}{\lambda}}_{p_0^D} + \underbrace{\frac{\lambda}{1+\lambda}}_{p_1^D} \bar{\theta} + \underbrace{\frac{1}{1+\lambda}}_{p_2^D} \theta \tag{17}$$

(D3) In the learning regime ($\gamma \neq 0$)*,*

$$\text{[Inflation]} \quad : \quad \pi^D(\bar{\theta}, \theta) = \underbrace{\pi^* + \frac{b}{\lambda}}_{p_0^D} + \underbrace{\frac{\lambda}{\lambda + 1 - \gamma^D}}_{p_1^D} \bar{\theta} + \underbrace{\frac{1-\gamma^D}{\lambda + 1 - \gamma^D}}_{p_2^D} \theta \tag{18}$$

$$\text{[Learning]} \quad : \quad \gamma^D = \frac{\lambda - (1+\lambda)\frac{z}{\sigma_\epsilon}}{\lambda - \frac{z}{\sigma_\epsilon}} \tag{19}$$

The proof of this proposition is provided in Section B.2 of the online Appendix. The discretionary policymaker can track both the agents' actions and the realized shock flexibly. Nonetheless, this flexibility leads to a bias in information acquisition, which will be discussed in Section 4.4, as well as the inflation bias, which appears in π in the form of $\frac{b}{\lambda}$.

Unlike with commitment, there is no learning incentive effect, since γ is considered as an exogenous variable by the policymaker. Hence, the discretionary policymaker takes into account two effects — (1) the output gap is an increasing function of p_1; (2) the inflation gap is a decreasing function of p_1. Thus in both learning regimes, the optimal $p_1 \in (0,1)$. Particularly, in the learning regime, the discretionary policymaker does not take the equilibrium condition $\mathcal{L}_O = z^2 + b^2$ (independent of π) into consideration. Instead, she chooses π to minimize $\mathcal{L} = \mathcal{L}_O + \mathcal{L}_I$ for a given γ. Proposition (D3) indicates that $p_1^D < p_1^C = 1$. Consequently, $\gamma^D < \gamma^C$.

4.3 Adaptation Monetary Policy

The adaptation policymaker chooses π after she observes agents' learning decision γ, but before she knows aggregate action a. Obviously, the learning incentive effect disappears. Also, she is less flexible than the discretionary policymaker. Following the discussion in Section B.3 of the online Appendix, the equilibrium condition is:

Proposition 4. *Equilibrium under Adaptation Monetary Policy*

(A1) There is private learning on aggregate condition if and only if $\left\{\left(\frac{z}{\sigma_\epsilon}\right)^2 \leq \frac{\lambda}{4(1+\sigma_\epsilon^2/\sigma_\theta^2)}\right\}$ *and* $\left\{\frac{z}{\sigma_\epsilon} < min\left\{\frac{\lambda}{2(1+\sigma_\epsilon^2/\sigma_\theta^2)}, \frac{\lambda}{(\lambda+1+\sigma_\epsilon^2/\sigma_\theta^2)}\right\} \quad or \quad \frac{z}{\sigma_\epsilon} > \frac{\lambda}{2(1+\sigma_\epsilon^2/\sigma_\theta^2)}\right\}$.

(A2) In the no-learning regime ($\gamma = 0$),

$$\textit{[Inflation]} \quad : \quad \pi^A(\bar{\theta}, \theta) = \underbrace{\pi^*}_{p_0^A} + \underbrace{\frac{\lambda}{\lambda + 1 + \sigma_\epsilon^2/\sigma_\theta^2}}_{p_1^A}\bar{\theta} + \underbrace{\frac{1}{\lambda + 1 + \sigma_\epsilon^2/\sigma_\theta^2}}_{p_2^A}\theta \tag{20}$$

(A3) In the learning regime ($\gamma \neq 0$),

$$\text{[Inflation]} \quad : \quad \pi^A(\bar{\theta}, \theta) = \underbrace{\pi^*}_{p_0^A} + \underbrace{\frac{\lambda}{\lambda + (1-\gamma^A)^2(1+\sigma_\epsilon^2/\sigma_\theta^2)}}_{p_1^A}\,\bar{\theta} + \underbrace{\frac{(1-\gamma^A)^2}{\lambda + (1-\gamma^A)^2(1+\sigma_\epsilon^2/\sigma_\theta^2)}}_{p_2^A}\,\theta$$

$$\text{[Learning]} \quad : \quad \gamma^A = 1 - \frac{\lambda \pm \sqrt{\lambda^2 - 4\left(\frac{z}{\sigma_\epsilon}\right)^2 (1+\sigma_\epsilon^2/\sigma_\theta^2)\lambda}}{2\frac{z}{\sigma_\epsilon}(1+\sigma_\epsilon^2/\sigma_\theta^2)} \tag{21}$$

In the learning regime, two equilibrium γ^A exist. However, if $\frac{z}{\sigma_\epsilon} < min\{\frac{\lambda}{2(1+\sigma_\epsilon^2/\sigma_\theta^2)}, \frac{\lambda}{(\lambda+1+\sigma_\epsilon^2/\sigma_\theta^2)}\}$ or $\frac{\lambda}{2(1+\sigma_\epsilon^2/\sigma_\theta^2)} < \frac{z}{\sigma_\epsilon} < \frac{\lambda}{(\lambda+1+\sigma_\epsilon^2/\sigma_\theta^2)}$, only the one with a minus sign in front of the square root exists.

Unlike the commitment policymaker, the adaptation policymaker loses the learning incentive effect. Unlike the discretionary policymaker, she does not know a when making policy decisions. However, in the no-learning regime, the equilibrium condition under adaptation is the same as under commitment. Both adaptation and commitment policymakers commit to action. Without private learning, whether or not the policymaker commits to information is irrelevant. In the learning regime, the policymaker takes γ as given and notices different effects of p_1 on the output gap and the inflation gap. Similar to the discretionary policymaker, she does not take the equilibrium condition $\mathcal{L}_O = z^2 + b^2$ into consideration. Thus, proposition (A3) indicates that $p_1^A < p_1^C = 1$. Consequently, $\gamma^A < \gamma^C$.

4.4 Policy Biases

The presented model introduces two biases: the classical inflation bias and the novel information bias. The discretionary rule has an *inflation bias*, which comes from the result that p_0^D is larger than p_0^C and p_0^A. Additionally, discretion and adaptation introduce an *information bias*, which comes from the choice of p_1 and p_2. In the learning regime, $max\{p_1^D, p_1^A\} < p_1^C$ and $min\{p_2^D, p_2^A\} > p_2^C$. This bias indicates insufficient learning on aggregate shock $\bar{\theta}$. Both γ^D and γ^A are smaller than γ^C. It is a bias in the acquisition of information (how much agents choose to learn). In the no-learning regime, $p_1^D > p_1^C = p_1^A$ and $p_2^D > p_2^C = p_2^A$. With discretionary policy, agents set higher weight on the noisy public signal θ. The information bias is in the response to information (how much agents respond to the noisy signal).

This additional effect is not related to b in the output loss $\mathcal{L}_O$. If $b = 0$, the inflation bias disappears, but the information bias still exists. Alternately, this additional effect comes from the imperfect information structure. Under perfect information with $b = 0$, central bank has no trade-off; discretion is the same as commitment or adaptation. However, under imperfect information with $b = 0$, the output gap $\pi - a$ is not 0, which means there is still output-inflation trade-off. Because the trade-off falls either with information known (a is a function of expected shocks) or the fundamental ($\bar{\theta}$), agents will respond to them ex ante and ex post. It is an endogenous information acquisition bias in the Barro-Gordon sense, that is, a deviation between the real aggregate shock and what agents expected. A discretionary policymaker tracks agents' action a flexibly ex post, thus she has an incentive to decrease p_1 and increase p_2.[7] Ex ante, agents expect this discretionary behavior, choose a low level of γ and is sensitive to θ. The information bias is both on acquisition of information and also how agents respond to information. In the no-learning region, there is no bias in the acquisition of information, but there is a bias in the response on the noisy signal. Similarly, the adaptation rule has the information bias in the learning regime.

Remarkably, in equilibrium the output loss $\mathcal{L}_O = z^2 + b^2$ is independent of the monetary policy. Nonetheless, only the commitment policymaker recognizes this equilibrium condition and chooses to minimize the inflation loss $\mathcal{L}_I$ only. Both discretion and adaptation take γ as exogenous and set optimal policy based on the logic discussed in this subsection. Because $max\{p_1^D, p_1^A\} < 1$ and $min\{p_2^D, p_2^A\} > 0$, $max\{\mathcal{L}_I^D, \mathcal{L}_I^A\} > \mathcal{L}_I^C$. The information bias causes the total loss to increase.

5 Policy Comparison

This section discusses the relative efficiency of the three policies separately in the learning and no-learning regimes.

[7] According to $a = p_0 + p_1\gamma\bar{\theta} + [p_1(1-\gamma) + p_2]\theta$ (Equation (9)), a is more sensitive to θ and less sensitive to $\bar{\theta}$ than π. In order to decrease $\pi - a$, a discretionary policymaker will choose a smaller p_1 and a larger p_2 than a commitment policymaker.

5.1 Efficiency Comparison in the No-Learning Regime

Table 1 presents the decomposition of the total loss in the no-learning regime. In this regime, commitment and adaptation are the same. They are strictly better than discretion, i.e.

$$\text{Commitment} \sim \text{Adaptation} \succ \text{Discretion}.$$

The values of p_0, p_1 and p_2 are larger under the discretionary policy than under the other two policies. The larger p_0 leads to inflation bias $\frac{b^2}{\lambda}$ in $\mathcal{L}_I^D$. The larger p_1 and p_2 introduce an information bias. As a result, the term $\frac{\lambda\sigma_\epsilon^2}{(1+\lambda)^2}$ in $\mathcal{L}_I^D$ is larger than $\mathcal{L}_I^C$. Also, $\mathcal{L}_O^D$ is larger than $\mathcal{L}_O^C$. On the one hand, the discretionary policymaker is more flexible in tracking a and $\bar{\theta}$, which eliminates σ_θ^2 in the total loss. On the other hand, a is more sensitive to the noisy signal θ under discretion than under the other two policies, thus discretion generates the information bias.

5.2 Efficiency Comparison in the Learning Regime

Table 2 lists the decomposition of the total loss in the learning regime. Clearly, the total loss under commitment ($\mathcal{L}^C$) is the smallest, i.e.

$$\begin{aligned} \text{Commitment} &\succ \text{Discretion}, \\ \text{Commitment} &\succ \text{Adaptation}. \end{aligned}$$

As presented in the second column of Table 2, all three policies have the same output loss $\mathcal{L}_O$. Given $\mathcal{L}_O = [p_1(1-\gamma)(\bar{\theta}-\theta)+b]^2$ (Equations (33) and (36)) and the equilibrium condition of γ (Equation (11)), the output loss equals to z^2+b^2, which is independent of p_0, p_1 and p_2 (refer to Section B of the online Appendix). However, only the commitment policymaker takes this equilibrium condition into consideration when choosing policies. She chooses π to minimize the inflation loss $\mathcal{L}_I$, while the other two policies minimize the total loss for a given γ.

Hence, the difference of the social loss under the three policies comes from the inflation loss $\mathcal{L}_I$. Both discretion and adaptation have the information bias, which results in a non-zero $\mathcal{L}_I$. Under discretion, the information bias generates $\frac{z^2}{\lambda}$, while inflation bias generates $\frac{b^2}{\lambda}$. Thus, the efficiency gap between commitment and discretion in an imperfect information framework is larger than that in a perfect information framework. Moreover, the comparison between discretion and adaptation depends on the parameter values. $\mathcal{L}^A$ is larger than $\mathcal{L}^D$ under some circumstances. These circumstances will be discussed in the next section.

In summary, commitment is the most efficient monetary policy in the two learning regimes. Moreover, under the same parameter setting, the equilibrium condition under commitment is more likely to be the one in the learning regime. Given that learning is socially beneficial, commitment is always better than discretion and adaptation.

6 When is There Learning and When is Adaptation Better Than Discretion?

Adaptation commits to policy action but not to information. Contrary to the conventional wisdom that almost all forms of commitment is better than discretion, adaptation is worse than discretion in the learning regime under some circumstances. This subsection shows how the relative efficiency depends on the property of the model and when those cases are achieved through the study of the key parameters, policy weight on inflation (λ), the relative importance of idiosyncratic shock to agents (z), and the volatility of the aggregate shock $\bar{\theta}$ (σ_θ^2).

To illustrate the properties of the key parameters, Figures 2, 3, and 4 plot the equilibrium learning effort γ and the total loss $\mathcal{L}$ as functions of λ while the other parameters are varied. In each figure, the six sub-figures plot $\gamma^C(\lambda)$, $\gamma^D(\lambda)$, $\gamma^A(\lambda)$, $\mathcal{L}^C(\lambda)$, $\mathcal{L}^D(\lambda)$, and $\mathcal{L}^A(\lambda)$ separately. In each sub-figure, different lines are plotted with different parameter settings. Firstly, to illustrate the effect of z, Figure 2 plot $\gamma(\lambda)$ and $\mathcal{L}(\lambda)$ with different values of z, while the rest of the parameters are kept as constant. Specifically, Figure 2 (a) plots $\gamma^C(\lambda)$ with four different values of z. Later, to depict the effect of σ_θ^2, Figure 3 plots $\gamma(\lambda)$ and $\mathcal{L}(\lambda)$ functions with different values of σ_θ^2. Finally, Figure 4 plots $\gamma(\lambda)$ and $\mathcal{L}(\lambda)$ functions while the values of σ_ϵ^2 are varied.

Additionally, Figure 5 plots the difference between $\mathcal{L}^D$ and $\mathcal{L}^A$, as a function of λ, while the values of z, σ_θ^2 and σ_ϵ^2 are varied one by one.

6.1 The Effect of the Policy Weight on Inflation (λ)

λ is the weight attached to price stability in the central bank's objective function (1). It controls the relative importance of the output loss and the inflation loss. When λ is 0, the inflation-output trade-off faced by the central bank disappears, so does the inflation and information biases. As a result, all three policies are the same.[8] Similarly, as λ approaches infinity, the three policies also converge.[9] For $0 < \lambda < \infty$, the following proposition holds.

Proposition 5. *As the weight on the inflation loss (λ) in the central bank's objective function* (1) *increases, the aggregate learning effort (γ) weakly increases; the inflation bias and the information bias shrink. Therefore,*

1. *There is a threshold λ_0, s.t. for $\lambda \leq \lambda_0$, there is no private learning on aggregate shock ($\bar{\theta}$), i.e. $\gamma = 0$. This threshold varies across policies. The total loss is a strictly increasing function of λ when $\lambda < \lambda_0$, and it is a constant or a decreasing function when $\lambda > \lambda_0$.*

2. *Adaptation is worse than discretion for λ between $(\underline{\lambda}^*, \bar{\lambda}^*)$. The existence of such an interval depends on the values of z, σ_ϵ^2 and σ_θ^2.*

When λ increases, the policymaker cares more about the inflation loss $(\pi - \bar{\theta} - \pi^*)^2$ and less about the output loss $(\pi - a - b)^2$. Therefore, p_1 increases towards 1 when λ increases and agents' learning incentive is higher. Thus, there is a threshold λ_0, such that when $\lambda > \lambda_0$, $\gamma > 0$.

The top rows of Figures 2, 3, and 4 plot the equilibrium learning effort γ as a function of λ while the other parameters are varied. Across all three policies, γ is a weakly increasing function of λ. $\gamma(\lambda)$ is discontinuous at λ_0 in the sub-figure (a) of each figure, while $\frac{\partial \gamma}{\partial \lambda}$ is discontinuous at λ_0 in sub-figures (b) and sub-figures (c). In the no-learning regime, $\lambda \leq \lambda_0$ and $\gamma = 0$. In the learning regime, $\lambda > \lambda_0$ and $\gamma > 0$.

Furthermore, both the inflation and the information biases decrease as λ increases in the learning regime. The three polices converge. The bottom rows of Figures 2, 3, and 4 plot the total loss $\mathcal{L}$ as a function of λ. Notably, $\frac{\partial \mathcal{L}}{\partial \lambda}$ is discontinuous at λ_0, where the economy switches from the no-learning regime to the learning regime. First, $\mathcal{L}$ increases in the no-learning regime, because a higher λ results in a higher p_1 and a larger gap between π and a. Second, $\mathcal{L}$ weakly decreases in the learning

[8] When λ is zero, all the three policies choose $p_1 = 0$.

