Exchange Rates, Risk Premia, and Inflation Indexed Bond Yields

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I. Introduction

Drawing on and extending Clarida (2012; 2013) and Luo (2013) this chapter derives and empirically estimates a structural relationship between the nominal exchange rate, national price levels, and observed yields on long maturity inflation - indexed bonds. This relationship can be interpreted as defining the *fair value* of the exchange rate that will prevail in any model or real world economy in which inflation indexed bonds are traded. *Fair value* is the level of the nominal exchange rate that equates the known real return to holding a long maturity home currency inflation indexed bond to the expected real return to holding an inflation indexed bond payable in foreign currency. We derive a novel, empirically observable real time measure of the *risk premium* that can open up a wedge between the observed level of the nominal exchange rate and its fair value and we relate our measure of the long horizon real risk premium to the Fama (1984) measure of the short horizon nominal risk premium.

We take our theory to a daily data set spanning the period January 2001 – February 2011 and study high frequency, real time decompositions of pound, euro, and yen exchange rates into their fair value and risk premium components. The relative importance of these two factors varies depending on the sub sample studied. However, sub samples in which we find correlations of 0.30 to 0.60 between daily exchange rate changes and

daily changes in fair value are not uncommon. We also show that on average in our data set, for all three exchange rates studied, a 1 percentage point *rise* in the foreign currency risk premium is contemporaneously associated with a roughly 50 basis point *rise* in the inflation indexed bond return differential in favor of the foreign country and an 50 basis point *appreciation* of the dollar. That is, the dollar tends to *strengthen o*n a day when our measure of *the foreign currency risk premium* on the pound, euro, or yen *rises*. This implies that our measure of the *level* of the risk premium should help forecast subsequent *changes* in the nominal exchange rate and present empirical evidence of this forecastability is present in our sample. In particular we show that indeed, a rise in foreign currency risk premium today help to forecast subsequent dollar depreciation.

II. An Equilibrium Implication

Consider a US investor who, among the many assets he can hold, can hold inflation indexed sovereign bonds denominated in dollars and pounds. Let ρ_t denote the dollar price a zero coupon inflation indexed bond that pays off *1* dollar in *n* years multiplied by cumulative realized US inflation over the next *n* years. The realized nominal gross return on this investment, if held to maturity, will be

1) $R^{hh}_{t,n} = (1/\ \rho_t)P_{t+n}/P_t$

where P_t is the CPI. Let ρ^*_t denote the pound price of a zero coupon inflation indexed bond that pays off 1 pound in n years multiplied by cumulative realized UK inflation over the next n years. The realized nominal gross return to a US investor on this investment, if held to maturity, will be

2) $R^{h,f}_{t,n} = (1/S_t\rho^*_t)S_{t+n}\ P^*_{t+n}/P^*_t$

where S_t is the dollar price of a pound. The realized *real* gross return on holding a US inflation indexed bond to maturity is

3) $RR^{hh}_{t,n} = R^{hh}_{t,n}\ (P_t/P_{t+n}) = (1/\ \rho_t) \equiv exp(nr_{t,n})$

Thus, of course, the US inflation indexed bond offers a known, non-stochastic realized real return $r_{t,n}$ if held to maturity. But what is the realized real return to a US investor of holding a UK inflation indexed bond?

4) $\quad RR^{hf}_{t,n}=R^{hf}_{t,n}\ (P_t/P_{t+n}) = \{(1/S_t\ \rho^*_t)\ (S_{t+n}\ P^*_{t+n}/P^*_t)\}\ P_t/P_{t+n} = \ exp(nr^*_{t,n})Q_{t+n}/Q_t$

where $Q_t = \ S_t\ P^*_t/P_t$ is the CPI real exchange rate. For a US investor the UK inflation indexed bond is not riskless if held to maturity: even though $r^*_{t,n}$ is known, the US investors bears real exchange rate risk. Notice that this is not an assumption: it follows from the nature of the stochastic nominal cash flows to a US investor for holding a UK linker to maturity.

Define $exp\ \theta_{t,n}$ as the ratio of expected real return to a US investor of holding a UK linker versus known real return to a US investor of holding a TIP

5) $\quad exp\ \theta_{t,n} \equiv E_t\ RR^{hf}_{t,n}/RR^{hh}_{t,n} \ = (exp\ nr^*_{t,n}\ /\ exp\ nr_{t,n})\ E_t\ Q_{t+n}/Q_t$

where $E_t\ Q_{t+n}$ is the expected level of the CPI real exchange rate in n years. When $\theta_{t,n} > (<)\ 0$ the expected real return to holding the UK linker exceeds (is less than) the known real return to holding the US TIP. Equation (5) has a particularly convenient interpretation when the horizon n is long enough so that $E_t\ Q_{t+n} = Q$, the assumed constant unconditional mean of the real exchange rate defined by long run relative PPP. In this case, multiplying through by Q_t and re arranging we see that:

6) $\quad S_t = (P_t/P^*_t\ )\ exp\ n(r^*_{t,n} - r_{t,n})\ Q\ exp(-\theta_{t,n})$

or

7) $\quad S_t = \tilde{S}_t\ exp - \theta_{t,n}$

Thus period by period, the observed spot exchange rate is the product of the *fair value* $\tilde{S}_t$ - the level of the spot exchange rate that equates the known real return to investing in the US TIP to the expected real return

to investing in the UK linker - and the *risk premium exp(-θₜ,ₙ)*, the ratio of the known real return to investing in the US TIP to the expected return to investing in the UK linker (the reciprocal of the risk premium on the uncovered UK linker investment). Taking logs of both sides and letting *q = ln Q*

8) $\quad s_t = p_t - p^*_t + n(r^*_{t,n} - r_{t,n}) + q - \vartheta_{t,n}$

We see that in first differences we must have

$$\Delta s_t = \Delta(p_t - p^*_t) + n\Delta(r^*_{t,n} - r_{t,n}) - \Delta\vartheta_{t,n}$$

A rise in the endogenous θₜ,ₙ is an increase in the risk premium on a UK investment which is just an increase the expected excess return a US investor earns on a UK linker investment compared with the known real return he earns on a US TIP. Thus from (6) a rise in expected real return on the UK linker investment must be brought about by some combination of *a rise in UK – US real interest differential and a contemporaneous real appreciation of the dollar relative to the pound*. An appreciation of the dollar relative to the pound *today* contributes to a positive expected *future* excess return on an inflation hedged pound investment because it sets up the expectation of a subsequent (real) depreciation of the dollar relative to the pound as is evident from (6).

When $S_t = \widetilde{S}_t$ the known real return to a US investor in investing in the US TIP is equal to the expected real return to the US investor in the UK linker. It is this sense in which $\widetilde{S}_t$ is the fair value of the nominal exchange rate. When the exchange rate is equal to fair value by this definition, expected real returns that can be achieved by holding inflation indexed bonds are equalized across countries. Thus our notion of fair value is related to the level of the exchange rate implied by the ex-ante real version of uncovered interest parity over a short holding period for nominal bonds. However as we illustrate below, the two are distinct ways to assess currency valuation.

An Asset Pricing Interpretation

We make a minimal number of assumptions to provide an asset pricing interpretation of Equation (7). We do not assume complete markets or a representative agent. We do not assume that we know the model, let alone the parameters, that link the present value of macro fundamentals to exchange rate valuation. Under our assumptions our framework is consistent with almost any underlying model in which inflation indexed bonds are traded. We assume that, in a global financial equilibrium, there is a functional relationship between the nominal (US dollar) price today of an asset that delivers a random dollar cash flow at some date in future (for concreteness, 10 years hence) and no cash flow at any date other than t+n:

$$\rho_t = F_t(N_{t+n}; \Omega_{t,t+n})$$

where $\Omega_{t,t+n}$ is the conditional probability distribution of the random nominal cash flow from the asset that pays off in n years. We specialize F so that

$$\rho_t = E_t(m_{t,n}N_{t+n}; \Omega_{t,t+n})$$

So today's price of an asset with random nominal cash flow in n years is the conditional expectation of the product of that cash flow and the random variable $m_{t,n}$.

Assumption: $m_{t,n}$ is homogenous in the price levels P_t and P_{t+n}

$$m_{t,n} = z_{t,n}\frac{P_t}{P_{t+n}}$$

This is a standard property in many asset pricing models (Cochrane (2001). For example in Lucas (1982) we have

$$m_{t,n} = \beta^n \frac{U'(C_{t+n})}{U'(C_t)} \frac{P_t}{P_{t+n}}$$

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Again, we do not require a representative agent, complete markets, or really any additional structure on $z_{t,n}$. This is an intuitive restriction on nominal asset prices that says that the *real* price of the asset today depends upon the *real* value of the cash flow it delivers state by state at maturity and not the price level itself at t+n itself (after, of course, controlling for factors other than the price level itself that are included in $z_{t,n}$.).

With this background, consider how to price a zero coupon inflation indexed bond that pays off 1 dollar in n years multiplied by cumulative realized inflation over the next n years.

$$\rho_t = E_t(m_{t,n} \cdot 1 \cdot \frac{P_{t+n}}{P_t}) = E_t(z_{t,n} \cdot 1)$$

Or, dividing by ρ_t

$$1 = \{\exp nr_{t,n}\} E_t(z_{t,n})$$

Where $r_{t,n}$ is the continuous compounded known real return on the inflation indexed bond.

