Propagation of Financial Shocks in an Input-Output Economy with Trade and Financial Linkages of Firms

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December 23, 2015

Abstract

Firms are connected through the production network. At the same time, the production linkages coincide with financial linkages because of delays to input payments. This paper investigates how these interconnected production and financial linkages lead to the propagation of financial shocks both upstream and downstream. First, I show that financial shocks can propagate upstream if there are financial linkages of firms and financial frictions in trade. Second, I find, based on the input-output matrix and the bond yield data in the U.S., upstream propagation of financial shocks is stronger than downstream propagation. Third, I elaborate a DSGE model that can capture this pattern of shocks and generate quantitative predictions. Fourth, I demonstrate that credit policies would have a stronger impact if liquidity were transferred to downstream sectors after aggregate liquidity shocks.

1 Introduction

An economy is an entangled network of specialized productions that are interconnected through inter-firm trade within and across sectors. In the course of this trade, the products

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of one firm may be purchased and consumed by another firm as inputs. In addition, there is often a waiting period between the moment in which a cost is incurred and the later point at which cash flow materializes. In this way, the production network also creates and sustains a trade credit network. An idiosyncratic financial shock may therefore spread through the production and financial linkages of firms. It is necessary to determine how the interaction between the production network and the trade credit network affects the propagation of financial shocks.

The automotive industry crisis of 2008-2010 offers an example of how a financial shock to one firm impacts its suppliers and customers. First, a financial shock to a firm may reduce its demand for the goods and services of its suppliers. For example, General Motors Co. significantly reduced its demand owing to a severe liquidity problem.[1] Consequently, American Axle & Manufacturing Holdings Inc., one of the major suppliers of GM, experienced a net loss of $112.1 million in the fourth quarter of 2008.[2,3] Second, when a firm experiences a financial shock, it may postpone repaying trade credit to its suppliers and reduce the provision of trade credit to its customers.[4] For example, General Motors Co. and Chrysler Group LLC. had in excess of $21 billion in domestic trade payables as of September 30, 2008.[5] The failure to meet these payments soon crippled GM's suppliers during the crisis.

This paper builds an input-output model with trade credit to facilitate the study of the propagation of financial shocks. To begin with, a simple network model is presented in an effort to explain the mechanism behind the propagation of financial shocks, specifically a static model with a vertical production network structure and financial linkages among firms. To capture financial frictions, I assume that firms have a working capital requirement on inputs, and thus need to pay wages and intermediate input costs in advance of production. To satisfy this requirement, firms acquire loans as well as trade credit from suppliers. Accordingly, firms are financially interlocked through trade: suppliers can use payments from customers as working capital, while at the same time the balance sheets of trading parties become interlocked through accounts-payable and accounts-receivable.

My model shows that the production and financial linkages among firms serve to propagate financial shocks both upstream and downstream. A positive borrowing cost shock to

[1] A combination of tight credit and declining sales caused the crisis in the U.S. auto industry from 2008 to 2010. This paper focuses on the impact of credit contraction in the production network.

[2] "American Axle CEO: '2008 Is the Year From Hell'." 30 January 2009, *Dow Jones International News*.

[3] Similarly, in the 1997 Korean financial crisis, Kia Motors suffered liquidity problems that affected its 16,000 domestic suppliers. Source: "South Korea's Kia Buckles, And Suppliers Begin to Break." 22 August 1997, *The Wall Street Journal*.

[4] Trade credit is the single most important source of external finance for firms (Boissay and Gropp (2007)). Trade credits are short-term loans that a supplier provides to a customer upon purchase of its product, and that thus constitute a form of deferred payment. A discussion of trade credit is provided in Appendix A.

[5] "US CREDIT - A GM failure risks more debt losses, default chain." 11 December 2008, *Reuters News*.

loans increases the input cost, reduces intermediate input demand and affects intermediate input suppliers. This is upstream propagation. The same shock increases the production price, which creates supply impact and affects product consumers. This is downstream propagation. The mechanisms behind these processes are as follows.

Two countervailing effects determine the extent of upstream propagation following a positive borrowing cost shock: the income effect decreases intermediate input demand, since production falls on account of the financial distress; the substitution effect, on the other hand, increases intermediate input demand. Because of delays to intermediate input payments, the marginal cost of intermediate input is less sensitive to borrowing cost shocks than is the marginal cost of labor. Consequently, firms will substitute labor with intermediate inputs after the shock. An important aspect of trade credit is that it amplifies the substitution effect and attenuates upstream propagation of a shock.

In like manner, two countervailing effects determine the extent of downstream propagation following a shock: the cost effect increases product price, while the discount effect decreases product price. Products are sold to downstream firms, which make early payments, and to households, which make late payments. Because producers value early payments, there is a price discount for downstream firms. Here again, trade credit plays a significant role, this time by weakening the discount effect and amplifying downstream propagation. The relative strength of upstream versus downstream propagation therefore depends on the level of trade credit.

Next I examine whether upstream or downstream propagation of financial shocks is revealed in the data, which in my analysis suggests that upstream propagation of financial shocks is stronger than downstream propagation. Using changes in sectoral bond yields as an indicator of idiosyncratic financial shocks to each sector, I find that sectors are more sensitive to financial shocks that hit their customers than to shocks that hit their suppliers. A 1% change in the bond yield of the customers of a sector generates a negative 0.17% output change in the focal sector. The same shock hit the sector's suppliers, by contrast, has no significant impact on the focal sector.

Afterwards, I use a DSGE model to numerically examine the transmission of idiosyncratic financial shocks. The input-output structure is standard (and follows that of Long and Plosser (1983) or Acemoglu et al. (2012)). In the economy, firms issue claims in order to purchase capital and acquire loans in order to satisfy their working capital requirements. Financial intermediaries supply liquidity to firms and face balance sheet constraints that are endogenously determined, in keeping with the framework developed by Gertler and Karadi (2011). Moreover, firms share liquidity with other firms through trade credit, just as in my static model, while they retain some flexibility for adjusting the level of trade credit.

By analyzing the model, I find the following results. This model indicates that the upstream propagation of financial shocks is stronger than the downstream propagation. It also demonstrates that trade credit increases the output correlation of firms. Furthermore, compared with a representative firm model, aggregate variables in the network model are more responsive to shocks of all kinds. The input-output structure generates a strong amplification effect on the aggregate impact of shocks.

Finally, the policy implication of my model is that credit policies would have a stronger impact if liquidity were transferred to downstream sectors after aggregate liquidity shocks. According to Gertler and Karadi (2011), the severity of a recession could be mitigated if policymakers were to supply liquidity during extreme economic conditions.[6] Such was the thinking behind the policy whereby Federal Reserve loans to private entities and other central banks reached $1.5 trillion by the end of 2008 in the wake of the Lehman Brothers bankruptcy (Price (2012)). However, which types of sectors or institutions should policymakers supply liquidity to? The implications of credit policies that channel liquidity to specific sectors or firms cannot be well studied by means of a representative firm model, though they can be investigated using my model.[7] My findings here suggest that it would be optimal if policymakers target liquidity in downstream sectors following an aggregate liquidity shock,[8] since the financial condition of downstream sectors is more important systemically, given that financial shocks mainly propagate upstream. For example, credit policies that supply liquidity to the construction sector would have stronger impacts than those to the utility sector after a uniform liquidity contraction across sectors.

Literature review. To my knowledge, this is the first paper that studies the input-output structure and trade-credit network in a general equilibrium framework. This project fits into three strands of literature, namely (1) production networks; (2) trade credit; and (3) financial frictions. Long and Plosser (1983) inaugurated the study of sectoral co-movements

[6]There are well-documented instances when just this policy was implemented. Thus, for example, the Federal Reserve granted the Continental Illinois National Bank and Trust Company access to the discount window from May 1984 through February 1985 despite the company's effective insolvency, and in an earlier instance, in 1970, it indirectly channeled credit to the commercial papers market in order to relieve the financial stress on the Penn Central Transportation Co. (Price (2012)). The bailout of General Motors Co. in 2009 undoubtedly saved that firm's suppliers from bankruptcy. Nonetheless, during the 1997 Korean financial crisis, Kia's suppliers complained that the liquidity injected into the banking system by Korea's central bank had failed to reach them. Source: "Auto Suppliers Turn Around: New Credit, Quick Restructurings of GM and Chrysler Sustained Parts Makers." 26 December 2009, The Wall Street Journal. "South Korea's Kia Buckles, And Suppliers Begin to Break." 22 August, 1997, *The Wall Street Journal*.

[7]Using a heterogeneous agent model, Reis (2009) suggests that providing credit to traders in securities markets can restore liquidity with fewer government funds than extending credit to the originators of loans.

[8]Naturally, political barriers could complicate supplying liquidity to specific firms or sectors, though the fact is that policymakers do provide liquidity in this way during extreme economic conditions (such as the examples provided in footnotes 5 and 6).

using a network model, sowing the seeds of a rich literature that has focused on the aggregate volatility generated by idiosyncratic shocks.[9] As presented by di Giovanni et al. (2014), there are two effects that are at work. First, idiosyncratic shocks have sizeable aggregate effects if there are strong input-output linkages between firms, which is the linkages effect (Bak et al. (1993), Horvath (1998, 2000), Dupor (1999), Shea (2002), Conley and Dupor (2003), Foerster et al. (2011), Acemoglu et al. (2012, 2015b), Jones (2011, 2012) etc.). Second, idiosyncratic shocks can directly contribute to aggregate fluctuations, which is the direct effect (Jovanovic (1987), Gabaix (2011), Carvalho and Gabaix (2013)). Of particular note here is the study by Acemoglu et al. (2015a) of the propagation of supply and demand shocks through input-output and geographic networks. My predictions for the propagation of these types of shocks are consistent with their work. My emphasis, however, is on the transmission of financial shocks in the input-output structure.

In research on the production network, financial frictions were not considered until the study by Bigio and La'O (2013), following which Su (2014) and this paper represent contributions to network-based approaches to the amplification of financial shocks. Aside from the questions these three papers focus on, the nature of the financial friction in these three papers is also different. Unlike Bigio and La'O (2013), my approach takes into account financial linkages of firms and establishes a micro foundation for the financial sector. And while Su (2014) presents a network model that accommodates financial frictions in the capital input, he does not account for financial frictions in trade, for which reason his model is allocationally equivalent to a horizontal one and can only describe the downstream propagation of financial shocks.

Second, several theories have been put forward to explain why suppliers provide trade credit to customers (e.g. Peterson and Rajan (1997), Burkart and Ellingsen (2004), Cunat (2007)). My analysis follows that of Kiyotaki and Moore (1997) in emphasizing the role of trade credit in the propagation of shocks. Regarding the third strand of inquiry, financial frictions have been extensively studied in the literature on the 2008 financial crisis (e.g. Gertler and Karadi (2011), Cúrdia and Woodford (2011)). This paper builds on these previous approaches in order to elaborate a heterogeneous firm model for the study of the impact of idiosyncratic and aggregate financial shocks.

In what follows, Section 2 presents a simple static model to illustrate the propagation of shocks in a chain economy. In Section 3, I discuss my empirical findings on the propagation of financial shocks. Section 4 presents my DSGE model, and Section 5 calibrates the model

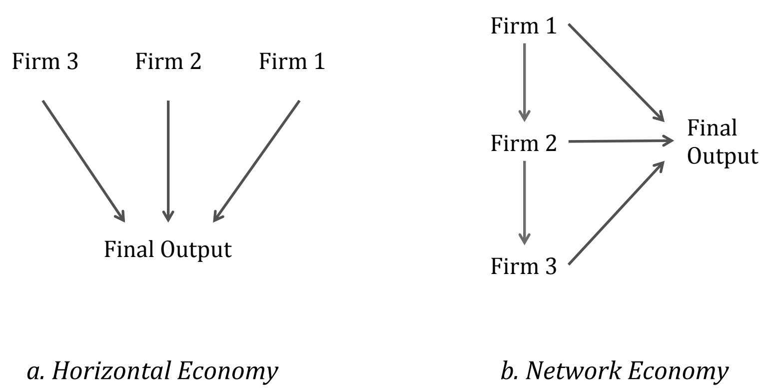
[9] There is also a branch of literature on the propagation of shocks in financial networks, e.g. Allen and Gale (2000), Acemoglu et al. (2015c). The focus of this paper is the propagation of shocks in production networks.

and forms quantitative predictions. Credit policy implications are the subject of Section 6. Section 7 concludes the paper with a summary of my findings and a discussion of their implications.

2 A Simple Model

In this section I consider a simple economy in order to study the mechanism of the propagation of financial shocks. In my model here, firms are connected vertically in a production chain. The interconnected production and financial linkages among the firms lead to the propagation of financial shocks both upstream and downstream.

Figure 1: Horizontal VS Network Economy



2.1 A horizontal economy with financial frictions

Before proceeding with this discussion, it will be useful to consider a *horizontal* economy as the benchmark. In this simple economy, three firms produce three types of specialized intermediate products $\{m_1, m_2, m_3\}$ with prices $\{p_1, p_2, p_3\}$ correspondingly. The flow of goods in this economy is illustrated in Figure 1 panel a. Under these circumstances, there are no connections among the firms, and labor l_i is the sole input of Firm i. Intermediate sectors thus have the following production functions,

$$m_1 = z_1 l_1^{\alpha_1}, \quad m_2 = z_2 l_2^{\alpha_2}, \quad m_3 = z_3 l_3^{\alpha_3},$$

where z_i denotes technology. To capture financial frictions, I assume a working capital requirement on labor exits. Firms in this case need to borrow in order to pay for their labor cost at the beginning of each production period. I further assume a small open economy in which firms obtain credit from the rest of the world with a fixed interest rate R. Policymakers in addition impose a borrowing tax e_i on each unit of the credit obtained by firm i.[10] Thus, the borrowing fee of firm i is $R_i = R + e_i$. For simplicity, I assume that $R = 1$. The tax revenue, which equals to the total borrowing cost, is distributed to households through lump-sum transfers.

In this economy, the final output is

$$Y = \Xi m_1^{\zeta_1^h} m_2^{\zeta_2^h} m_3^{\zeta_3^h}, \tag{1}$$

where Ξ is a constant, ζ_i^h denotes the final share of product i. A representative household solves the following problem,

$$\begin{aligned} \max_{C,L} \quad & \log(C) - L, \\ s.t. \quad & C = wL + \Psi, \end{aligned} \tag{2}$$

where Ψ represents the tax revenue distributed to households, and w is the wage rate. Households make choices of consumption C and labor supply L subject to their budget constraint. The market clearing condition of goods follows $Y = C$. The labor market clearing condition follows $L = \sum_{i=1}^{3} l_i$.

Given $\{z_i, R_i\}$ for $i \in \{1, 2, 3\}$, a *competitive equilibrium* in this horizontal economy is represented by a group of endogenous variables $\{m_i,\ l_i,\ L,\ Y,\ C,\ \Psi,\ p_i,\ w\}$ for $i \in \{1, 2, 3\}$, such that (1) each firm maximizes its own profits, (2) each household maximizes its own utility, and (3) goods and labor markets clear.

The equilibrium results are presented in Appendix B.1.

2.2 A network economy with financial frictions

Unlike in a horizontal economy, firms in a network are interconnected through trade. Thus, consider a production network in which three firms are vertically linked. The input-output structure is illustrated in Figure 1 panel b. For the sake of simplicity, Firm 1 is the furthest upstream, while Firm 3 is the furthest downstream. All three firms supply input that figures in the production of the final goods. Each firm produces output using Cobb-Douglas

[10] The borrowing tax comes through the banking system implicitly. This assumption provides one justification for idiosyncratic borrowing costs. In the general model presented in Section 4, I will provide a micro foundation for the firm specific interest rates.

technologies production functions given by the following,

$$m_1 = z_1 l_1^{\alpha_1}, \quad m_2 = z_2 l_2^{\alpha_2} m_{21}^{1-\alpha_2}, \quad m_3 = z_3 l_3^{\alpha_3} m_{32}^{1-\alpha_3},$$

where $m_{i+1,i}$ denotes the intermediate inputs of Firm $i+1$ for $i \in \{2, 3\}$, and $\alpha_1 = 1$. Firms have a working capital requirement on labor and intermediate inputs. Again, firms pay an exogenous borrowing fee R_i on each unit of the borrowed funds. Producers could, however, defer a proportion of their intermediate input payments using trade credit. A proportion θ_i of intermediate input payments is paid after production without borrowing cost. Let $p_{i+1,i}$ denote the price of product i paid by Firm $i+1$ for $i \in \{2, 3\}$.

The final output of the economy is,

$$Y = \zeta_1^{-\zeta_1} \zeta_2^{-\zeta_2} \zeta_3^{-\zeta_3} y_1^{\zeta_1} y_2^{\zeta_2} y_3^{\zeta_3},$$

where y_i represents output i used in the final production, and ζ_i represents the share of production i in the final product. Unlike intermediate firms, final producers pay input payments at the end of the production period with price p_i.

Households under these conditions solve the same problem in like manner as they do in the horizontal economy (Equation 2). The goods market clearing conditions are: $m_1 = m_{21} + y_1$, $m_2 = m_{32} + y_2$, $m_3 = y_3$, and $Y = C$. The labor market clearing condition is $L = \sum_{i=1}^{3} l_i$.

Given $\{z_i, R_i\}$ for $i \in \{1, 2, 3\}$, a *competitive equilibrium* in the network economy is a group of endogenous variables $\{m_i,\ l_i,\ y_i,\ p_i\}$ for $i \in \{1, 2, 3\}$ and $\{m_{21},\ m_{32},\ L,\ Y,\ C,\ \Psi,\ p_{21},\ p_{32},\ w\}$, such that, (1) each firm maximizes profits, (2) each household maximizes its utility, (3) goods and labor markets clear.

2.2.1 Financial linkages of firms

In this economy, firms pay a proportion $(1 - \theta_i)$ of their intermediate input purchases at the beginning of the production period and pay the balance at the end. Therefore, each firm i has accounts-payable that equal θ_i of its total intermediate input costs, and has accounts-receivable that equal θ_j of its sales to firm j. When $\theta_i = 0$, intermediate input costs have to be paid fully before production; when $\theta_i = 1$, there are no financial frictions in trade. Early payments received by suppliers can then be used to pay their input costs. Thus, firms are interconnected through both the production network and the trade credit network. The financial linkages of these three firms are presented in Table 1. Obviously, upstream firms provide additional credit to downstream firms, compared to which upstream firms naturally have more working capital.