[9] When λ is ∞, the timing of the game becomes irrelevant and all the three policies set $\pi = \pi^* + \bar{\theta}$.

regime. As discussed in Section 5, $\mathcal{L}_O$ is constant and independent of p_1 in this regime, but $\mathcal{L}_I$ is a weakly decreasing function of p_1. p_1 is an increasing function of λ, thus $\mathcal{L}_I$ is a decreasing function of λ.

Additionally, as plotted in Figure 5, when $\mathcal{L}^D - \mathcal{L}^A$ is smaller than zero, discretion is more efficient than adaptation. There is an interval $(\underline{\lambda}^*, \bar{\lambda}^*)$ (marked by 'x' in the Figure) such that this condition is true. Also, the $(\underline{\lambda}^*, \bar{\lambda}^*)$ interval changes when the values of the parameters are varied.

6.2 The Effect of the Relative Importance of the Individual Circumstance (z)

The parameter z, which equals $\frac{\omega\phi}{\alpha e^k}$, governs the relative importance of idiosyncratic condition versus aggregate condition to agents. When z is large, agents care less about the monetary policy and allocate larger attention to idiosyncratic shock. Thus z is another important variable that affects the equilibrium condition.

Proposition 6. *As the idiosyncratic condition becomes more important to agents (z increases), the aggregate learning effort (γ) decreases; the information bias increases.*

1. *The threshold of the learning regime and the no-learning regime, λ_0, is an increasing function of z. The total loss increases in both regimes.*

2. *For a given level of σ_ϵ^2, σ_θ^2 and as z increases, adaption is more likely to be less efficient than discretion. $(\underline{\lambda}^*, \bar{\lambda}^*)$ exits when z is larger than a threshold level z^*. This nonempty interval shifts to the right, and its length increases as z increases. When z approaches ∞, $\bar{\lambda}^*$ converges to ∞.*

When z increases, monetary policy is less important to agents. Agents care less about aggregate shock $\bar{\theta}$. Therefore, γ decreases as z increases leading to the increase of the inflation bias. Agents are less likely to put learning effort in the aggregate shock, thus λ_0 increases. This is illustrated in the top row of Figure 2. For example, as z increases, $\gamma^C(\lambda)$ in sub-figure (a) shifts down. Also, λ_0 shifts to the right. Moreover, $\pi - a$ is a decreasing function of γ. Thus, $\mathcal{L}$ of all the three policies increase. As illustrated in the bottom row of Figure 2, $\mathcal{L}(\lambda)$ shifts up as z increases.

Furthermore, Figure 5 (a) plots $\mathcal{L}^D - \mathcal{L}^A < 0$ as a function of λ with four different values of z. The length of the $(\underline{\lambda}^*, \bar{\lambda}^*)$ interval increases as z increases. The advantage of being flexible is stronger when z is large, because the discrepancy between a and π increases. Therefore, discretion is likely to be more efficient than adaptation. $\bar{\lambda}^*$ is an increasing function of z and it approaches to infinity as z approaches to ∞.

6.3 The Effect of the Aggregate Shock Volatility (σ_θ^2)

The volatility of $\bar{\theta}$ is represented by σ_θ^2. Similar to λ, σ_θ^2 affects the inflation-output trade-off. Also, the inflation loss $\mathcal{L}_I$ is an increasing function of σ_θ^2.

Proposition 7. *As the aggregate shock volatility (σ_θ^2) increases, γ^C and γ^D are not affected while γ^A increases; the information bias decreases.*

1. *As σ_θ^2 increases, λ_0^C and λ_0^A decrease, while λ_0^D is not affected. $\mathcal{L}^C$ increases in the no-learning regime and is not affected in the learning regime. $\mathcal{L}^A$ increases in the no-learning regime and decreases in the learning regime. $\mathcal{L}^D$ is not affected.*

2. *As σ_θ^2 increases, adaptation is more likely to be more efficient than discretion. The interval $(\underline{\lambda}^*, \bar{\lambda}^*)$ shifts towards zero, and its length decreases.*

As σ_θ^2 increases, the inflation loss increases. Hence, p_1 tends to increase and the information bias shrinks. Also, the economy is more likely to be in the learning regime, so λ_0 decreases. As illustrated in the top row of Figure 3, γ is a weakly increasing function of σ_θ^2. As shown in Figure 3 (a), γ^C is not affected in the learning regime, but λ_0^C decreases. That's because the commitment policymaker chooses $p_1^C = 1$ in the learning regime, which is independent of σ_θ^2. Nonetheless, neither γ^D nor λ_D is affected by σ_θ^2 (Figure 3 (b)), because discretion has the flexibility to track $\bar{\theta}$ ex post. In Figure 3 (c), γ^A weakly increases as σ_θ^2 increases, because the adaptation policymaker cares more about the information loss when σ_θ^2 gets larger, thus the information bias shrinks. Also, λ_0^A decreases.

Under the same intuition, the total loss is affected by σ_θ^2 in the following way: (1) in the learning regime , $\mathcal{L}^C$ is independent of σ_θ^2 (Figure 3 (d)) while $\mathcal{L}^A$ decreases with σ_θ^2 (Figure 3 (f)); (2) in the no-learning regime, both $\mathcal{L}^C$ and $\mathcal{L}^A$ increase due to the increase of the inflation loss; (3) $\mathcal{L}^D$ is independent of σ_θ^2 (Figure 3 (e)).

Furthermore, Figure 5 (b) plots $\mathcal{L}^D - \mathcal{L}^A < 0$ as a function of λ with four different values of σ_θ^2. As σ_θ^2 increases, discretion is less likely to be more efficient than adaptation, because $\mathcal{L}^A$ decreases while $\mathcal{L}^D$ is unaffected. The $(\underline{\lambda}^*, \bar{\lambda}^*)$ interval shifts to the left, and its length decreases.

7 Further Issues: Going Deeper on the Objective Function and the Role of Public Signal

Two issues play key roles in the intuition so far. One is public signaling. The other one is the wedge between the central bank and the private sector. To understand the intuition more deeply, this section discusses the following issues. First, should there be a signal or not? If so, how precise should it be? Second, the wedge in the form of other shocks is studied. What if there is a shock on output instead of inflation? Furthermore, the equilibrium condition under the Adam (2007) specification is discussed. The last part goes deeper on the objective function. Particularly, it considers debt as a reason we worry about discretion and inflation in a new Keynesian framework.

7.1 Should There Be A Public Signal?

If there is no public signal, the only way agents learn about an aggregate shock is through private learning. This is a special case of the general model with $\theta = 0$ or $\sigma_\epsilon^2 = \infty$.[10] Thus, agents' prior on $\bar{\theta}$ has a mean of 0. Similar to the standard model, the monetary policy is assumed to be an affine function of $\bar{\theta}$, i.e.

$$\pi = p_0 + p_1\bar{\theta}. \tag{22}$$

The aggregate agents' action is

$$a = p_0 + p_1\gamma\bar{\theta}. \tag{23}$$

The information acquisition solution under this set up is solved in Section A.2 of the online Appendix. The equilibrium γ satisfies condition (30). The optimal learning effort changes from Equation (10) to

$$x_i^* = max\{k + \ln(\frac{\sigma_\theta}{\phi}) + \ln(\frac{(1-\alpha)\gamma + \alpha}{\omega}) + \ln p_1, 0\}. \tag{24}$$

[10] Note that this special case is different from the general case with public signaling. In the general case, $\sigma_\epsilon^2 \leq \sigma_\theta^2$. Otherwise, the public signal is meaningless. In this special case, σ_ϵ^2 is ∞.

σ_ϵ is replaced by σ_θ from equation (10). $x_i^* > 0$ if $z < \sigma_\theta p_1$. Combining with the equilibrium condition of γ (30), one obtains,

$$\gamma = \begin{cases} 1 - \frac{z}{\sigma_\theta p_1} \quad \text{and} \quad x \neq 0, & \text{if } z < \sigma_\theta p_1 \ , \\ 0 \quad \text{and} \quad x = 0, & \text{if } z \geq \sigma_\theta p_1 \ . \end{cases} \tag{25}$$

The optimal choice of monetary policy, the equilibrium learning effort, and the total loss are summarized in Table A1 in Section C of the online Appendix. In this economy, commitment is strictly better than discretion and adaptation. On the one hand, without a public signal, the learning incentive effect is still important. On the other hand, the information acquisition bias is stronger.

Moreover, it is not always better to send a public signal. In the no-learning regime, the loss is smaller if the policymaker releases a signal. Clearly, although the public signal is noisy, it provides more accurate information than the prior of the shock. In the learning regime, the loss under commitment and discretion are not affected. Nonetheless, the loss under adaptation is smaller without a signal. Without public signal, agents have a larger learning incentive on $\bar{\theta}$ due to strong information inaccuracy. Although the prior is less accurate, the increase of γ is so large that the social loss is smaller than that in an economy with public signal. Subsequently, $\mathcal{L}^A$ decreases and the information bias is smaller. This property is confirmed in the next subsection, in which case an increase of σ_ϵ^2 is studied. However, the loss under discretion is independent of the accuracy of θ in the learning regime (Table 2).[11] Due to its flexibility, the discretionary policymaker has the ability to eliminate the impact of the signal inaccuracy on the total loss ex post. This is the main reason that discretion can be more efficient than adaptation when there is a public signal.

Consequently, in the learning regime, $\mathcal{L}^D$ is not affected by the existence of a public signal, while $\mathcal{L}^A$ is smaller without a public signal. Adaptation is always better than discretion in the economy without a public signal. The range, in which discretion is better than adaptation, completely disappears.

7.2 How Precise Should the Public Announcement Be?

How precise should the public announcement be? This is a question studied in the literature such as Morris and Shin (2002), Blinder et al. (2008) and Chahrour (2013). This subsection shows

[11] Table 2 shows that $\mathcal{L}^D$ is independent of σ_ϵ^2.

that more transparent public communication is not always better.

If there is a public signal θ, the precision is controlled by σ_ϵ^2. This parameter has a strong effect on agents' learning incentives and the trade-off of the central bank.

Proposition 8. *As the volatility of the signal error (σ_ϵ^2) goes up, γ increases; the information bias weakly decreases.*

1. *The learning regime and the no-learning regime threshold λ_0 is a decreasing function of σ_ϵ^2. The total loss increases in the no-learning regime. In the learning regime, $\mathcal{L}^C$ and $\mathcal{L}^D$ does not change, while $\mathcal{L}^A$ decreases.*

2. *If σ_ϵ^2 increases, it is less likely adaptation is worse than discretion. The interval $(\underline{\lambda}^*, \bar{\lambda}^*)$ shifts towards zero. and its length decreases.*

As σ_ϵ^2 increases, the public signal is less accurate. Agents have a larger learning incentive on $\bar{\theta}$, thus γ increases (equation (11)) and the threshold λ_0 decreases. These effects are illustrated in the top row of Figure 4. As σ_ϵ^2 increases, γ curve shifts up; λ_0 shifts to the left.

However, the total loss does not always increase under a less accurate signal system. On the one hand, in the no-learning regime, the losses of all three policies increase, as shown in the bottom row of Figure 4. In this case, σ_ϵ^2 increases the information error and the output gap. On the other hand, in the learning regime, the loss weakly decreases as σ_ϵ^2 increases. Under commitment or discretion, the total loss is not affected. However, under adaptation, the total loss decreases (Proposition A3). Markedly, only the adaptation rule depends on σ_ϵ^2 at equilibrium (Table 2). $\mathcal{L}_I^A$ is a decreasing function of γ^A and an increasing function of σ_ϵ^2. Notably, a less accurate public signal increases agents' learning incentives. Equation (11) shows that γ is an increasing function of σ_ϵ^2. The increase in the learning incentive is so strong that the aggregate impact of σ_ϵ^2 on $\mathcal{L}_I^A$ is negative. So, the total loss under adaptation decreases.[12] Therefore, it is more likely that adaptation is more efficient than discretion. As shown in Figure 5 (c), the interval $(\underline{\lambda}^*, \bar{\lambda}^*)$ shifts to the left and its length decreases.

In conclusion, it is not always optimal to maximize the accuracy of the public announcement. The key intuition is that more accurate signal will abate the learning incentives of agents when the communication technique is not perfect.

[12]Similarly, the total loss under adaptation without a public signal is smaller than it is with a public signal, as presented in Section 7.1.

7.3 The Wedge: Other Shocks

The key ingredient to generate interesting results in the standard model is the wedge between the central bank and agents, that the policymaker has a trade-off between tracking agents' aggregate action (output loss) versus tracking aggregate shock (inflation loss), while agents have a learning trade-off. The policymaker would like agents to maximize their learning effort on aggregate shock, but agents need a learning incentive to do that.

Instead of having a shock $\bar{\theta}$ enters into the inflation loss $\mathcal{L}_I$, it can enter into the output loss $\mathcal{L}_O$. In this case, the central bank's objective (Equation (1)) changes to,

$$Min \quad (\pi - a - \bar{\theta} - b)^2 + \lambda(\pi - \pi^*)^2.$$

Here $\bar{\theta}$ can be interpreted as an output shock. Different from the previous economy, both a and $\bar{\theta}$ affect the output loss, while the inflation loss depends on the inflation deviation from its target. The original wedge disappears. The central bank doesn't want agents to maximize their learning effort on $\bar{\theta}$. Both commitment and adaptation set inflation equals to its target ($\pi = \pi^*$), so there is no private learning on $\bar{\theta}$ (discussed in Section D of the online Appendix). Furthermore, discretion puts a higher policy weight on $\bar{\theta}$, which introduces larger learning incentives. Nonetheless, a larger policy weight on $\bar{\theta}$ increases the total loss. Firstly, the output loss $\mathcal{L}_O$ is independent of monetary policy.[13] Secondly, a larger p_1 increases the inflation deviation from π^*, thus $\mathcal{L}_I$ increases. Therefore, although discretion introduces larger learning incentives, this brings an information bias in the opposite direction. Discretion is less efficient than commitment and adaptation.

7.4 The Adam (2007) Specification

So far, the central bank's objective function and the trade-off follow the classical Barro-Gordon framework. What if the central bank has a different social objective function? This subsection redoes the above analysis using the Adam (2007) specification.

The derivation of the model under this specification is presented in Section E of the online Appendix. Firstly, in order to add the endogenous information acquisition structure and learning trade-off, a firm-specific taxation or revenue shock u_i is added into the Adam (2007) model. The

[13] Similar as the standard model, the part $(\pi - a)^2$ in $\mathcal{L}_O$ is independent on p_i at equilibrium, which equals to z^2.

aggregate shock $\bar{\theta}$ comes from the stochastic labor supply shifter. The central bank's objective function is the same as it is in Adam (2007),

$$Min \quad (\pi - a - \bar{\theta})^2 + \tilde{\lambda} \int (a_i - a)^2 di, \tag{26}$$

where $\pi - a$ equals to y (output) and $\bar{\theta}$ equals to y^* (efficient output) in the Adam (2007) notation. π represents the nominal output; a corresponds to the aggregate price level; $\bar{\theta}$ is the efficient output level.

Hence, the central bank faces a trade-off between output volatility and price dispersion. The key change is that the inflation loss, $\lambda(\pi - \bar{\theta})^2$, is replaced by the loss from price dispersion, $\tilde{\lambda} \int (a_i - a)^2 di$. Essentially, $\int (a_i - a)^2 di$, is independent of the monetary policy p_0, p_1 and p_2, as discussed in Section E.3 of the online Appendix,

$$\int (a_i - a)^2 di = \phi^2 [1 - \exp(-2k)]. \tag{27}$$

Therefore, the policymaker only cares about the output loss and the trade-off disappears. Both a and $\bar{\theta}$ enters into the output loss function.

Furthermore, agents' objective function is slightly different from (3), with an additional term $\alpha\bar{\theta}$, because the labor supply shifter also affects the price decision of firms. Agents try to minimize,

$$Min \quad \mathbb{E}_i [a_i - (1 - \alpha)a - \alpha(\pi - \bar{\theta}) - \omega u_i]^2.$$

As a result, commitment, discretion and adaptation policies are identical. All of them choose p_1 equals one. The learning incentive effect and the information bias disappear in this set up.