US investors can also obtain US dollar cash flows by investing in a UK inflation indexed bond and selling the pound proceeds for dollars in n years. Let S_t be the dollar price of a pound and * represent a UK variable. Let $Q_t = S_t P^*_t / P_t$ define the real exchange rate and Q its unconditional mean $Q = E_t Q_\infty$. Then we have

$$S_t \rho^*_t = E_t(m_{t,n} \cdot 1 \cdot S_{t+n} \cdot \frac{P^*_{t+n}}{P^*_t})$$

Or dividing though:

$$1 = \exp nr*_{t,n} E_t(z_{t,n} \cdot 1 \cdot \frac{Q_{t+n}}{Q_t})$$

With these building blocks we now derive a structural exchange rate equation that will hold in any model that seeks to describe a world in which long maturity inflation indexed bonds trade. Since such bonds trade in many countries (US, UK, France, Canada, Japan) this should apply to a large number of models. We see that

$$9) \quad S_t = \frac{P_t}{P*_t} \frac{\exp nr*_{t,n}}{\exp nr_{t,n}} \frac{Q E_t(z_{t,n} \cdot 1 \cdot \frac{Q_{t+n}}{Q})}{E_t(z_{t,n})}$$

Comparing with equation (6) we see that $\theta_{t,n}$ is given by

$$10) \quad \exp - \vartheta_{t,n} = \frac{E_t(z_{t,n} \cdot 1 \cdot \frac{Q_{t+n}}{Q})}{E_t(z_{t,n})}$$

Although not necessary for what follows, we gain additional insight by looking at the log normal case in which we have equation (11).

$$11) \quad -\vartheta_t = \operatorname{cov}_{t,n}(\ln z_{t,n}, q_{t+n} - q) + \operatorname{var}_{t,n}(q_{t+n} - q) + E_t(q_{t+n} - q)$$

We note that the first term in the above expression is the conditional covariance between the stochastic discount factor and real exchange rate that prevails when the zero coupon inflation linked bonds mature. This can be interpreted as a risk premium that opens up a wedge between known real return (to a US investor) of holding a long maturity TIP and the stochastic real return to a US investor of holding a UK linker. When this covariance is negative, an unhedged position in a UK linker pays off less (because of realized real appreciation of dollar relative to the pound) when the stochastic discount factor is high. Thus a *positive theta* corresponds to a *positive risk premium on the UK linker*. That is, the known

real return on the US linker is less than the expected real return to the US investor, inclusive of expected appreciation of the pound, of holding a UK linker when θ is positive. An increase in the expected excess return on the UK linker will require some combination of an increase in $r^*_{t,n} - r_{t,n}$ and a jump appreciation of the dollar.

In what follows we shall assume for ease of exposition that expected deviations from PPP at a 10 year horizon are sufficiently close to zero so as to be ignored. Importantly, however, researchers who have a view on long horizon PPP deviations can include that view directly and use it as an input to the accounting framework we develop below. Thus, in what follows, we shall refer to $\theta_{t,n}$ as the risk premium.

It is worth noting that the complete markets assumption which we don't require to derive (7) or (9) would put a number of additional restrictions on the joint behavior of exchange rates and bond yields, both inflation indexed and nominal. For example, under complete markets, Backus et. al. (2001) show that

12) $\quad \dfrac{S_{t+1}}{S_t} = \dfrac{m_{t,1}}{m^*_{t,1}}$

We see that in our notation this would also imply

13) $\quad \dfrac{Q_{t+1}}{Q_t} = \dfrac{z_{t,1}}{z^*_{t,1}}$

These are elegant, powerful implications but we do not impose them on the data or use them to interpret real time exchange rate fluctuations.

III. Comparison with the Literature

Building on earlier work by Hansen and Hodrick (1980) and Cumby and Obstfeld (1981), Fama (1984) is the classic study of the risk premium to holding a long position in a foreign currency nominal bond for one period (but see also Clarida, Davis, and Pedersen (2009) for a recent analysis of

what can – and can't be leaned – from a Fama regression). In the literature (see Engel 2010 for a review and extension) this concept of the risk premium is usually defined by the first order log approximation

14) $\quad rp_{t,1} = E_t s_{t+1} - s_t + i*_{t,1} - i_{t,1}$

where lower case i denotes the short term nominal interest rate. There is of course a long and proud tradition in the international finance literature, beginning with Frankel (1978), of empirically relating real exchange rates to real interest differentials (Shafer and Loopesko (1983); Campbell and Clarida (1987); Clarida – Gali (1994) are early examples). For the most part, this literature pre dates the widespread introduction of long maturity inflation indexed bonds and of necessity solves forward the real version of the deviations from UIP equation.

15) $\quad rp_{t,1} = E_t q_{t+1} - q_t + er*_{t,1} - er_{t,1}$

where $er_{t,1} = i_{t,1} - E_t\pi_{t,1}$ is the ex ante short term real interest rate on nominal bonds at home and similarly abroad. Solving forward and assuming $\ln Q_t$ is stationary we obtain (see Engle (2010) for a lucid discussion and Brunermeier, Nagel, and Pedersen (2008) for an interpretation of the forward solution for the nominal exchange rate under uncovered interest parity):

16) $\quad q_t = \sum_{i=0}^{\infty}(_t er*_{t+i,1} - _t er_{t+i,1} - \mu) - \sum_{i=0}^{\infty}(_t rp_{t+i,1} - \lambda) + E \ln Q_{\infty}$

Note that convergence of these non-discounted present value equation requires the unconditional mean of the ex-ante real rate differential μ to equal the mean of the Fama risk premium λ. Comparing terms we see

17) $\quad \vartheta_{t,n} = \sum_{i=0}^{n} {}_t rp_{t=I,1} - \lambda + \{n(r*_{t,n} - r_{t,n}) - \sum_{i=n}^{n}(_t er*_{t+i,1} - _t er_{t+i,1} - \mu)\} + q - E \ln Q_{t+n}$

where for concreteness we suppose that

18) $\quad \sum_{i=n}^{\infty}(_t er*_{t+i,1} - _t er_{t+i,1} - \mu) = 0$

and similarly for the Fama premium after n periods. We see that our measure of the risk premium $\theta_{t,n}$ is the sum of three terms: the n period sum of the expected Fama risk premiums; the difference (in brackets) between home and foreign long maturity inflation indexed bond returns and the expected real returns to rolling one period nominal bonds; and a Jensen's inequality term.

IV. Data

Our data set is comprised of daily observations on spot exchange rates, inflation indexed bond yields, and monthly observations on consumer price indexes for the US, UK, and Euro area for the period January 2001 through January 2011 and for Japan since December 2004 shortly after inflation indexed bonds were introduced. We convert monthly CPI levels to daily observations via interpolation. Given the low and relative stable rate of inflation for these countries over this period, the approximation of the unobserved daily inflation differential with the observed per day monthly average inflation differential introduces measurement error, but this error is small relative to the observed daily volatility of exchange rates and inflation indexed bond yields.

Our theoretical model is derived in terms of the yields on inflation indexed zero coupon bonds. Inflation indexed bonds are typically issued in coupon form. However, in the US there is a market in which inflation indexed coupon Tips are stripped of their coupons and trade in zero coupon form. In our empirical analysis we will use daily data on constant 10 years to maturity yields on zero coupon Tips provided by Barclays. For the UK, we use the data on zero coupon linker yields provided by the Bank of England. For the Euro, we use estimates of the zero coupon inflation indexed yield curve for French and German inflation indexed bonds provided by Morgan Stanley. For Japan, no data on zero coupon inflation indexed yields could be found so we use the observed yield on coupon bearing inflation indexed bonds.

One final point to discuss is how we calibrate the constant term in Equation (6) for fair value. This constant term is not important for much of what we do since we will often seek to account for percent *changes* in observed nominal exchange rates in terms of percent *changes* in fair value and changes in the risk premium. For these exercises, the constant drops out. However, in drawing the some of the graphs we will wish to preserve the levels information, and will select the constant term equal to the average real exchange rate during the sample period depicted. In Section VI we relax this assumption that the expectation at each date of the real exchange rate 10 years forward from that date is constant. The net effect of relaxing this assumption is to increase somewhat the contribution of fair value in accounting for nominal exchange rate fluctuations.

V. Empirical Results

We now use the framework developed above to interpret the behavior of the Euro, Pound, and Yen exchange rates over the past 10 years. There are no econometric estimates to present because our framework (Equation 7) provides day by day a real time decomposition the change in the exchange into the change in the fair value and the change in risk premium. Our framework allows – indeed we expect to find – periods in which shocks to the risk premium are large and die out slowly while there may be other periods in which exchange rate movements, contrary to the original Meese-Rogoff (1983) finding that exchange rate changes are difficult to explain even given ex post realizations of fundamentals, are well accounted for by shifts in our measure of fair value derived above.

We present our main findings in a series of charts. For each exchange rate, the charts will help us to identify as well as quantify the importance of shocks to fair value and shocks to the risk premiums in accounting for exchange rate fluctuations over different periods as well as over various horizons of interest. As our sample includes the global financial crisis and its aftermath (at least through January 2011!), we are particularly interested to determine and quantify the shifts in risk premium and fair value that occurred over this period.