Table 1: Financial linkages of firms in the network model

	Accounts-payable	Accounts-receivable	Net working capital from trade
Firm 1	0	$\theta_2 p_{21} m_{21}$	$(1-\theta_2)p_{21}m_{21}$
Firm 2	$\theta_2 p_{21} m_{21}$	$\theta_3 p_{32} m_{32}$	$-(1-\theta_2)p_{21}m_{21} + (1-\theta_3)p_{32}m_{32}$
Firm 3	$\theta_3 p_{32} m_{32}$	0	$-(1-\theta_3)p_{32}m_{32}$

Financial linkages among firms play important roles in the sensitivity of these firms to shocks. On the one hand, the working capital of upstream firms is more sensitive to economic fluctuations, which finding is consistent with those of Kalemli-Ozcan et al. (2012). A shock to a downstream firm would impact the early payments to its upstream firms, which would in turn cause fluctuations in the working capital of these upstream firms. Working capital increases relatively more for upstream firms during booms and declines relatively more for upstream firms during recessions. On the other hand, trade credit increases the correlation of firms; firms share liquidity with surrounding firms through trade credit. Thus Raddatz (2008) shows empirically that trade credit increases sectoral output correlations. A detailed theoretical explanation is provided in Section 4.

2.2.2 Solve the problem

In this economy, the firms maximize their profit. For example, Firm 2 solves the following problem,

$$\max_{l_2,m_{32},m_{21},y_2} [(1-\theta_3)R_2+\theta_3]p_{32}m_{32} + p_2 y_2 - w l_2 R_2 - [(1-\theta_2)R_2+\theta_2]p_{21}m_{21}$$

$$s.t. \quad z_2 l_2^{\alpha_2} m_{21}^{1-\alpha_2} \geq m_{32} + y_2$$

The first term represents the benefit of selling products to Firm 3. The marginal benefit of selling one unit of m_{32} is $[p_{32} + (R_2-1)(1-\theta_3)p_{32}]$, where the second term represents the benefit derived from borrowing cost saving. Through first order conditions, I find that

$$p_{32} = p_2/[\theta_3 + (1-\theta_3)R_2]. \tag{3}$$

Close reality, price and trade credit are bundled together in the contract. The price is low when θ_3 is small.

The equilibrium results of this economy are presented in Appendix B.1.

Proposition 1. *(a) If there are no financial frictions in trade (i.e. $\theta_i = 1\ \forall i$), the network economy can be allocationally equivalent to the horizontal economy. In either economy, the aggregate output follows,*

$$Y = \Theta z_1^{V_1} z_2^{V_2} z_3^{V_3} R_1^{-V_1} R_2^{-\alpha_2 V_2} R_3^{-\alpha_3 V_3},$$

if $\Xi = \Theta$ and $\zeta_i^h = V_i$.[11] *The labor allocation is give by $l_i = \frac{\alpha_i V_i}{R_i}$.*

(b) If there are financial frictions in trade (i.e. $\theta_i \in [0, 1)\ \forall i$), the network economy cannot be allocationally equivalent to the horizontal economy. In the network economy, the aggregate output follows,

$$\begin{aligned} Y \;\; = \;\; & \Theta z_1^{V_1} z_2^{V_2} z_3^{V_3} R_1^{-V_1} R_2^{-\alpha_2 V_2} R_3^{-\alpha_3 V_3} \\ & \left(\frac{[\theta_2 + (1-\theta_2)R_1]}{[\theta_2 + (1-\theta_2)R_2]}\right)^{V_1-\zeta_1} \left(\frac{[\theta_3 + (1-\theta_3)R_2]}{[\theta_3 + (1-\theta_3)R_3]}\right)^{V_2-\zeta_2}. \end{aligned}$$

The labor allocation is given by Equations 41-43 in Appendix B.1.

In the absence of financial frictions in inter-firm trade, an economy with input-output connections is isomorphic to a horizontal economy with a certain construction of parameters.[12] However, when there are financial frictions in inter-firm trade, an economy with input-output connections cannot be isomorphic to a horizontal economy. The labor allocation under these circumstances depends on the financial condition of other firms in the network economy, while in the case of a horizontal economy it depends only on the firm's own financial condition (according to Equations 41-43). Moreover, idiosyncratic shocks in a network economy spill over from one firm to another along the production chain; output sensitivity to shocks differs according to the network locations of these firms, as is discussed in next subsection.

2.2.3 The propagation of financial shocks

Having established the network model, I will now discuss how financial shocks propagate through the economy. Specifically, given that production processes are sequential, it is crucial to determine whether shocks propagate upstream or downstream. Propagation is *upstream* if a firm-specific shock spills over to the firm's suppliers. Propagation is *downstream* if a

[11] $\Theta = [\alpha_2^{\alpha_2}(1-\alpha_2)^{1-\alpha_2}]^{\zeta_2}[\alpha_3^{\alpha_3}(1-\alpha_3)^{(1-\alpha_3)}\alpha_2^{\alpha_2(1-\alpha_3)}(1-\alpha_2)^{(1-\alpha_2)(1-\alpha_3)}]^{\zeta_3}$, $V_1 = \zeta_3(1-\alpha_2)(1-\alpha_3) + \zeta_2(1-\alpha_2) + \zeta_1$, $\quad V_2 = \zeta_3(1-\alpha_3) + \zeta_2$, $\quad V_3 = \zeta_3$.

[12] This result is close to the finding by Bigio and La'O (2013) that, without frictions, an economy's network structure is irrelevant. Baqaee (2015) presents a similar result. Notably, I show that the network structures are irrelevant provided that there are no frictions in inter-firm trade.

firm-specific shock affects the firm's customers. In terms of the model just elaborated, the question becomes whether a borrowing cost R_2 shock will impact Firm 1 or Firm 3.

Proposition 2. *(a) Financial shocks generate strong upstream and downstream propagations in a network economy with trade credit. The elasticity of output i with respect to the borrowing cost of Firm 2 is,*

$$\frac{\partial m_i}{\partial R_2}\frac{R_2}{m_i} = f_i(R_1, R_2, R_3),$$

with $f_i \leq 0$ (refer to Appendix B.2). If $\theta_i = 0\,\forall i$, there is no downstream propagation and $f_3 = 0$. If $\theta_i = 1\,\forall i$, there is no upstream propagation and $f_1 = 0$.

(b) The upstream propagation of a R_2 shock is more likely to dominate the downstream propagation as trade credit decreases. Suppose that

(i) All firms face the same borrowing cost: $R_i = \bar{R}\,\forall i$.

(ii) All firms use the same amount of trade credit: $\theta_i = \theta\,\forall i$.

Upstream propagation is stronger than downstream propagation if,

$$\theta < \frac{(1-\alpha_2)\zeta_2\bar{R}}{(1-\alpha_2)\zeta_2\bar{R} + \alpha_2(1-\alpha_3)[\zeta_1 + (1-\alpha_2)(\zeta_2 + (1-\alpha_3)\zeta_3)]}.$$

The proof of Proposition 2 is presented in Appendix B.2.

Upstream propagation. Two factors affect the intermediate input demand of Firm 2 after a positive R_2 shock, which factors are in turn related to the upstream propagation and the sensitivity of m_1 to R_2. On the one hand, the income effect decreases intermediate input demand, since production falls on account of the financial distress. On the other hand, the substitution effect increases the intermediate input demand. Trade credit defers part of the intermediate input payments. The marginal cost of labor for Firm 2 is wR_2. The marginal cost of intermediate inputs of Firm 2 is $p_{21}[(1-\theta_2)R_2 + \theta_2]$, which is less sensitive to R_2 shocks. Thus, Firm 2 will substitute labor with intermediate inputs after a positive R_2 shock. The substitution effect attenuates the negative impact to m_1. Notably, as θ_2 increases, this substitution effect becomes stronger and the upstream propagation effect is diminished. In the extreme case that $\theta_i = 1\,\forall i$, there is no upstream propagation because the income and the substitution effect cancel each other out in the Cobb-Douglas production structure.

Downstream propagation. Two factors affect the product prices of Firm 2, which factors are in turn related to the downstream propagation and the sensitivity of m_3 to R_2. On the one hand, the cost effect increases product price p_{32}. On the other hand, the discount effect decreases p_{32}. When R_2 is high, Firm 2 gives a large discount to Firm 3, which pays in advance (Equation 3). The discount effect attenuates the negative impact to m_3. It is again notable that, as θ_3 decreases, the discount effect becomes stronger and the downstream propagation effect is diminished. In the extreme case that $\theta_i = 0 \, \forall i$, the downstream propagation effect disappears.

The relative strength of upstream versus downstream propagation depends of course on parameter values. With a moderate amount of trade credit, the upstream propagation effect is stronger than the downstream effect. Moreover, trade credit takes the form of external loans that firms receive from their suppliers, and this kind of credit relaxes the financial constraints of the firm to which it is extended. On the one hand, accounts-payable alleviates the financial constraint on the firm and attenuates the transmission of financial shocks. On the other hand, accounts-receivable amplifies the financial stress of the firm and strengthens the transmission of financial shocks.

2.2.4 The importance of firms - the influence vector

The propagation of shocks is closely related to the systemic importance of firms in the economy. Consider a general input-output structure, where the production function is,

$$m_i = z_i l_i^{\alpha_i} (\Pi_{j=1}^{N} m_{ij}^{\omega_{ij}})^{1-\alpha_i}, \tag{4}$$

and $(1-\alpha_i)\omega_{ij}$ captures the direct use of product j for the production of i. All the other conditions in the network economy are the same as before. The equilibrium level of the output of firms is a log-linear function of the productivities and borrowing fees in the economy. Again, the financial shocks propagate both upstream and downstream. Refer to Appendix B.3 for detail.

Let $\mathbf{z}$ denote a vector of the log productivity deviation from steady state level ($\tilde{z}_i$). Let $\mathbf{R}$ denote a vector of the log borrowing fee deviation from steady state level ($\tilde{R}_i$). Let ζ denote a vector of the final share (ζ_i). Let Ω denote the direct input-output matrix with entry $(1-\alpha_i)\omega_{ij}$, i.e.

$$\Omega = \begin{bmatrix} (1-\alpha_1)\omega_{11} & (1-\alpha_1)\omega_{12} & \cdots & (1-\alpha_1)\omega_{1N} \\ (1-\alpha_2)\omega_{21} & (1-\alpha_2)\omega_{22} & \cdots & (1-\alpha_2)\omega_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ (1-\alpha_N)\omega_{N1} & (1-\alpha_N)\omega_{N2} & \cdots & (1-\alpha_N)\omega_{NN} \end{bmatrix}. \tag{5}$$

Proposition 3. *Assume that the labor shares of all firms are equal $\alpha_i = \alpha \, \forall i$ and the steady state borrowing costs of all firms are $\bar{R}$. Then, in the competitive equilibrium of the network* *economy, the log deviation of the aggregate output (GDP) from its steady state is given by*

$$\tilde{Y} = v'\mathbf{z} + (v^R)'\mathbf{R},$$

where v and v^R are N-dimensional vectors given by

$$v \equiv (I - \Omega')^{-1}\zeta, \quad (6)$$

$$v^R \equiv -(I_1 - I_2\Omega)'v, \quad (7)$$

where I_1 and I_2 are constants. [13]

Hence, the equilibrium value of the aggregate output is a log-linear function of the productivities and borrowing costs in the economy. The coefficients for these shocks are the *influence vectors*. The influence vectors capture how firm-level changes in productivity and borrowing costs propagate to other firms and ultimately affect aggregate output. The influence vector of productivity is v, which is similar to the influence vector presented in the existing network literature (e.g. Acemoglu et al. (2012) and Bigio and La'O (2013)).[14] Nonetheless, the influence vector of the borrowing fee is v^R, which is different from the conventional influence vector v. Moreover, each element v_i of v corresponds to the well-known notion of *Bonacich centrality* (e.g. Acemoglu et al. (2015b)), which captures the systemic importance of a firm's productivity. Accordingly, I define each element v_i^R of v^R as *financial centrality*, which captures the systemic importance of a firm's financial condition.

Notably, the last term in the bracket ($I_2\Omega$) attenuates the impact of the borrowing condition of upstream sectors on the aggregate output; the term is positive when $\theta \neq 1\ \forall i$, and disappears when $\theta = 1\ \forall i$. This term exists because of the financial frictions in trade and financial linkages among firms. It is a decreasing function of θ_i, and is maximized at $\theta_i = 0$. In economies with small θ_i, the financial condition of downstream sectors has a stronger impact on the aggregate economy and is more important systemically than that of the upstream sectors. This feature is also related to the fact that the upstream propagation of financial shocks is strong in the network model.

[13] I_1 is a diagonal matrix with diagonal values $\alpha + (1-\alpha)\frac{(1-\theta_i)\bar{R}}{[(1-\theta_i)\bar{R}+\theta_i]}$. I_2 is a diagonal matrix with diagonal values $\frac{(1-\theta_i)\bar{R}}{[(1-\theta_i)\bar{R}+\theta_i]}$. Refer to Appendix B.3 for detail. The log deviation of the firm output (m_i) from steady state level is also presented in Appendix B.3.

[14] Remarkably, the influence vector cannot be equal to the vector of equilibrium shares of sales when there is market imperfection, i.e. $v_i \neq \frac{p_i m_i}{\sum_{j=1}^{N} p_j m_j}$. This result is consistent with Hulten (1978) and Bigio and La'O (2013).

2.3 Conclusions to be drawn from the simple model

In the case of a simple network model with financial frictions and trade credit, then, it is to be noted that financial constraint generates a strongly negative impact on aggregate output. Moreover, depending on the level of financial frictions in trade, the propagation of financial shocks could be either upstream or downstream.[15] In particular, I want to call attention to the fact that financial frictions in trade and financial linkages of firms can generate strong upstream propagation of financial shocks. Supply shocks, by contrast, always propagate downstream, and demand shocks always propagate upstream, regardless of the level of financial frictions in trade (as discussed in Appendix B.2).

While my prediction of the propagation of supply and demand shocks is consistent with the empirical findings in the existing literature (like Shea (2002), Acemoglu et al. (2015a)), the question remains regarding which propagation pattern of financial shocks is actually revealed in the data, so I will take up this question in the next section.

3 Empirical Findings

In what follows, I present my primary empirical findings on the propagation of financial shocks.

3.1 Upstream versus downstream

A theoretical model could offer a precise prediction for the form of the propagation of shocks, though the direction taken by shocks would remain unclear in the data, because firm i could be both a supplier and a customer of firm j. The concepts of the upstream and downstream propagation, as well as the relative upstreamness and downstreamness of a firm, therefore need to be defined within the context of a more general network structure.

To begin with, $(1-\alpha_i)\omega_{ij}$ captures the amount of j used as an input in producing \$1 worth of i output as mentioned in the Section 2.2.4. Thus, the input-output matrix Ω (in Equation 5) only captures the direct use of inputs in the production network. Instead, the n-step indirect use of inputs can be captured by Ω^n. Thus, the geometric summations of Ω reflect both the direct and indirect use of one product for the production of another product. I define this quantity by

$$\mathcal{H} \equiv \Omega(I-\Omega)^{-1}, \tag{8}$$

[15] Effects propagate only downstream when there are no financial frictions in trade, and only upstream when there is a working capital requirement on all intermediate inputs; propagation occurs in both directions when trade credit is between 0 and 1.

with entry denoted by h_{ij}. It also equals Ω times the Leontief inverse of the economy. Therefore, h_{ij} presents the total use (or demand) of j for the production of i. It is closely related to the downstream impact from j to i. h_{ij} is used in the next subsection to construct a measure of shocks transmitted downstream.

Meanwhile, $\hat{\Omega}$ is the matrix with entries $(1-\alpha_i)\hat{\omega}_{ij}$, where $\hat{\omega}_{ij} \equiv \omega_{ij}\frac{p_i m_i}{p_j m_j}$, that denotes sales from j to i normalized by sales of industry j. The matrix

$$\hat{\mathcal{H}} \equiv \hat{\Omega}(I-\hat{\Omega})^{-1} \tag{9}$$

has entries $\hat{h}_{ij}$, which variable captures the total supply from j to i. It is closely related to the upstream impact from i to j. $\hat{h}_{ij}$ is used to construct a measure of shocks transmitted upstream in the next subsection.

Moreover, $h_i = \sum_j h_{ij}$ captures the total usage of intermediate inputs for the production of i. The larger the value of h_i, the further downstream the firm is in the network. $\hat{h}_i = \sum_j \hat{h}_{ji}$ captures the total supply of intermediate input i. The larger $\hat{h}_i$ is, the further upstream the firm is located in the network.

3.2 Empirical approach

The propagation of financial shocks is assessed using a method that compares the relative strength of upstream versus downstream propagation in a manner similar to the method employed by Acemoglu et al. (2015a), which takes the following form:

$$\Delta m_{it} = \beta^{up} UP_{it-1} + \beta^{down} DOWN_{it-1} + \beta^{si} shock_{it-1} + \beta^{m}\tilde{m}_{it-1} + \epsilon_{it}, \tag{10}$$

where i indexes sub-sectors, ϵ_{it} is an error term. Δm_{it} is sectoral output change. UP_{it} and $DOWN_{it}$ stand for shocks working through the network. Specifically, UP_{it} measures the shocks to an industry's customers that flow up the production chain, while $DOWN_{it}$ measures the shocks to suppliers of an industry that flow down the production chain. These shocks are calculated as follows:

$$UP_{it} = \sum_j \hat{h}_{ji} \cdot shock_{jt}, \tag{11}$$

$$DOWN_{it} = \sum_j h_{ij} \cdot shock_{jt}. \tag{12}$$

Thus, UP_{it} is a measure of shocks to the customers of industry i that is weighted according to the entry of $\hat{\mathcal{H}}$. $DOWN_{it}$ is a measure of shocks to suppliers of industry i that is weighted likewise according to the entry of $\mathcal{H}$. $shock_{it}$ is the idiosyncratic part of the financial shock. The focus of this regression is β^{up} and β^{down}.

I lag variables related to financial shocks on the right hand side of Equation 10 in order to avoid concerns about contemporaneous joint determination.[16] In my baseline results, I allow only a single lag of the dependent variable. I controlled for additional lags in robustness checks.