Remarkably, the trade-off between output volatility and price dispersion disappears under this specification. In addition to Adam (2007), other papers, such as Morris and Shin (2002), Angeletos and Pavan (2004), Hellwig (2005), also study the optimal transparency level of public announcement through the analysis of the output volatility and price dispersion trade-off. To the contrary, with the private sector learning trade-off, the social loss from price dispersion only depends on the aggregate learning capacity (Equation (27)) and is independent of the monetary policy. Thus the trade-off of the central bank disappears. The model presented in this paper captures this interesting effect which is also important. Consequently, the results on the efficiency of the public announcement transparency in the existing literature are not necessarily true any more. Moreover, this property also applies to other specifications, such as the one discussed in the next subsection. The only requirement is that the information structure is similar to the one presented in this paper.

7.5 Public Debt in the Wedge

Public debt and taxation is another reason Barro-Gordon themselves justify the commitment problem. Additionally, Calvo and Guidotti (1990) explores the government's incentive to use inflation to reduce the real value of its nominal liabilities. Adopting the intuition discussed in the existing literature, this subsection provides a simple exercise with debt.

Consider that the economy is endowed with some external nominal debt denoted in the domestic currency, which can be positive or negative (see Section F of the online Appendix for detail). With a positive debt for example, the government should repay the debt through the selling of real goods. The real goods for debt repayment are collected through a lump-sum tax on real income of households. However, the policymaker could control the real value of their debt through price level manipulation. When the price is high, the real value of the repaid debt is low. The aggregate shock of this economy comes from the external debt level.

The production sector consists of a continuum of monopolistically competitive firms $i \in [0, 1]$. Each firm makes a price decision a_i, which is equivalent to agents' actions in the standard model. Each firm also faces an idiosyncratic revenue shock u_i. After a straightforward but tedious derivation shown in the online Appendix, firms' action changed slightly from Equation (4) to,

$$a_i = \mathbb{E}_i[(1 - \alpha)a + \alpha(\pi - b) - \xi(\pi - \bar{\theta}) - u_i],$$

where the new component is $-[\xi(\pi - \bar{\theta}) + \alpha b]$. The aggregate shock $\bar{\theta}$ is related to the external debt level.

Nonetheless, the central bank's objective function is the same as Equation (1).

In this setup, the wedge and the trade-off are the same as in the standard model. Consequently, the efficiency comparison result of the three policies is the same (see Section F.6 of the online Appendix for details). Commitment is still the most efficient policy. It has a learning incentive effect to control the optimal level of private learning on $\bar{\theta}$.[14] In comparison, discretion and adaptation generate the information bias, as they set policy after agents' learning decisions. Nevertheless, the discretionary policy is flexible in tracking a and $\bar{\theta}$ and is more efficient than adaptation under certain conditions, such as the case presented in Figure 6. Therefore, the two effects (the learning incentive effect and the flexible policy setting effect) and the interesting results in the standard model still apply.

8 Conclusion

This paper considered an economy where the policymaker cannot perfectly communicate with agents about the aggregate economic condition while agents face an information capacity constraint. It studied three kinds of policies — commitment, discretion, and adaptation — differing on the timing of the policy with respect to the two actions of agents. In addition to the usual inflation bias studied in the classical literature, there are two mechanisms that's at work in the choice of monetary policy. First, committing a policy before agents' learning decision provides learning incentives to the private sector. Second, flexibility generates the information bias, which induces insufficient information acquisition. In conclusion, commitment is the most efficient policy while the superiority of discretion over adaptation depends on the circumstances. Moreover, it is not always optimal to achieve maximum transparency in the public communication and it is not always efficient to send a public signal.

Thus, communication technique and its interaction with the central bank's actions are important aspects most policymakers need to consider, which warrants more research. This paper makes two contributions. First, it extends the rational inattention literature to take into account of commitment and discretion. Second, it provides new insight to the classic central bank question — how transparent the policy announcement should be.

Finally, it is notable that the model presented in this paper can be enriched with different information structures to study the trade-off of the central bank and the optimal conduct of monetary policy. This model can also be modified to address several important questions in asset pricing and contract design. The information structure and the wedge play roles in the principle-agent problem.

[14] One difference is that the optimal policy weight on the aggregate shock, p_1^C, is not 1.

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Table 1: Loss Decomposition in the No-Learning Regime ($\gamma = 0$)

Total Loss	Output Loss ($\mathcal{L}_O$)	Inflation Loss ($\mathcal{L}_I$)
Commitment: $\mathcal{L}^C$	$\frac{\lambda^2\sigma_\epsilon^2}{(\lambda+1+\sigma_\epsilon^2/\sigma_\theta^2)^2} + b^2$	$\frac{\lambda(1+\sigma_\epsilon^2/\sigma_\theta^2)\sigma_\epsilon^2}{(\lambda+1+\sigma_\epsilon^2/\sigma_\theta^2)^2}$
Discretion: $\mathcal{L}^D$	$\frac{\lambda^2\sigma_\epsilon^2}{(\lambda+1)^2} + b^2$	$\frac{\lambda\sigma_\epsilon^2}{(1+\lambda)^2} + \frac{b^2}{\lambda}$
Adaptation: $\mathcal{L}^A$	$\frac{\lambda^2\sigma_\epsilon^2}{(\lambda+1+\sigma_\epsilon^2/\sigma_\theta^2)^2} + b^2$	$\frac{\lambda(1+\sigma_\epsilon^2/\sigma_\theta^2)\sigma_\epsilon^2}{(\lambda+1+\sigma_\epsilon^2/\sigma_\theta^2)^2}$

Table 2: Loss Decomposition in the Learning Regime ($\gamma \neq 0$)

Total Loss	Output Loss ($\mathcal{L}_O$)	Inflation Loss ($\mathcal{L}_I$)
Commitment: $\mathcal{L}^C$	$z^2 + b^2$	0
Discretion: $\mathcal{L}^D$	$z^2 + b^2$	$\frac{z^2}{\lambda} + \frac{b^2}{\lambda}$
Adaptation: $\mathcal{L}^A$	$z^2 + b^2$	$(1-\gamma^A)^2\frac{z^2}{\lambda} + (1-\gamma^A)^2\frac{z^2}{\lambda}\frac{\sigma_\epsilon^2}{\sigma_\theta^2}$

Figure 1: Timing of the Game

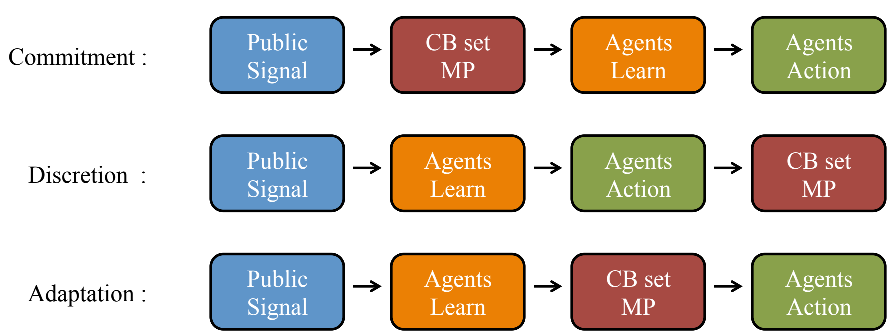
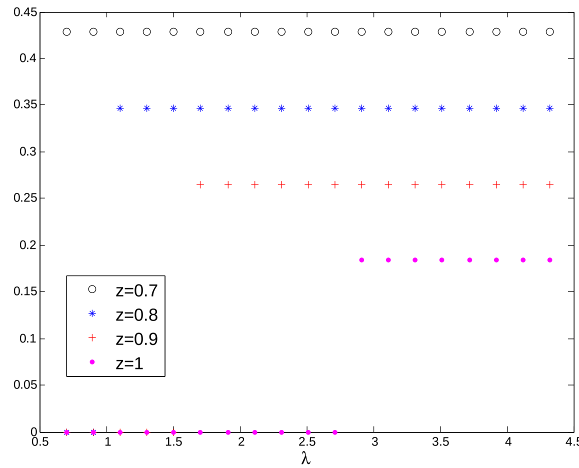
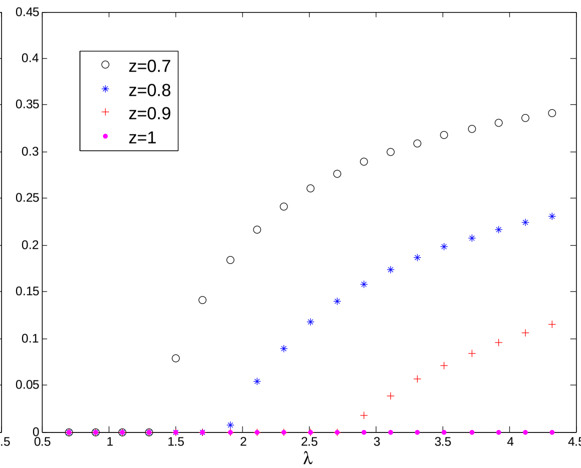


Figure 2: The effect of the idiosyncratic condition importance to agents (z)

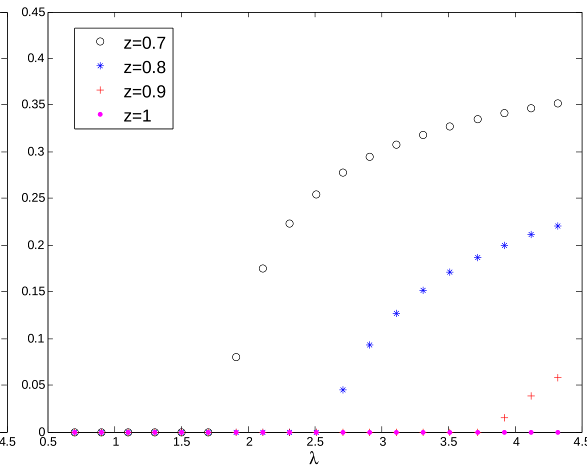
(a) learning effort under commitment (γ^C)



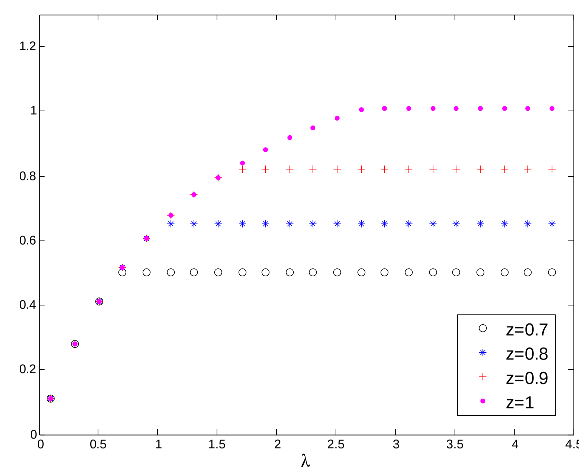
(b) learning effort under discretion (γ^D)



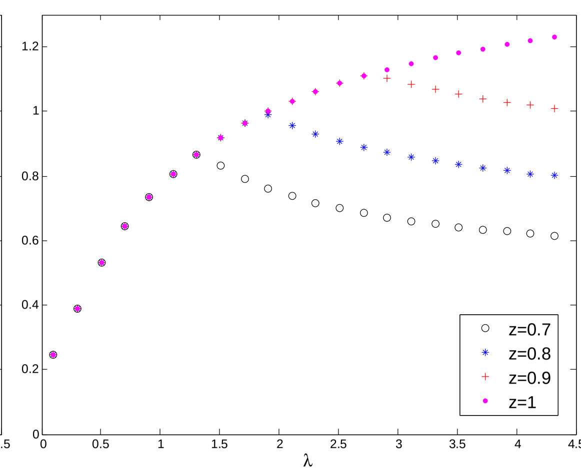
(c) learning effort under adaptation (γ^A)



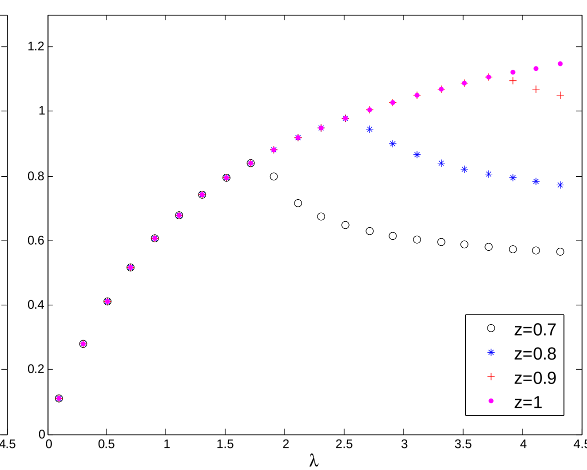
(d) total loss under commitment $(\mathcal{L}^C)$



(e) total loss under discretion $(\mathcal{L}^D)$



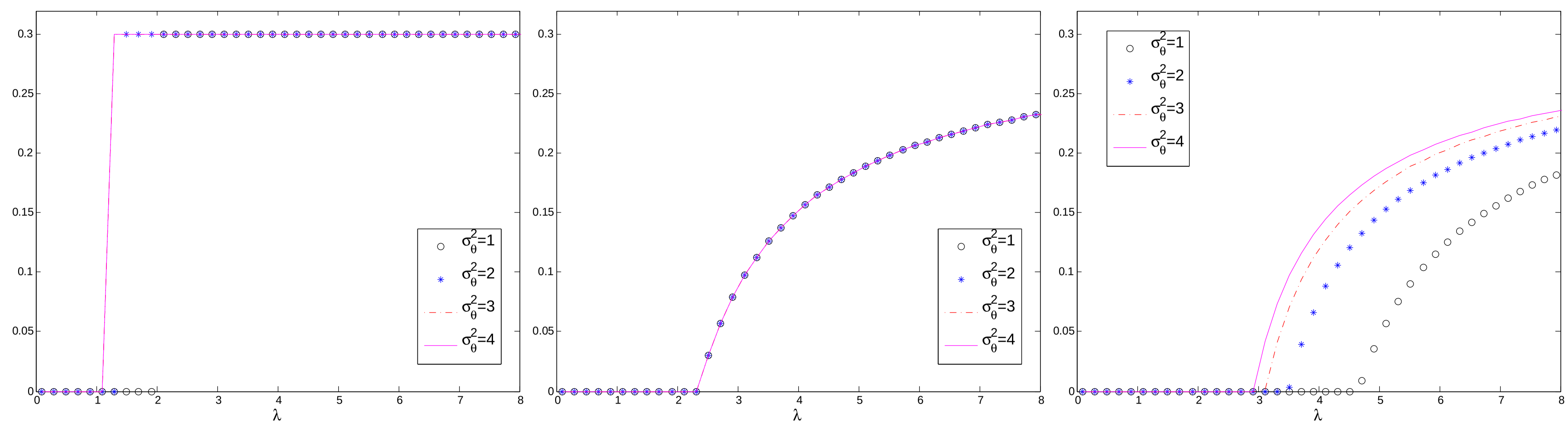
(f) total loss under adaptation $(\mathcal{L}^A)$



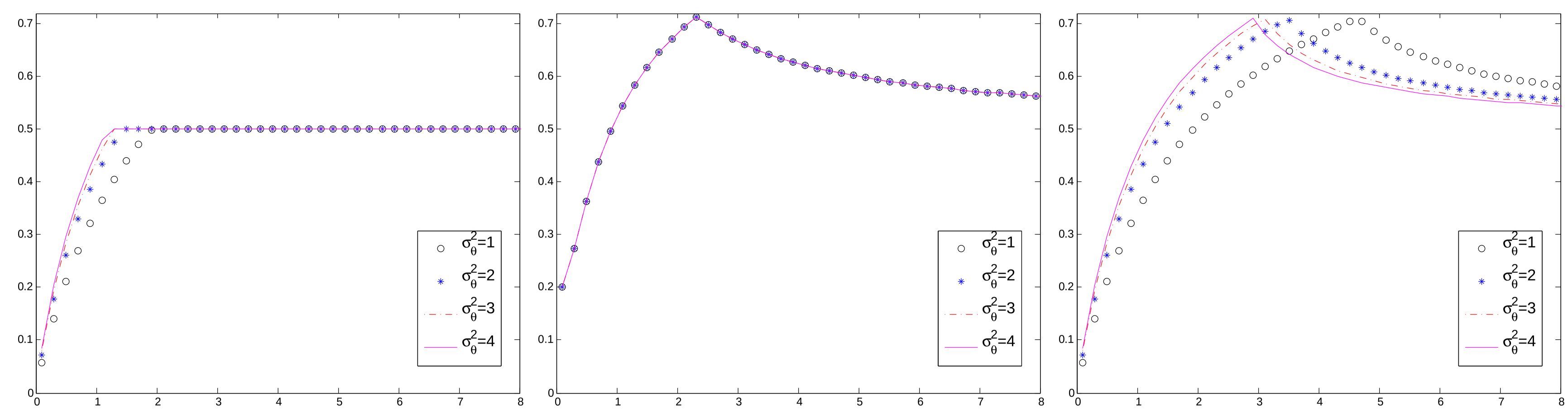
Note: In this figure, $b = 0.1, \sigma_\epsilon^2 = 1.5$, and $\sigma_\theta^2 = 4$. Each line is plotted as a function of λ, with different values of z. In (a), $\gamma(\lambda)^C$ is discontinuous at λ_0. In (b) and (c), $\frac{\partial \gamma^D}{\partial \lambda}$ and $\frac{\partial \gamma^A}{\partial \lambda}$ are discontinuous at λ_0. Correspondingly, in (d), (e) and (f), $\frac{\partial \mathcal{L}}{\partial \lambda}$ are discontinuous at λ_0. When $\lambda > \lambda_0$ $(\lambda < \lambda_0)$, the economy is in the learning (no-learning) regime.