Euro
Chart 1
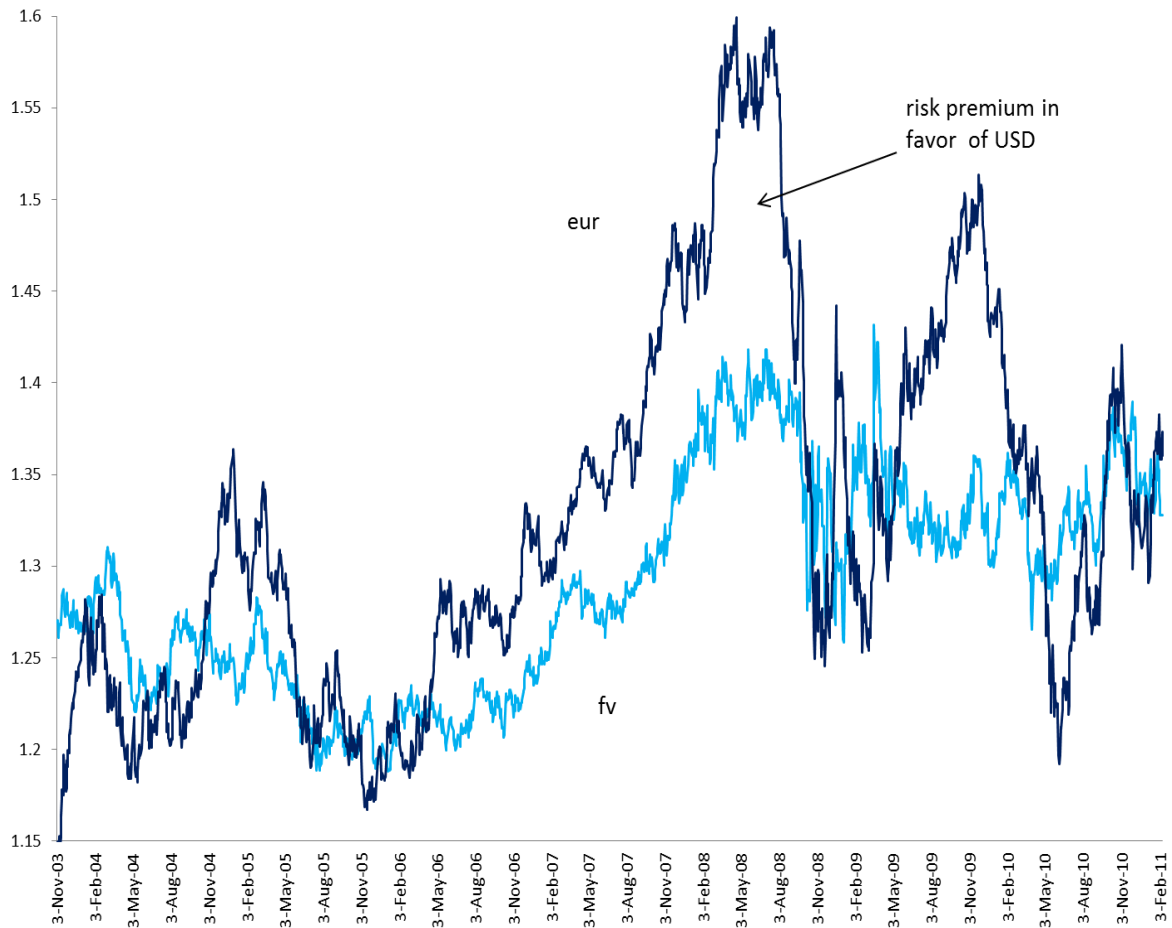
In Chart 1, and in all subsequent charts, the dark blue line depicts the spot exchange rate, in this case the US dollar price of a Euro, the aqua – blue line is the fair value defined by Equation 11.   The amount by which the exchange rate EUR exceeds FV measures the risk premium *in favor of the USD* that is reflected in the EUR spot exchange rate. This corresponds to $-\theta_{t,n}$. The amount by which the exchange rate EUR falls short of FV measures the risk premium *in favor of the EUR* that is reflected in the EUR spot exchange rate.   This corresponds to $\theta_{t,n}$.

Our framework we believe provides a compelling qualitative as well as a plausible quantitative account of the swings in Euro exchange rate since 2005. As can be seen from the chart, the broad move in the Euro from 1.25 in the summer of 2005 to 1.45 in the spring of 2008 is well

accounted for, both in direction and in magnitude, by the rise in the fair value during that period. According to our model, the next move in the Euro from 1.45 to the 'brutal' level of 1.60 reached in the summer of 2008 was due almost entirely an equal move in the risk premium, in favor of the dollar and thus against the Euro.

Since the onset the global financial crisis in September 2008, movements in the Euro have been dominated by fluctuations in risk premium with fair value fluctuating in a rather narrow range centered at roughly 1.37. In October 2008, our measure of the risk premium swings in favor of the Euro (e.g. it appreciated the dollar price of the Euro to such an extent it set up the expectation of a depreciation and thus capital gain on a Euro investment). The risk premium swings back in favor of the dollar in the second half of 2009 as the dollar depreciates in tandem with the Fed's quantitative easing programs announced in March of that year. Since 2010, our framework indicates that the foreign exchange market has required a positive risk premium to hold the Euro. This period of course coincides the crisis in the Euro periphery.

Of course, it is important to confirm that the visual impression conveyed by the chart is evident in the actual empirical correlation between the Euro exchange rate and our measure of fair value.

Chart 2: Correlation in Daily Changes in Eur and *FV* (60 day window)
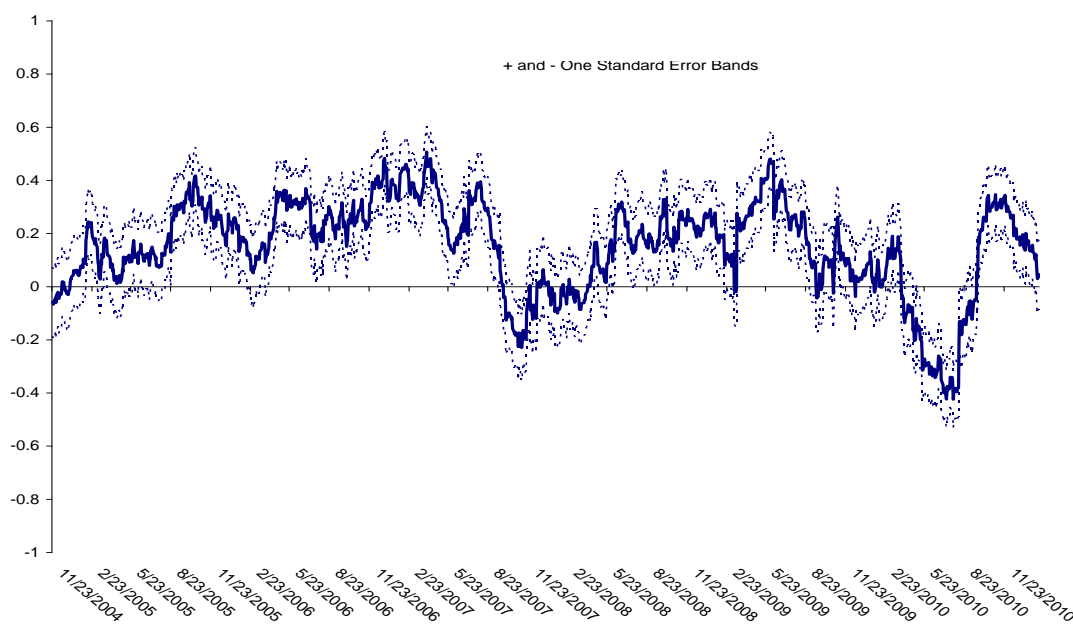
Chart 2 depicts the correlation (over rolling 60 day windows) between daily changes in Euro exchange rates and daily changes in our measure of fair value which of course is dominated by daily changes in real interest rate differentials between Europe and US inflation indexed bonds. We see that periods in which the correlation is in the range of 0.3 to 0.4 are not uncommon. We also see that in periods in which shocks to the risk premium are seen to dominate, the correlation between the Euro and *fv* falls to zero or is even negative. One is tempted to identify periods in which the exchange rate is well accounted for by movements in *fv* (such as 2005 to 2008 in Chart 1) as periods in which 'fundamentals' mostly matter for exchange rate determination, in contrast to periods since September 2008 in which 'fundamental' are pushed aside and 'risk aversion' appears to take over. But within the strict logic of our framework, this temptation would not be justified. Fundamentals may drive the risk premium as well, but without imposing much more additional structure on $m_{t,n}$ we can't really say more. However, unlike the traditional approach (Fama (1984)) in which an unobserved currency risk premium must be inferred by extracting the forecastable component from realized returns on currency carry trades, our framework provides an econometric free measure of the relevant risk premium given observed yields on inflation indexed bonds and the spot exchange rate.
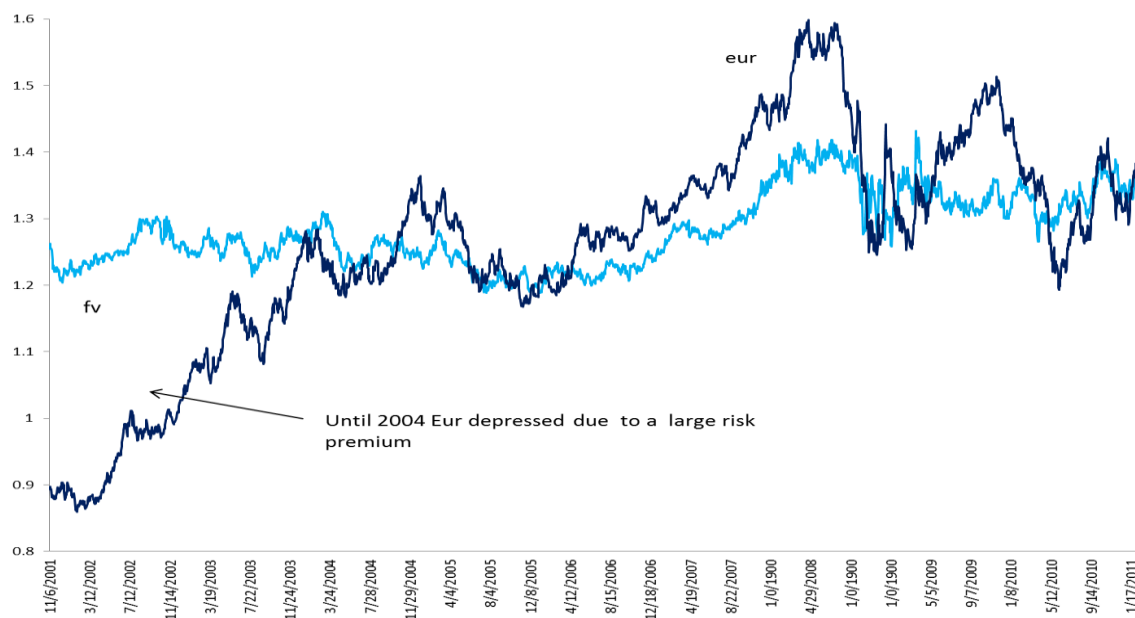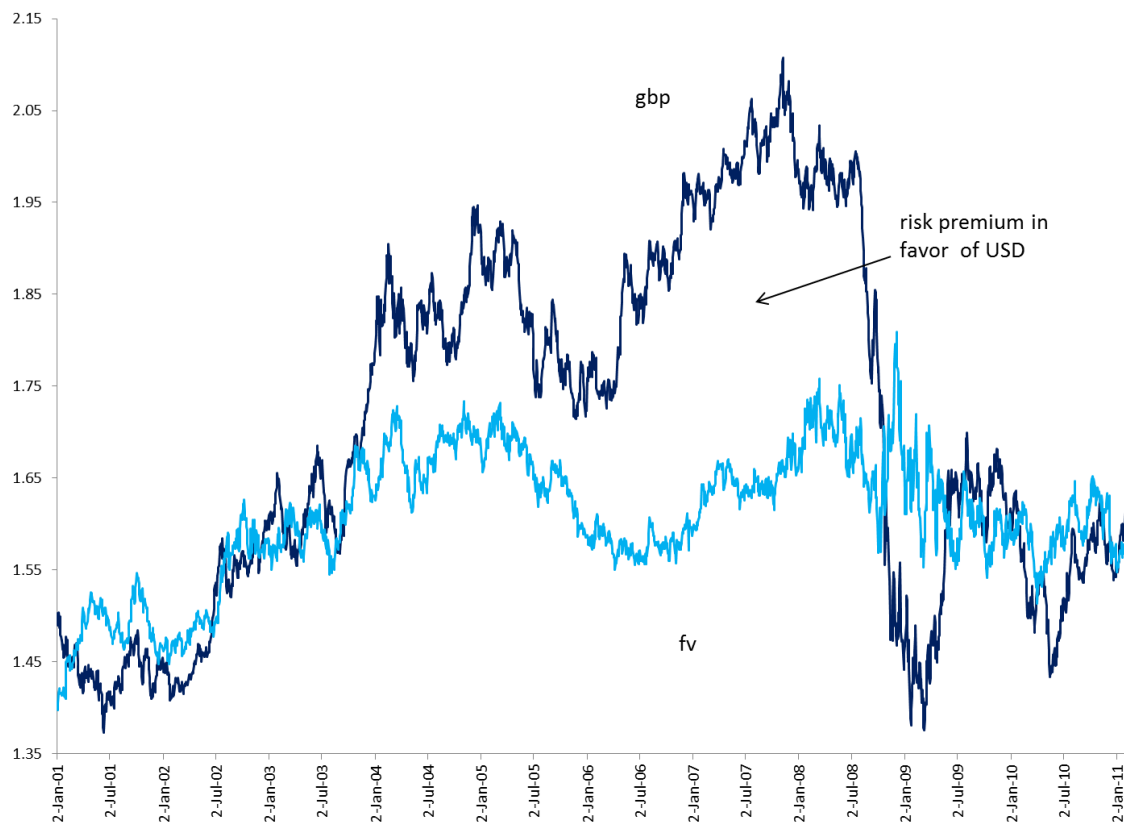
Chart 3