3.3 Data sources

The industry-level data for manufacturing was obtained from the Industrial Production Index by the Federal Reserve Board of the United States. This index reports the level of production for the period 1986-2015 on a monthly basis. Changes in industrial production (Δm_{it}) are measured at different frequencies (such as monthly, quarterly or biannually). Here I utilize the data at the 4-digit NAICS level.

To measure the linkages among industries ($h_{ij}, \hat{h}_{ji}$), I use the Input-Output Table created by the Bureau of Economic Analysis. This table reports the usages of industry i's output in industry j's production, as well as the direct usage of industry i's output in the final consumption.

TRACE (the Trade Reporting and Compliance Engine) collects corporate bond tick data from 2002 to 2015, and this dataset is used to measure idiosyncratic financial shocks at the sectoral level ($shock_{it}$). The first step in doing so was to normalize the data to a monthly frequency tied to the last observation of each month. Industry-specific financial shocks are calculated using TRACE data of bond yields that are measured as the mean of bond yield changes by industries. In the second step, I regress monthly sectoral bond yield changes on Federal Funds rate change and Aaa bond index change; the residual of this regression is the instrument of sectoral idiosyncratic financial shocks.

3.4 Empirical results

My primary empirical results are presented in Table 2. Regression (1) reports the results of Equation 10 at the biannual frequency. Regressions (2) and (3) are event studies of Lehman Brothers' bankruptcy, carried out as cross section regressions. Notably, regression

[16] The bias in the panel regression is not strong in this case, given my sample size.

Table 2: UP vs. DOWN

	Panel	Cross Section	
	biannual (2003.01-2014.12)	6 months after Lehman Bankruptcy	
	(1)	(2)	(3)
UP$_i$	**-0.0929****	**-0.3042*****	**-0.1724****
	[0.0441]	[0.0733]	[0.0695]
DOWN$_i$	**0.1645**	**-0.1969**	**-0.0635**
	[0.1687]	[0.2715]	[0.2550]
Shock$_i$	-0.4395***	-0.4008*	-0.4469**
	[0.1221]	[0.2337]	[0.2022]
$\epsilon_{i,2008}^{demand}$			0.1835***
			[0.0383]
$\epsilon_{i,2008}^{supply}$			0.0305
			[0.0686]
Δm_{it-1}	0.0891*	0.6062***	0.5047***
	[0.0502]	[0.0543]	[0.0607]
Time Fixed Effect	X		
Sector Fixed Effect	X		
Observations	1357	61	61
# of sectors	59	-	-

Note: *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels respectively. (1) Panel regression reports standard errors clustered by sector and are unweighted. Independent variables are lagged by one period. Two sectors are dropped due to high serial correlation between UP_{it} and $DOWN_{it}$. In cross section regressions (2) and (3), dependent variable is the sectoral output change 6 months after Lehman's collapse (i.e. from 2008.09 to 2009.02). Independent variables are the respective changes of each variable 3-month after Lehman's collapse (i.e. from 2008.09 to 2008.11). $\epsilon_{i,2008}^{demand}$ and $\epsilon_{i,2008}^{supply}$ are measured using commodity output changes from the end of 2007 to the end of 2008.

(3) controls for demand and supply shocks to each sector.[17] ϵ_{2008}^{demand} represents demand side shocks that came from sectoral output changes of an industry's customers in 2008.

[17] Ideally, supply and demand side shocks should be controlled in the regression. However, sectoral level supply and demand side shocks remain unavailable for all sectors in the economy or at frequencies higher than yearly. I constructed $\epsilon_{it}^{demand} = \sum_j h_{ij} \Delta output_{jt}$ as instruments for supply shocks and $\epsilon_{it}^{supply} = \sum_j \hat{h}_{ji} \Delta output_{jt}$ as instruments for demand shocks. $\Delta output_{jt}$ is constructed based on the yearly change in sectoral output at the 4-digit NAICS level using the total commodity output in the BEA Input-Output table.

ϵ_{2008}^{supply} represents supply side shocks that come from sectoral output changes by an industry's suppliers in the same year. Regressions with different frequencies and robustness checks are reported in Appendix C.2.[18]

Clearly, upstream effects that come from financial shocks to an industry's customers strongly influence the output of the focal sector. Downstream effects that come from financial shocks to an industry's suppliers, on the other hand, are less significant.[19] According to regression (3), a 1% increase in one sector's bond yield generates a -0.45% change in that sector's output. A 1% increase in the bond yield of one sector's customers would impact the focal sector's output by -0.17%.

4 A General Network Model With Trade Credit

Section 2 explains that financial shocks can propagate both upstream and downstream in an economy with production and trade credit networks. Notably, financial frictions in trade and financial linkages of firms are key elements in my model to generate upstream propagation. However, the relative strength of upstream versus downstream propagation depends on parameters. In Section 3, I find that the upstream propagation of financial shocks is stronger than the downstream propagation in reality. In this section, I am ready to build a general network model that can capture this pattern quantitatively, and then use it as a laboratory to understand the credit policy. Specifically, I consider a DSGE model with an input-output structure in which firms are linked financially. This section illustrates one way to incorporate financial frictions and the trade credit network into a dynamic input-output model that can be used to calibrate and study the U.S. economy in Section 5.

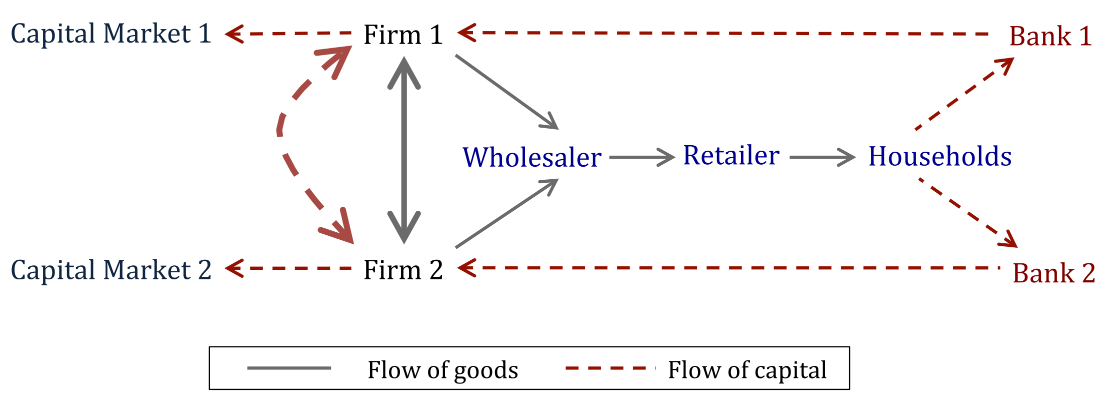
The input-output structure of the model follows that of Long and Plosser (1983) and Acemoglu et al. (2012). The financial friction is introduced through the working capital requirements of the production sector. The financial intermediaries that face endogenously determined balance sheet constraints follow the framework of Gertler and Karadi (2011). Furthermore, firms are connected financially through the trade credit network, as discussed by Kiyotaki and Moore (1997). Figure 2 illustrates the economic structure assumed by this paper. The solid grey arrows represent the flow of goods while the dashed red arrows represent the flow of capital. Different from a representative or horizontal economy, the

[18]Regression 10 with different frequencies is presented in Table 8. Also, cross section regressions with dependent variables that have been measured at different horizons are reported in Table 9. Lastly, as an index of robustness, Table 10 reports the regression results with additional lags of independent variables.

[19]On the financial side, Hertzel et al. (2008) provide empirical evidence that firms' equity price returns respond to events affecting their customers and suppliers, such as bankruptcy filings. Specifically, customers of firms that file for bankruptcy generally do not experience a contagion effect, while suppliers of such firms do. This result is in line with my empirical finding on the real activities.

economy studied in this paper has an input-output structure and financial linkages of firms (represented by the thick grey arrow and the thick red arrow).

Figure 2: The model structure



4.1 Intermediate Goods Firms.

There are N sectors in the economy. Each sector produces one type of product. Firms within each sector are homogeneous and competitive. The intermediate goods are consumed by other firms within and across sectors, and are used by wholesalers. Each sector produces output using the following Cobb-Douglas production function,

$$m_{it} = z_{it}^{\alpha_i} l_{it}^{\alpha_i} (\xi_{it} k_{it-1})^{\beta_i} \left(\Pi_{i=1}^{N} m_{ij,t}^{\omega_{ij}} \right)^{\gamma_i}, \tag{13}$$

where z_{it} denotes technology, ξ_{it} denotes the quality of capital. $m_{ij,t}$ denotes the amount of product j used by sector i. The exponent ω_{ij} denotes the share of good j in the total intermediate input use of sector i. Assume constant returns to scale $\alpha_i + \beta_i + \gamma_i = 1$ and $\sum_j \omega_{ij} = 1$. Up to now, I have presented a standard input-output network model. I will now introduce the working capital requirement and financial frictions into this model.

At the end of period t, an intermediate goods firm acquires capital k_{it} from the capital market i for the production in period $t+1$. The firm issues S_{it} claims equal to k_{it}, and prices each claim at the capital price of Q_{it} in order to acquire funds for capital from financial intermediaries at the beginning of the period. Thus, the total amount of funds obtained for the capital purchase is $Q_{it}k_{it}$, which equals $Q_{it}S_{it}$. Given that firms earn zero profits, firms resell the capital to the capital market and pay out the ex-post return to capital and the sales of capital to the financial intermediaries at the end of period $t+1$. Accordingly, the stochastic return of a given financial intermediary's investment on a capital asset of firm i is,

$$R_{ik,t+1} = \xi_{i,t+1} \frac{u_{i,t+1} + Q_{it+1} - \delta}{Q_{it}}, \tag{14}$$

where $u_{i,t+1}$ is the capital utilization rate at period $t+1$, δ is the depreciation rate of capital. Assume that the replacement price of the depreciated capital is unity, so the value of the capital stock after production is $(Q_{it+1} - \delta)\xi_{it+1}k_{it}$.

Furthermore, I assume that firms dealing in intermediate goods face a working capital requirement on labor and intermediate inputs. In particular, the labor expenditure and the intermediate input purchases need to be paid in full in advance of production. Firms therefore need additional banking credit. They do so via loans L_{it} from banks at the beginning of period t, which pays a non-contingent interest rate R_{iLt} at the beginning of period t. I simplify the model by further assuming that there is no friction in the process of obtaining funds. There is no information friction or moral hazard problems between firms and banks.

Additionally, suppliers provide liquidity to their customers in the form of trade credit. Firms in sector i only need to pay $(1-\theta_i)p_{jt}m_{ijt}$ to their suppliers in sector j at the beginning of period t and to clear their accounts payable $\theta_i p_{jt} m_{ijt}$ at the end of period t.[20]

Moreover, I introduce a flexible trade credit adjustment feature into the model. Firm i could adjust its level of accounts-payable θ_{ij} while supplier j takes θ_{ij} as a given.[21] Nonetheless, there is a quadratic trade credit adjustment cost

$$C(\theta_{ij}, \bar{\theta}_i) = \varsigma(\theta_{ij} - \bar{\theta}_i)^2 \tag{15}$$

per dollar unit of input purchases, where $\bar{\theta}_i$ is the steady state trade credit of firms in sector i. The cost is zero when $\theta_{ij} = \bar{\theta}_i$. ς controls the size of the cost. Trade credit adjustment is more flexible when ς is small (i.e. $\partial C/\partial\varsigma > 0$).

[20] As discussed by Kiyotaki and Moore (1997), a supply contract and a debt contract between two trading parties are bundled together. The deferment of part of the purchase can be considered as customers borrowing from suppliers. Alternatively, the suppliers borrow from customers because they are paid something in advance of product delivery. In this model, I simply assume that all firms are produced simultaneously and that products are delivered simultaneously in the network. The distinctions between borrower and lender are not important for my argument.

[21] It is well observed in reality that customers have stronger bargaining power on trade credit.

Thus, each firm solves the following problem,

$$\max_{l_{it},\{m_{ijt}\}_{j=1}^{N},\{m_{jit}\}_{j=1}^{N},y_{it},\theta_{ijt}} \quad \sum_j [(1-\theta_{jit})R_{iLt}+\theta_{jit}]p_{jit}m_{jit}+p_{it}y_{it}-w_t l_{it}R_{iLt}-u_{it}\xi_{it}k_{it-1}$$
$$-\sum_j [(1-\theta_{ijt})R_{iLt}+\theta_{ijt}]p_{ijt}m_{ijt}-\sum_j C(\theta_{ijt},\bar{\theta}_i)p_{ijt}m_{ijt}$$
$$+\Phi_{it}\left[(z_{it}l_{it})^{\alpha_i}(\xi_{it}k_{it-1})^{\beta_i}(\Pi_j m_{ijt}^{\omega_{ij}})^{\gamma_i}-\sum_j m_{jit}-y_{it}\right]. \tag{16}$$

Φ_{it} is the Lagrangian multiplier and corresponds to the marginal benefit of producing one unit of product. The total working capital requirement for intermediate inputs is $\sum_{j=1}^{N}(1-\theta_{ijt})p_{ijt}m_{ijt}$. Equivalently, firms in sector i also receive $(1-\theta_{jit})p_{jit}m_{jit}$ from their customers in sector j at the beginning of t and have account receivables $\theta_{jit}p_{jit}m_{jit}$. The total working capital gain from trade is $\sum_{j=1}^{N}(1-\theta_{jit})p_{jit}m_{jit}$. Firms are linked through production and trade credit networks.

Managers make two-step decisions. Initially, they decide the level of trade credit for each intermediate input purchase, θ_{ijt}. They then choose $\{l_{it}, m_{ijt}, m_{jit}, y_{it}\}$ given $\{\theta_{ijt}, \theta_{jit}\}$. A firm's problem is solved through backward induction.

The first-order conditions of the problem that occurs in the context of the second choosing step are,

$$\partial m_{jit}: \qquad p_{jit}[(1-\theta_{jit})R_{iLt}\theta_{jit}] = \Phi_{it} \tag{17}$$
$$\partial l_{it}: \qquad \alpha_i \Phi_{it} m_{it} = w l_{it} R_{iLt} \tag{18}$$
$$\partial m_{ijt}: \qquad [(1-\theta_{ijt})R_{iLt}+\theta_{ijt}]\, p_{ijt}m_{ijt} = \Phi_{it}\gamma_i\omega_{ij}m_{it} \tag{19}$$
$$\partial y_{it}: \qquad \Phi_{it} = p_{it} \tag{20}$$

Notably, $p_{ijt} = p_{jt}/[(1-\theta_{ijt})R_{jLt}+\theta_{ijt}]$. The price of product j purchased by sector i, p_{ijt}, depends on θ_{ijt}. The higher the trade credit, the more expensive the product price becomes. Producers naturally value early payments, and a manager, recognizing this, chooses trade credit level θ_{ijt} in the first step so as to minimize the unit intermediate input cost. The trade-off for adjusting trade credit is as follows. The benefit of increasing θ_{ijt} is the reduction of banking loans and interest costs. The cost of increasing θ_{ijt} is the increase in intermediate input price p_{ijt} given that $\partial p_{ijt}/\partial \theta_{ijt} > 0$. There is moreover a trade credit adjustment cost per dollar purchases, $C(\theta_{ijt}, \bar{\theta}_i)$. Thus, the manager solves the following problem,

$$\min_{\theta_{ijt}} [(1-\theta_{ijt})R_{iLt}+\theta_{ijt}]\, p_{ijt} + C(\theta_{ijt},\bar{\theta}_i)p_{ijt}$$

$$s.t. \quad p_{ijt} = p_{jt}/[(1-\theta_{ijt})R_{jLt} + \theta_{ijt}]$$

Therefore, the optimal level of trade credit is,

$$\theta_{ijt} = f(\varsigma, \bar{\theta}_i, R_{it}, R_{jt}) \tag{21}$$

with

$$\partial\theta_{ijt}/\partial R_{it} > 0,$$
$$\partial\theta_{ijt}/\partial R_{jt} < 0,$$
$$|\partial\theta_{ijt}/\partial\varsigma| < 0,$$
$$\theta_{ijt} = \bar{\theta}_i, \quad \text{when} \quad R_{it} = R_{jt}.$$

Further, trade credit is more sensitive to the relative financial condition of the two trading parties when ς is low. In the extreme case that $\varsigma = 0$, trade credit becomes fully flexible. Under these conditions, $\theta_{ijt} = 1$ when $R_{iLt} > R_{jLt}$ and $\theta_{ijt} = 0$ when $R_{iLt} < R_{jLt}$.

Proposition 4. *Trading parties share liquidity through the trade credit mechanism. θ_{ijt} is an increasing function of R_{iLt} and a decreasing function of R_{jLt}. Sectoral correlation is high when trade credit adjustment is flexible.*

When firm i finds that bank loans are becoming costly, it increases trade credit. When its suppliers are suffering financially, firm i shrinks its accounts-payable. This finding provides a theoretical foundation on the fact that financially distressed firm may postpone repaying trade credit to its suppliers and reduce the provision of trade credit to its customers as discussed in the introduction. This response consistent with the finding by Gao (2014) that trade credit plays an important role as an inter-firm financing channel by allowing firms to share liquidity with each other. It is also in line with the empirical finding that an increase in the use of trade credit along the product chain that links two sectors results in an increase in correlation between them (Raddatz (2008)). In addition, this model predicts that sectoral correlation is even higher when trade credit adjustment is flexible. An idiosyncratic liquidity shock could thus spill over to surrounding firms vigorously. If one firm has a liquidity problem, its accounts-payable would increase. Consequently, the financial stress is transmitted to its suppliers.

4.2 Retailers and Wholesale Firms.

Wholesale firms in the economy produce wholesale product Y_{wt}, which is a composite of the products produced by each sector. The wholesale output and the price are represented by:

$$Y_{wt} = \Pi_{i=1}^{N} \zeta_i^{-\zeta_i} y_{it}^{\zeta_i}, \quad P_{wt} = \Pi_{i=1}^{n} p_{it}^{\zeta_i},$$

where y_{it} is the amount of products produced by sector i and used in the production of wholesale goods. ζ_i governs the share of output i used in the production of final goods.