Figure 3: The effect of the aggregate shock volatility (σ_θ^2)

(a) learning effort under commitment (γ^C) (b) learning effort under discretion (γ^D) (c) learning effort under adaptation (γ^A)

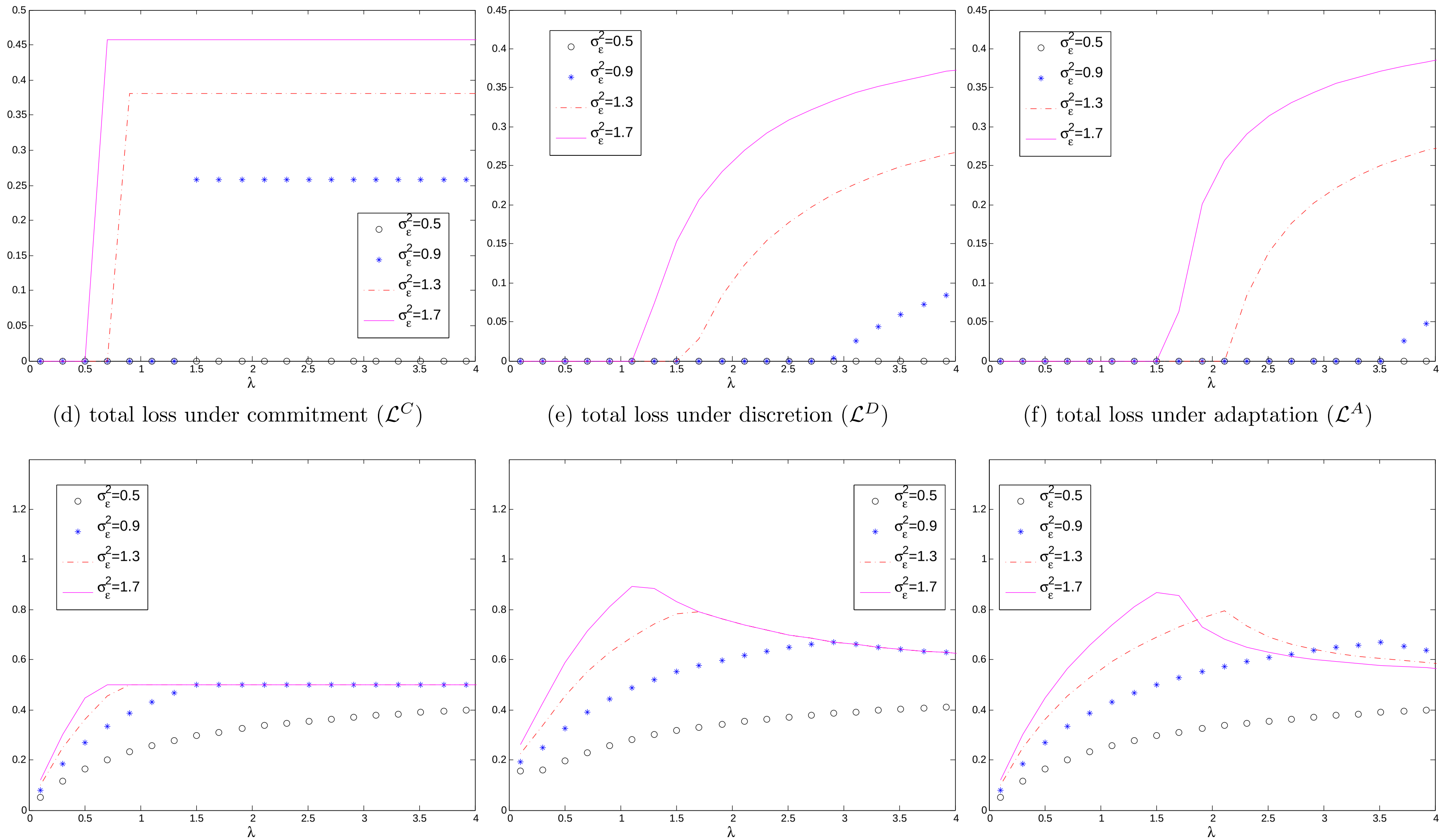


(d) total loss under commitment ($\mathcal{L}^C$) (e) total loss under discretion ($\mathcal{L}^D$) (f) total loss under adaptation ($\mathcal{L}^A$)



Note: In this figure, $b = 0.1$, $z = 0.7$, and $\sigma_\epsilon^2 = 1$. Each line is plotted as a function of λ, with different values of σ_θ^2. In (a), $\gamma(\lambda)^C$ is discontinuous at λ_0. In (b) and (c), $\frac{\partial \gamma^D}{\partial \lambda}$ and $\frac{\partial \gamma^A}{\partial \lambda}$ are discontinuous at λ_0. Correspondingly, in (d), (e) and (f), $\frac{\partial \mathcal{L}}{\partial \lambda}$ are discontinuous at λ_0. When $\lambda > \lambda_0$ ($\lambda < \lambda_0$), the economy is in the learning (no-learning) regime.

Figure 4: The effect of the public signal error volatility (σ_ϵ^2)



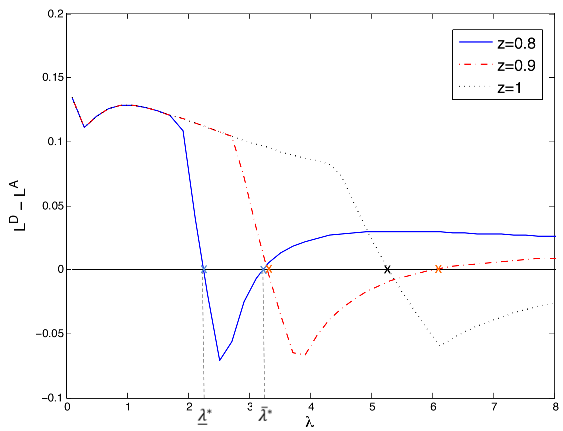
(a) learning effort under commitment (γ^C) (b) learning effort under discretion (γ^D) (c) learning effort under adaptation (γ^A)

(d) total loss under commitment ($\mathcal{L}^C$) (e) total loss under discretion ($\mathcal{L}^D$) (f) total loss under adaptation ($\mathcal{L}^A$)

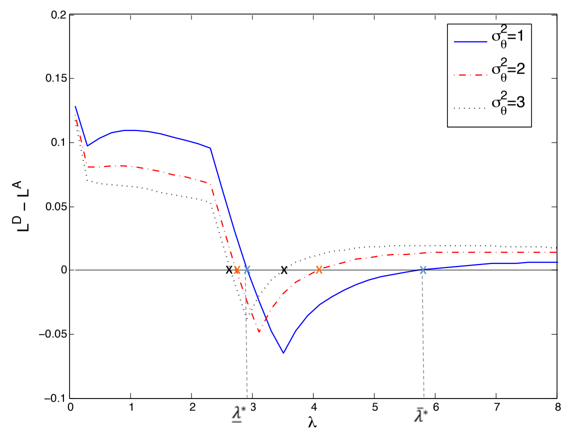
Note: In this figure, $b = 0.1, z = 0.7, \sigma_\theta^2 = 4$. Each line is plotted as a function of λ, with different values of σ_ϵ^2. In (a), $\gamma(\lambda)^C$ is discontinuous at λ_0. In (b) and (c), $\frac{\partial \gamma^D}{\partial \lambda}$ and $\frac{\partial \gamma^A}{\partial \lambda}$ are discontinuous at λ_0. Correspondingly, in (d), (e) and (f), $\frac{\partial \mathcal{L}}{\partial \lambda}$ are discontinuous at λ_0. When $\lambda > \lambda_0$ ($\lambda < \lambda_0$), the economy is in the learning (no-learning) regime.

Figure 5: When discretion is more efficient than adaptation

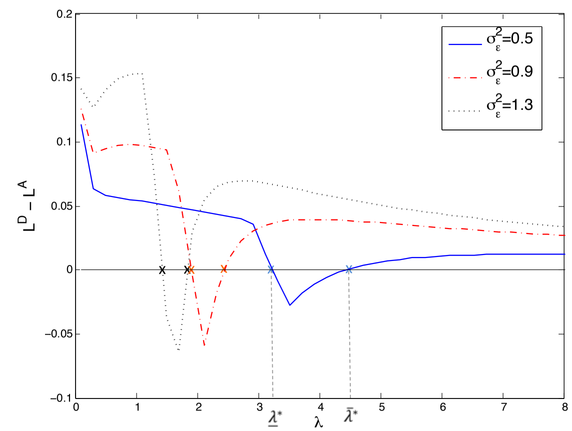
(a) $\mathcal{L}^D - \mathcal{L}^A$, varying z



(b) $\mathcal{L}^D - \mathcal{L}^A$, varying σ_θ^2

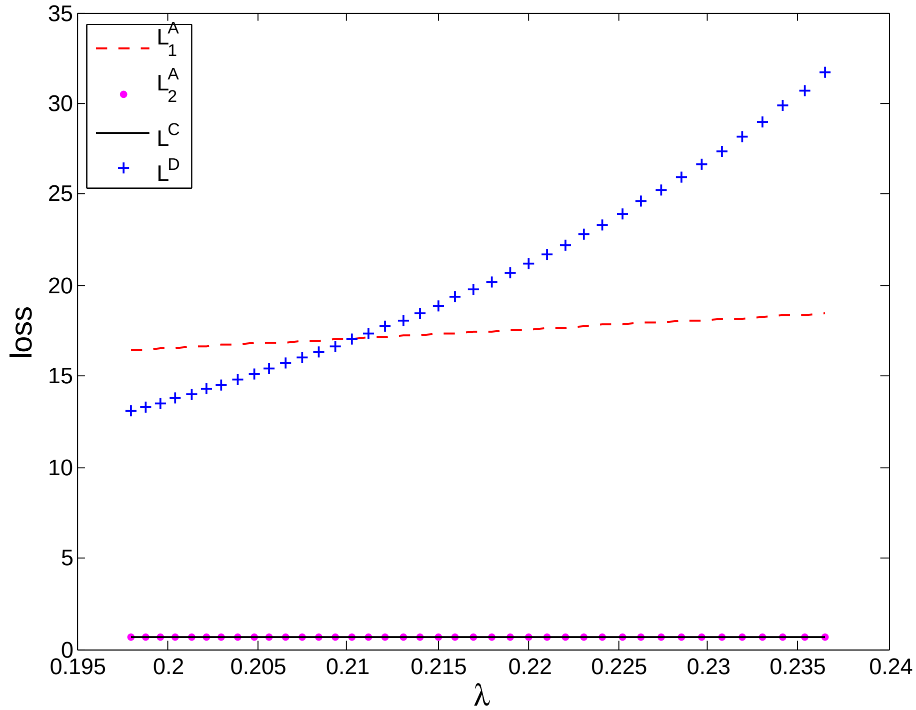


(c) $\mathcal{L}^D - \mathcal{L}^A$, varying σ_ϵ^2



Note: This figure plots the difference between the total loss under discretion and the total loss under adaptation as a function of λ, varying the other three parameters. (a) $b = 0.1, \sigma_\epsilon^2 = 1.5, \sigma_\theta^2 = 4$, z has three different values. (b) $b = 0.1, z = 0.7, \sigma_\epsilon^2 = 1$, σ_θ^2 has three different values. (c) $b = 0.1, z = 0.7, \sigma_\theta^2 = 4$, σ_ϵ^2 has three different values. In each sub-figure, when $\mathcal{L}^D - \mathcal{L}^A < 0$, discretion is better than adaptation. For each plot, there is an interval $(\underline{\lambda}^*, \bar{\lambda}^*)$ (marked by 'x'), such that for λ in this interval $\mathcal{L}^D - \mathcal{L}^A < 0$.

Figure 6: Total loss as a function of λ (with public debt in the wedge)



Note: In this simulation, $\eta = 4$ (as in the standard menu cost model), $\sigma_\theta^2 = 1000$, $\sigma_\epsilon^2 = 100$ and $\frac{\phi}{\exp(k)} = 0.1$. There are two equilibria under adaptation, since the equilibrium γ^A is calculated from a quadratic function. In this case, $\mathcal{L}_1^A$ is the total loss with the low γ^A while $\mathcal{L}_2^A$ is the total loss with the high γ^A. $\mathcal{L}_2^A$ is very close to $\mathcal{L}^C$. $\mathcal{L}_1^A$ is larger than $\mathcal{L}^D$ when λ is small.

A Agents Expected $\bar{\theta}$

A.1 Learning with public signal

Agents do not have perfect information on the aggregate shock, $\bar{\theta}$ ($\bar{\theta} \sim \mathcal{N}(0, \sigma_\theta^2)$), but they receive a noisy public signal, θ ($\theta \sim \mathcal{N}(\bar{\theta}, \sigma_\epsilon^2)$). In addition, through private learning, agents get a private signal θ_i,

$$\theta_i \sim \mathcal{N}(\bar{\theta}, \sigma_i^2),$$

where the magnitude of σ_i^2 depends on the learning effort of agents. From agents' perspective, Bayes rule implies that the posterior distribution of $\bar{\theta}$ is,

$$\begin{aligned} P(\bar{\theta}|\theta_i, \theta) &\propto P(\theta_i|\bar{\theta})P(\theta|\bar{\theta}) \\ &\propto exp(-\frac{(\sigma_\theta^2 + \sigma_i^2)\bar{\theta}^2 - 2\bar{\theta}(\sigma_\epsilon^2\theta_i + \sigma_i^2\theta)}{2\sigma_i^2\sigma_\epsilon^2}), \end{aligned}$$

with mean,

$$\mu_{\bar{\theta}} = \frac{\sigma_\epsilon^2}{\sigma_\epsilon^2 + \sigma_i^2}\theta_i + \frac{\sigma_i^2}{\sigma_\epsilon^2 + \sigma_i^2}\theta,$$

and variance,

$$var_{\bar{\theta}} = \frac{\sigma_\epsilon^2\sigma_i^2}{\sigma_\epsilon^2 + \sigma_i^2} \equiv \sigma_\epsilon^2 exp(-x_i),$$

where $x_i = ln(\frac{\sigma_\epsilon^2 + \sigma_i^2}{\sigma_i^2})$. The variable, x_i, is the (logarithm of the) reduction in variance from devoting attention to learning more about $\bar{\theta}$. Thus, the expected $\bar{\theta}$ for agents is

$$\hat{\theta}_i = \mathbb{E}_i(\bar{\theta}) = (1 - e^{-x})\theta_{it} + e^{-x}\theta. \tag{28}$$

According to Equation (3), Equation (28) and Equation (9), the optimal reaction a_i follows,

$$a_i = p_0 + p_1[(1-\alpha)\gamma + \alpha]\hat{\theta}_i + [p_1(1-\alpha)(1-\gamma) + p_2]\theta. \tag{29}$$

Integrating over the unit mass of agents,

$$a = p_0 + p_1[(1-\alpha)\gamma + \alpha](1 - e^{-x})\bar{\theta} + \{p_1[(1-\alpha)\gamma + \alpha]e^{-x_i} + [p_1(1-\alpha)(1-\gamma) + p_2]\}\theta.$$