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Pound

Chart 4

Chart 4 depicts our decomposition of the GBP exchange rate into is fair value and risk premium components. From 2001 through summer of 2005, the appreciation of the pound from 1.50 to 1.75 is almost fully accounted for by an equal rise in our estimate of fair value from the inflation indexed bond market. However, our framework accounts for the subsequent move up from 1.75 to 2.05 reached in January 2008 almost entirely by the emergence of a substantial risk premium in favor of the dollar (i.e. a risk premium that set up expectation of a higher return on a US inflation linked bonds). This risk premium is eliminated and shifts in favor of the GBP in September 2008 and has remained in place since. Since 2009, our estimate of *fair value* has stayed in a narrow range centered around 1.65.

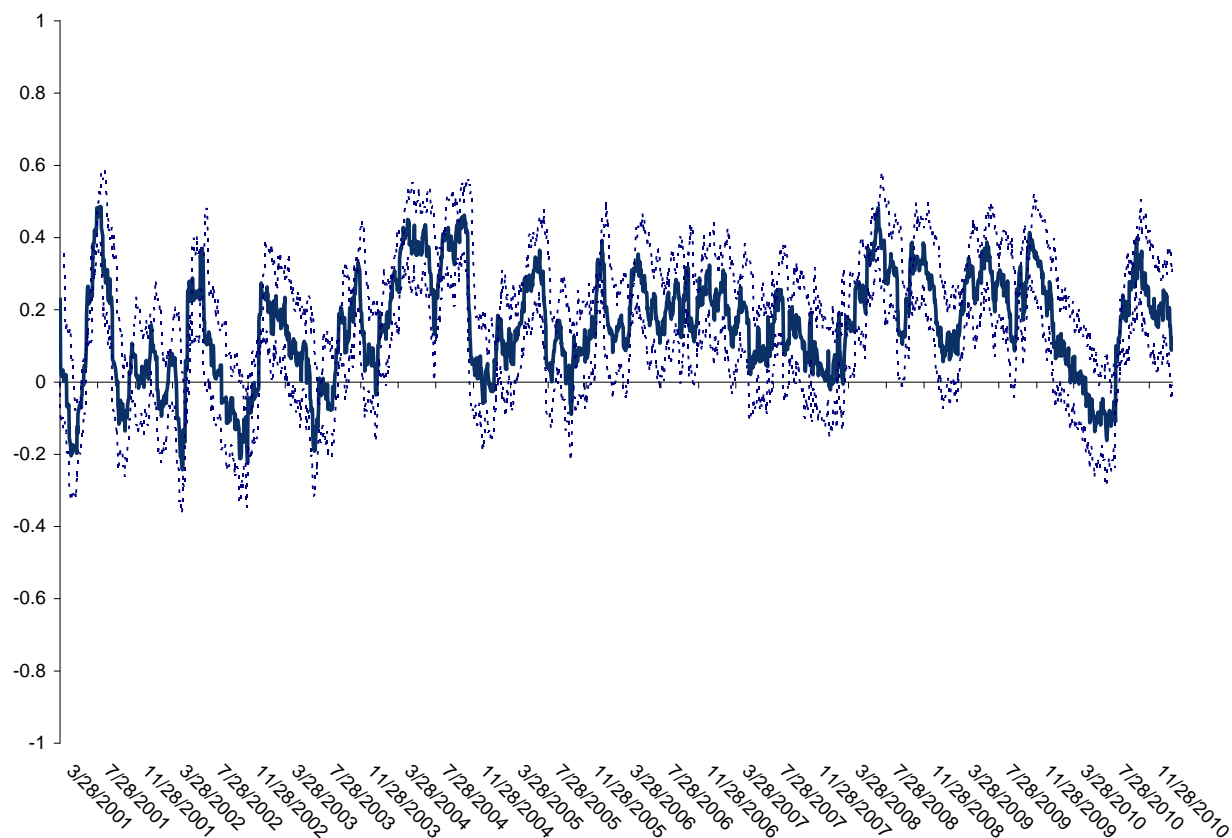Chart 5: Correlation in Daily Changes in GBP and *FV* (60 day window)

Chart 5 depicts the correlation (over rolling 60 day windows) between daily changes in GBP exchange rates and daily changes in our measure of fair value. Again we see that periods in which the correlation between daily changes is in the range of 0.3 to 0.5 are not uncommon. We also see that in periods in which shocks to the risk premium are seen to dominate, the correlation between the GBP and *fv* falls to zero or is even negative. This implies that large shocks to the risk premium in favor of the pound (or in Chart 1 the Euro) tend to require both depreciations of the exchange rate relative to the dollar – to set up the expectation of future appreciation - as well as a rise in the real interest rate differential in favor of the pound (or the Euro).
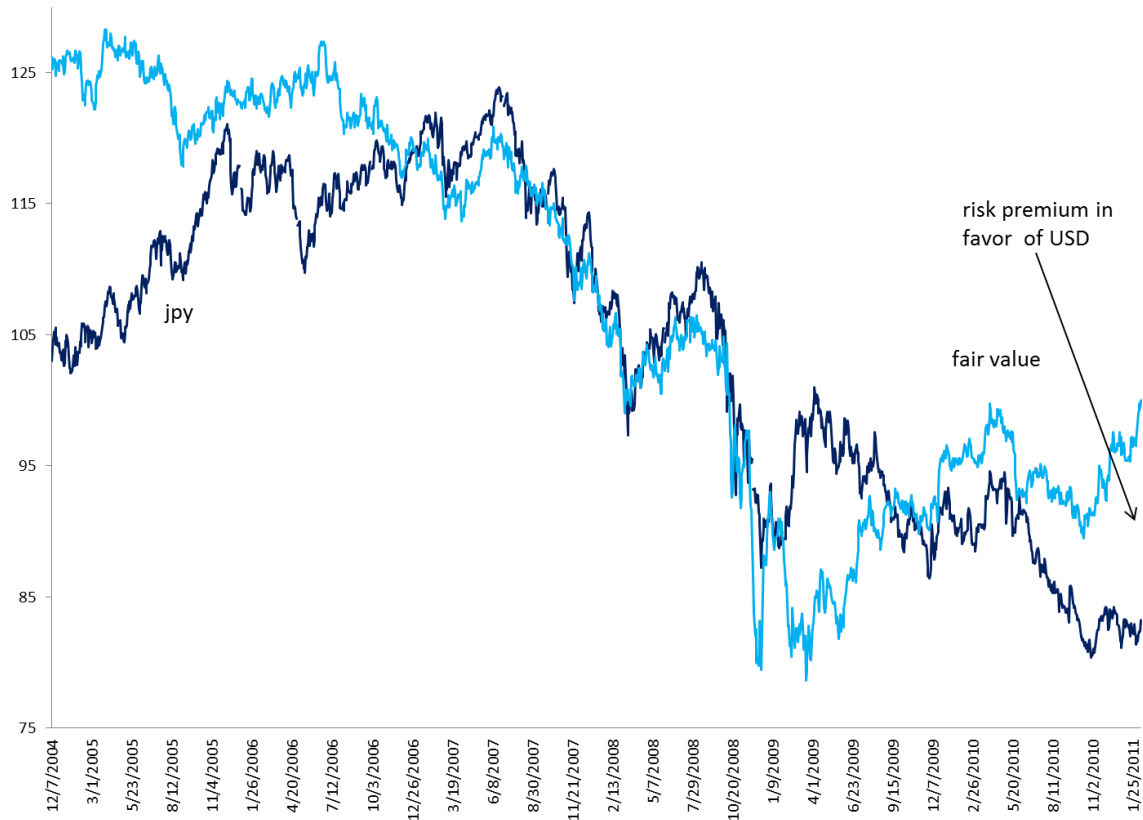
Yen