Consumption goods are sold by a set of monopolistically competitive retailers uniformly distributed from 0 to 1, who can costlessly differentiate the single final good assembled by wholesale firms. One unit of wholesale output Y_w is required to make a unit of retail output with marginal cost P_w. The final output composite is,

$$Y_t = \left(\int_0^1 Y_{rt}^{\frac{\epsilon-1}{\epsilon}} dr \right)^{\frac{\epsilon}{\epsilon-1}},$$

where Y_{rt} is the retail output of retailer r. Retailers face nominal rigidities following Calvo model. They could freely adjust their price with probability $1-\gamma$ each period; the problem is identifying the optimal price P_{rt}^*. By tedious but straightforward derivation in Appendix D.1, I have

$$\frac{P_{rt}^*}{P_t} = \frac{\epsilon}{\epsilon-1} \frac{\sum_{i=0}^{\infty} \beta^i \gamma^i \Lambda_{t+i,t} P_{wt+i} \left(\frac{P_{t+i}}{P_t}\right)^{\epsilon} Y_{t+i}}{\sum_{i=0}^{\infty} \beta^i \gamma^i \Lambda_{t+i,t} \left(\frac{P_{t+i}}{P_t}\right)^{\epsilon-1} Y_{t+i}}.$$

Thus, the innovation of the aggregate price level is,

$$P_t^{1-\epsilon} = (1-\gamma) P_{rt}^{*1-\epsilon} + \gamma (P_{t-1})^{1-\epsilon}. \tag{22}$$

4.3 Capital Producer.

At the end of period t, competitive capital producers in each sector buy capital from their respective capital markets, and repair and build new capital. Capital producers make new capital using input of final output and are subject to adjustment costs. They sell new capital k_{it} to firms in sector i at the price Q_{it}, so their problem is to,

$$\max_{I_{it}} \quad Q_{it} I_{it} - \left[1 + \frac{\eta_I}{2} \left(\frac{I_{it}}{I_{i,t-1}} - 1 \right)^2 \right] I_{it}. \tag{23}$$

Thus, the price of capital goods is equal to the marginal cost of investment goods production as follows,

$$Q_{it} = 1 + \frac{1}{2}\eta_I(\frac{I_{it}}{I_{i,t-1}} - 1)^2 + \frac{I_{it}}{I_{i,t-1}}\eta_I(\frac{I_{it}}{I_{i,t-1}} - 1) - \mathbb{E}_t[\beta\Lambda_{t+1}(\frac{I_{i,t+1}}{I_{it}})^2\eta_I(\frac{I_{i,t+1}}{I_{it}} - 1)].$$

The capital innovation is:

$$k_{it} = e^{\psi_{it}}(1-\delta)k_{i,t-1} + I_{it}.$$

4.4 Households.

There is a continuum of identical households with a fraction $1-u$ of workers and a fraction u of bankers. Over time, an individual switch between a worker and a banker with probability $(1-\tau)$. In other words, a banker at time t stays as a banker at time $t+1$ with probability τ.

Workers supply labor l_t to the production sector and return their wages to households. Bankers manage financial intermediaries and transfer profits back to households. Households consume C_t and save. They save by depositing funds in banks or by purchasing government debt. Both deposits and government debt are one-period riskless assets that pay the real return of R_t. I consider these two assets perfect substitutes and denote them by B_t. The households' welfare function is,

$$\max \mathbb{E}_0 \sum_{i=0}^{\infty} \beta^i \left[\ln(C_{t+i} - hC_{t+i-1}) - \frac{\chi}{1+\psi} l_{t+i}^{1+\psi} \right]. \tag{24}$$

The budget constraint they face is,

$$C_t = w_t l_t + \Pi_t + T_t + R_{t-1}B_{t-1} - B_t, \tag{25}$$

where w_t is real wage, l_t denotes the aggregate labor and equals $\sum_i l_{it}$ in equilibrium, Π_t represents profits distributed from bankers and capital producing firms, T_t represents government transfers. Let ϱ_t denotes the marginal utility of consumption. Denote the stochastic discount factor $\Lambda_{t,t+1} \equiv \frac{\varrho_{t+1}}{\varrho_t}$.

4.5 Financial Intermediaries.

The structure of financial intermediaries generally follows that of Gertler and Karadi (2011), but with the following modifications. Financial intermediaries (banks) are segmented into N groups. Each production sector i is connected to a unique banking group i. Hence, banks of group i cannot finance firms in sector $j \neq i$.[22] I further assume that there is no lending among banks. Each period, banks use their own net worth N_{it} and the deposit B_{it} from households in order to finance their purchases of financial claims S_{it} and loans L_{it}. The intermediary balance sheet of banks in group i is,

$$Q_{it}S_{it} + L_{it} = N_{it} + B_{it}. \tag{26}$$

The return of S_{it} is realized by the end of period $t+1$ with a stochastic return R_{ikt+1}. L_{it} is matured by the end of period t with a non-contingent return R_{iL_t}. Also, banks pay a non-contingent return R_t to households at period $t+1$. Their net worth then evolves according to

$$N_{it+1} = R_{ik,t+1}Q_{it}S_{it} + R_{iL,t}L_{it} - R_tB_{it} \tag{27}$$

The financial intermediary's objective is to maximize:

$$V_{it}(N_t) = \max \mathbb{E}_t \sum_{j=0}^{\infty}(1-\tau)\tau^j\beta^{j+1}\Lambda_{t,t+1+j}(N_{i,t+1+j}), \tag{28}$$

given that the probability of a banker becoming a worker in the next period is $(1-\tau)$. For the intermediary to operate, the risk premium must be positive, i.e. $E_t\beta^i\Lambda_{t,t+1}(R_{ikt+1+i}-R_{t+i}) \geq 0$ and $E_t\beta^i\Lambda_{t,t+1}(R_{iLt+i} - R_{t+i}) \geq 0$. However, in order to prohibit intermediaries expanding their assets indefinitely when risk premium is positive, I introduce the following incentive constraint,

$$V_{it} \geq \lambda_i(Q_{it}S_{it} + L_{it}). \tag{29}$$

Bankers will lose their expected terminal wealth V_{it} if they divert assets, while their gain from such an action is $\lambda_i(Q_{it}S_{it} + L_{it})$.

The binding incentive constraint in equilibrium implies that the leverage ratio $(\frac{Q_{it}S_{it}+L_{it}}{N_{it}})$ denoted by ϕ_{it} equals,

$$\phi_{it} = \frac{\eta_{it}}{\lambda_i - d_{it}\nu_{kit} - (1-d_{it})\nu_{lit}}, \tag{30}$$

with

$$\nu_{kit} = \mathbb{E}_t[(1-\tau)\beta\Lambda_{t,t+1}(R_{ik,t+1} - R_t) + \beta\Lambda_{t,t+1}\tau x_{kit,t+1}\nu_{kit+1}],$$

$$\nu_{lit} = \mathbb{E}_t[(1-\tau)\beta\Lambda_{t,t+1}(R_{iLt} - R_t) + \beta\Lambda_{t,t+1}\tau x_{lit,t+1}\nu_{lit+1}],$$

[22] This is consistent with the finding by Chodorow-Reich (2014) that bank-borrower relationships are sticky.

and

$$\eta_{it} \equiv \mathbb{E}_t[(1-\tau)\Lambda_{t,t+1}R_t + \beta\Lambda_{t,t+1}\tau z_{it,t+1}\eta_{it+1}],$$

where $d_{it} = \frac{Q_{it}S_{it}}{Q_{it}S_{it}+L_{it}}$ is the share of risky assets, $x_{kit,t+j} \equiv \frac{Q_{i,t+j}S_{i,t+j}}{Q_{it}S_{it}}$ and $x_{lit,t+j} \equiv \frac{L_{i,t+j}}{L_{it}}$ are the gross growth rate in assets between t and $t+j$, and $z_{it,t+j} \equiv \frac{N_{it+j}}{N_{it}}$ is the gross growth rate of net worth. ν_{it} is the expected discounted marginal gain that a unit enjoys by expanding its assets, when net worth remains constant, and is an increasing function of the risk premium. η_{it} is the expected discounted value of having an additional unit of net worth, when the asset remains constant. As derived in Appendix D.3, the first order conditions of the problem facing banks imply the no-arbitrage condition,

$$\mathbb{E}_t(H_{t,t+1}R_{ik,t+1}) = \mathbb{E}_t(H_{t,t+1})R_{iLt},$$

where $H_{t,t+1} = \beta\Lambda_{t,t+1}[(1-\tau) + \tau(\nu_{kit+1}d_{it+1}\phi_{it+1} + \nu_{lit+1}(1-d_{it+1})\phi_{it+1} + \eta_{it+1})]$. The evolution of bankers' net worth can be expressed as,

$$N_{it} = [(R_{ikt} - R_{t-1})d_{it-1}\phi_{it-1} + (R_{iLt-1} - R_{t-1})(1-d_{it-1})\phi_{it-1} + R_{t-1}]N_{it-1}. \tag{31}$$

Notably, the sensitivity of N_{it} to the excess return is increasing in the leverage ratio ϕ_{it-1}. Therefore, ϕ_i is an important factor affecting the sectoral sensitivity to financial shocks.

The total net worth of bankers in group i includes the net worth of existing bankers N_{iet} together with the net worth of new bankers N_{int},

$$N_{it} = N_{eit} + N_{nit},$$

where $N_{eit} = \tau[(R_{ikt} - R_{t-1})d_{it-1}\phi_{it-1} + (R_{iLt-1} - R_{t-1})(1-d_{it-1})\phi_{it-1} + R_{t-1}]N_{it-1}$. A financial shock represents an unexpected contraction of the existing bankers' net worth.

Assuming further that a fraction of $\omega_i/(1-\tau)$ of the total assets of exiting bankers $((1-\tau)(Q_{it}S_{it-1} + L_{it-1}))$ is transferred to new bankers, I have

$$N_{nit} = \omega_i(Q_{it}S_{it-1} + L_{it-1}).$$

4.6 Monetary policy and Government Expenditure

Monetary policy follows a simple Taylor rule with interest rate smoothing. The nominal interest rate follows,

$$i_t = (1-\rho)[i + \kappa_\pi\pi_t + \kappa_y(\log Y_t - \log Y_t^*)] + \rho i_{t-1} + \epsilon_t^i, \tag{32}$$

where i denotes the steady state nominal rate, π_t represents inflation and Y_t^* is the natural level of output.

Government expenditure G is financed by lump sum taxes.

4.7 Equilibrium

The competitive equilibrium in the general model is defined as follows:
A *competitive equilibrium* consists of a collection of quantities $\{m_{it}, m_{ijt}, y_{it}, l_{it}, l_t, k_{it}, \theta_{ijt}, Y_{wt}, Y_t, I_{it}\, C_t, G_t, S_{it}, N_{it}, N_{eit}, N_{nit}, L_{it}, \}$ and a sequence of prices $\{R_{ik,t}, R_{iL,t}, R_t, i_t, Q_{it}, p_{it}, p_{ijt}, P_{wt}, \pi_t, w_t, \}$ for $i \in \{1, ..., N\}$ and $j \in \{1, ..., N\}$, such that

1. Intermediate firms maximize profits (16). Capital producers maximize profit (23). Wholesalers and retailers maximize profits.

2. Households maximize utility (24).

3. Financial intermediaries maximize expected wealth (28).

4. Goods, labor, capital and credit markets clear.

The equilibrium results can be obtained by solving Equations (50)-(75) in Appendix D.4.

5 Quantitative Predictions

5.1 Calibration

I have calibrated the general model using the U.S. data at the 2-digit NAICS level. I exclude the governmental sector, as well as the finance and insurance subsectors from the FIRE (finance, insurance and real estate) sector.[23]

Conventional Parameters. Conventional parameters (such as the discount factor and the Calvo parameter) follow the calibration by Gertler and Karadi (2011) and are listed in Table 3.

Input-output Structure, Labor, Capital, Intermediate Input Share, and Final Output Share. The input-output structure ω_{ij} is calibrated using the BEA input-output table. Labor share α_i, capital share β_i and intermediate input share γ_i are listed in Table 4. α_i and β_i

[23]The objective of the government is not profit maximization. The finance and insurance firms cannot be formulated by the production problem presented in Section 4 because their investment strategies and balance sheet composites are complicated. Moreover, the trade credit of these two types of firms is not the same as that of goods producers.

are calibrated using the BEA GDP by Industry Value-added Components Table (1998-2013) and the calibration method follows Su (2014) (refer to Appendix D.5).

Banking Parameters. ω_i (the proportional transfer to the entering bankers) and λ_i (the fraction of capital that can be diverted) are calibrated to match ϕ_i (the leverage of each sector) and R_{iL} (the banking lending rate). I assume the steady state R_{iL} to be identical across sectors.[24] It is calibrated to hit the steady state credit spread. The leverage level of each sector is measured using the Compustat Data, which is listed in Table 4. The survival rate of bankers τ adopts the value set by Gertler and Karadi (2011), which hits the average horizon of bankers within a given decade.

Trade credit. The trade credit level θ_{ij} corresponds to accounts-payable over cost of goods sold.[25] Table 6 in Appendix A presents accounts-payable over cost of goods sold from all sectors, based on Compustat data (measured as the median of each sector in each time period, and averaged from 2000 to 2014).[26] Additionally, the Quarterly Financial Report (QFR) collects quarterly aggregate statistics on the financial results and positions of U.S. corporations. QFR is more comprehensive than Compustat, but it only covers four industry sectors, mining, manufacturing, wholesale and retail. All four of these measures of the standardized trade credit are relatively stable over the QFR sample period (2000q4-2014q4) as presented in Table 7 in Appendix A.[27] Moreover, these measures using QFR data approximate the measures using Compustat data (compare Table 6 and Table 7). Therefore, I calibrate $\bar{\theta}_i$ using the Compustat measure. The trade credit adjustment cost ς is estimated to match the standard deviation of accounts payable over total sales in the QFR data.

5.2 The propagation of financial shocks

Before calibrating the model completely, I would like first to compare the propagation of financial shocks predicted by the model with my empirical findings.

Factors that are specific to certain firms, such as intermediate input share, capital share, leverage and trade credit, all have strong impacts on the sensitivity of these firms' output to shocks. In order to focus on the propagation effect along the production chain, I simulate a 9-firm circle network model. Firm i uses intermediate inputs $i-1$ (as illustrated in Figure

[24] No strong empirical evidence shows that sectoral interest rates differ significantly in the long run.

[25] Although accounts-payable is a stock variable, it is close to its flow value because the length of the trade credit period is generally less than a quarter. I obtained accounts-payable and cost of goods sold from quarterly financial reports.

[26] The financial and insurance sectors have accounts payable over cost of goods sold at a level of 60. The balance sheets of these financial sectors are complicated, and the definition of accounts-payable in those sectors differs from that used in other production sectors.

[27] I standardize accounts payable and accounts receivable based on total sales and total assets.

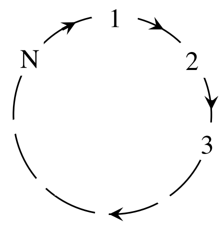
Table 3: Parameters

conventional parameters		
β	0.99	discount rate
h	0.815	habit parameter
ϕ	0.276	inverse Frisch elasticity of labor supply
χ	3.4108	relative utility weight of labor
η_I	1.728	investment adjustment cost
γ	0.779	probability of keeping price fixed
κ_π	1.5	inflation coefficient of the taylor rule
κ_y	0.5/4	output gap coefficient of the taylor rule
G/Y	0.2	steady state proportion of government expenditures
unconventional parameters		
θ	0.972	survival rate of bankers
ς	0.1	trade credit adjustment cost

Table 4: Sectoral Level Parameters

sector	labor share α_i	capital share β_i	intermediate share γ_i	final share ζ_i	trade credit $\bar{\theta}_i$	leverage ϕ_i
Agriculture	0.24	0.18	0.58	0.01	0.43	1.70
Mining	0.23	0.39	0.37	0.01	1.00	1.53
Utility	0.17	0.41	0.42	0.02	0.49	3.24
Construction	0.39	0.13	0.48	0.07	0.43	2.35
Manufacturing	0.20	0.16	0.64	0.21	0.49	1.50
Wholesale	0.37	0.33	0.31	0.04	0.44	2.30
Retail	0.41	0.27	0.32	0.10	0.42	1.99
Transportation	0.35	0.16	0.50	0.02	0.30	2.33
Information	0.25	0.30	0.45	0.04	0.64	1.58
real estate	0.24	0.48	0.28	0.15	0.64	2.27
PBS	0.50	0.13	0.37	0.06	0.37	1.53
Education	0.53	0.08	0.40	0.17	0.25	1.77
Arts	0.38	0.18	0.44	0.07	0.21	1.92
Other services	0.49	0.13	0.38	0.04	0.37	2.33
Average	0.33	0.45	0.22	-	0.45	2.00

Figure 3: Supply chain of a circle economy



3). The only difference among the firms is their relative location along the supply chain. A symmetric network model allows me to focus on the propagation effect of shocks in the model. I calibrate the economy using the parameters in Table 3 and the average labor share, capital share, trade credit and leverage ratios in the U.S. economy (as presented in the last row of Table 4).[28]

I impose an unexpected banking net worth shock to bankers linked to Firm 5, ϵ_{Ne5}, namely a contraction of the existing bankers' net worth N_{e5}; specifically, I assume that their net worth declines by one percent and that the decline is transferred to households. One immediate consequence is a rise in the cost of borrowing (R_{5Lt}) for Firm 5. The borrowing costs of other firms in the economy are also affected. This negative financial shock impacts the outputs of all firms, 5 included.

Table 5: Model vs. Data: the propagation of financial shocks

	Data	Model
$\text{UP}_{i,t=0}$	-0.1724**	-0.1979***
$\text{DOWN}_{i,t=0}$	-0.0635	-0.1294 ***
$\text{Shock}_{i,t=0}$	-0.4469**	-0.4360***

Notes: *** indicate statistical significance at the 1% levels. The dependent variable is the first 6-month output change $\tilde{m}_{i,t=6m}$ after the shock. $\text{Shock}_{i,t=0}$ is the borrowing cost R_{iL} change upon the impact of ϵ_{Ne5}. $\text{UP}_{i,t=0}$ and $\text{DOWN}_{i,t=0}$ follow Equations 11 and 12. The first column copies regression result (3) in Table 2.