Applying Equation (9), the condition for equilibrium, γ, is,

$$[(1-\alpha)\gamma+\alpha](1-e^{-x})=\gamma. \tag{30}$$

After substituting Equation (29) into Equation (3), agents

$$Min \quad \omega^2\phi^2 e^{-2k}e^{x}+[(1-\alpha)\gamma+\alpha]^2 p_1^2\sigma_\epsilon^2 e^{-x}. \tag{31}$$

Thus, the optimal learning effort on the aggregate shock is

$$x_i^*=max\{k+\ln(\frac{\sigma_\epsilon}{\phi})+\ln(\frac{(1-\alpha)\gamma+\alpha}{\omega})+\ln p_1,0\}. \tag{32}$$

After substituting this condition into Equation (30), the equilibrium, γ, can be written as a function of the exogenous parameters and the monetary policy variable p_1:

$$\gamma=[(1-\alpha)\gamma+\alpha]-e^{-k}\frac{\phi\omega}{\sigma_\epsilon p_1},$$

or

$$(1-\gamma)p_1=\frac{\phi\omega}{\sigma_\epsilon\alpha e^{-k}}=\frac{z}{\sigma_\epsilon}.$$

A.2 Learning without public signal

Without public signal, the Bayesian updated belief depends on the prior of $\bar\theta$ $((\bar\theta\sim\mathcal{N}(0,\sigma_\theta^2))$ and the private signal $(\theta_i\sim\mathcal{N}(\bar\theta,\sigma_i^2))$. Bayes rule implies that the posterior distribution of $\bar\theta$ is,

$$\begin{aligned} P(\bar\theta|\theta_i) &\propto P(\theta_i|\bar\theta)P(\bar\theta) \\ &\propto exp(-\frac{(\sigma_\theta^2+\sigma_i^2)\bar\theta^2-2(\sigma_\theta^2\theta_i)\bar\theta}{2\sigma_i^2\sigma_\theta^2}). \end{aligned}$$

with mean,

$$\mu_{\bar\theta}=\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_i^2}\theta_i,$$

and variance,

$$var_{\bar\theta}=\frac{\sigma_\theta^2\sigma_i^2}{\sigma_\theta^2+\sigma_i^2}\equiv\sigma_\theta^2 exp(-x),$$

where $x = ln(\frac{\sigma_\theta^2+\sigma_i^2}{\sigma_i^2})$. Thus, agents' expected $\bar{\theta}$ is

$$\hat{\theta}_{it} = \mathbb{E}_{it}(\bar{\theta}) = (1 - e^{-x})\theta_{it}.$$

B The Equilibrium Condition

B.1 Commitment Monetary Policy

A commitment rule is set before agents make decisions on γ and a. The commitment policymaker can provide learning incentives to agents. She takes this effect into consideration when making decisions. Substituting Equation (9) and Equation(10) into Equation (1) delivers the following: in the learning regime, the policymaker

$$Min \quad \mathbb{E}[(1-\gamma)p_1(\bar{\theta}-\theta) - b]^2 + \lambda[p_0 - \pi^* + (p_1+p_2-1)\bar{\theta} + p_2(\theta - \bar{\theta})]^2, \tag{33}$$

$$s.t. \quad p_1 > z/\sigma_\epsilon \quad and \quad p_1(1-\gamma) = z/\sigma_\epsilon,$$

in the no-learning regime, the policymaker

$$Min \quad \mathbb{E}[p_1(\bar{\theta}-\theta) - b]^2 + \lambda[p_0 - \pi^* + (p_1+p_2-1)\bar{\theta} + p_2(\theta - \bar{\theta})]^2, \tag{34}$$

$$s.t. \quad p_1 \leq z/\sigma_\epsilon.$$

The first optimization problem, Equation (33), is the one in the learning regime, i.e. $\gamma \neq 0$. The objective of the policymaker is subject to the equilibrium condition of γ, which depends on p_1 and satisfies Equation(11). The second optimization problem, Equation (34), is the one in the no-learning regime. Substituting Equation (9) and $\gamma = 0$ into Equation(1) delivers Equation (34). There is no learning if $p_1 \leq z/\sigma_\epsilon$. If there are two equilibria — one in the learning regime, and the other one in the no-learning regime — the policymaker will choose the one with a lower total loss.

In the learning regime, by applying Equation (11), Equation (33) becomes

$$Min \quad z^2 + b^2 + \lambda(p_0 - \pi^*)^2 + \lambda(p_1+p_2-1)^2\sigma_\theta^2 + \lambda p_2^2\sigma_\epsilon,$$

$$s.t. \quad p_1 > z/\sigma_\epsilon.$$

The first order conditions are,

$$\partial p_0 : \quad p_0 = \pi^*,$$

$$\partial p_1 : \quad p_1 = 1,$$

$$\partial p_2 : \quad p_2 = 0.$$

Therefore,

$$\gamma = 1 - \frac{z}{\sigma_\epsilon}.$$

This equilibrium holds if $\frac{z}{\sigma_\epsilon} < 1$.

In the no-learning regime, Equation (34) becomes

$$Min \quad p_1^2\sigma_\epsilon^2 + b^2 + \lambda(p_0 - \pi^*)^2 + \lambda(p_1 + p_2 - 1)^2\sigma_\theta^2 + \lambda p_2^2\sigma_\epsilon,$$

$$s.t. \quad p_1 \leq z/\sigma_\epsilon.$$

The first order conditions are,

$$\partial p_0 : \quad p_0 = \pi^*,$$

$$\partial p_1 : \quad p_1 = \frac{\lambda}{\lambda + 1 + \sigma_\epsilon^2/\sigma_\theta^2},$$

$$\partial p_2 : \quad p_2 = \frac{1}{\lambda + 1 + \sigma_\epsilon^2/\sigma_\theta^2}.$$

And,

$$\gamma = 0.$$

This equilibrium holds if $\frac{\lambda}{\lambda+1+\sigma_\epsilon^2/\sigma_\theta^2} < \frac{z}{\sigma_\epsilon}$.

In summary, under commitment:

1. if $\frac{\lambda}{\lambda+1+\sigma_\epsilon^2/\sigma_\theta^2} < 1 < \frac{z}{\sigma_\epsilon}$, the economy is in the no-learning regime ($\gamma = 0$);

2. if $\frac{z}{\sigma_\epsilon} < \frac{\lambda}{\lambda+1+\sigma_\epsilon^2/\sigma_\theta^2}$, the economy is in the learning regime ($\gamma \neq 0$);

3. if $\frac{\lambda}{\lambda+1+\sigma_\epsilon^2/\sigma_\theta^2} < \frac{z}{\sigma_\epsilon} < 1$, two equilibria exit — one with $\gamma \neq 0$ and the other one with $\gamma = 0$;

   - if the commitment rule follows $\pi^C(\bar{\theta}, \theta) = \pi^* + \bar{\theta}$, then $\gamma = 0$ and $\mathcal{L}^C(\gamma = 0) = z^2 + b^2$;
   - if the commitment rule follows $\pi^C(\bar{\theta}, \theta) = \pi^* + \frac{\lambda}{\lambda+1+\sigma_\epsilon^2/\sigma_\theta^2}\bar{\theta} + \frac{1}{\lambda+1+\sigma_\epsilon^2/\sigma_\theta^2}\theta$, then $\gamma \neq 0$ and $\mathcal{L}^C(\gamma \neq 0) = \frac{\lambda\sigma_\epsilon^2}{\lambda+1+\sigma_\epsilon^2/\sigma_\theta^2} + b^2$;

   however, if $\left(\frac{z}{\sigma_\epsilon}\right)^2 < \frac{\lambda}{\lambda+1+\sigma_\epsilon^2/\sigma_\theta^2}$, $\mathcal{L}^C(\gamma = 0) < \mathcal{L}^C(\gamma \neq 0)$ and the policymaker will choose $\pi^C(\bar{\theta}, \theta) = \pi^* + \bar{\theta}$. Thus, the economy is in the no-learning regime.

Therefore, Proposition 2 holds.

B.2 Discretionary

The discretionary rule is set after agents make decisions on γ and a. Backward induction is used to solve for the equilibrium. The policymaker

$$Min_\pi \quad \mathbb{E}\left\{[(\pi - a) - b]^2 + \lambda\left(\pi - \pi^* - \bar{\theta}\right)^2\right\}.$$

The first order condition delivers

$$\partial\pi : \pi^D(\bar{\theta}, \theta) = \frac{a + b + \lambda\pi^* + \lambda\bar{\theta}}{1+\lambda}. \tag{35}$$

Substituting it into Equation (8) and Equation (9) delivers

$$p_0 = \pi^* + \frac{b}{\lambda},$$

$$p_1 = \frac{\lambda}{\lambda + 1 - \gamma^D},$$

$$p_2 = \frac{1 - \gamma^D}{\lambda + 1 - \gamma^D}.$$

Applying Equation (30), the equilibrium γ is,

$$\gamma^D = \frac{\lambda - (1+\lambda)\frac{z}{\sigma_\epsilon}}{\lambda - \frac{z}{\sigma_\epsilon}}.$$

This holds, if $\frac{z}{\sigma_\epsilon} < \frac{\lambda}{1+\lambda}$. Otherwise, if $\frac{z}{\sigma_\epsilon} \geq \frac{\lambda}{1+\lambda}$, $\gamma^D = 0$, and the policymaker chooses the following:

$$p_0 = \pi^* + \frac{b}{\lambda},$$

$$p_1 = \frac{\lambda}{\lambda+1},$$

$$p_2 = \frac{1-\gamma^D}{\lambda+1}.$$

Thus, Proposition 3 holds.

B.3 Adaptation Monetary Policy

The adaptation rule is set after agents make decisions on γ, but before they make decisions on a. In the final step, the aggregate action, a, from Equation (9), depends on p_0, p_1, p_2 and γ. In the penultimate step, the policymaker chooses p_0, p_1, and p_2 to

$$Min \quad \mathbb{E}[(1-\gamma)p_1(\bar{\theta}-\theta) - b]^2 + \lambda[p_0 - \pi^* + (p_1+p_2-1)\bar{\theta} + p_2(\theta-\bar{\theta})]^2, \tag{36}$$

$$s.t. \quad \gamma = \bar{\gamma} \neq 0,$$

in the learning regime, or

$$Min \quad \mathbb{E}[p_1(\bar{\theta}-\theta) - b]^2 + \lambda[p_0 - \pi^* + (p_1+p_2-1)\bar{\theta} + p_2(\theta-\bar{\theta})]^2, \tag{37}$$

$$s.t. \quad \gamma = \bar{\gamma} = 0,$$

in the no-learning regime. It is clear that the optimization problem in the no-learning regime is the same as the one under the commitment rule.

In the learning regime ($\bar{\gamma} \neq 0$), Equation (36) can be rewritten as

$$Min \quad (1-\bar{\gamma})^2 p_1^2 \sigma_\epsilon^2 + b^2 + \lambda(p_0-\pi^*)^2 + \lambda(p_1+p_2-1)^2\sigma_\theta^2 + \lambda p_2^2 \sigma_\epsilon^2.$$

The first order conditions are as follows:

$$\partial p_0 : \quad p_0 = \pi^*,$$

$$\partial p_1 : \quad p_1 = \frac{\lambda}{\lambda + (1-\gamma^A)^2(1+\sigma_\epsilon^2/\sigma_\theta^2)},$$

$$\partial p_2 : \quad p_2 = \frac{(1-\gamma^A)^2}{\lambda + (1-\gamma^A)^2(1+\sigma_\epsilon^2/\sigma_\theta^2)}.$$

Substituting p_1 into Equation (11) delivers

$$\gamma_1^A = 1 - \frac{\lambda - \sqrt{\lambda^2 - 4\left(\frac{z}{\sigma_\epsilon}\right)^2 (1+\sigma_\epsilon^2/\sigma_\theta^2)\lambda}}{2\frac{z}{\sigma_\epsilon}(1+\sigma_\epsilon^2/\sigma_\theta^2)},$$

$$\gamma_2^A = 1 - \frac{\lambda - \sqrt{\lambda^2 + 4\left(\frac{z}{\sigma_\epsilon}\right)^2 (1+\sigma_\epsilon^2/\sigma_\theta^2)\lambda}}{2\frac{z}{\sigma_\epsilon}(1+\sigma_\epsilon^2/\sigma_\theta^2)}.$$

- γ_1^A is meaningful if $0 < \gamma_1^A < 1$. Thus, the condition for the existence of γ_1^A is $\{\frac{\lambda}{2(1+\sigma_\epsilon^2/\sigma_\theta^2)} < \frac{z}{\sigma_\epsilon} \quad \& \quad \left(\frac{z}{\sigma_\epsilon}\right)^2 \leq \frac{\lambda}{4(1+\sigma_\epsilon^2/\sigma_\theta^2)}\}$, or $\left(\frac{z}{\sigma_\epsilon}\right)^2 < \frac{\lambda}{4(1+\sigma_\epsilon^2/\sigma_\theta^2)} < \frac{\lambda}{2(1+\sigma_\epsilon^2/\sigma_\theta^2)} < \frac{z}{\sigma_\epsilon}$.
- γ_2^A is meaningful if $0 < \gamma_2^A < 1$. Thus, the condition for the existence of γ_2^A is $Max\left\{\frac{\lambda}{2(1+\sigma_\epsilon^2/\sigma_\theta^2)}, \frac{\lambda}{\lambda+1+\sigma_\epsilon^2/\sigma_\theta^2}\right\} < \left(\frac{z}{\sigma_\epsilon}\right)^2 < \frac{\lambda}{4(1+\sigma_\epsilon^2/\sigma_\theta^2)}$.

In the no-learning regime ($\bar{\gamma} = 0$), the equilibrium condition is the same as under commitment.

Therefore, Proposition 4 holds.

C Without Public Signal

Without a public signal, the objective function of the policymaker is

$$Min \quad \mathbb{E}\{[p_1(1-\gamma)\bar{\theta} + b]^2 + \lambda[p_0 - \pi^* + (p_1 - 1)\bar{\theta}]^2\}, \tag{38}$$

where γ can be positive or zero. The procedure to solve for the equilibrium condition is the same as the one presented in the previous section. Table A1 lists π and the total losses under the three policies.

Table A1: Optimal Policy without Public Signal

	Commitment	Discretionary	Adaptation
No-Learning Regime			
$\pi(\gamma=0)$	$\pi^* + \frac{\lambda}{1+\lambda}\bar{\theta}$	$\pi^* + \frac{b}{\lambda} + \frac{\lambda}{1+\lambda}\bar{\theta}$	$\pi^* + \frac{\lambda}{1+\lambda}\bar{\theta}$
$\mathcal{L}(\gamma=0)$	$\frac{\lambda}{1+\lambda}\sigma_\theta^2 + b^2$	$\frac{\lambda}{1+\lambda}\sigma_\theta^2 + b^2 + \frac{b^2}{\lambda}$	$\frac{\lambda}{1+\lambda}\sigma_\theta^2 + b^2$
Learning Regime			
$\pi(\gamma\neq 0)$	$\pi^* + \bar{\theta}$	$\pi^* + \frac{b}{\lambda} + \frac{\lambda}{\lambda+1-\gamma^D}\bar{\theta}$	$\pi^* + \frac{\lambda}{\lambda+(1-\gamma^A)^2}\bar{\theta}$
$\mathcal{L}(\gamma\neq 0)$	$z^2 + b^2$	$z^2 + b^2 + \frac{z^2}{\lambda} + \frac{b^2}{\lambda}$	$z^2 + b^2 + (1-\gamma^A)^2\frac{z^2}{\lambda}$

D Shock on Output Target

If the aggregate shock is on output, the objective function of the policymaker is

$$Min \quad \mathbb{E}(\pi - a - b - \bar{\theta})^2 + \lambda(\pi - \pi^*)^2. \tag{39}$$

Commitment. The commitment policymaker takes the equilibrium condition of γ from Equation (11) into consideration. Thus, she minimizes

$$Min \quad z^2 + b^2 + \sigma_\theta^2 + \lambda[(p_0 - \pi^*)^2 + (p_1 + p_2)^2\sigma_\theta^2 + p_2^2\sigma_\epsilon^2],$$

$$s.t. \quad p_1 > z/\sigma_\epsilon;$$

or,

$$Min \quad p_1^2\sigma_\epsilon^2 + b^2 + \sigma_\theta^2 + \lambda[(p_0 - \pi^*)^2 + (p_1 + p_2)^2\sigma_\theta^2 + p_2^2\sigma_\epsilon^2],$$

$$s.t. \quad p_1 \le z/\sigma_\epsilon.$$

The only equilibrium is the one in the no-learning regime. The policy under commitment is

$$\pi^C = \pi^*.$$

Agents do not make learning effort on $\bar{\theta}$, i.e. $\gamma = 0$. The loss under the commitment rule is $\mathcal{L}^C = b^2 + \sigma_\theta^2$.