Chart 6

Chart 6 depicts our decomposition of the JPY exchange rate into is fair value and risk premium components. From 2005 through summer of 2010, the appreciation of the yen from 120 to 88 is almost fully accounted for by an equal shift in our estimate of fair value. During most of this period there was also a modest and not very volatile risk premium in favor of the yen. This risk premium widened in the fall of 2008 but was almost entirely eliminated by the summer of 2009. Since that time, we estimate that a risk premium in favor of the dollar opened up as the yen continued to appreciate notwithstanding a shift in fair value in the direction of a weaker yen. Our last data point is February 11, 2011. Finally Chart 7 confirms that, if anything, changes in the yen and our measure of fair value have been more highly correlated than we found for the Euro and the pound.

Chart 7: Correlation in Daily Changes in JPY and *FV* (60 day window)
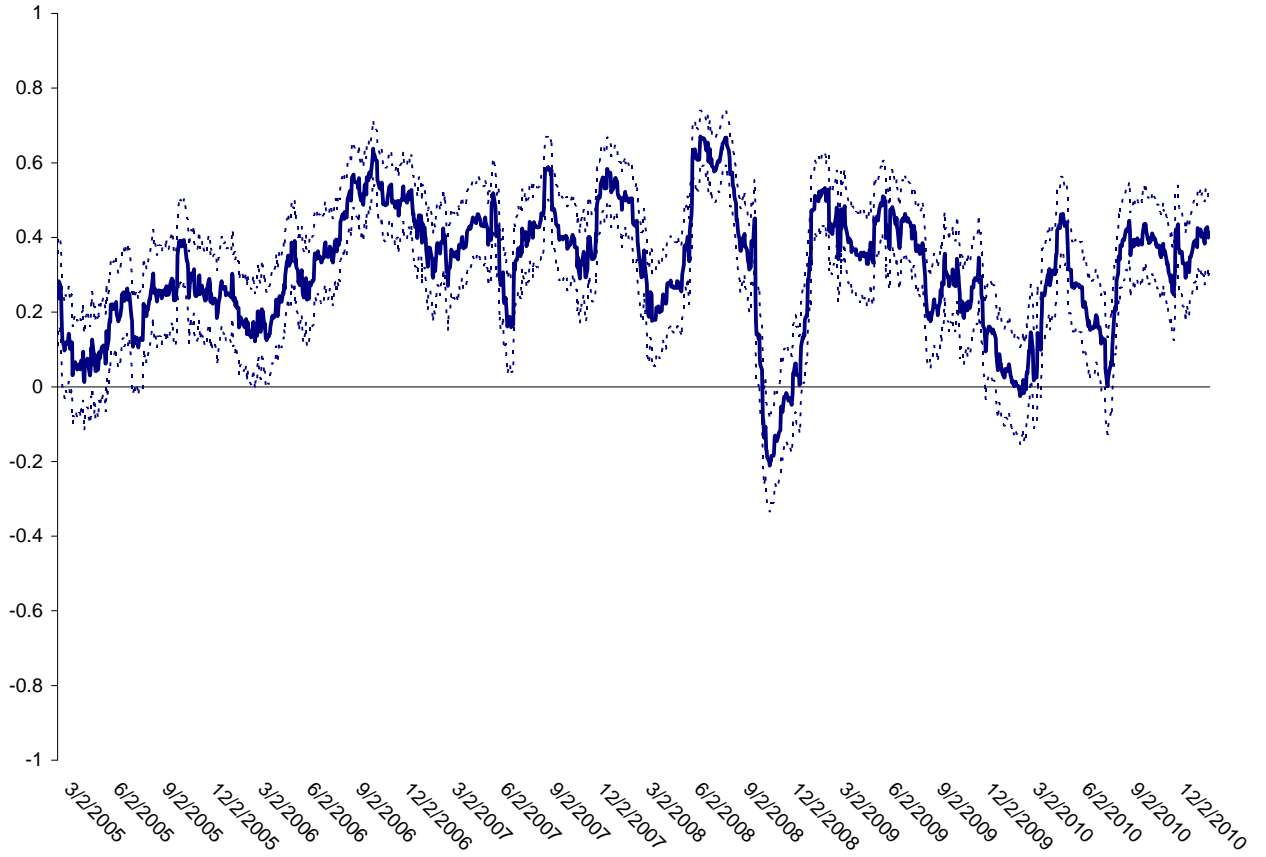
VI. Risk Premium Estimates Conditional on the Half Life of PPP Deviations

      Up until now, we have assumed that expected deviations of the real exchange rate from relative PPP at horizon of 10 years are equal to 0. Empirically some researchers (Rogoff (1996)) find slower speeds of adjustment – for example estimating half lives of 3 to 5 years. In this section, we estimate the time varying risk premium conditional on assumption of a half life of five years.

      Following section II, we re-arrange Equation 5 as,

19) $\exp(\tilde{\theta}_{t,n}) = \dfrac{exp(nr^*_{t,n})}{exp(nr_{t,n})} \mathrm{E}_t \left(\dfrac{Q_{t+n}}{exp\ (\mathrm{E}(lnQ))} \right) \left(\dfrac{exp\ (\mathrm{E}(lnQ))}{Q_t} \right)$

where $\mathrm{E}(lnQ)$ is the expected constant unconditional mean of the real exchange rate defined by long run relative PPP in log form. To estimate the

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risk premium, we assume that lnQ_t follows an AR(1) process with autocorrelation ρ and variance σ^2,

$$lnQ_{t+1} - ElnQ = \rho(lnQ_t - ElnQ) + u_{t+1},$$

where ρ equals to 0.87, if the half life is five years. Thus, the risk premium can be expressed as,

$$\exp(\tilde{\theta}_{t,n}) = \frac{exp(nr_{t,n}^*)}{exp(nr_{t,n})}\left(\frac{expE(lnQ)}{Q_t}\right)exp[\varrho^n(lnQ_t - E(lnQ))]\,exp\left[\tfrac{1}{2}\sigma^2(1 + \rho^2 \cdots \rho^{2(n-1)})\right].$$

Multiplying through by Q_t and re-arranging, we obtain,

$$S_t = \frac{P_t}{P_t^*}\frac{exp(nr_{t,n}^*)}{exp(nr_{t,n})}\exp[E(\ln Q)]\exp[\varrho^n(\ln Q_t - E(\ln Q))]$$
$$\exp\left[\tfrac{1}{2}\sigma^2(1 + \rho^2 \cdots \rho^{2(n-1)})\right]\exp(-\tilde{\theta}_{t,n})$$

or

$$S_t = \tilde{\tilde{S}}_t\exp(-\tilde{\theta}_{t,n})$$

where

$$20)\qquad \tilde{\tilde{S}}_t = \frac{P_t}{P_t^*}\frac{exp(nr_{t,n}^*)}{exp(nr_{t,n})}\exp[E(\ln Q)]\exp[\varrho^n(\ln Q_t$$
$$- E(\ln Q))]\exp\left[\tfrac{1}{2}\sigma^2(1 + \rho^2 \cdots \rho^{2(n-1)})\right].$$

Thus, the spot exchange rate is the product of the fair value $\tilde{\tilde{S}}_t$ and the risk premium $\exp(-\tilde{\theta}_{t,n})$ conditional on the half life PPP deviation. Taking natural logs and in first differences, we must have

$$\Delta s_t = \pi_t - \pi_t^* + n\Delta(r_{t,n}^* - r_{t,n}) - \Delta\tilde{\theta}_{t,n} + \rho^n\Delta(\ln Q_t - E\ln Q).$$

Hence, a rise in expected real return on the UK linker investment must be brought about by some combination of a rise in UK-US real interest differential, a contemporaneous real appreciation of the dollar

relative to the pound and the real exchange rate deviation from its long run mean.

Using the data set described in Section V, we decompose the spot exchange rate into the risk premium and fair value conditional on the half life of PPP deviation to be five years. In Chart 8 – Chart 10, the dark blue line depicts the spot exchange rate; the aqua – blue line is the fair value defined by Equation 11; the red line is the fair value ($\tilde{\tilde{S}}_t$) defined by Equation 20 conditional on the half life PPP deviation. The risk premium ($\exp\left(-\theta_{t,n}\right)$, $\exp\left(-\tilde{\theta}_{t,n}\right)$) is illustrated as the gap between the spot exchange rate and the fair value. When the exchange rate EUR (Chart 8) or GBR (Chart 9) exceeds fair value, the risk premium is in favor of the USD. In Chart 10, risk premium is in favor of the dollar when spot exchange rate YEN is lower than the fair value, since yen is denoted as yen price per dollar.