I run regression 10 using the simulated data. The primary result is presented in Table

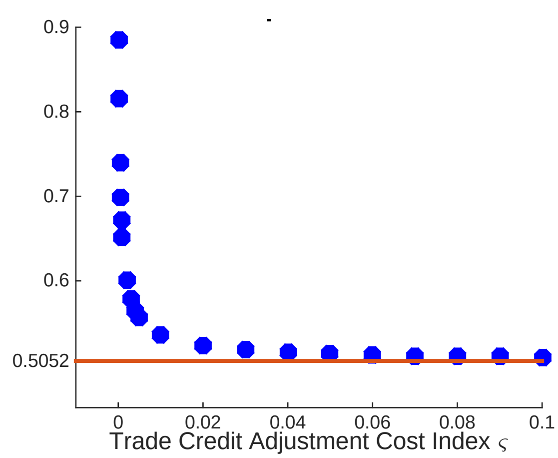
[28]Final share is set to be 1/9.

5. In the regression, the dependent variable represents the change in each firm's output six months after the shock.[29] The independent variables are the loan rate change at the moment the shock is felt $Shock_{i,t=0} = \tilde{R}_{iL,t=0}$, the shocks transmitted from customers $UP_{i,t=0} = \sum_j \hat{h}_{ji}\tilde{R}_{jL,t=0}$ and the shocks transmitted from suppliers $DOWN_{i,t=0} = \sum_j h_{ij}\tilde{R}_{jL,t=0}$. In general, my model performs reasonably well with respect to the impact of the interest rate shock and the propagation of shocks in the production network.

5.3 The role of trade credit

As discussed in Section 4, there is evidence that firms share liquidity with surrounding firms through trade credit in the real economy. Consequently, trade credit increases output correlation among firms in the production network. The general model captures this pattern in the following way. Figure 4 plots the output correlation among firms against the level of trade credit adjustment cost using data simulated from the circle model with idiosyncratic banking network shocks $\epsilon_{N_{ei}}$. The output correlation of firms and the level of trade credit adjustment cost vary inversely to one another: the more flexible the trade credit adjustment, the higher the output correlation among firms. The trade credit adjustment mechanism therefore amplifies the propagation of idiosyncratic financial shocks.

Figure 4: Firm output correlation



[29] Regression results of the simulated data at different horizons are presented in Table 11 in Appendix D.6.

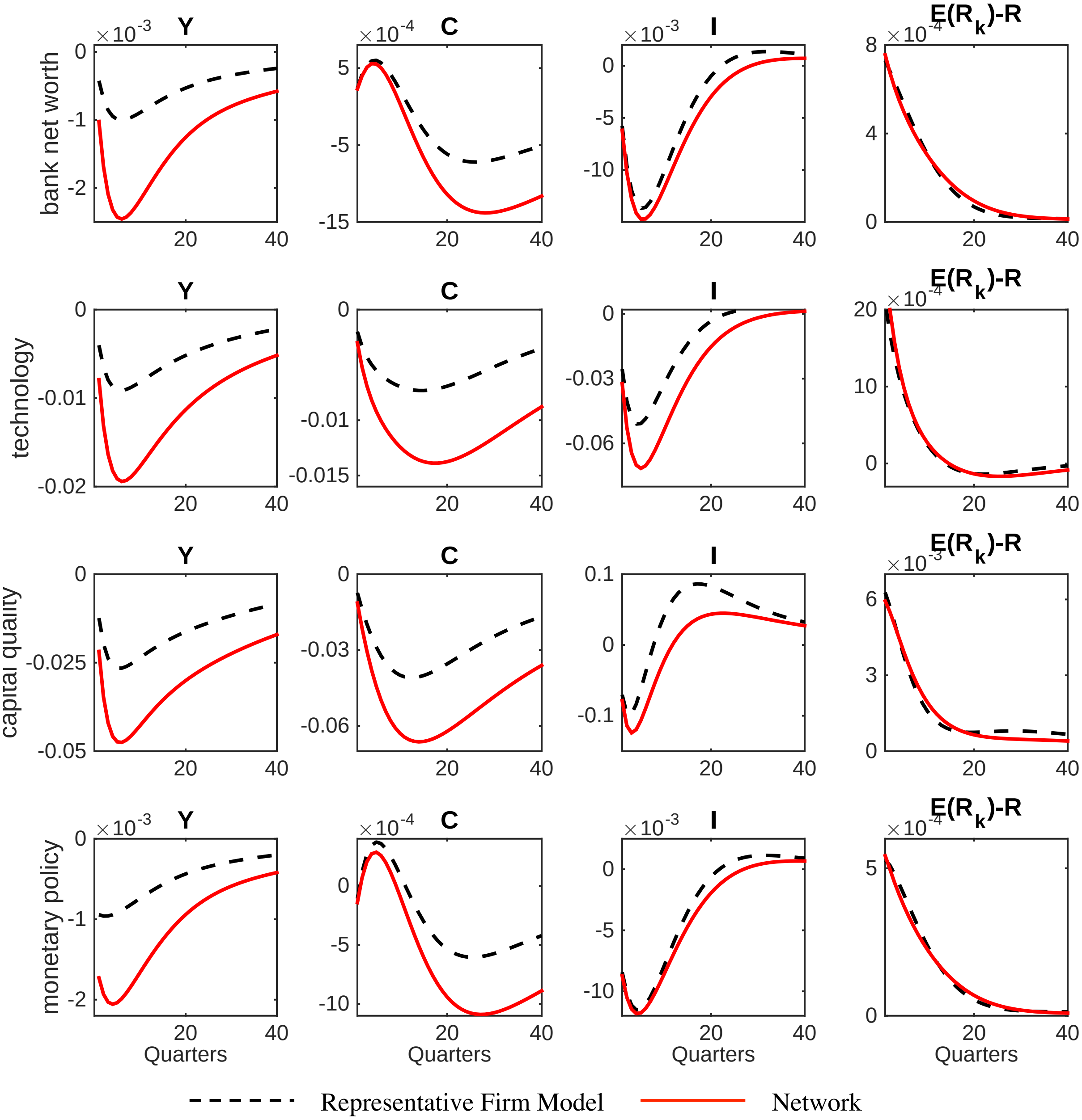
5.4 Amplification effect of the network structure

After confirming that the general model could capture empirical patterns presented in Section 3, I proceed to calibrate the model fully using the 14-sector data in Table 4. I begin by comparing the network model with a representative firm model. Consider a representative firm model with the same type of financial frictions described in Section 4. The parameters in this economy are calibrated using data in Table 3 and the last row of Table 4. As illustrated in Figure 5, the sensitivity of aggregate variables to shocks is very different in these two types of economies. Figure 5 plots the impulse responses of aggregate output Y, consumption C, investment I and premium $(E(R_{ik}) - R)$ after an aggregate banking net worth Ne shock, an aggregate technology shock z, a capital quality shock ξ and monetary policy shock i.[30] Ne shock is an unanticipated negative 1% reduction in the existing banker's net worth within each sector. The technology z shock is a negative 1% innovation in TFP of each sector, which is an AR1 process with autocorrelation 0.95. ξ shock is a negative 1% change in capital quality of each sector and is AR1 with autocorrelation 0.66. Monetary policy i shock is an unanticipated 10 basis-point increase in the short-term interest rate.

Remarkably, the input-output structure generates an amplification effect on the aggregate impact of shocks. Compared with a representative firm model (indicated by the dashed black line), a network economy (indicated by the solid red line) is more responsive to all manner of shocks. Consider, for example, an aggregate banking net worth shock: the responses of the premium $(E(R_k) - R)$ in these two types of models are not significantly different from each other. Because the financial intermediaries have the same type of horizontal structure in these two models, a uniform contraction of Ne impacts the interest rate R_k and R_L through the same mechanism within each banking sector. Consequently, the responses of the premium are similar. Nonetheless, the response of Y in the network model is more than two times stronger than it is in a representative firm model. Unlike bankers, firms in the network economy are interconnected. The input-output linkages among firms amplify the negative impact of the high borrowing cost and the liquidity shock. Similarly, the technology or capital quality shocks propagate upstream and downstream in the production network and ultimately affect aggregate output. The network structure amplifies the aggregate impact of the shock.

[30] $(E(R_k) - R)$ is calculated as the average premium across sectors; I is the total investment in the economy.

Figure 5: Aggregate variables response to banking net worth (Ne), technology (z), capital quality (ξ) and monetary (i)



6 Credit Policy Implication

Suppose that, during a severe financial crisis, a policymaker is willing to facilitate lending. Let S^p_{it} and L^p_{it} be the value of assets intermediated by the financial intermediaries, and let S^g_{it} and L^g_{it} be the value of assets intermediated by the government. Under these circumstances, the total value of intermediated assets of sector i is,

$$\begin{aligned} S_{it} &= S^p_{it} + S^g_{it}, \\ L_{it} &= L^p_{it} + L^g_{it}. \end{aligned}$$

To facilitate lending, the policymaker issues government debt to households that pay the riskless rate R_t, and it lends funds to non-financial firms at the banking credit rate R_{iLt}. For the policymaker, unlike for the financial intermediaries, there are no frictions on issuing debt and purchasing private assets. Whereas there are N types of assets according to the issuing sector, the policymaker is flexible in targeting liquidities. The policymaker could then either lend funds to all sectors proportionally (according to their market size), or could target liquidity to specific sectors. The total value of assets intermediated by the government would be,

$$CP_t = \sum_i^N CP_{it} = \sum_i^N (Q_{it} S^g_{it} + L^g_{it}) \tag{33}$$

Suppose the credit policy follows that

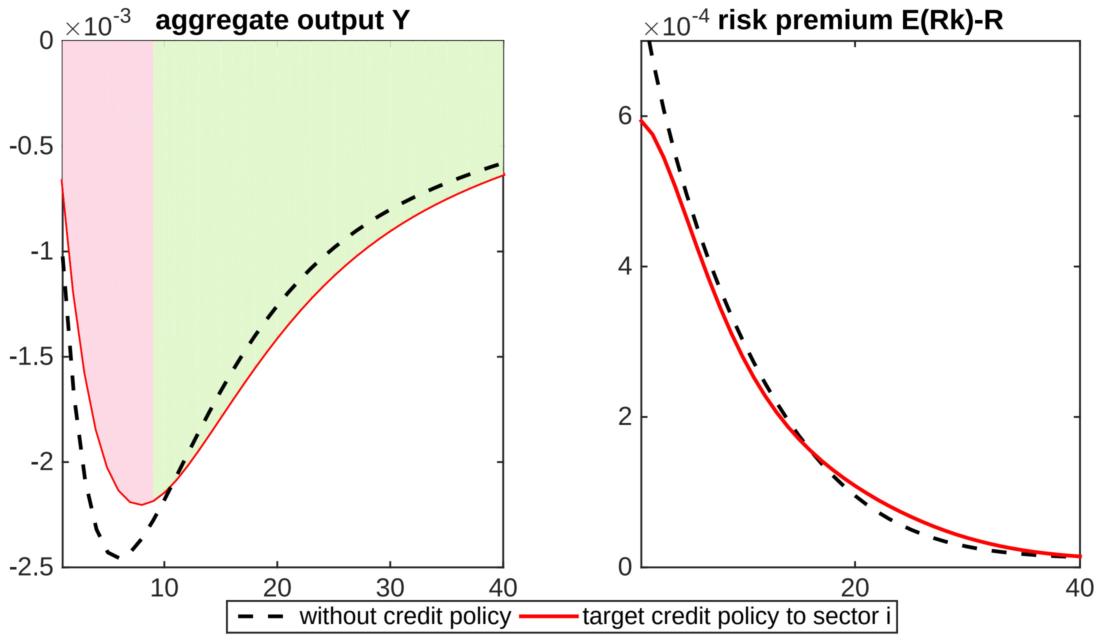
$$CP_{t+1} = \rho_{CP} CP_t + \epsilon^{CP}_{t+1}, \quad \text{with} \quad \epsilon^{CP}_t = -\vartheta \epsilon^{Ne}_t, \tag{34}$$

where ϑ is a constant and ϵ^{Ne}_t is the banking net worth shock. Accordingly, the total value of assets intermediated by the policymaker would be proportional to the contraction of the banking net worth, and CP_t follows an AR1 process. Moreover, following Gertler and Karadi (2011), this central bank intermediation involves the loss of efficiency; specifically, there is an efficiency cost of o for every unit of central bank credit that is supplied.

I compare credit policies by targeting liquidity to different sectors. Let $\tilde{Y}_{CPoni}$ denote the aggregate output change after an aggregate financial shock (Ne shock) and a sector-specific credit policy shock (i.e. $CP_t = CP_{it}$, $CP_{jt} = 0, \forall j \neq i$). The comparison of $\tilde{Y}_{CPoni}$ across i makes clear which sector i the central bank should target with loans during the crisis.

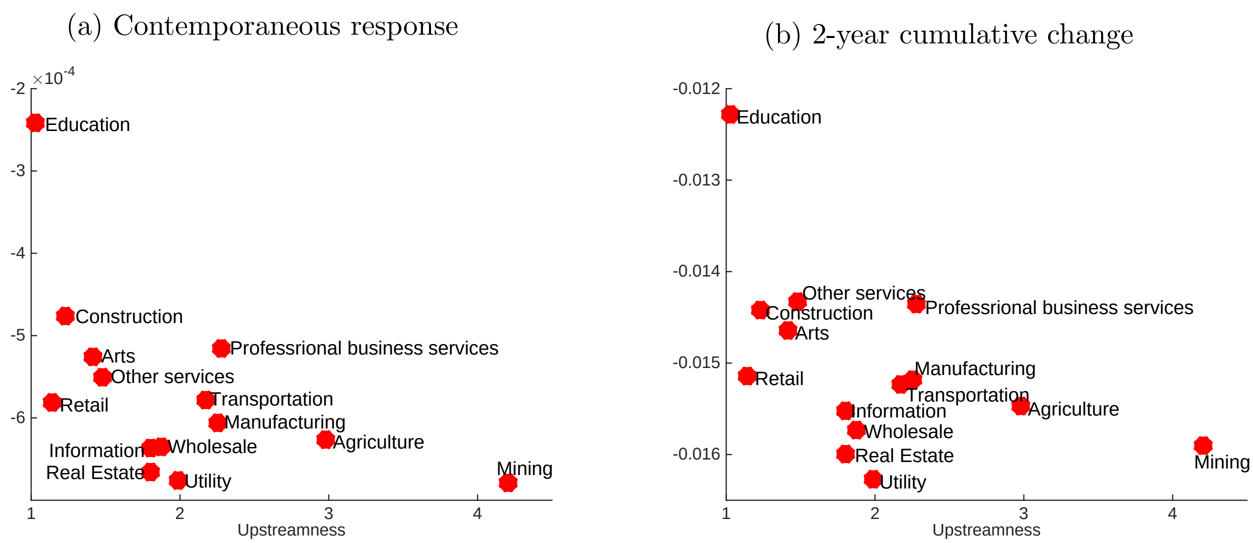
I simulate the model with $\vartheta = 5\%$ and $o = 0.01$. Figure 6 plots the response of the aggregate output and the premium after a -1% Ne shock and a credit policy targeting the real estate sector. Compared with an economy that lacks credit policies, this economy

Figure 6: Aggregate variables response to banking net worth (Ne) and credit policy (CP) shocks



Ne shock is a negative 1% change in the banking net worth of each sector. Credit policy is a 5% of the Ne shock targeting real estate with a quarterly autoregressive factor of 0.8.

Figure 7: Y_{CPoni}



In panel (a) contemporaneous response, each red dot represents Y_{CPoni} upon the impact of the Ne and credit policy shocks. The real estate dot corresponds the cross point of the red curve with the y-axis in the left panel of Figure 6. In panel (b) 2-year cumulative change, each red dot represents $\sum_{t=0}^{t=8} Y_{CPoni,t}$. The real estate dot corresponds to the size of the pink area in the left panel of Figure 6.

experiences less output contraction and a smaller premium increase.

Given that government has the ability to target liquidity to different sectors, the question arises as to which sector or sectors should be treated as priorities following aggregate financial shocks.[31] Figure 7 plots $\tilde{Y}_{CPoni}$ in the short run and in the long run against the relative upstreamness of each sector.[32] It turns out that credit policies have greater impact when liquidity is supplied to downstream sectors, which is not surprising, given that financial shocks propagate upstream. Moreover, this result is consistent with Proposition 3 that the financial condition of the downstream sector is more important systemically.

7 Conclusions

This paper investigates how the interaction between the production network and trade credit network affect the propagation of financial shocks. Using the U.S. input-output matrix and the bond yield data, I find strong upstream propagation of financial shocks. A 1% increase in the bond yield of one sector's customers (weighted by their input-output linkages) could impact the focal sector's output by -0.17%. To capture this pattern in an input-output model, it has been shown to be important to introduce financial friction in trade and interlocked balance sheets of trading parties. I have incorporated financial frictions and the trade credit network into an input-output model and have thereby been able to generate quantitative predictions consistent with my empirical findings. In addition, I have demonstrated that credit policies have a greater impact when liquidity is supplied to downstream sectors after an aggregate liquidity shock. For downstream sectors are more important systemically, given that financial shocks propagate upstream.

This paper represents the first theoretical study to introduce a trade credit network into an input-output model within a general equilibrium framework. Financial market loans and trade credit are the two most important sources of external finance for firms. Through the trade credit mechanism, each firm shares liquidity with surrounding firms, and as a consequence idiosyncratic shocks spread through trade and financial linkages of firms.

My findings here may serve to stimulate further research. Thus, for example, interesting results might be observed in a model based on a micro-founded trade credit structure. If one

[31] I focus on the credit policy implications after those of aggregate financial shocks, where the liquidity contraction across sectors is uniformly distributed. In the case of unbalanced financial shocks across sectors, it is obvious that the policymaker should supply liquidity to sectors that have experienced the strongest liquidity shock.

[32] Different measures of the long run change of $\tilde{Y}_{CPoni}$ have been examined. For example, the scatter plot of the 10-year cumulative change of $\tilde{Y}_{CPoni}$ and the upstreamness of sectors behaves in a manner qualitatively similar to that found in Panel (b) of Figure 7. The 2-year cumulative change of $\tilde{Y}_{CPoni}$ corresponds to the total size of the pink and green areas in the left panel of Figure 6.

were to allow a trade credit default, a strong liquidity shock to one firm could cause a cascade of defaults throughout the trade credit network, setting off an avalanche of production failure and generating a persistent aggregate output contraction. The work discussed here also suggests that inter-bank lending could play an important role in recovery from idiosyncratic shocks. In reality, there exists both a complicated production network and an entangled financial network. An idiosyncratic financial shock not only transfers through the input-output network, but also spills over in the banking network. It is my hope that this paper will encourage further theoretical and empirical studies on the role of financial frictions in the input-output economy.