Discretion. The discretionary rule is as follows:

$$\pi^D = \pi^* + \frac{b}{\lambda} + \frac{1}{\lambda + 1 - \gamma^D}\bar{\theta} + \frac{(1-\gamma^D)}{\lambda}\frac{1}{\lambda + 1 - \gamma^D}\theta.$$

Applying Equation (30), the equilibrium γ is

$$\gamma^D = \frac{1 - (1+\lambda)\frac{z}{\sigma_\epsilon}}{1 - \frac{z}{\sigma_\epsilon}}.$$

This holds if $\frac{z}{\sigma_\epsilon} < \frac{1}{1+\lambda}$. Otherwise, if $\frac{z}{\sigma_\epsilon} \ge \frac{1}{1+\lambda}$, $\gamma^D = 0$. The loss under the discretionary rule is $\mathcal{L}^D = \frac{(1+\lambda)}{\lambda}(b^2 + \sigma_\theta^2 + z^2)$ in the learning regime, and $\mathcal{L}^D = \frac{(1+\lambda)}{\lambda}(b^2 + \sigma_\theta^2) + \frac{\sigma_\epsilon^2}{\lambda(1+\lambda)}$ in the no-learning regime.

Adaptation. The adaptation policymaker minimizes

$$Min \quad [p_1^2(1-\gamma)^2\sigma_\epsilon^2 + b^2 + \sigma_\theta^2] + \lambda[(p_0 - \pi^*)^2 + (p_1 + p_2)^2\sigma_\theta^2 + p_2^2\sigma_\epsilon^2].$$

The adaptation rule is the same as the commitment rule, in that

$$\pi^A = \pi^*.$$

The total loss under the adaptation rule is $\mathcal{L}^A = b^2 + \sigma_\theta^2$.

In summary, the commitment and adaptation rules are the same and are better than the discretionary rule. With a larger p_1, the discretionary policymaker gives agents larger learning incentives. However, this will increase the total loss. Discretion introduces an information bias, which results in excessive information acquisition.

E The Adam (2007) Specification

E.1 Derivation of the Household Sector and the Production Sector

The household sector is exactly the same as the one presented by Adam (2007). Households

$$\max_{Y,L} U(Y) - \nu V(L),$$

$$s.t. \quad PY = WL + \Pi - T.$$

The aggregate consumption is Y and labor supply is L. W denotes wage; Π is the monopolicy profits from firms; T is nominal transfers from the government; P is the price index; ν denotes a stochastic labor supply shifter.

The production sector consists of a continuum of monopolistically competitive firms, with production function,

$$Y^i = L^i,$$

and

$$Y = \left(\int_0^1 (Y^i)^{\eta-1/\eta} di \right)^{\eta/\eta-1},$$

where η is the elasticity of substitution between varieties. The only difference with Adam (2007) is that the production sector has an idiosyncratic taxation or revenue shock (t_i). The profit maximization problem of firm i is

$$\max_{P^i} \mathbb{E}[(1+t_i)P^i Y^i(P^i) - WY^i(P^i)|I^i]. \tag{40}$$

In Adam (2007), $t_i = \tau = \frac{1}{\eta-1}$, which is a constant output subsidy. We change it to be an idiosyncratic taxation or revenue shock around the steady state value $\frac{1}{\eta-1}$, in order to introduce agents' learning trade-off and idiosyncratic shocks into the model. Linearizing the first order condition of Equation (40) around the steady state delivers

$$\frac{P^i - \bar{P}}{\bar{P}} = \mathbb{E}\left[\frac{P - \bar{P}}{\bar{P}} + \frac{V''(\bar{Y})U'(\bar{Y}) - V'(\bar{Y})U''(\bar{Y})}{(U'(\bar{Y}))^2}\bar{Y}(\frac{Y - \bar{Y}}{\bar{Y}} - \frac{Y^* - \bar{Y}}{\bar{Y}}) - t_i + O(1)|I^i]\right].$$

The lower case letters are used to denote the percentage deviation from their steady state values, i.e. $x \equiv \frac{X - \bar{X}}{\bar{X}}$. p^i corresponds to a_i in the standard model. To be consistent with the notation in Section 2 to Section 6, p^i is relabeled by a^i and the nominal expenditure is denoted by $\pi \equiv y + p$. Also, we denote $\alpha \equiv \frac{V''(\bar{Y})U'(\bar{Y}) - V'(\bar{Y})U''(\bar{Y})}{(U'(\bar{Y}))^2}\bar{Y}$, and denote $u_i \equiv -t_i$. Additionally, the aggregate shock comes from the stochastic labor supply shifter (ν). We relabel the aggregate shock, $y^* \equiv \frac{Y^* - \bar{Y}}{\bar{Y}}$, by $\bar{\theta}$. Thus, each firm or agent chooses an action a_i to

$$\min \quad \mathbb{E}_i[a_i - (1-\alpha)a - \alpha(\pi - \bar{\theta}) - u_i]^2$$

The agents' objective function is very similar to Equation (3), except that there is an additional term, $\alpha\bar{\theta}$.

E.2 Agents' Learning Equilibrium and The Loss Function

Equilibrium γ. Using the same information structure as the one presented in Section 2 to Section 6, and following the procedure in A.1, the following is obtains at equilibrium,

$$[(1-\alpha)\gamma + \alpha](1 - exp(-x)) = \gamma, \tag{41}$$

and

$$\gamma = \begin{cases} 1 - \frac{\phi}{\alpha \exp(k)\sigma_\epsilon(p_1 - 1)}, & \text{when } \frac{\phi}{\alpha \exp(k)\sigma_\epsilon} < (p_1 - 1), \\ 0, & \text{when } \frac{\phi}{\alpha \exp(k)\sigma_\epsilon} \geq (p_1 - 1). \end{cases} \tag{42}$$

The loss function. The welfare-based monetary policy objective in this specification is exactly the same as the one presented by Adam (2007) (Equation (66), see Adam (2007) Appendix A.2 for details). Using my notation, the welfare loss is

$$(\pi - a - \bar{\theta})^2 + \lambda \int_0^1 (a_i - a)^2 di.$$

The social welfare loss contains output volatility and price dispersion. Nonetheless, with the information structure presented in this paper (as presented in Appendix A), price dispersion is independent of monetary policy (p_0, p_1 and p_2). Thus the trade-off of the central bank disappears. The formal proof for this property is presented below.

Price dispersion is independent of p_0, p_1 and p_2. Rather, price dispersion comes from agents' heterogeneous beliefs on $\bar{\theta}$ and their idiosyncratic shocks,

$$\begin{aligned}
\int (a_i - a)^2 di &= \{(p_1 - 1)[(1-\alpha)\gamma + \alpha](1 - exp(-x))(\theta_i - \bar{\theta})\}^2 + \int \mathbb{E}_i(u_i)^2 di \\
&= (p_1 - 1)^2 \gamma^2 (\theta_i - \bar{\theta})^2 + \int \mathbb{E}_i(u_i)^2 di.
\end{aligned} \tag{43}$$

Firstly,

$$\int \mathbb{E}_i(u_i)^2 di = \phi^2(1 - \exp(-2k)\exp(x)) = \phi^2(1 - \exp(-2k)(1 + \frac{\gamma}{\alpha(1-\gamma)})).$$

Secondly,

$$\int (\theta_i - \bar{\theta})^2 di = \frac{\exp(-x)\sigma_\epsilon^2}{1 - \exp(-x)} = \frac{\alpha(1-\gamma)}{\gamma}\sigma_\epsilon^2.$$

By substituting Equation (42), one obtains

$$(p_1 - 1)^2 \gamma^2 (\theta_i - \bar{\theta})^2 = \left(\frac{\gamma}{\alpha(1-\gamma)} \frac{\phi}{\exp(k)\sigma_\epsilon}\right)^2 \frac{\alpha(1-\gamma)}{\gamma}\sigma_\epsilon^2 = \phi^2 \frac{\gamma}{\alpha(1-\gamma)} \exp(-2k).$$

Therefore, price dispersion is equal to

$$\int (a_i - a)^2 di = \phi^2[1 - \exp(-2k)],$$

which is independent of monetary policy (p_0, p_1, p_2).

E.3 The Optimal Monetary Policy

The central bank in this set up,

$$\min \quad (\pi - a - \bar{\theta})^2.$$

Across all the three policies, it is optimal to set $p_1 = 1$ while p_0, p_2 are free parameters.

F An Alternative Model with Public Debt

This section presents an alternative micro founded model using debt as another justification of the commitment versus discretion problem.

F.1 Households

Households solve the following optimization problem,

$$\max_{C,L} \quad U(C) - \nu V(L),$$

$$s.t. \quad CP + B = WL + \Pi,$$

where C is consumption; L is labor; P is the price index of the aggregate consumption good; W denotes the competitive wage; Π is the monopoly profits from firms; T denotes nominal transfers from the monetary authority; $\nu > 0$ is a labor supply shock with $E(\nu) = 1$. The log utility equation $U(C) = \log(C)$ is used for simplicity. In addition, there is an external nominal debt (or loan) B, which needs to be repaid to foreign lenders. B is denoted in domestic currency and has to be repaid through selling real goods. The government imposes a lump-sum taxation on real income (τY) to repay the debt, such that $B = \tau Y P$, where τ represents the taxation percentile. The policymaker can manipulate P in such a way that when P is high, the real value of the debt is low. The market clearing condition is $C + \frac{B}{P} = Y$, which implies $C = (1-\tau)Y$. The aggregate shock comes from the variation of τ, which is related to the external debt B. The rational for the level of τ in steady state is discussed in Section F.3.

Thus, the household maximizes $U((1-\tau)Y) - \nu V(L)$.

F.2 Firm

The production sector consists of a continuum of monopolistically competitive firms, $i \in [0,1]$. Firms in this model correspond to agents (a_i) in the standard model (Section 2 to Section 6). Each firm has a linear technology with labor as the only production input,

$$Y^i = L^i.$$

Intermediate goods, Y^i, is aggregated to the aggregate output, Y, according to a Dixit-Stiglitz aggregator, such that

$$Y = \left(\int (Y^i)^{(\eta-1)/\eta} di \right)^{\eta/(\eta-1)},$$

where $\eta > 1$ is the aggregate price elasticity of firms' product demand. The demand function of each firm is

$$Y^i(P^i) = \left(\frac{P^i}{P} \right)^{-\eta} Y,$$

where

$$P = \left(\int (P^i)^{1-\eta} di \right)^{1/(1-\eta)},$$

is the aggregate price index. Each firm chooses P^i to maximize their profit as follows:

$$\max_{P^i} \mathbb{E}[(1+t_i)P^i Y^i(P^i) - W Y^i(P^i) | I^i],$$

where t_i is the idiosyncratic taxation or revenue shock, with steady state value, $\bar{t} = 0$. The first order condition is,

$$\mathbb{E} \quad \left[(1+t_i)(1-\eta) \left(\frac{P^i}{P} \right)^{-\eta} + \eta \frac{W}{P} \left(\frac{P^i}{P} \right)^{-\eta-1} | I^i \right] = 0.$$

The optimization for households delivers

$$\frac{W}{P} = \frac{\nu V'(\hat{Y})}{U'(C)} = \frac{\nu V'(\hat{Y})}{U'((1-\tau)Y)},$$

where $\hat{Y} = \int Y^i di$. There exists a symmetric deterministic steady state with $P^i = \bar{P}$, $Y^i = \bar{Y}$, and $\bar{\nu} = 1$. $\bar{Y}$ and $\bar{\tau}$ solves

$$\frac{V'(\bar{Y})}{U'((1-\bar{\tau})\bar{Y})} = \frac{\eta - 1}{\eta}.$$

The central bank can choose any level of $\bar{P}$. Consider $\bar{P}$ such that $\bar{B}/\bar{P}\bar{Y} \equiv \bar{\tau} = 1/\eta$. Thus the previous function implies,

$$\frac{V'(\bar{Y})}{(1-\bar{\tau})U'((1-\bar{\tau})\bar{Y})} = 1.$$

In addition, the efficient output level is such that

$$\frac{\nu V'(Y^*)}{(1-\bar{\tau})U'((1-\bar{\tau})Y^*)} = 1.$$

The variable ν induces variations in the efficient level of output. Linearizing around the steady state ν delivers

$$\nu - 1 = -\frac{V''(\bar{Y})U'((1-\bar{\tau})\bar{Y}) - (1-\bar{\tau})V'(\bar{Y})U''((1-\bar{\tau})\bar{Y})}{(1-\bar{\tau})(U'((1-\bar{\tau})\bar{Y}))^2}\bar{Y}\frac{Y^* - \bar{Y}}{\bar{Y}}.$$

Thus, linearizing the first order condition on the optimal pricing delivers

$$p_i = p + \alpha(y - y^*) - \xi(\pi - \bar{\theta}) - u_i + O(1), \tag{44}$$

where $\alpha \equiv \frac{\eta}{\eta-1}\frac{V''(\bar{Y})U'((1-\bar{\tau})\bar{Y}) - (1-\bar{\tau})V'(\bar{Y})U''((1-\bar{\tau})\bar{Y})}{(U'((1-\bar{\tau})\bar{Y}))^2}\bar{Y}$ and $\xi \equiv \frac{1}{1-\eta}$. All the lower case letters denote the log deviation from their long run steady state level, i.e. $n = \ln(\frac{N}{\bar{N}})$. The idiosyncratic shock, t_i, is relabeled by u_i. $\pi \equiv p + y$ is the aggregate nominal expenditure.

The derivation in this section holds, with or without variation of ν. When there is no labor supply shock, $\nu = 1$ and $Y^* = \bar{Y}$ (or $y^* = 0$). When there is ν shock, we assume that it is a common knowledge to both the central bank and the firms. We denote $b = y^*$ to be consistent with the notation in Section 2 to Section 6. When there is no ν shock, $b = 0$. $\hat{\tau} = ln(\frac{\tau}{\bar{\tau}})$ can be approximated by $(-\pi + \frac{b-\bar{b}}{\bar{b}})$. The aggregate shock, $\bar{\theta} \equiv \frac{b-\bar{b}}{\bar{b}}$, is a bijection of the external debt, B.

F.3 A Discussion on $\bar{\tau}$

In general, taxation is on the production revenue. Such as the discussion in Adam (2007) and Woodford (2003), τ is a profit subsidy to counter react with the monopoly power. Reconsidering the profit function and assuming there is a taxation or subsidy, τ^a, on the revenue, one finds that firms

$$\max \quad (1+\tau^a)P_i'Y_i - W'Y_i.$$

The first order condition is

$$(1-\eta)\left(\frac{P_i'}{P'}\right)^{-\eta} + \eta\frac{W'}{P'}\frac{1}{(1+\tau^a)}\left(\frac{P_i'}{P'}\right)^{-\eta-1} = 0,$$

where $\frac{W'}{P'} = \frac{V'(Y)}{U'(Y)}$. The optimal taxation level is

$$\tau^a = \frac{1}{\eta - 1}.$$

However, in my model, taxation is on the real income. There is distortion because taxation affects the consumption level. Consumption will change from Y to $(1-\tau)Y$. Thus, the wage price ratio is,[15]

$$\frac{W}{P} = \frac{V'(Y)}{U'((1-\tau)Y)} = (1-\tau)\left(\frac{V'(Y)}{U'(Y)}\right).$$

To achieve a similar effect as the one where the taxation is on revenue, we need $\frac{W}{P} = \frac{1}{(1+\tau^a)}\frac{W'}{P'} = \frac{1}{(1+\tau^a)}\frac{V'(Y)}{U'(Y)}$. Therefore, we choose the steady state taxation $\bar{\tau}$, so that

$$(1-\bar{\tau}) = \frac{1}{(1+\tau^a)}.$$

And,

$$\bar{\tau} = 1/\eta.$$

[15] With log utility, $(1-\tau)U'((1-\tau)Y) = U'(Y)$.