Euro

Chart 8
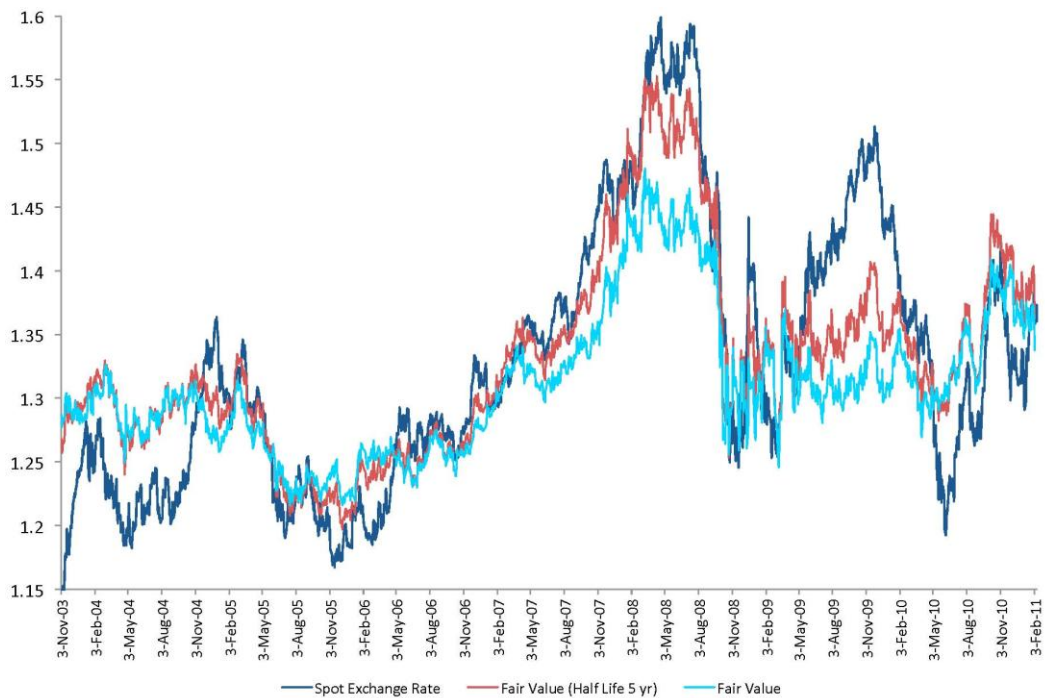
It is clear that the risk premium conditional on the half life PPP deviation ($\tilde{\theta}_{t,n}$) is smaller than the risk premium without PPP adjustment ($\theta_{t,n}$) in general. This phenomenon makes sense intuitively, because the assumption of a constant long run real exchange rate would attribute the change of the long run real exchange rate to be part of the risk premium. However, the difference between $\tilde{\tilde{S}}_t$ and $\tilde{S}_t$, or the difference between $\tilde{\theta}_{t,n}$ and $\theta_{t,n}$, is not significant, especially in first difference terms.

Pound
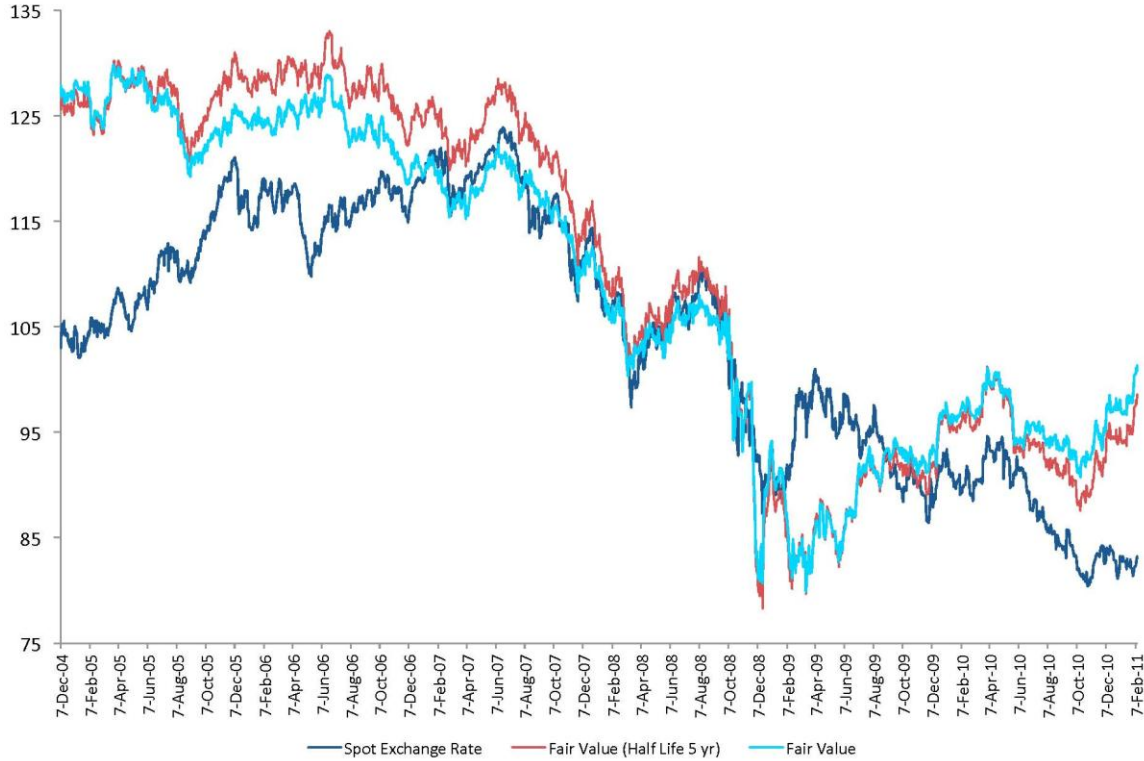
Chart 9

Yen

Chart 10

Spot Exchange Rate — Fair Value (Half Life 5 yr) — Fair Value
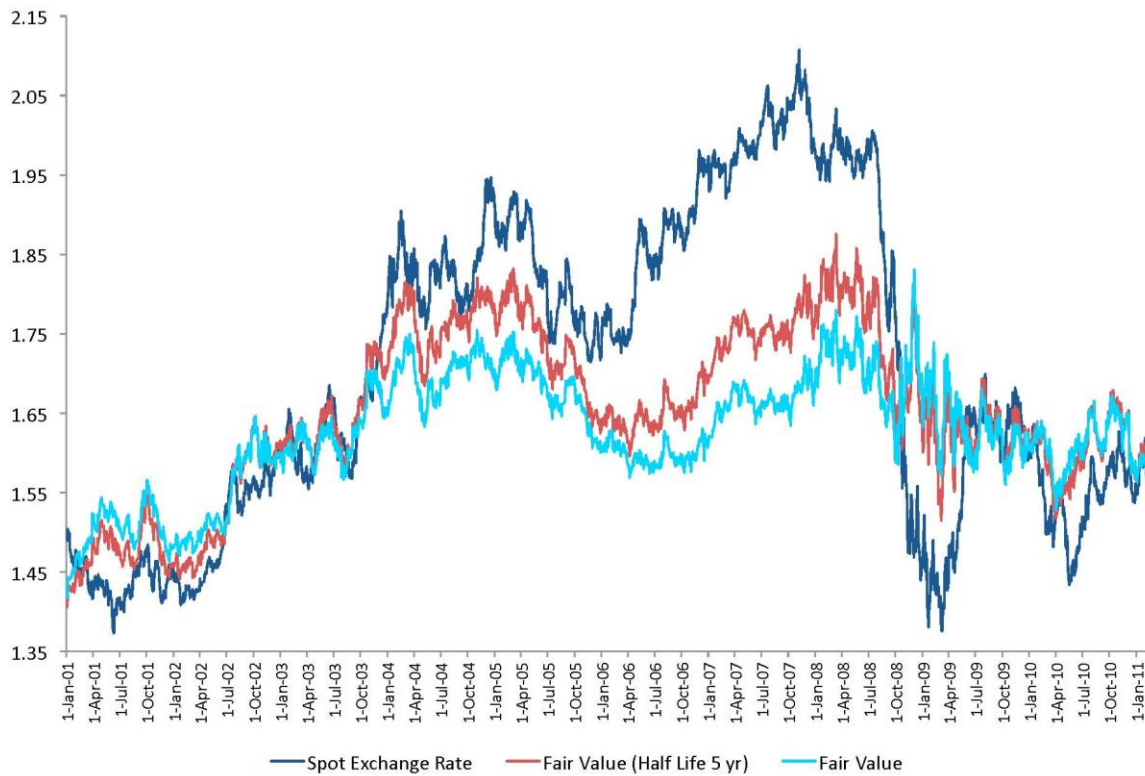
VII. Regressions of $(r^*_{t,n} - r_{t,n})$ on $\Delta\theta_{t,n}$ and Δs_t on $\Delta\theta_{t,n}$

Recall in our framework, period by period, we have

$$21) \quad \Delta s_t = \pi_t - \pi_t^* + n\Delta\left(r^*_{t,n} - r_{t,n}\right) - \Delta\theta_{t,n}$$

A positive shock to $\theta_{t,n}$ is an increase in the risk premium on a UK investment which increases the expected excess return a US investor earns on a UK investment. This must be brought about by some combination of *a rise in UK – US inflation indexed yield differential and or an appreciation of the dollar relative to the pound*. In daily data, we can recover $\Delta\theta_{t,n}$ since every other term in equation (19) is observable (up to the negligible error in approximating unobserved daily inflation differentials with the per day

monthly average inflation differential). We can thus for each country regress $\Delta n(r^*_{t,n} - r_{t,n})$ on $\Delta\theta_{t,n}$ and Δs_t on $\Delta\theta_{t,n}$ to quantify how much of a change in the risk premium is on average reflected in indexed bond yields and how much is reflected in the nominal exchange rate. When we do so, we find that a striking feature in the data is that, *for all three exchange rates, roughly half of a given rise in the risk premium is reflected in a rise in the inflation indexed bond return differentials in favor of the foreign country and the remaining half is reflected in an appreciation of the dollar*. These regression results are presented below in the following charts.