References

Acemoglu, Daron, Ufuk Akcigit, and William Kerr (2015a), "Networks and the macroeconomy: An empirical exploration." NBER Macro Conference.

Acemoglu, Daron, Vasco M. Carvalho, Asuman Ozdaglar, and Alireza Tahbaz-Salehi (2012), "The network origins of aggregate fluctuations." *Econometrica*, 80.

Acemoglu, Daron, Asuman Ozdaglar, and Alireza Tahbaz-Salehi (2015b), "Networks, shocks, and systemic risk." In *The Oxford Handbook on the Economics of Networks* (Yann Bramoullé, Andrea Galeotti, and Brian Rogers, eds.), Oxford University Press, Oxford.

Acemoglu, Daron, Asuman Ozdaglar, and Alireza Tahbaz-Salehi (2015c), "Systemic risk and stability in financial networks." *American Economic Review*, 105.

Allen, Franklin and Douglas Gale (2000), "Financial contagion." *Journal of Political Economy*, 108.

Bak, Per, Kan Chen, José Scheinkman, and Michael Woodford (1993), "Aggregate fluctuations from independent sectoral shocks: self-organized criticality in a model of production and inventory dynamics." *Ricerche Economiche*, 47, 3–30.

Baqaee, David R. (2015), "Cascading failures in production networks." Working paper.

Bigio, Saki and Jennifer La'O (2013), "Financial frictions in production networks." Working paper.

Boissay, Frédéric and Reint Gropp (2007), "Trade credit defaults and liquidity provision by firms." European Central Bank, Working Paper Series No. 753.

Burkart, Mike and Tore Ellingsen (2004), "In-kind finance: A theory of trade credit." *American Economic Review*, 94.

Carvalho, Vasco M. and Xavier Gabaix (2013), "The great diversification and its undoing." *American Economic Review*, 103.

Chodorow-Reich, Gabriel (2014), "The employment effects of credit market disruptions: Firm-level evidence from the 20089 financial crisis." *The Quarterly Journal of Economics*, 129.

Conley, Timothy G. and Bill Dupor (2003), "A spatial analysis of sectoral complementarity." *Journal of Political Economy*, 111.

Cunat, Vicente (2007), "Trade credit: Suppliers as debt collectors and insurance providers." *Review of Financial Studies*, 20, 491–527.

Cúrdia, Vasco and Michael Woodford (2011), "The central-bank balance sheet as an instrument of monetary policy." *Journal of Monetary Economics*, 58, 54–79.

di Giovanni, Julian, Andrei A. Levchenko, and Isabelle Méjean (2014), "Firms, destinations, and aggregate fluctuations." *Econometrica*, 82, 13031340.

Dupor, Bill (1999), "Aggregation and irrelevance in multi-sector models." *Journal of Monetary Economics*, 43.

Foerster, Andrew T., Pierre-Daniel G. Sarte, and Mark W. Watson (2011), "Sectoral versus aggregate shocks: A structural factor analysis of industrial production." *Journal of Political Economy*, 119, 1–38.

Gabaix, Xavier (2011), "The granular origins of aggregate fluctuations." *Econometrica*, 79.

Gao, Janet (2014), "Business networks, firm connectivity, and firm policies." Working paper.

Gertler, Mark and Peter Karadi (2011), "A model of unconventional monetary policy." *Journal of Monetary Economics*, 58, 17–34.

Hertzel, Michael G., Zhi Li, Micah S. Officer, and Kimberly J. Rodgers (2008), "Officer and inter-firm linkages and the wealth effects of financial distress along the supply chain." *Journal of Financial Economics*, 87, 374–387.

Horvath, Michael (1998), "Cyclicality and sectoral linkages: aggregate fluctuations from independent sectoral shocks." *Review of Economic Dynamics*, 1.

Horvath, Michael (2000), "Sectoral shocks and aggregate fluctuations." *Journal of Monetary Economics*, 45.

Hulten, Charles R. (1978), "Growth accounting with intermediate inputs." *The Review of Economic Studies*, 45.

Jones, Charles I. (2011), "Intermediate goods and weak links: A theory of economic development." *American Economic Journal: Macroeconomics*, 3, 1–28.

Jones, Charles I. (2012), "Misallocation, economic growth, and input-output economics." In *Advances in Economics and Econometrics* (M. Arellano D. Acemoglu and E. Dekel, eds.), Cambridge University Press, New York.

Jovanovic, Boyan (1987), "Micro shocks and aggregate risk." *Quarterly Journal of Economics*, 102.

Kalemli-Ozcan, Sebnem, Hyun Song Shin Se-Jik Kim, Bent E. Sørensen, and Sevcan Yesiltas (2012), "Financial shocks in production chains." Unpublished manuscript.

Kiyotaki, Nobuhiro and John Moore (1997), "Credit chains." Unpublished manuscript.

Long, John B. and Charles I. Plosser (1983), "Real business cycles." *Journal of Political Economy*, 91.

Nilsen, Jeffrey H. (2002), "Trade credit and the bank lending channel." *Journal of Money, Credit and Banking*, 34.

Peterson, Michell A. and Raghuram G. Rajan (1997), "Trade credit: Theories and evidence." *Review of Financial Studies*, 10.

Price, David A. (2012), "When the fed conducts credit policy." Federal Reserve Region Focus.

Raddatz, Claudio (2008), "Credit chains and sectoral comovement: Does the use of trade credit amplify sectoral shocks?" Policy Research Working Paper Series 4525, The World Bank.

Reis, Ricardo (2009), "Interpreting the unconventional u.s. monetary policy of 2007-09." *Brookings Papers on Economic Activity*, 40, 119–165.

Shea, John (2002), "Complementarities and comovements." *Journal of Money, Credit, and Banking*, 34, 412433.

Su, Hsuan-Li (2014), "Financial frictions, capital misallocation, and input-output linkages." Working paper.

A Appendix: Trade Credit

Trade credit is a short-term loan a supplier provides to its customer upon a purchase of its product. It is the single most important source of external finance for firms (Boissay and Gropp (2007)). Two types of trade credit rules are common (Nilsen (2002)). One type is a one part contract, Net-30. Suppliers give buyer 30-day interest free loans. The other type is a two part contract, 2/10 Net 30. If customers pay within 10 days of delivery, then they qualify a 2% discount; otherwise they can pay up to 30 days after delivery.

Empirical evidence shows that the implicit interest rate in a trade credit agreement is usually very high as compared with the rate on bank credit.[33] Peterson and Rajan (1997) for example, conservatively estimate trade credit cost and find it more expensive than 99.8% of the loans. The high interest rate on trade credit arises because of an insurance premium and a default premium. Suppliers provide financing help through trade credit or defer payments when customers have already exhausted their bank credit line. In my model, I do not consider trade credit interests. However, my results about the propagation of financial shocks still hold in a set up with positive trade credit interest rates.

Several theories have been put forth to explain why suppliers provide credit to customers even though those firms cannot get additional banking credit. On the one hand, suppliers would like to keep business relationship as the cost of losing customers is high. On the other hand, suppliers may have an information advantage over banks and have a comparative advantage in liquidating collateral that the borrower may put up to secure the loan. Moreover, the transaction argument states that trade credit reduces the transaction cost of paying bills. Also, there are papers study the role of trade credit in mitigating supply chain moral hazard. This paper does not focus on explaining the existence and the optimal choice of trade credit, but it studies the impact of trade credit on the propagation of shocks.

Notably, a trade credit debtor in bankruptcy almost surely default on the claims held by its trade creditors (Boissay and Gropp (2007)). Shocks to the liquidity of some firms caused by the default of the customers, may in turn cause default or postponement of accounts payables on their suppliers and propagate upstream through the production chain. A distress at a single firm may induce a cascade of defaults throughout the production chain. The trade credit propagation mechanism will amplify the impact of idiosyncratic shocks on aggregate output. Thus, this mechanism creates a big multiplier effect. Although this feature is not included in this paper, it is worth investigating in my future research.

[33] Be aware that price may be in the form of intrinsic interest.

Table 6: Trade Credit (Compustat)

	AP/COGS	AP/S	AR/S	AP/TA	AR/TA
Agriculture	0.43	0.30	0.41	0.05	0.04
Mining	1.00	0.50	0.60	0.05	0.05
Utilities	0.49	0.36	0.51	0.04	0.06
Construction	0.43	0.34	0.88	0.10	0.20
Manufacturing	0.49	0.32	0.57	0.07	0.13
Wholesale trade	0.44	0.33	0.41	0.18	0.21
Retail trade	0.42	0.28	0.09	0.14	0.05
Transportation	0.30	0.22	0.37	0.04	0.09
Information	0.64	0.26	0.57	0.05	0.09
FIRRL	60.42	25.68	0.68	0.56	0.07
PBS	0.37	0.23	0.66	0.06	0.20
Educational	0.25	0.16	0.57	0.05	0.15
Arts	0.21	0.15	0.05	0.04	0.01
Other services	0.37	0.23	0.22	0.06	0.07

Table 7: Trade Credit (QFR)

	AP/S	AR/S	AP/TA	AR/TA
Mining	0.49	0.64	0.05	0.06
Manufacturing	0.31	0.45	0.07	0.10
Wholesale trade	0.33	0.36	0.20	0.22
Retail trade	0.28	0.14	0.16	0.08

B Appendix: Simple Network Models

B.1 Equilibrium results

The horizontal economy:

Notably, the labor demand function in this economy follows,

$$\alpha_i p_i m_i = R_i w l_i, \quad i \in \{1, 2, 3\}. \tag{35}$$

The equilibrium output of firms are functions of their own technology level z_i and the interest rate R_i, i.e.

$$m_i^h = z_i (R_i^{-1} \alpha_i \zeta_i^h)^{\alpha_i}, \quad i \in \{1, 2, 3\}. \tag{36}$$

The equilibrium aggregate output becomes,

$$Y^h = \Xi (m_1^h)^{\zeta_1^h} (m_2^h)^{\zeta_2^h} (m_3^h)^{\zeta_3^h} \tag{37}$$

The network economy:

The equilibrium sectoral outputs of the TC-network economy are,

$$m_1 = \frac{z_1}{R_1} \left\{ \zeta_1 + (1-\alpha_2) \frac{[\theta_2 + (1-\theta_2)R_1]}{[\theta_2 + (1-\theta_2)R_2]} \left[\zeta_2 + (1-\alpha_3)\zeta_3 \frac{[\theta_3 + (1-\theta_3)R_2]}{[\theta_3 + (1-\theta_3)R_3]} \right] \right\} \tag{38}$$

$$m_2 = z_2 z_1^{1-\alpha_2} \alpha_2^{\alpha_2} (1-\alpha_2)^{1-\alpha_2} R_1^{-(1-\alpha_2)} R_2^{-\alpha_2} \left(\frac{[\theta_2 + (1-\theta_2)R_1]}{[\theta_2 + (1-\theta_2)R_2]} \right)^{1-\alpha_2} \left[\zeta_2 + (1-\alpha_3)\zeta_3 \frac{[\theta_3 + (1-\theta_3)R_2]}{[\theta_3 + (1-\theta_3)R_3]} \right] \tag{39}$$

$$m_3 = z_3 z_2^{1-\alpha_3} z_1^{(1-\alpha_2)(1-\alpha_3)} \alpha_3^{\alpha_3} (1-\alpha_3)^{1-\alpha_3} \alpha_2^{\alpha_2(1-\alpha_3)} (1-\alpha_2)^{(1-\alpha_2)(1-\alpha_3)} \zeta_3 R_1^{-(1-\alpha_2)(1-\alpha_3)} R_2^{-\alpha_2(1-\alpha_3)} R_3^{-\alpha_3} \left(\frac{[\theta_2 + (1-\theta_2)R_1]}{[\theta_2 + (1-\theta_2)R_2]} \right)^{(1-\alpha_2)(1-\alpha_3)} \left(\frac{[\theta_3 + (1-\theta_3)R_2]}{[\theta_3 + (1-\theta_3)R_3]} \right)^{1-\alpha_3} \tag{40}$$

The labor allocation is,

$$l_1 = \left\{ \zeta_1 + (1-\alpha_2) \frac{[\theta_2 + (1-\theta_2)R_1]}{[\theta_2 + (1-\theta_2)R_2]} \left[\zeta_2 + (1-\alpha_3)\zeta_3 \frac{[\theta_3 + (1-\theta_3)R_2]}{[\theta_3 + (1-\theta_3)R_3]} \right] \right\} \cdot \frac{\alpha_1}{R_1}, \tag{41}$$

$$l_2 = \left\{ \zeta_2 + (1-\alpha_3)\zeta_3 \frac{[\theta_3 + (1-\theta_3)R_2]}{[\theta_3 + (1-\theta_3)R_3]} \right\} \cdot \frac{\alpha_2}{R_2}, \tag{42}$$

$$l_3 = \zeta_3 \cdot \frac{\alpha_3}{R_3}, \tag{43}$$

The aggregate output is,

$$\begin{aligned} Y &= \Theta z_1^{V_1} z_2^{V_2} z_3^{V_3} R_1^{-V_1} R_2^{-\alpha_2 V_2} R_3^{-\alpha_3 V_3} \\ & \left(\frac{[\theta_2 + (1-\theta_2)R_1]}{[\theta_2 + (1-\theta_2)R_2]}\right)^{V_1-\zeta_1} \left(\frac{[\theta_3 + (1-\theta_3)R_2]}{[\theta_3 + (1-\theta_3)R_3]}\right)^{V_2-\zeta_2}. \end{aligned} \tag{44}$$

B.2 Propagation of Shocks

B.2.1 Supply shocks

I begin the analysis of supply shocks by analyzing their transmission. In particular, I want to consider technology shocks in the network economy. Clearly, the technologies employed by upstream firms have significant impact on the outputs of their downstream firms (according to Equation 38-40).

Proposition 5. *In the network model with financial frictions, supply shocks generate strong downstream propagation. Moreover, downstream firms with larger shares of intermediate inputs* $(1-\alpha_{i+1})$ *are more affected by this downstream propagation. The elasticity of sectoral output with respect to the technology of firm 2* (z_2) *is,*

$$\frac{\partial m_1}{\partial z_2}\frac{z_2}{m_1} = 0, \quad \frac{\partial m_2}{\partial z_2}\frac{z_2}{m_2} = 1, \quad \frac{\partial m_3}{\partial z_2}\frac{z_2}{m_3} = (1-\alpha_3),$$

Technology shocks mainly generate downstream propagation because they lead to significant changes in intermediate input prices. Moreover, the share of intermediate inputs in the production function ($(1-\alpha_i)$) controls the power of this downstream propagation. Intuitively, the larger the intermediate input usage of a firm, the more sensitive the firm is to shocks that flow down the production chain.

B.2.2 Demand shocks

To explain the propagation of demand shocks in a production chain, I modify the non-production side of the model slightly, as follows: 1) the aggregate consumption of households is measured as a composite of the three specialized products: $C = c_1^{\zeta_1} c_2^{\zeta_2} c_3^{\zeta_3}$; 2) a government that collects lump-sum taxes to finance its expenditures $\{g_1, g_2, g_3\}$ is posited. The budget constraints of households thus become $\sum_i p_i c_i + T = wL + \Psi$. The budget constraint of the government becomes $\sum_i p_i g_i = T$. The market clearing conditions become

$$m_1 = m_{21} + c_1 + g_1, \quad m_2 = m_{32} + c_2 + g_2, \quad m_3 = c_3 + g_3.$$

By the zero profit condition,

$$p_1 = \frac{w}{z_1},$$

$$p_2 = \frac{1}{z_2}\left(\frac{1}{\alpha_2}\right)^{\alpha_2}\left(\frac{1}{1-\alpha_2}\right)^{(1-\alpha_2)} w^{\alpha_2} p_1^{(1-\alpha_2)},$$

$$p_3 = \frac{1}{z_3}\left(\frac{1}{\alpha_3}\right)^{\alpha_3}\left(\frac{1}{1-\alpha_3}\right)^{(1-\alpha_3)} w^{\alpha_3} p_2^{(1-\alpha_3)}.$$

Normalize w to be 1. I have sectoral prices, p_1, p_2, p_3 are independent of the demand shock. Consumption $\{c_1, c_2, c_3\}$ is independent of the demand shock. m_3 doesn't change. Δm_2 equals Δg_2.

From this starting point, the impact of demand shocks such as government expenditure shocks (g_i shocks) to the economy can be analyzed. Solving for the equilibrium of this economy generates the following result.

Proposition 6. *In the network model, when government expenditure is taken into account, demand shocks generate strong upstream propagation. Moreover, firms that have a larger proportion of output purchased by the impacted firms ($\rho_i \equiv \frac{m_{i+1,i}}{m_i}$) are naturally more sensitive to the demand shock. The sensitivity of firm outputs to the government expenditure shock to product 2 (g_2) is,*

$$\frac{\partial m_1}{\partial g_2} = f(\rho_1), \quad \frac{\partial m_2}{\partial g_2} = 1, \quad \frac{\partial m_3}{\partial g_2} = 0,$$

with $\frac{\partial f(\rho_1)}{\partial \rho_1} > 0$.

Consumption and price levels of the economy remain constant. The affected firms adjust their production levels, and thus their input demands, after the government expenditure shock. As a consequence, demand-side shocks mainly create upstream propagation. Moreover, the share of sectoral output used as intermediate inputs ρ_i governs the strength of this upstream propagation. Intuitively, the larger this share is, the more sensitive the firm is to shocks that flow up the production chain.