F.4 The Loss Function

This subsection derives the central bank's objective function. Let's denote $\Omega \equiv U((1-\tau)Y) - \nu V(Y)$ and define $\bar{C} \equiv (1-\bar{\tau})\bar{Y}$. The social welfare deviation from its steady state level is,

$$\begin{aligned}
\Omega - \bar{\Omega} \;=\; & -U'(\bar{C})\bar{Y}(\tau-\bar{\tau}) + U'(\bar{C})(1-\bar{\tau})(Y-\bar{Y}) \\
& + \frac{1}{2}U''(\bar{C})\bar{Y}^2(\tau-\bar{\tau})^2 + \frac{1}{2}U''(\bar{C})(1-\bar{\tau})^2(Y-\bar{Y})^2 - [U''(\bar{C})(1-\bar{\tau})\bar{Y} + U'(\bar{C})](\tau-\bar{\tau})(Y-\bar{Y}) \\
& - V'(\bar{Y})(\hat{Y}-\bar{Y}) - \frac{1}{2}V''(\bar{Y})(\hat{Y}-\bar{Y})^2 - V'(\bar{Y})(\hat{Y}-\bar{Y})(\nu-1) + O(2) + t.i.p.
\end{aligned}$$

Using $(1-\bar{\tau})U'((1-\bar{\tau})\bar{Y}) = V'(\bar{Y})$ and $V'(\bar{Y})(\nu-1) = [U''(\bar{C})(1-\bar{\tau})^2 - V''(\bar{Y})]\bar{Y}y^*$, one obtains,

$$U'(\bar{C})(1-\bar{\tau})(Y-\bar{Y}) - V'(\bar{Y})(\hat{Y}-\bar{Y}) = V'(\bar{Y})[(Y-\bar{Y}) - (\hat{Y}-\bar{Y})],$$

$$V'(\bar{Y})(\hat{Y}-\bar{Y})(\nu-1) = [U''(\bar{C})(1-\bar{\tau})^2 - V''(\bar{Y})](Y^*-\bar{Y})(\hat{Y}-\bar{Y}).$$

Using this relation and the fact that $Y = \hat{Y} + O(1) + t.i.p.$, and adding $\frac{1}{2}(U''(\bar{C})(1-\bar{\tau})^2 - V''(\bar{Y}))(Y^* - \bar{Y})^2$ (which is independent of the policy), one obtains,

$$\frac{1}{2}U''(\bar{C})(1-\bar{\tau})^2(Y-\bar{Y})^2 - \frac{1}{2}V''(\bar{Y})(\hat{Y}-\bar{Y})^2 - V'(\bar{Y})(\hat{Y}-\bar{Y})(\nu-1)$$

$$= \frac{1}{2}(U''(\bar{C})(1-\bar{\tau})^2 - V''(\bar{Y}))\bar{Y}^2\left(\frac{(\hat{Y}-\bar{Y})}{\bar{Y}} - \frac{(Y^*-\bar{Y})}{\bar{Y}}\right)^2.$$

Moreover, from the definition of Y and $\bar{Y}$, one obtains

$$\begin{aligned}
Y - \bar{Y} &= (\hat{Y}-\bar{Y}) - \frac{1}{2}\frac{1}{\eta\bar{Y}}\int (Y^j - \hat{Y})^2 dj + O(2) + t.i.p., \\
Y^i &= Y - \frac{\eta\bar{Y}}{\bar{P}}(P^i - P) + O(1) + t.i.p., \\
(Y^i - Y)^2 &= \left[\frac{\eta\bar{Y}}{\bar{P}}(P^i - P)\right]^2 + O(2) + t.i.p..
\end{aligned}$$

Thus,

$$V'(\bar{Y})[(Y-\bar{Y}) - (\hat{Y}-\bar{Y})] = -\frac{1}{2}V'(\bar{Y})\eta\bar{Y}\int \left(\frac{P^j-\bar{P}}{\bar{P}} - \frac{\hat{P}-\bar{P}}{\bar{P}}\right)^2 dj.$$

With log utility, one obtains $U'(\bar{C})+U''(\bar{C})\bar{C}=0$. Therefore, the cross product term, $(\tau-\bar{\tau})(Y-\bar{Y})$, disappears. The terms related to τ are

$$\frac{1}{2}U''(\bar{C})\bar{Y}^2(\tau-\bar{\tau})^2-U'(\bar{C})\bar{Y}(\tau-\bar{\tau}).$$

Adding a constant term $\frac{1}{2}\left(\frac{U'(\bar{C})^2}{U''(\bar{C})(2-\bar{\tau})}\right)$, the last term of $\Omega-\bar{\Omega}$ becomes,

$$\frac{1}{2}U''(\bar{C})\bar{Y}^2\bar{\tau}^2(2-\bar{\tau})(\hat{\tau}+\frac{1-\bar{\tau}}{\bar{\tau}(2-\bar{\tau})})^2,$$

where $\hat{\tau}=\ln(\frac{\tau}{\bar{\tau}})$.[16] Recall that $\tau=\frac{B}{PY}$ and $\bar{\tau}=\frac{\bar{B}}{\bar{P}\bar{Y}}$, so

$$\hat{\tau}=\ln\left(\frac{\frac{B}{PY}}{\frac{\bar{B}}{\bar{P}\bar{Y}}}\right)=-\ln(\frac{PY}{\bar{P}\bar{Y}})+\ln(\frac{B}{\bar{B}})\approx(-\pi+\frac{b-\bar{b}}{\bar{b}}).$$

Denoting $\bar{\theta}=\frac{b-\bar{b}}{\bar{b}}$ as the aggregate shock, the last term of $\Omega-\bar{\Omega}$ becomes

$$\frac{1}{2}U''(\bar{C})\bar{Y}^2\bar{\tau}^2(2-\bar{\tau})(\pi-\bar{\theta}-\pi^*)^2.$$

The variable π^* does not denote the optimal nominal GDP level; it is just the constant term in the quadratic function, and $\pi^*=-\frac{1-\bar{\tau}}{\bar{\tau}(2-\bar{\tau})}$.

Therefore, the central bank's objective function is

$$\begin{aligned}\Omega-\bar{\Omega}= \quad & \frac{1}{2}[U''(\bar{C})(1-\bar{\tau})^2-V''(\bar{Y})]\bar{Y}^2(y-y^*)^2+\frac{1}{2}U''(\bar{C})\bar{Y}^2\bar{\tau}^2(2-\bar{\tau})(\pi-\bar{\theta}-\pi^*)^2\\ & -\frac{1}{2}V'(\bar{Y})\eta\bar{Y}\int(p^i-p)^2di.\end{aligned}$$

It is similar to the central bank's objective function in the standard Barro-Gordon economy, Equation (1), except that there is a third term related with price dispersion. However, price dispersion is independent of monetary policy (see the proof in the next subsection). Thus, the social

[16] Following the second order Taylor series expansion in Rotemberg and Woodford (1998), $\frac{1}{2}U''(\bar{C})\bar{Y}^2(\tau-\bar{\tau})^2-U'(\bar{C})\bar{Y}(\tau-\bar{\tau})\approx\frac{1}{2}[U''(\bar{C})\bar{Y}^2-U'(\bar{C})\bar{Y}]\bar{\tau}^2\hat{\tau}^2-U'(\bar{C})\bar{Y}\bar{\tau}\hat{\tau}$.

welfare loss function is the same as in the standard model, denoted by Equation (1),

$$(\pi - a - b)^2 + \lambda(\pi - \pi^* - \bar{\theta})^2,$$

where $b \equiv y^*$, $\pi \equiv y + a$ and $\lambda \equiv \frac{U''(\bar{C})\bar{\tau}^2(2-\bar{\tau})}{(U''(\bar{C})(1-\bar{\tau})^2 - V''(\bar{Y}))} > 0$.[17]

From now on, to be consistent with the standard model, we relabel p by a and relabel y^* by b. So, the optimal reaction of agents or the price level of each firm (Equation (44)) is,

$$a_i = \mathbb{E}_i[(1-\alpha)a + \alpha(\pi - b) - \xi(\pi - \theta) - u_i], \tag{45}$$

where $y = \frac{Y-\bar{Y}}{\bar{Y}}$,$a = \frac{P-\bar{P}}{\bar{P}}$. It is very similar to Equation (4), except that there is an additional term, $-\xi(\pi - \theta) - \alpha b$.

F.5 Agents' Optimal Learning Decision

So far, the objective functions of agents and the central bank have been set up. Now we will repeat the procedure from Section 2 to Section 5 and solve for the equilibrium learning effort (γ) and the optimal monetary policy (p_0, p_1, and p_2). The information structure and the monetary policy structure are exactly the same as before.

First, we solve for the optimal reaction of agents. With perfect information, the aggregate action (a, Equation (45)) is,

$$a = \pi - b - \frac{\xi}{\alpha}(\pi - \bar{\theta}).$$

Under imperfect information, the initial guess about a is,

$$a = (1 - \frac{\xi}{\alpha})[P_0 + P_1(\gamma\bar{\theta} + (1-\gamma)\theta] + P_2\theta] + \frac{\xi}{\alpha}[\gamma\bar{\theta} + (1-\gamma)\theta] - b, \tag{46}$$

[17] $b = 0$ when there is no labor supply shock.

using $E(\bar{\theta}) = \gamma\bar{\theta} + (1-\gamma)\theta$. Therefore,

$$a_i = (1-\frac{\xi}{\alpha})P_0 + \left[(\frac{1-\alpha}{\alpha}\gamma+1)[(\alpha-\xi)P_1+\xi\right]\mathbb{E}_i(\bar{\theta}) + \left\{\frac{1-\alpha}{\alpha}(1-\gamma)[(\alpha-\xi)P_1+\xi] + (1-\frac{\xi}{\alpha})P_2\right\}\theta - b + E(u_i).$$

At equilibrium $\int a_i = a$, so,

$$[(\frac{1-\alpha}{\alpha}\gamma+1)[(\alpha-\xi)P_1+\xi](1-\exp(-x)) = [(\alpha-\xi)P_1+\xi]\frac{\gamma}{\alpha}.$$

The objective function for agents transforms to

$$\min \quad E(u_i)^2 + X_1^2 E(\hat{\theta}_i - \bar{\theta})^2,$$

with $X_1 \equiv [(\frac{1-\alpha}{\alpha}\gamma+1)[(\alpha-\xi)P_1+\xi]$. It is equivalent to,

$$\min \quad \phi^2 e^{-2k} e^x + X_1^2 e^{-x}\sigma_\epsilon^2.$$

Therefore, the optimal learning effort on the aggregate shock is

$$x = \max\{k + ln(\frac{\sigma_\epsilon}{\phi}) + ln(X_1), 0)\},$$

or, in exponential form,

$$\exp(-x) = \frac{\phi}{\exp(k)\sigma_\epsilon}\frac{1}{[(\frac{1-\alpha}{\alpha}\gamma+1)[(\alpha-\xi)P_1+\xi]}.$$

At equilibrium, $X_1(1-e^{-x}) = [(\alpha-\xi)P_1+\xi]\frac{\gamma}{\alpha}$ and $X_1 e^{-x} = (1-\gamma)[(\alpha-\xi)P_1+\xi]$. Therefore,

$$[(\alpha-\xi)P_1+\xi](1-\gamma) = \frac{\phi}{\exp(k)\sigma_\epsilon}.$$

And the initial guess on a is confirmed. The condition that there is private learning on aggregate shock $(\bar{\theta})$ is

$$\gamma > 0 \quad if \quad [(\alpha-\xi)P_1+\xi] > \frac{\phi}{\exp(k)\sigma_\epsilon}.$$

Price Dispersion is independent of p_0, p_1 and p_2. Similar to the discussion in Appendix E.3, price dispersion comes from the heterogeneous beliefs of agents about $\bar{\theta}$ and their idiosyncratic shocks.

$$\begin{aligned}\int (a_i - a)^2 di &= \{X_1(1-e^{-x})(\theta_i - \bar{\theta})\}^2 + \int \mathbb{E}_i(u_i)^2 di \\ &= [(\alpha-\xi)P_1 + \xi]^2 \frac{\gamma^2}{\alpha^2}(\theta_i - \bar{\theta})^2 + \int \mathbb{E}_i(u_i)^2 di.\end{aligned} \tag{47}$$

Firstly,

$$\int \mathbb{E}_i(u_i)^2 di = \phi^2(1 - \exp(-2k)\exp(x)) = \phi^2(1 - \exp(-2k)(1 + \frac{\gamma}{\alpha(1-\gamma)})).$$

Secondly,

$$\int (\theta_i - \bar{\theta})^2 di = \frac{\exp(-x)\sigma_\epsilon^2}{1-\exp(-x)} = \frac{\alpha(1-\gamma)}{\gamma}\sigma_\epsilon^2.$$

Thus,

$$[(\alpha-\xi)P_1 + \xi]^2 \frac{\gamma^2}{\alpha^2}(\theta_i - \bar{\theta})^2 = \left(\frac{\gamma}{\alpha(1-\gamma)}\frac{\phi}{\exp(k)\sigma_\epsilon}\right)^2 \frac{\alpha(1-\gamma)}{\gamma}\sigma_\epsilon^2 = \phi^2 \frac{\gamma}{\alpha(1-\gamma)}\exp(-2k).$$

Therefore, price dispersion is equal to

$$\int (a_i - a)^2 di = \phi^2[1 - \exp(-2k)],$$

which is independent of the monetary policy (p_0, p_1, p_2).

F.6 The Optimal Monetary Policy

Perfect Information. Under perfect information, the central bank's objective function is

$$\min \quad \left(\frac{\xi}{\alpha}\right)^2 (\pi - \bar{\theta})^2 + \lambda(\pi - \pi^* - \bar{\theta})^2.$$

The optimal commitment and discretionary policies are as follows:

$$\pi^C = \bar{\theta} + \frac{\lambda \pi^*}{\lambda + \frac{\xi^2}{\alpha^2}},$$

$$\pi^D = \bar{\theta} + \frac{\lambda\pi^*}{\lambda + \frac{\xi}{\alpha}},$$

with $\xi < 0$ and $\pi^D > \pi^C$.

Discretion is worse than commitment due to the inflation bias.