Chart 11: *Regression of 10Δ(r* - r) on Δϑ for Euro*

Slope = 0.4437
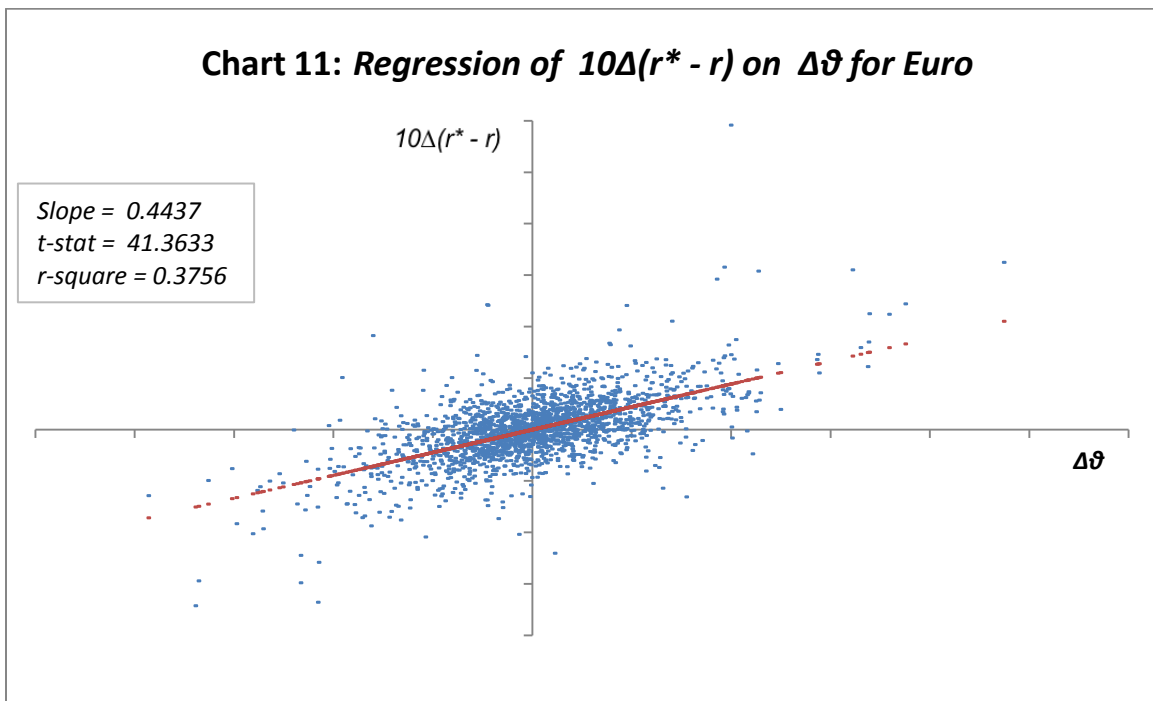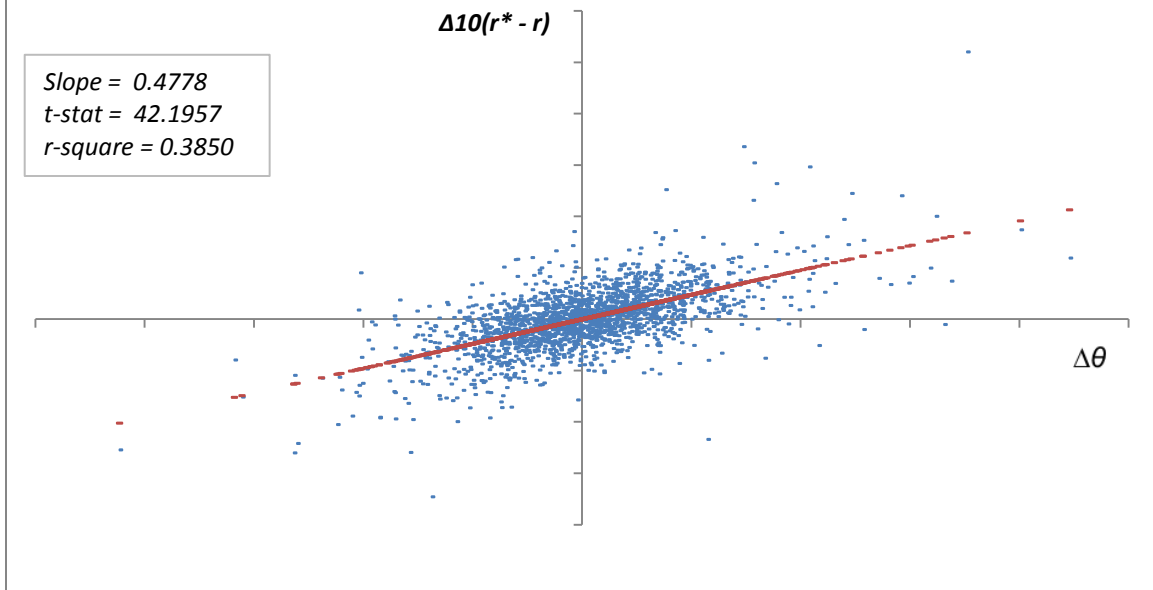t-stat = 41.3633
r-square = 0.3756

Chart 11 presents the regression of the change of the *Euro – US* inflation indexed yield differential $\Delta n(r^*_{t,n} - r_{t,n})$ on the change of euro risk premium $\Delta\theta_{t,n}$. Blue dots are the observations, while the red line presents linear regression result. The coefficient is 0.44, which means around half of the rise in risk premium is reflected in a rise in the inflation indexed bond return differentials in favor of the Euro. Additionally, Chart 12 plots the regression result of the change of the euro Δs_t on $\Delta\theta_{t,n}$. The regression coefficient is -0.53, which means the other half of the rise in risk premium is reflected in an appreciation of the dollar.
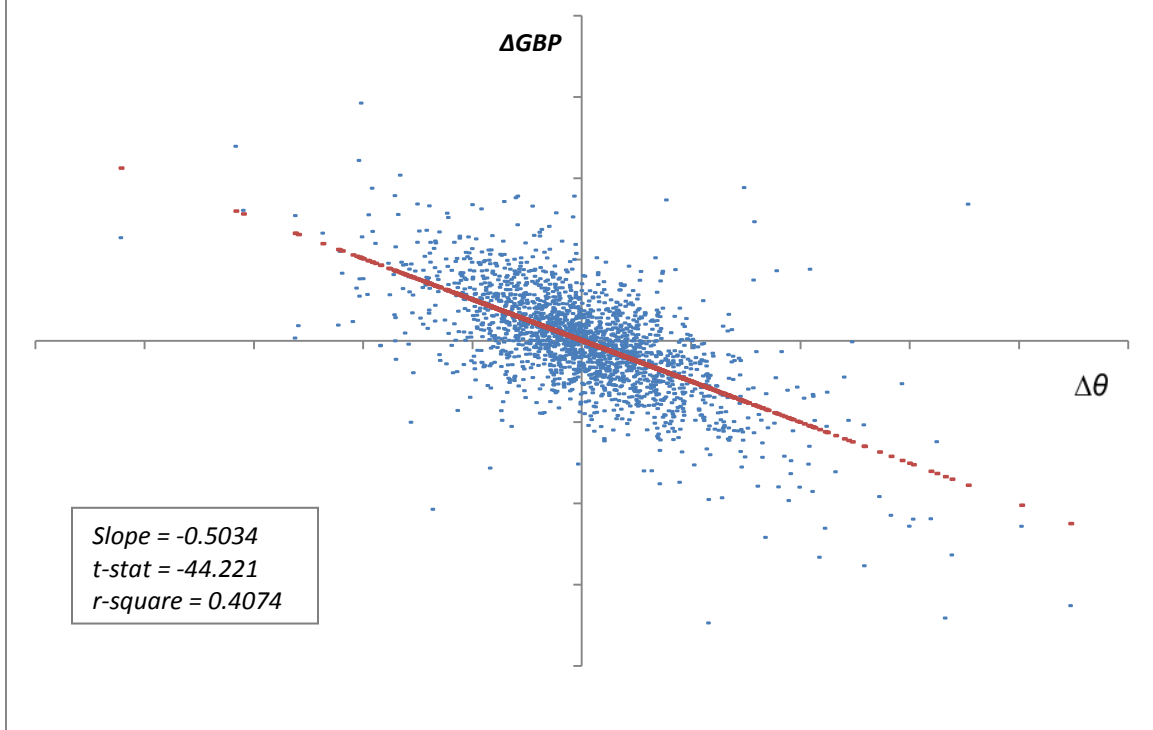
Chart 12: Regression of ΔEur on Δϑ