B.2.3 Financial shocks

The elasticities of sectoral output with respect to the interest rate of firm 2 (R_2) are,

$$
\begin{aligned}
\frac{\partial m_1}{\partial R_2}\frac{R_2}{m_1} &= \frac{(1-\alpha_2)\zeta_2[\theta_2+(1-\theta_2)R_1]}{[\theta_2+(1-\theta_2)R_2]^2}\frac{\left[-(1-\theta_2)+(1-\alpha_3)\zeta_3\frac{\theta_2-\theta_3}{[\theta_3+(1-\theta_3)R_3]}\right]R_2}{\left\{\zeta_1+(1-\alpha_2)\frac{[\theta_2+(1-\theta_2)R_1]}{[\theta_2+(1-\theta_2)R_2]}\left[\zeta_2+(1-\alpha_3)\zeta_3\frac{[\theta_3+(1-\theta_3)R_2]}{[\theta_3+(1-\theta_3)R_3]}\right]\right\}} \\
\frac{\partial m_2}{\partial R_2}\frac{R_2}{m_2} &= -\alpha_2-(1-\alpha_2)\frac{(1-\theta_2)R_2}{\theta_2+(1-\theta_2)R_2}+\frac{(1-\alpha_3)\zeta_3(1-\theta_3)R_2}{\zeta_2+(1-\alpha_3)\zeta_3\frac{\theta_3+(1-\theta_3)R_2}{\theta_3+(1-\theta_3)R_3}}, \\
\frac{\partial m_3}{\partial R_2}\frac{R_2}{m_3} &= -(1-\alpha_3)\left[\alpha_2+(1-\alpha_2)\frac{(1-\theta_2)R_2}{\theta_2+(1-\theta_2)R_2}-\frac{(1-\theta_3)R_2}{\theta_3+(1-\theta_3)R_2}\right].
\end{aligned}
$$

If $\theta_i=0,\forall i$,

$$
\frac{\partial m_1}{\partial R_2}\frac{R_2}{m_1}=-(1-\alpha_2)\frac{\zeta_2 z_1\Upsilon_1}{R_2 m_1},\quad \frac{\partial m_2}{\partial R_2}\frac{R_2}{m_2}=-\frac{\zeta_2 z_2 z_1^{1-\alpha_2}\Upsilon_2}{R_2 m_2},\quad \frac{\partial m_3}{\partial R_2}\frac{R_2}{m_3}=0.
$$

If $\theta_i=1,\forall i$,

$$
\frac{\partial m_1}{\partial R_2}\frac{R_2}{m_1}=0,\quad \frac{\partial m_2}{\partial R_2}\frac{R_2}{m_2}=-\alpha_2,\quad \frac{\partial m_3}{\partial R_2}\frac{R_2}{m_3}=-\alpha_2(1-\alpha_3).
$$

B.3 The influence vector

Proof of Proposition 3. By the zero profit condition,

$$
p_i=\alpha_i^{\alpha_i}(1-\alpha_i)^{1-\alpha_i}\Pi(\omega_{ij})^{\omega_{ij}(1-\alpha_i)}\frac{1}{z_i}w^{\alpha_i}R_i^{\alpha_i}\Pi_{j=1}^{N}\left(p_j\frac{[(1-\theta_i)R_i+\theta_i]}{[(1-\theta_i)R_j+\theta_i]}\right)^{\omega_{ij}(1-\alpha_i)}.
$$

Log linearize:

$$
\begin{aligned}
\alpha_i\tilde{w} &= \tilde{z}_i+\tilde{p}_i-(1-\alpha_i)\sum\omega_{ij}\tilde{p}_j \\
&\quad -\left[\alpha_i+(1-\alpha_i)\frac{(1-\theta_i)\bar{R}}{[(1-\theta_i)\bar{R}+\theta_i]}\right]\tilde{R}_i+(1-\alpha_i)\frac{(1-\theta_i)\bar{R}}{[(1-\theta_i)\bar{R}+\theta_i]}\sum\omega_{ij}\tilde{R}_j.
\end{aligned}
$$

Define the following vectors,

$$
\mathbf{z}=\begin{bmatrix}\tilde{z}_1\\ \tilde{z}_2\\ \vdots\\ \tilde{z}_N\end{bmatrix},\quad \mathbf{R}=\begin{bmatrix}\tilde{R}_1\\ \tilde{R}_2\\ \vdots\\ \tilde{R}_N\end{bmatrix},\quad \mathbf{p}=\begin{bmatrix}\tilde{p}_1\\ \tilde{p}_2\\ \vdots\\ \tilde{p}_N\end{bmatrix}\quad \Omega=\begin{bmatrix}(1-\alpha_1)\omega_{11} & (1-\alpha_1)\omega_{12} & \cdots & (1-\alpha_1)\omega_{1N}\\ (1-\alpha_2)\omega_{21} & (1-\alpha_2)\omega_{22} & \cdots & (1-\alpha_2)\omega_{2N}\\ \vdots & \vdots & \ddots & \vdots\\ (1-\alpha_N)\omega_{N1} & (1-\alpha_N)\omega_{N2} & \cdots & (1-\alpha_N)\omega_{NN}\end{bmatrix}
$$

$$I_1 = \begin{bmatrix} \alpha_1 + (1-\alpha_1)\frac{(1-\theta_1)\bar{R}}{[(1-\theta_1)\bar{R}+\theta_1]} & 0 & \cdots & 0 \\ 0 & \alpha_2 + (1-\alpha_2)\frac{(1-\theta_2)\bar{R}}{[(1-\theta_2)\bar{R}+\theta_2]} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \alpha_N + (1-\alpha_N)\frac{(1-\theta_N)\bar{R}}{[(1-\theta_N)\bar{R}+\theta_N]} \end{bmatrix}$$

$$I_2 = \begin{bmatrix} \frac{(1-\theta_1)\bar{R}}{[(1-\theta_1)\bar{R}+\theta_1]} & 0 & \cdots & 0 \\ 0 & \frac{(1-\theta_2)\bar{R}}{[(1-\theta_2)\bar{R}+\theta_2]} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{(1-\theta_N)\bar{R}}{[(1-\theta_N)\bar{R}+\theta_N]} \end{bmatrix}$$

Assume $\alpha_i = \alpha_j$, thus

$$\alpha \mathbf{1} \tilde{w} = \mathbf{z} - [I_1 - I_2\Omega]\mathbf{R} + (I - \Omega)\mathbf{p},$$

where $\mathbf{1}$ is a column vector of N ones.

Multiplying this equation by $((I - \Omega')^{-1}\zeta)'$ obtains

$$\tilde{w} = v'[\mathbf{z} - (I_1 - I_2\Omega)\mathbf{R}].$$

with

$$v \equiv (I - \Omega')^{-1}\zeta.$$

$\zeta'\mathbf{p}$ corresponds to the aggregate price index P, which is constant.

From households' optimization problem, we have $Y = w$. Thus,

$$\tilde{Y} = v'[\mathbf{z} - (I_1 - I_2\Omega)\mathbf{R}].$$

Solve y_i and m_i. Henceforth, I solve for y_i and m_i. From firms' first order conditions I obtain,

$$\frac{(1-\theta_{ij})R_i + \theta_{ij}}{(1-\theta_{ij})R_j + \theta_{ij}} p_j m_{ij} = (1-\alpha_i)\omega_{ij} p_i m_i,$$

$$w l_i R_i = \alpha_i p_i m_i.$$

From the final producer's first order conditions I have,

$$p_i y_i = \zeta_i Y = \zeta_i w.$$

Log linearize the production function and substitute the first three equations, I obtain,

$$\begin{aligned}\tilde{m}_i &= \tilde{z}_i + \alpha_i(\tilde{m}_i + \tilde{p}_i - \tilde{w} - \tilde{R}_i) + (1-\alpha_i)\sum_j \omega_{ij}[\tilde{m}_i + \tilde{p}_i - \tilde{p}_j \\ &\quad - \frac{(1-\theta_i)\bar{R}}{[(1-\theta_i)\bar{R}+\theta_i]}\tilde{R}_i + \frac{(1-\theta_i)\bar{R}}{[(1-\theta_i)\bar{R}+\theta_i]}\tilde{R}_j],\end{aligned}$$

$\Rightarrow$

$$0 = \tilde{z}_i + \alpha_i(-\tilde{y}_i - \tilde{R}_i) + (1-\alpha_i)\sum_j \omega_{ij}\left\{\tilde{y}_j - \tilde{y}_i - \frac{(1-\theta_i)\bar{R}}{[(1-\theta_i)\bar{R}+\theta_i]}\tilde{R}_i + \frac{(1-\theta_i)\bar{R}}{[(1-\theta_i)\bar{R}+\theta_i]}\tilde{R}_j\right\}.$$

Thus,

$$\mathbf{y} = [I-\Omega]^{-1}[\mathbf{z} - (I_1 - I_2\Omega)]\mathbf{R},$$

where $\mathbf{y}$ is a vector of $\tilde{y}_i$.

From goods market clearing condition, I obtain

$$\begin{aligned} m_i &= y_i + \sum_j m_{ji} \\ &= y_i + \sum_j (1-\alpha_j)\omega_{ji} m_j \frac{y_i}{y_j}\frac{\zeta_j}{\zeta_i}\frac{[(1-\theta_j)R_i+\theta_j]}{[(1-\theta_j)R_j+\theta_j]}, \\ \zeta_i\frac{m_i}{y_i} &= \zeta_i + \sum_j (1-\alpha_j)\omega_{ji}\zeta_j \frac{m_j}{y_j}\frac{[(1-\theta_j)R_i+\theta_j]}{[(1-\theta_j)R_j+\theta_j]}.\end{aligned}$$

Denote $t_i = \frac{m_i}{y_i}$, I obtain

$$\zeta_i \bar{t}_i \tilde{t}_i = \sum_j (1-\alpha_j)\omega_{ji}\zeta_j\bar{t}_j\left[\tilde{t}_j + \frac{(1-\theta_j)\bar{R}}{[(1-\theta_j)\bar{R}+\theta_j]}(\tilde{R}_i - \tilde{R}_j)\right].$$

Further replace ω_{ji} by $\hat{\omega}_{ji} \equiv \frac{1}{1-\alpha_j}\frac{\bar{m}_{ji}\bar{p}_{ji}}{\bar{m}_i\bar{p}_i} = \omega_{ji}\frac{\bar{m}_j\bar{p}_j}{\bar{m}_i\bar{p}_i}$, then[34]

$$\tilde{t}_i = \sum_j (1-\alpha_j)\hat{\omega}_{ji}\left[\tilde{t}_j + \frac{(1-\theta_j)\bar{R}}{[(1-\theta_j)\bar{R}+\theta_j]}(\tilde{R}_i - \tilde{R}_j)\right].$$

[34]Notice that at $\omega_{ji} = \frac{1}{1-\alpha_j}\frac{\bar{m}_{ji}\bar{p}_{ji}}{\bar{m}_j\bar{p}_j}$.

Denote,

$$\hat{\Omega} = \begin{bmatrix} (1-\alpha_1)\hat{\omega}_{11} & (1-\alpha_1)\hat{\omega}_{12} & \cdots & (1-\alpha_1)\hat{\omega}_{1N} \\ (1-\alpha_2)\hat{\omega}_{21} & (1-\alpha_2)\hat{\omega}_{22} & \cdots & (1-\alpha_2)\hat{\omega}_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ (1-\alpha_N)\hat{\omega}_{N1} & (1-\alpha_N)\hat{\omega}_{N2} & \cdots & (1-\alpha_N)\hat{\omega}_{NN} \end{bmatrix}$$

Thus

$$\mathbf{t} = [I - \hat{\Omega}']^{-1}[I_{\sum} - \hat{\Omega}' I_2]\mathbf{R},$$

where $I_{\sum}$ is a diagonal matrix with diagonal element $\sum_j (1-\alpha_j)\hat{\omega}_{ji} \frac{(1-\theta_j)\bar{R}}{[(1-\theta_j)\bar{R}+\theta_j]}$. Given that, $\tilde{t}_i = \tilde{m}_i - \tilde{y}_i$, so $\mathbf{t} = \mathbf{m} - \mathbf{y}$.

Thus, the vector of the log deviation of firm output from the steady state ($\tilde{m}_i$) follows,

$$\mathbf{m} = [I - \Omega]^{-1}\mathbf{z} - \left\{ [I - \hat{\Omega}']^{-1}[-I_{\sum} + \hat{\Omega}' I_2] + [I - \Omega]^{-1}(I_1 - I_2\Omega)] \right\} \mathbf{R}. \tag{45}$$

The first term in the brace captures upstream propagation of the interest rate shock, while the second term captures downstream propagation. This equation indicates that financial shocks propagate both upstream and downstream, while supply side shocks (such as technology shocks) only propagate downstream.

C Appendix: Empirical Results & Robustness Checks

C.1 The U.S. input-output matrix

Figure 8 plots the input-output matrix (Ω) of the U.S..

C.2 Empirical results

Figure 8: The U.S. Input-Output Matrix

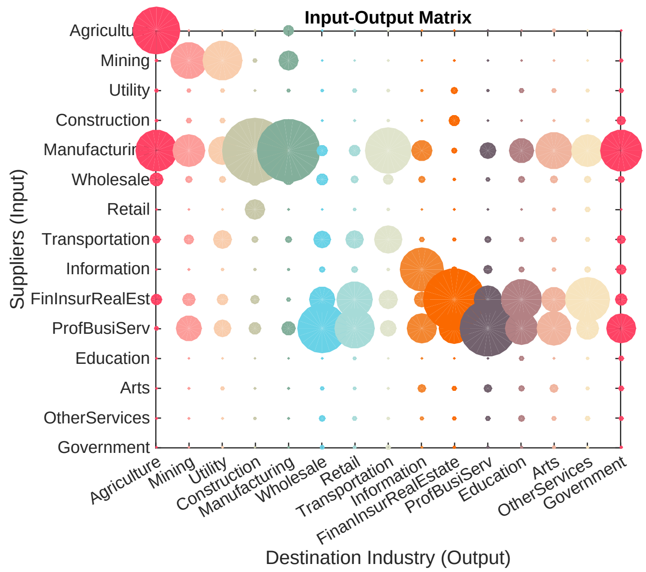


Table 8: UP vs. DOWN - panel

	dependent variable: $\tilde{m}_{it}$ (2002.09-2014.12)			
	(1) monthly	(2) quarterly	(3) 6-month	(4) yearly
UP$_{it-1}$	**-0.0876****	**-0.1704*****	**-0.0929****	**-0.1108***
	[0.0338]	[0.0301]	[0.0441]	[0.0564]
DOWN$_{it-1}$	**0.0479**	**0.1779**	**0.1645**	**0.0374**
	[0.0605]	[0.1252]	[0.1687]	[0.1927]
Shock$_{it-1}$	0.0685	-0.0917	-0.4395***	-0.2775**
	[0.0480]	[0.1058]	[0.1221]	[0.1230]
$\tilde{m}_{it-1}$	-0.1270***	0.0027	0.0891*	-0.0851
	[0.0358]	[0.0424]	[0.0502]	[0.0701]
Observations	8,673	2,832	1,357	649
Group	59	59	59	59

Notes: Estimations include time and sector fixed effects, report standard errors clustered by sector and are unweighted. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels respectively.

Table 9: UP vs. DOWN - cross section regression

	3-month (2008.11)		6-month (2009.02)		9-month (2009.05)		1-year (2009.08)	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
UP$_{2008.09-11}$	**-0.1639*****	**-0.1038****	**-0.3042*****	**-0.1724****	**-0.0295**	**-0.0614**	**-0.0301**	**-0.0131**
	[0.0427]	[0.0469]	[0.0733]	[0.0695]	[0.0560]	[0.0644]	[0.0590]	[0.0667]
DOWN$_{2008.09-11}$	**0.0634**	**0.1847**	**-0.1969**	**-0.0635**	**0.0723**	**-0.0203**	**0.2950**	**0.3638**
	[0.1598]	[0.1671]	[0.2715]	[0.2550]	[0.1958]	[0.2138]	[0.2077]	[0.2211]
Shock$_{2008.09-11}$	-0.0452	-0.0801	-0.4008*	-0.4469**	-0.4579**	-0.4015**	-0.1053	-0.1468
	[0.1242]	[0.1266]	[0.2337]	[0.2022]	[0.1806]	[0.1906]	[0.1977]	[0.2075]
ϵ_{2008}^{demand}		0.0844**		0.1835***		-0.0204		-0.0021
		[0.0319]		[0.0383]		[0.0373]		[0.0399]
ϵ_{2008}^{supply}		-0.0185		0.0305		0.0653		-0.0870
		[0.0497]		[0.0686]		[0.0589]		[0.0564]
$\tilde{m}_{it-1}$	0.7204***	0.6927***	0.6062***	0.5047***	1.0016***	0.9699***	0.9451***	0.9987***
	[0.0849]	[0.0929]	[0.0543]	[0.0607]	[0.0439]	[0.0642]	[0.0451]	[0.0584]
Observations	61	61	61	61	61	61	61	61

Note: I regress the output changes three months, six months, nice months, and one year after the Lehman Bankruptcy on the cumulative sectoral bond yield changes from Sep. 2008 to Nov. 2011. The lag output variable to the output change of the previous three months, six months, nine months, and one year respectively.

Table 10: UP vs. DOWN: additional lags

	dependent variable: $\tilde{m}_{it}$ (2002.09-2014.12)			
	(1)	(2)	(3)	(4)
	monthly	quarterly	6-month	yearly
L.upshock	-0.0557	-0.1963***	-0.1029**	-0.0985*
	[0.0358]	[0.0343]	[0.0415]	[0.0576]
L2.upshock	-0.0905**	0.0281	-0.0612	0.0454
	[0.0352]	[0.0341]	[0.0421]	[0.0392]
L.downshock	0.0428	0.2187	0.1686	0.0610
	[0.0670]	[0.1327]	[0.1747]	[0.2002]
L2.downshock	-0.1389**	-0.1045	-0.0254	-0.0164
	[0.0655]	[0.1106]	[0.1149]	[0.1471]
L.ownshock	0.0479	-0.0641	-0.4374***	-0.3317**
	[0.0489]	[0.1189]	[0.1238]	[0.1362]
L2.ownshock	0.0778	-0.2173**	-0.0790	-0.1668
	[0.0659]	[0.0859]	[0.0902]	[0.1218]
L.dip	-0.0078	0.0087	-0.0804**	-0.0473
	[0.0277]	[0.0412]	[0.0364]	[0.0418]
L2.dip	-0.1306***	0.0394	0.0768	-0.1039
	[0.0385]	[0.0291]	[0.0509]	[0.0724]
Observations	8,673	2,832	1,357	649
Group	59	59	59	59

Notes: Estimations include time and sector fixed effects, report standard errors clustered by sector and are unweighted. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels respectively.