Imperfect Information. Under imperfect information, the central bank's objective is to minimize

$$\min \quad (\pi - a - b)^2 + \lambda(\pi - \pi^* - \bar{\theta})^2.$$

F.6.1 Commitment

In the learning regime ($\gamma \neq 0$), the policymaker minimizes

$$\frac{\phi^2}{\exp(2k)\alpha^2} + (\lambda + \frac{\xi^2}{\alpha^2})P_2^2\sigma_\epsilon^2 - 2\frac{\phi}{\exp(k)\alpha}\frac{\xi}{\alpha}P_2\sigma_\epsilon + (\lambda + \frac{\xi^2}{\alpha^2})(P_1 + P_2 - 1)^2\sigma_\theta^2 + \frac{\xi^2}{\alpha^2}P_0^2 + \lambda(P_0 - \pi^*)^2.$$

The first order conditions are,

$$P_0^C = \frac{\lambda\pi^*}{\lambda + \frac{\xi^2}{\alpha^2}},$$

$$P_1^C = \frac{\lambda + \frac{\xi^2}{\alpha^2} - \frac{\phi}{\alpha\exp(k)\sigma_\epsilon}\frac{\xi}{\alpha}}{\lambda + \frac{\xi^2}{\alpha^2}},$$

$$P_2^C = \frac{\frac{\phi}{\alpha\exp(k)\sigma_\epsilon}\frac{\xi}{\alpha}}{\lambda + \frac{\xi^2}{\alpha^2}}.$$

The equilibrium learning effort on the aggregate shock is

$$\gamma^C = 1 - \frac{\left(\frac{\phi}{\exp(k)\sigma_\epsilon}\right)(\lambda + \frac{\xi^2}{\alpha^2})}{\alpha(\lambda + \frac{\xi^2}{\alpha^2}) - (\alpha - \xi)\left(\frac{\phi}{\alpha\exp(k)\sigma_\epsilon}\frac{\xi}{\alpha}\right)}.$$

The total loss is

$$\mathcal{L}^C = \frac{\phi^2}{\exp(2k)\alpha^2} + (\lambda + \frac{\xi^2}{\alpha^2})\left(\frac{\frac{\phi}{\alpha\exp(k)\sigma_\epsilon}\frac{\xi}{\alpha}}{\lambda + \frac{\xi^2}{\alpha^2}}\right)^2\sigma_\epsilon^2 - 2\frac{\phi}{\alpha\exp(k)}\frac{\xi}{\alpha}\left(\frac{\frac{\phi}{\alpha\exp(k)\sigma_\epsilon}\frac{\xi}{\alpha}}{\lambda + \frac{\xi^2}{\alpha^2}}\right)\sigma_\epsilon + \lambda\frac{\xi^2}{\alpha^2}\frac{\pi^{*2}}{(\lambda + \frac{\xi^2}{\alpha^2})}.$$

In the no-learning regime ($\gamma = 0$), the policymaker minimizes

$$\frac{1}{\alpha^2}[(\alpha-\xi)P_1+\xi]^2\sigma_\epsilon^2+(\frac{\xi^2}{\alpha^2}+\lambda)(P_1+P_2-1)^2\sigma_\theta^2+(\frac{\xi^2}{\alpha^2}+\lambda)P_2^2\sigma_\epsilon^2-2[(1-\frac{\xi}{\alpha})P_1+\frac{\xi}{\alpha}]\frac{\xi}{\alpha}P_2\sigma_\epsilon^2+\frac{\xi^2}{\alpha^2}P_0^2+\lambda(P_0-\pi^*)^2.$$

The first order conditions are as follows:

$$P_0^C = \frac{\lambda\pi^*}{\lambda+\frac{\xi^2}{\alpha^2}},$$

$$P_1^C = \frac{(\lambda+\frac{\xi^2}{\alpha^2})^2\frac{\sigma_\epsilon^2}{\sigma_\theta^2} - (\lambda+\frac{\xi^2}{\alpha^2})\left\{\frac{\sigma_\epsilon^2}{\sigma_\theta^2}(1+\frac{\sigma_\epsilon^2}{\sigma_\theta^2})\frac{\xi}{\alpha}(1-\frac{\xi}{\alpha})+2\frac{\sigma_\epsilon^2}{\sigma_\theta^2}\frac{\xi^2}{\alpha^2}-\frac{\sigma_\epsilon^2}{\sigma_\theta^2}\frac{\xi}{\alpha}\right\}+(\frac{\sigma_\epsilon^2}{\sigma_\theta^2})^2(1-\frac{\xi}{\alpha})\frac{\xi^3}{\alpha^3}}{(\lambda+\frac{\xi^2}{\alpha^2})(1+\frac{\sigma_\epsilon^2}{\sigma_\theta^2})\left(\lambda+\frac{\xi^2}{\alpha^2}+\frac{\sigma_\epsilon^2}{\sigma_\theta^2}(1-\frac{\xi}{\alpha})^2\right)-\left(\lambda+\frac{\xi^2}{\alpha^2}-\frac{\sigma_\epsilon^2}{\sigma_\theta^2}(1-\frac{\xi}{\alpha})\frac{\xi}{\alpha}\right)^2},$$

$$P_2^C = \frac{(\lambda+\frac{\xi^2}{\alpha^2})\left[(1-\frac{\xi}{\alpha})+\frac{\xi}{\alpha}\right]}{(\lambda+\frac{\xi^2}{\alpha^2})^2+(\lambda+\frac{\xi^2}{\alpha^2})\left[(1+\frac{\sigma_\epsilon^2}{\sigma_\theta^2})(1-\frac{\xi}{\alpha})^2+2\frac{\xi}{\alpha}(1-\frac{\xi}{\alpha})\right]-\frac{\sigma_\epsilon^2}{\sigma_\theta^2}\frac{\xi^2}{\alpha^2}(1-\frac{\xi}{\alpha})^2}.$$

The total loss equals

$$\mathcal{L}^C = \frac{1}{\alpha^2}[(\alpha-\xi)P_1+\xi]^2\sigma_\epsilon^2+(\frac{\xi^2}{\alpha^2}+\lambda)(P_1+P_2-1)^2\sigma_\theta^2+(\frac{\xi^2}{\alpha^2}+\lambda)P_2^2\sigma_\epsilon^2$$

$$-2[(1-\frac{\xi}{\alpha})P_1+\frac{\xi}{\alpha}]\frac{\xi}{\alpha}P_2\sigma_\epsilon^2+\frac{\xi^2}{\alpha^2}P_0^2+\lambda(P_0-\pi^*)^2.$$

F.6.2 Discretion

The optimal discretionary rule is,

$$\pi = \frac{a+b+\lambda\pi^*+\lambda\bar{\theta}}{1+\lambda}.$$

In the learning regime ($\gamma \neq 0$), the policymaker chooses the following:

$$P_0^D = \frac{\lambda\pi^*}{\lambda+\frac{\xi}{\alpha}},$$

$$P_1^D = \frac{\lambda+\frac{\xi}{\alpha}\gamma}{\lambda+\frac{\xi}{\alpha}\gamma+1-\gamma},$$

$$P_2^D = \frac{1-\gamma}{\lambda + \frac{\xi}{\alpha}\gamma + 1 - \gamma}.$$

The equilibrium learning effort is,

$$\gamma^D = \frac{\lambda + \frac{\xi}{\alpha} - (1+\lambda)\frac{\phi}{\alpha \exp(k)\sigma_\epsilon}}{\lambda + \frac{\xi}{\alpha} - (1-\frac{\xi}{\alpha})\frac{\phi}{\alpha \exp(k)\sigma_\epsilon}},$$

using $(\alpha - \xi)P_1 + \xi = \alpha(\lambda + \frac{\xi}{\alpha})$.

The total loss is

$$\mathcal{L}^D = \frac{\phi^2}{\exp(2k)\alpha^2} + \left[(\frac{\xi^2}{\alpha^2} + \lambda) \left(\frac{1-\gamma}{\lambda + \frac{\xi}{\alpha}\gamma + 1 - \gamma} \right)^2 - 2\frac{\phi}{\alpha exp(k)\sigma_\epsilon}\frac{\xi}{\alpha}\left(\frac{1-\gamma}{\lambda + \frac{\xi}{\alpha}\gamma + 1 - \gamma} \right) \right] \sigma_\epsilon^2$$

$$+\lambda\frac{\xi^2}{\alpha^2}\frac{(\lambda+1)\pi^{*2}}{(\lambda + \frac{\xi}{\alpha})^2}.$$

In the no-learning regime ($\gamma = 0$), the discretionary policymaker sets

$$P_0^D = \frac{\lambda\pi^*}{\lambda + \frac{\xi}{\alpha}},$$

$$P_1^D = \frac{\lambda}{\lambda+1},$$

$$P_2^D = \frac{1}{\lambda+1}.$$

The total loss is,

$$\mathcal{L}^D = \frac{(\lambda + \frac{\xi}{\alpha})^2\sigma_\epsilon^2}{(\lambda+1)^2} + \frac{(\lambda + \frac{\xi^2}{\alpha^2})\sigma_\epsilon^2}{(\lambda+1)^2} - 2\frac{\lambda + \frac{\xi}{\alpha}}{\lambda+1}\frac{\xi}{\alpha}\frac{1}{\lambda+1}\sigma_\epsilon^2 + \lambda\frac{\xi^2}{\alpha^2}\frac{(\lambda+1)\pi^{*2}}{(\lambda + \frac{\xi}{\alpha})^2}.$$

F.6.3 Adaptation

In the learning regime ($\gamma \neq 0$), the adaptation policymaker minimizes

$$\frac{(1-\gamma)^2}{\alpha^2}[(\alpha-\xi)P_1+\xi]^2\sigma_\epsilon^2 + (\frac{\xi^2}{\alpha^2}+\lambda)P_2^2\sigma_\epsilon^2 - 2\frac{1-\gamma}{\alpha}\frac{\xi}{\alpha}[(\alpha-\xi)P_1+\xi]P_2\sigma_\epsilon^2 + (\frac{\xi^2}{\alpha^2}+\lambda)(P_1+P_2-1)^2\sigma_\theta^2$$

$$+\frac{\xi^2}{\alpha^2}P_0^2 + \lambda(P_0 - \pi^*)^2.$$

The adaptation rule is as follows:

$$P_0^A = \frac{\lambda \pi^*}{\lambda + \frac{\xi^2}{\alpha^2}},$$

$$P_1^A = \frac{(\lambda+\frac{\xi^2}{\alpha^2})^2\frac{\sigma_\epsilon^2}{\sigma_\theta^2} - (\lambda+\frac{\xi^2}{\alpha^2})\left\{(1-\gamma)^2\frac{\sigma_\epsilon^2}{\sigma_\theta^2}(1+\frac{\sigma_\epsilon^2}{\sigma_\theta^2})\frac{\xi}{\alpha}(1-\frac{\xi}{\alpha}) + 2(1-\gamma)\frac{\sigma_\epsilon^2}{\sigma_\theta^2}\frac{\xi^2}{\alpha^2} - (1-\gamma)\frac{\sigma_\epsilon^2}{\sigma_\theta^2}\frac{\xi}{\alpha}\right\} + (1-\gamma)^2(\frac{\sigma_\epsilon^2}{\sigma_\theta^2})^2(1-\frac{\xi}{\alpha})\frac{\xi^3}{\alpha^3}}{(\lambda+\frac{\xi^2}{\alpha^2})(1+\frac{\sigma_\epsilon^2}{\sigma_\theta^2})\left(\lambda+\frac{\xi^2}{\alpha^2}+(1-\gamma)^2\frac{\sigma_\epsilon^2}{\sigma_\theta^2}(1-\frac{\xi}{\alpha})^2\right) - \left(\lambda+\frac{\xi^2}{\alpha^2}-(1-\gamma)\frac{\sigma_\epsilon^2}{\sigma_\theta^2}(1-\frac{\xi}{\alpha})\frac{\xi}{\alpha}\right)^2},$$

$$P_2^A = \frac{(\lambda+\frac{\xi^2}{\alpha^2})(1-\gamma)\left[(1-\gamma)(1-\frac{\xi}{\alpha})+\frac{\xi}{\alpha}\right]}{(\lambda+\frac{\xi^2}{\alpha^2})^2 + (\lambda+\frac{\xi^2}{\alpha^2})\left[(1-\gamma)^2(1+\frac{\sigma_\epsilon^2}{\sigma_\theta^2})(1-\frac{\xi}{\alpha})^2 + 2(1-\gamma)\frac{\xi}{\alpha}(1-\frac{\xi}{\alpha})\right] - (1-\gamma)^2\frac{\sigma_\epsilon^2}{\sigma_\theta^2}\frac{\xi^2}{\alpha^2}(1-\frac{\xi}{\alpha})^2}.$$

The equilibrium γ is

$$\gamma^A = 1 - \frac{(\lambda+\frac{\xi^2}{\alpha^2})^2 - 2(\lambda+\frac{\xi^2}{\alpha^2})\left(\frac{\phi}{\alpha\exp(k)\sigma_\epsilon}\right)\frac{\xi}{\alpha}(1-\frac{\xi}{\alpha}) \pm \sqrt{(\lambda+\frac{\xi^2}{\alpha^2})^4 - 4\lambda(\lambda+\frac{\xi^2}{\alpha^2})^2(1+\frac{\sigma_\epsilon^2}{\sigma_\theta^2})(1-\frac{\xi}{\alpha})^2\left(\frac{\phi}{\alpha\exp(k)\sigma_\epsilon}\right)^2}}{2(\lambda+\frac{\xi^2}{\alpha^2})\left[(1+\frac{\sigma_\epsilon^2}{\sigma_\theta^2})(1-\frac{\xi}{\alpha})^2\left(\frac{\phi}{\alpha\exp(k)\sigma_\epsilon}\right) - \frac{\xi}{\alpha}(1-\frac{\xi}{\alpha})\right] - 2\frac{\sigma_\epsilon^2}{\sigma_\theta^2}\frac{\xi^2}{\alpha^2}(1-\frac{\xi}{\alpha})^2\left(\frac{\phi}{\alpha\exp(k)\sigma_\epsilon}\right)}.$$

The total loss is,

$$\mathcal{L}^A = \frac{(1-\gamma)^2}{\alpha^2}[(\alpha-\xi)P_1+\xi]^2\sigma_\epsilon^2 + (\frac{\xi^2}{\alpha^2}+\lambda)P_2^2\sigma_\epsilon^2 - 2\frac{1-\gamma}{\alpha}\frac{\xi}{\alpha}[(\alpha-\xi)P_1+\xi]P_2\sigma_\epsilon^2 + (\frac{\xi^2}{\alpha^2}+\lambda)(P_1+P_2-1)\sigma_\theta^2$$

$$+\frac{\xi^2}{\alpha^2}P_0^2 + \lambda(P_0-\pi^*)^2.$$

In the no-learning regime ($\gamma = 0$), the adaptation rule is the same as the commitment rule.

F.7 Intuition and Properties

The only difference with the standard model is the optimal reaction function of agents, Equation (45). However, all the interesting discoveries presented in Section 2-6 still hold: there is an information bias under discretion and adaptation; the commitment policy has the learning incentive effect while the discretionary policy has the flexible policy setting effect; the commitment rule is the most efficient one while the comparison between discretion and adaptation depends on the circumstances.

However, in this model, there is a gap between π and a, even without information frictions ($a =$

$(1-\frac{\xi}{\alpha})\pi + \frac{\xi}{\alpha}\bar{\theta} - b)$. The commitment policymaker will not set $p_1^C = 1$ or $p_2^C = 0$. Also, the equilibrium γ^C is smaller than the one in the standard model. From the point of view of the central bank, it is not true that the larger γ is, the better the economy is.

The policymaker minimizes

$$(\pi - a - b)^2 + \lambda(\pi - \pi^* - \bar{\theta})^2,$$

which is equal to

$$\left\{(1-\gamma)[(1-\frac{\xi}{\alpha})p_1 + \frac{\xi}{\alpha}] - \frac{\xi}{\alpha}p_2\right\}^2 \sigma_\epsilon^2 + \lambda p_2^2 \sigma_\epsilon^2 + (\lambda + \frac{\xi^2}{\alpha^2})(p_1 + p_2 - 1)^2 \sigma_\theta^2 + \frac{\xi^2}{\alpha^2} p_0^2 + \lambda(p_0 - \pi^*)^2,$$

with,

$$\begin{aligned}
\mathcal{L}^O &= \left\{(1-\gamma)[(1-\frac{\xi}{\alpha})p_1 + \frac{\xi}{\alpha}] - \frac{\xi}{\alpha}p_2\right\}^2 \sigma_\epsilon^2 + \frac{\xi^2}{\alpha^2}(p_1 + p_2 - 1)^2 \sigma_\theta^2 + \frac{\xi^2}{\alpha^2} p_0^2, \\
\mathcal{L}^I &= \lambda p_2^2 \sigma_\epsilon^2 + \lambda(p_1 + p_2 - 1)^2 \sigma_\theta^2 + \lambda(p_0 - \pi^*)^2.
\end{aligned}$$

Because γ could approach 1 but could not equal 1 in this model, the optimal p_1^C is not 1.

Biases. The discretionary policy has the *inflation bias*, because $p_0^D > p_0^C$.

In addition, the discretionary rule and the adaptation rule have the *information bias*. The values of p_1^D (p_2^D) and p_1^A (p_2^A) are different from p_1^C (p_2^C). The commitment policymaker gives agents learning incentives and controls γ through p_1. Unlike the benchmark model presented in Section 2 to Section 6, a larger γ may increase the total loss. If $p_1^C > 1$, the result shows that $min\{p_1^D, p_1^A\} > p_1^C$.[18] For example, if agents believe that $p_1 = p_1^C = 1.2$, a will put a larger weight on $\bar{\theta}$ than p_1, according to Equation (46) (since $\xi < 0$ and $p_1 > 0$). Under discretion or adaptation, the policymaker would like to track agents' actions closer, thus putting a larger weight on $\bar{\theta}$, i.e. $min\{p_1^D, p_1^A\} > 1.2$. However, agents expect this ex ante, so a becomes even more sensitive to $\bar{\theta}$. In this case, the information bias is excessive information acquisition (in the standard model, it is insufficient information acquisition). This information bias increases the total loss.

[18] In this new model, it is possible to have negative p_2 and $p_1 > 1$.

Efficiency. It is still true that the commitment rule is the most efficient monetary policy. Moreover, as in the standard model, adaptation could be worse than discretion, depending on the circumstances. As illustrated in Figure 6, $\mathcal{L}^A$ is bigger than $\mathcal{L}^D$ under the condition presented.[19]

[19] There are two equilibria under adaptation. In Figure 6, $\mathcal{L}_1^A$ is bigger than $\mathcal{L}^D$, when λ is small.