D Appendix: General Model

D.1 Derivation of Philips' Curve

For retailer f the problem is,

$$\max \quad \sum_{i=0}^{\infty} \beta^i \gamma^i \Lambda_{t+i,t} (\frac{P_{ft}^*}{P_{t+i}} - P_{mt+i}) Y_{ft+i}$$

because $Y_{ft} = \left(\frac{P_{ft}}{P_t}\right)^{-\epsilon} Y_t$

$$\max \quad \sum_{i=0}^{\infty} \beta^i \gamma^i \Lambda_{t+i,t} [P_{ft}^{*(1-\epsilon)} P_{t+i}^{\epsilon-1} - P_{mt+i} P_{ft}^{*(-\epsilon)} P_{t+i}^{\epsilon}] Y_{t+i}$$

F.O.C.$[(1-\epsilon) P_{ft}^{*(-\epsilon)} P_{t+i}^{\epsilon-1} + \epsilon P_{mt+i} P_{ft}^{*(-\epsilon-1)} P_{t+i}^{\epsilon}] Y_{t+i}$

$$\sum_{i=0}^{\infty} \beta^i \gamma^i \Lambda_{t+i,t} [P_{ft}^* P_{t+i}^{\epsilon-1} - \frac{\epsilon}{\epsilon-1} P_{mt+i} P_{t+i}^{\epsilon}] Y_{t+i}$$

$$P_{ft}^* = \frac{\epsilon}{\epsilon-1} \frac{\sum_{i=0}^{\infty} \beta^i \gamma^i \Lambda_{t+i,t} P_{mt+i} \left(P_{t+i}\right)^{\epsilon} Y_{t+i}}{\sum_{i=0}^{\infty} \beta^i \gamma^i \Lambda_{t+i,t} \left(P_{t+i}\right)^{\epsilon-1} Y_{t+i}}$$

or

$$\frac{P_{ft}^*}{P_t} = \frac{\epsilon}{\epsilon-1} \frac{\sum_{i=0}^{\infty} \beta^i \gamma^i \Lambda_{t+i,t} P_{mt+i} \left(\frac{P_{t+i}}{P_t}\right)^{\epsilon} Y_{t+i}}{\sum_{i=0}^{\infty} \beta^i \gamma^i \Lambda_{t+i,t} \left(\frac{P_{t+i}}{P_t}\right)^{\epsilon-1} Y_{t+i}}$$

$$\frac{P_{ft}^*}{P_t} = \frac{\epsilon}{\epsilon-1} \frac{F_t}{Z_t}$$

where,

$$F_t = Y_t P_{mt} + \beta \gamma \Lambda_{t,t+1} \Pi_{t+1}^{\epsilon} F_{t+1},$$

$$Z_t = Y_t + \beta \gamma \Lambda_{t,t+1} \Pi_{t+1}^{\epsilon-1} Z_{t+1}.$$

Aggregate price level:

$$P_t^{1-\epsilon} = \int_0^1 P_{ft}^{1-\epsilon} df$$

$$P_t^{1-\epsilon} = \int_0^{1-\gamma} P_{ft}^{*(1-\epsilon)} df + \int_{1-\gamma}^1 (P_{ft-1})^{(1-\epsilon)} df$$

$$P_t^{1-\epsilon} = (1-\gamma) P_{ft}^{*(1-\epsilon)} df + \int_{1-\gamma}^1 (P_{ft-1})^{(1-\epsilon)} df$$

$$P_t^{1-\epsilon} = (1-\gamma)P_{ft}^{*(1-\epsilon)} + \gamma(P_{t-1})^{(1-\epsilon)}$$

Denote

$$\pi_t^* \equiv \frac{P_{ft}^*}{P_{t-1}}$$

$$\pi_t^{1-\epsilon} = (1-\gamma)\pi_t^{*(1-\epsilon)} + \gamma$$

$$\pi_t^* = \frac{\epsilon}{1-\epsilon}\frac{F_t}{Z_t}\pi_t$$

D.2 Output loss from price dispersion

$$Y_{ft} = \left(\frac{P_{ft}}{P_t}\right)^{-\epsilon} Y_t$$

$$Y_{wt} = \int Y_{ft} df = \int \left(\frac{P_{ft}}{P_t}\right)^{-\epsilon} Y_t df = Y_t \int \left(\frac{P_{ft}}{P_t}\right)^{-\epsilon} df$$

$$D_t \equiv \int \left(\frac{P_{ft}}{P_t}\right)^{-\epsilon} df$$

$$D_t = \int_0^{1-\gamma} \left(\frac{P_{ft}^*}{P_t}\right)^{-\epsilon} df + \int_0^{\gamma} \left(\frac{P_{ft-1}}{P_{t-1}}\right)^{-\epsilon} \left(\frac{P_{t-1}}{P_t}\right)^{-\epsilon} df$$

$$D_t = (1-\gamma)\pi^{*-\epsilon}\pi^{\epsilon} + \gamma D_{t-1}\pi^{\epsilon}$$

$$Y_w = DY$$

D.3 Financial Intermediary

The intermediary balance sheet of banks in group i is,

$$Q_{it}S_{it} + L_{it} = N_{it} + B_{it}. \tag{46}$$

Their net worth evolves according to:

$$N_{it+1} = (R_{ik,t+1} - R_t)Q_{it}S_{it} + (R_{iL,t} - R_t)L_{it} + R_t N_{it} \tag{47}$$

Thus,

$$N_{it} = [(R_{ikt} - R_{t-1})d_{it-1}\phi_{it-1} + (R_{iLt-1} - R_{t-1})(1-d_{it-1})\phi_{it-1} + R_{t-1}]N_{it-1}.$$

Denote $z_{it-1,t} = \frac{N_{it}}{N_{it-1}}$ is the gross growth rate of net worth. Financial intermediary's objective is to maximize:

$$V_{it}(N_{it}) = \max \mathbb{E}_t \sum_{j=0}^{\infty} (1-\tau)\tau^j \beta^{j+1} \Lambda_{t,t+1+j}(N_{i,t+1+j}), \tag{48}$$

Thus, it can be written as,

$$V_{it}(N_{it}) = \nu_{kit} Q_{it} S_{it} + \nu_{lit} L_{it} + \eta_{it} N_{it}$$

with

$$\nu_{kit} = \mathbb{E}_t[(1-\tau)\beta\Lambda_{t,t+1}(R_{ik,t+1} - R_t) + \beta\Lambda_{t,t+1}\tau x_{kit,t+1}\nu_{kit+1}],$$

$$\nu_{lit} = \mathbb{E}_t[(1-\tau)\beta\Lambda_{t,t+1}(R_{iLt} - R_t) + \beta\Lambda_{t,t+1}\tau x_{lit,t+1}\nu_{lit+1}],$$

and

$$\eta_{it} \equiv \mathbb{E}_t[(1-\tau)\Lambda_{t,t+1}R_{t+1} + \beta\Lambda_{t,t+1}\tau z_{it,t+1}\eta_{it+1}].$$

$x_{kit,t+j} \equiv \frac{Q_{i,t+j}S_{i,t+j}}{Q_{it}S_{it}}$ and $x_{lit,t+j} \equiv \frac{L_{i,t+j}}{L_{it}}$ are the gross growth rate in assets between t and $t+j$, and $z_{it,t+j} \equiv \frac{N_{it+j}}{N_{it}}$ is the gross growth rate of net worth. ν_{it} is the expected discounted marginal gain of expanding assets by a unit, holding net worth constant. It is an increasing function of the risk premium. η_{it} is the expected discounted value of having an additional unit of net worth, holding asset constant.

Given the binding incentive constraint,

$$V_{it}(N_{it}) \geq \lambda_i(Q_{it}S_{it} + L_{it}),$$

I obtain leverage ratio,

$$\phi_{it} = \frac{\eta_{it}}{\lambda_i - d_{it}\nu_{kit} - (1-d_{it})\nu_{lit}}. \tag{49}$$

Proof of the non-arbitrage condition

$$V_{it}(N_{it}) = (1-\tau)\beta\mathbb{E}(\Lambda_{t,t+1}N_{it+1}) + \tau\beta\mathbb{E}(\Lambda_{t,t+1}V_{it+1})$$

I have,

$$\begin{aligned}[][\nu_{kit}d_{it}\phi_{it} + \nu_{lit}(1-d_{it})\phi_{it} + \eta_{it}]N_{it} &= \\ \beta\Lambda_{t,t+1}[(1-\tau) + \tau(\nu_{kit+1}d_{it+1}\phi_{it+1} + \nu_{lit+1}(1-d_{it+1})\phi_{it+1} + \eta_{it+1})] & \\ \cdot[(R_{ik,t+1} - R_t)d_{it}\phi_{it} + (R_{iL,t} - R_t)(1-d_{it})\phi_{it} + R_t]N_{it} &\end{aligned}$$

Denote $H_{t,t+1} = \beta\Lambda_{t,t+1}[(1-\tau) + \tau(\nu_{kit+1}d_{it+1}\phi_{it+1} + \nu_{lit+1}(1-d_{it+1})\phi_{it+1} + \eta_{it+1})]$. Thus,

$$\begin{aligned}\nu_{kit} &= H_{t,t+1}(R_{ik,t+1} - R_t) \\ \nu_{lit} &= H_{t,t+1}(R_{iLt} - R_t) \\ \eta_{it} &= H_{t,t+1}R_t\end{aligned}$$

Denote h_{it} as the Lagrangian multiplier. The bankers' problem can be written as,

$$\max_{d_{it},\phi_{it}}(1+h_{it})[\nu_{kit}d_{it}\phi_{it} + \nu_{lit}(1-d_{it})\phi_{it} + \eta_{it}] - h_{it}\lambda_{it}[\nu_{kit}d_{it}\phi_{it} + \nu_{lit}(1-d_{it})\phi_{it}]$$

The F.O.C. implies

$$\nu_{kit} = \nu_{lit}$$

Thus,

$$\mathbb{E}_t(H_{t,t+1}R_{ik,t+1}) = \mathbb{E}_t(H_{t,t+1})R_{iLt}$$

D.4 Equilibrium Conditions

Household:
Euler equation:

$$\mathbb{E}_t\beta\Lambda_{t,t+1}R_{t+1} = 1 \tag{50}$$

Stochastic discount rate:

$$\Lambda_{t,t+1} \equiv \frac{\varrho_{t+1}}{\varrho_t}$$

$$\varrho_t = (C_t - hC_{t-1})^{-1} - \beta h\mathbb{E}_t(C_{t+1} - hC_t)^{-1}$$

Labor market equilibrium:

$$\varrho_t w_t = \chi l_t^{\psi} \tag{51}$$

Financial intermediary:
Value of bank's capital

$$\nu_{kit} = \mathbb{E}_t[(1-\tau)\beta\Lambda_{t,t+1}(R_{ik,t+1} - R_t) + \beta\Lambda_{t,t+1}\tau x_{kit,t+1}\nu_{kit+1}]$$

$$\nu_{lit} = \mathbb{E}_t[(1-\tau)\beta\Lambda_{t,t+1}(R_{iLt} - R_t) + \beta\Lambda_{t,t+1}\tau x_{lit,t+1}\nu_{lit+1}]$$

Value of banks' net wealth

$$\eta_{it} = \mathbb{E}_t[(1-\tau)\Lambda_{t,t+1}R_{t+1} + \beta\Lambda_{t,t+1}\tau z_{it,t+1}\eta_{it+1}]$$

Optimal leverage

$$\phi_{it} = \frac{\eta_{it}}{\lambda_i - d_{it}\nu_{kit} - (1 - d_{it})\nu_{lit}}$$

Growth rate of banks' capital

$$z_{it,t+1} = [(R_{ikt} - R_{t-1})d_{it-1}\phi_{it-1} + (R_{iLt-1} - R_{t-1})(1 - d_{it-1})\phi_{it-1} + R_{t-1}]$$

Growth rate of banks' net wealth

$$x_{kit-1,t} = \frac{d_{it}\phi_t}{d_{it-1}\phi_{it-1}} z_{it,t+1}$$

$$x_{lit-1,t} = \frac{(1 - d_{it})\phi_{it}}{(1 - d_{it-1})\phi_{it-1}} z_{it,t+1}$$

Aggregate capital , net worth

$$Q_{it}S_{it} + L_{it} = \phi_{it}N_{it} \tag{52}$$

$$S_{it} = k_{it} \tag{53}$$

$$L_{it} = wl_{it} + \sum(1 - \theta_{ijt})p_{ijt}m_{ijt} - \sum(1 - \theta_{jit})p_{jit}m_{jit} \tag{54}$$

Risky asset share

$$d_{it} = \frac{Q_{it}S_{it}}{Q_{it}S_{it} + L_{it}}$$

Non-arbitrage condition,

$$\nu_{kit} = \nu_{lit}$$

Banks' net worth

$$N_{it} = N_{eit} + N_{nit} \tag{55}$$

Existing banks' net worth accumulation

$$N_{eit} = \tau[(R_{ikt} - R_{t-1})d_{it-1}\phi_{it-1} + (R_{iLt-1} - R_{t-1})(1 - d_{it-1})\phi_{it-1} + R_{t-1}]N_{it-1}e_{Nei} \tag{56}$$

New banks' net worth

$$N_{nit} = \omega(Q_{it}S_{it-1} + L_{it-1}) \tag{57}$$

The return of the capital

$$R_{ik,t+1} = e^{\psi_{t+1}} \frac{u_{it+1} + (1 - \delta)Q_{it+1}}{Q_{it}} \tag{58}$$

Production sector

Intermediate goods production function:

$$m_{it} = z_{it}^{\alpha_i} l_{it}^{\alpha_i} (e^{\psi_t} k_{it-1})^{\beta_i} \left(\Pi_{i=1}^{n} m_{ij,t}^{\omega_{ij}}\right)^{1-\alpha_i-\beta_i} \tag{59}$$

Labor demand

$$w_t l_{it} R_{iLt} = \alpha_i p_{i,t} m_{it} \tag{60}$$

Capital demand

$$u_{it} e^{\psi_t} k_{it-1} = \beta_i p_{i,t} m_{it}$$

Intermediate goods demand

$$[[(1-\theta_{ijt})R_{iLt} + \theta_{ijt}] + \varsigma(\theta_{ijt} - \bar{\theta}_i)^2] p_{ij,t} m_{ij,t} = \gamma_i \omega_{ij} p_{i,t} m_{it} \tag{61}$$

$$p_{ijt} = p_{jt}/[(1-\theta_{ijt})R_{jLt} + \theta_{ijt}] \tag{62}$$

Trade credit:

$$2\varsigma(\theta_{ijt} - \bar{\theta}_i)[(1-\theta_{ijt})R_{jLt} + \theta_{ijt}] + (R_{jLt} - 1)\varsigma(\theta_{ijt} - \bar{\theta}_i)^2 = (R_{iLt} - R_{jLt}) \tag{63}$$

Wholesaler:

$$Y_w = \Pi_{i=1}^{n} \zeta_i^{-\zeta_i} y_i^{\zeta_i} \tag{64}$$

$$y_i = \zeta_i \left(\frac{p_i}{P_w}\right)^{-1} Y_w \tag{65}$$

Capital goods producer

$$Q_{it} = 1 + \frac{1}{2}\eta_I(\frac{I_{it}}{I_{it-1}} - 1)^2 + \frac{I_{it}}{I_{it-1}}\eta_I(\frac{I_{it}}{I_{it-1}} - 1) - \mathbb{E}_t[\beta\Lambda_{t+1}(\frac{I_{it+1}}{I_{it}})^2\eta_I(\frac{I_{it+1}}{I_{it}} - 1)] \tag{66}$$

Retailer:

$$Y = Y_w D \tag{67}$$

where

$$D_t = \left\{(1-\gamma)\left(\frac{\epsilon}{\epsilon-1}\frac{F_t}{Z_t}\right)^{-\epsilon}\pi^{\epsilon} + \gamma D_{t-1}\pi^{\epsilon}\right\},$$

$$F_t = Y_t P_{mt} + \beta\gamma\Lambda_{t,t+1}\pi^{\epsilon} F_{t+1}$$

$$Z_t = Y_t + \beta\gamma\Lambda_{t,t+1}\pi^{\epsilon-1} Z_{t+1}$$

Price index

$$1 = \gamma\pi^{\epsilon-1} + (1-\gamma)\left(\frac{\epsilon}{\epsilon-1}\frac{F_t}{Z_t}\right)^{1-\epsilon} \tag{68}$$

Capital accumulation equation

$$k_{it} = e^{\psi_t}(1-\delta)k_{it-1} + I_{it} \tag{69}$$

Goods market clearing:

$$\sum_{j=1}^{n} m_{ij} + y_i = m_i \tag{70}$$

$$Y_t = C_t + G_t + \sum_i [I_{it} + \frac{\eta_I}{2}\left(\frac{I_{it}}{I_{it-1}} - 1\right)^2 I_{it}] \tag{71}$$

Labor market clearing:

$$\sum_{i=1}^{N} l_{it} = l_t \tag{72}$$

Fisher equation

$$i_t = R_t \mathbb{E}_t(\pi_{t+1}) \tag{73}$$

Markup:

$$X = 1/P_w$$

Interest rate rule:

$$i_t = i_{t-1}^{\rho_i}\left(\frac{1}{\beta}\pi_t^{\kappa_\pi}\left(\frac{X}{\frac{\epsilon}{\epsilon-1}}\right)^{\kappa_y}\right)^{(1-\rho_i)} e_i \tag{74}$$

Government expenditure:

$$G_t = \bar{G}e_g \tag{75}$$

D.5 Calibration of labor share and capital share

α_i and β_i are calibrated using the BEA GDP by Industry Value-added Components Table (1998-2013) and the calibration method follows Su (2014).

Labor share = Compensation of employees $+1/2$ of the noncorporate part of other gross operating surplus.

Capital share = Gross operating surplus + taxes on production and imports less subsidies $-1/2$ of the noncorporate part of other gross operating surplus.

D.6 Propagation of financial shocks

Table 11: UP vs. DOWN - model simulation

$\tilde{m}_{it}$	one quarter	6-month	3-quarter
$\text{UP}_{i,t=0}$	-0.1895***	-0.1979***	-0.1970***
$\text{DOWN}_{i,t=0}$	-0.0894***	-0.1293 ***	-0.1818***
$\text{Shock}_{i,t=0}$	-0.3961***	-0.4360***	-0.4968***

Notes: *** indicate statistical significance at the 1% levels. The dependent variables are the first quarter output change $\tilde{m}_{i,t=1q}$, the first 6-month output change $\tilde{m}_{i,t=6m}$ and the first year output change $\tilde{m}_{i,t=1y}$ respectively. $\text{Shock}_{i,t=0}$ is the borrowing cost R_{iL} change upon the impact of ϵ_{Ne5}. The constructions of $\text{UP}_{i,t=0}$ and $\text{DOWN}_{i,t=0}$ follows Equations 11 and 12.