ΔEur

Δϑ

Slope = -0.5268
t-stat = -50.8078
r-square = 0.4758
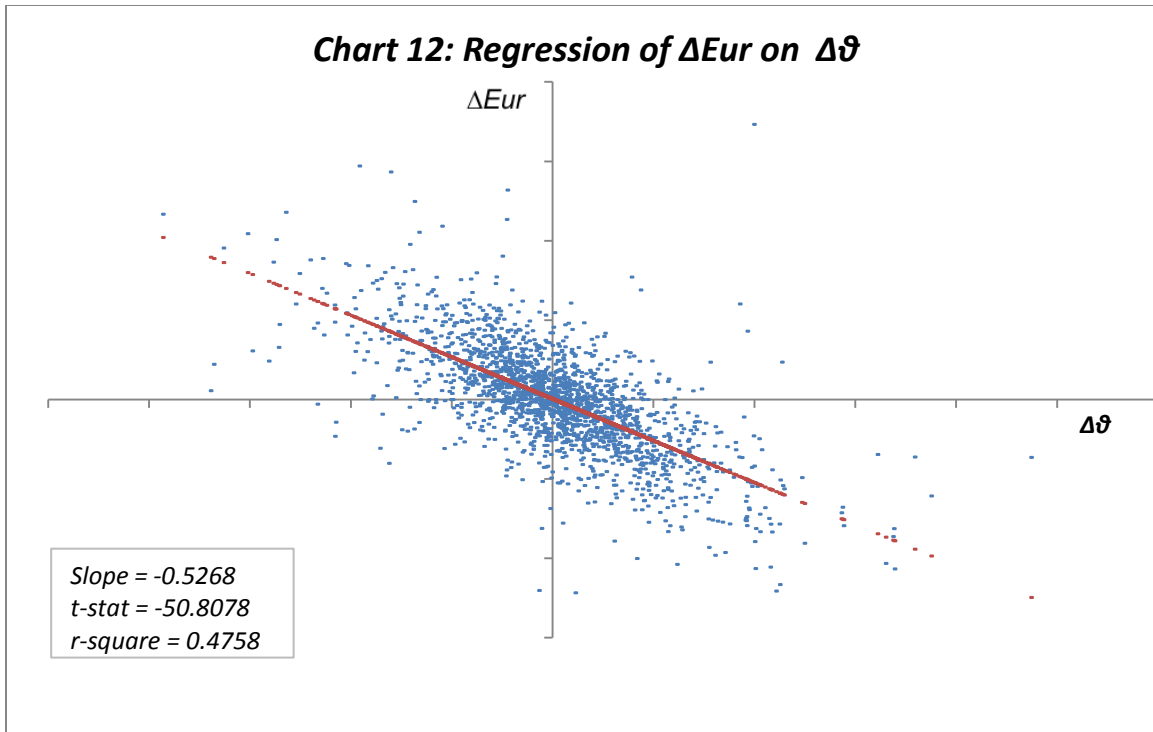
In Chart 13 and 14, we regress the change of the UK-US inflation indexed yield differential $\Delta n(r^{*}_{t,n} - r_{t,n})$ on the change of pound risk premium $\Delta\theta_{t,n}$ and the change of the pound Δs_t on $\Delta\theta_{t,n}$. Again the result confirms that roughly half of a given rise in the risk premium is reflected in a rise in the inflation indexed bond return differential in favor of the UK (the regression coefficient is 0.48) and the remaining half is reflected in an appreciation of the dollar (the regression coefficient is -0.50).
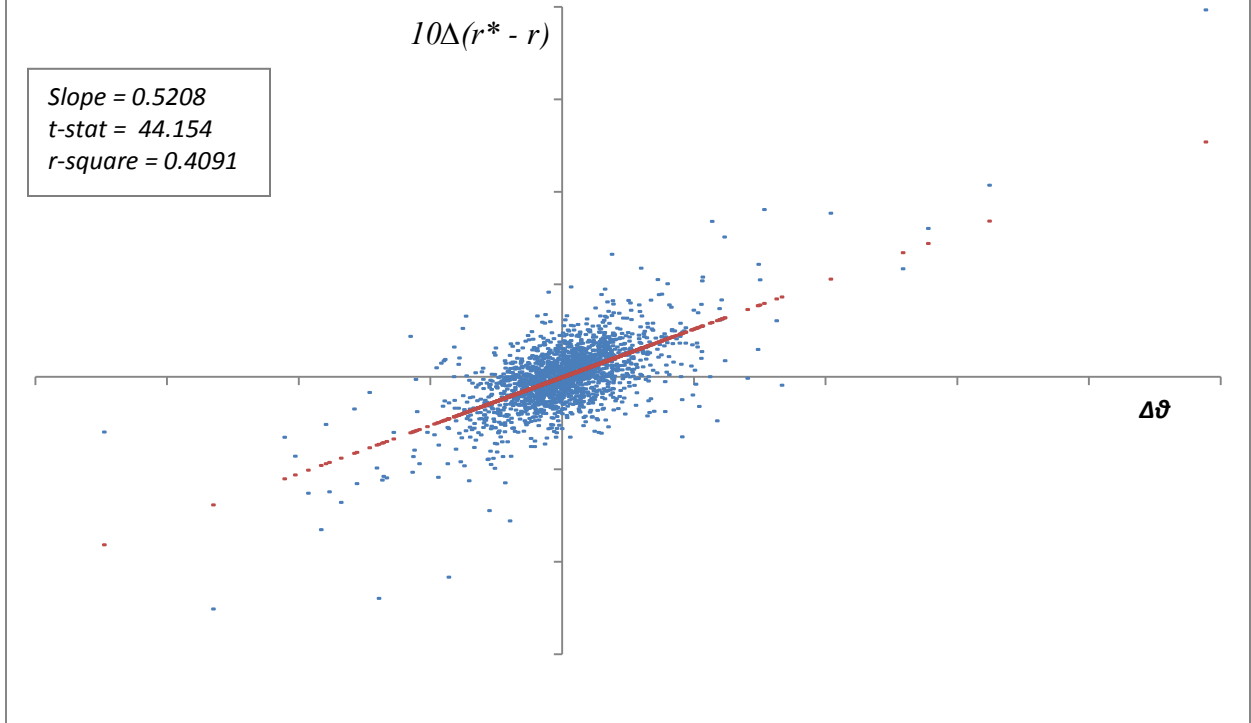
Chart 13: *Regression of Δ10(r* - r) on Δθ for GBP*

Δ10(r* - r)

Slope = 0.4778
t-stat = 42.1957
r-square = 0.3850

Δθ

Chart 14: *Regression of ΔGBP on Δϑ*

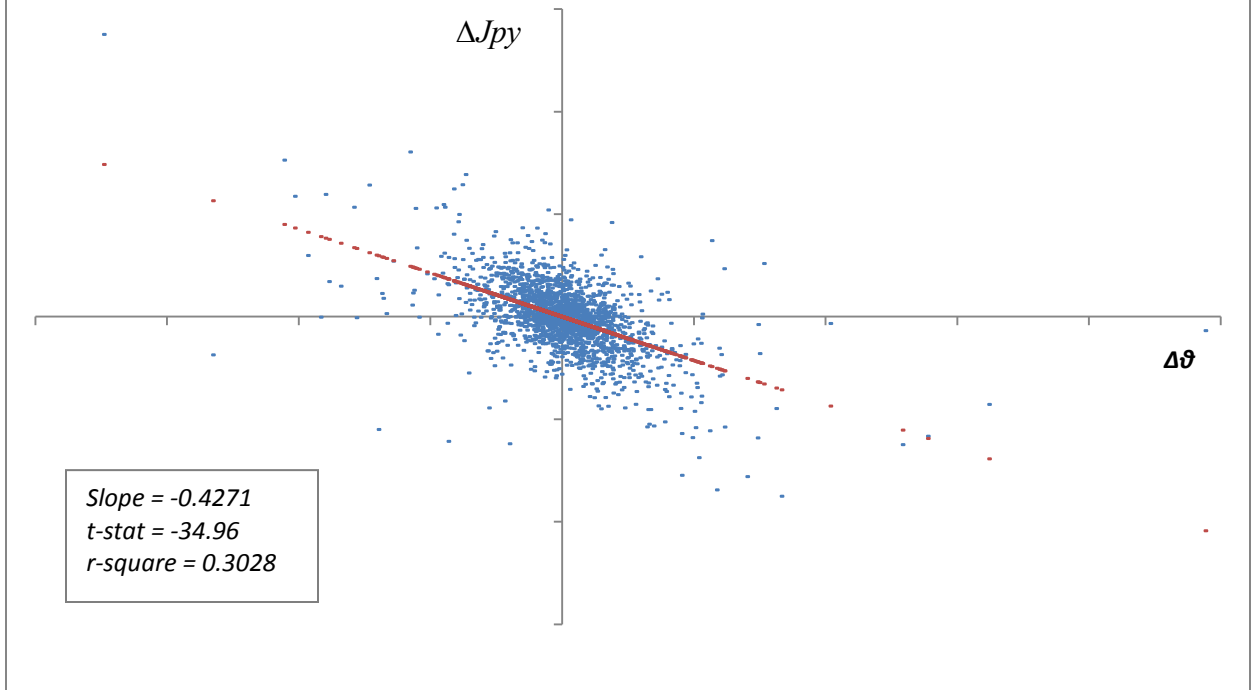ΔGBP

Slope = -0.5034
t-stat = -44.221
r-square = 0.4074

Δθ

Finally, Chart 15 and 16 further confirm this feature using inflation indexed bond market information of Japan and the US.

Chart 15: Regression of 10Δ(r* - r) on Δθ for Jpy

Slope = 0.5208
t-stat = 44.154
r-square = 0.4091

$10\Delta(r^* - r)$

$\Delta\vartheta$

Chart 16: Regression of ΔJpy on Δθ

ΔJpy

$\Delta\vartheta$

Slope = -0.4271
t-stat = -34.96
r-square = 0.3028

We note that while constant terms are included in each regression, none is statistically significant. We do not make any inference of cause and effect. All three variables are endogenous. But any macro model of risk and return in the foreign exchange market in a world with inflation indexed bonds should endeavor to match these correlations.

VIII Does the Risk Premium Forecast Subsequent Exchange Rate Changes?

As we saw in Section VII, only about half of a shock to the risk premium on the foreign currency is reflected in linker yield differentials. This implies that some of the ex ante risk premium is earned via an expected future *depreciation* of the dollar even though, as we document in Section VII on average the dollar appreciates contemporaneously when the risk premium rises. We now explore whether or not in our dataset we in fact estimate that a rise in the foreign currency risk premium today helps to forecast dollar depreciation in the future.

We would like to forecast the subsequent period spot exchange rate in response to a contemporaneous risk premium shock. Thus, we run the following regression on monthly data:

22) $\qquad s_{t+j} - s_t = \zeta_\theta + \beta_\theta^j \theta_t + u_{t+j} \qquad j \in \{1,2,\dots,12\}.$

Our interest is in estimating the sign and significance of the coefficient β_θ^j.

Regressions of the form (22) suffer the complications overlapping data and as well small sample bias since $\theta_{t,n}$ while pre-determined is not exogenous.. On the one hand, the explanatory variable $\theta_{t,n}$, is persistent in the data. On the other hand, the shock to the regressor is correlated with the shock to the exchange rate. Therefore, we implement the augmented regression method introduced in Hjalmarsson (2008) to correct for small-sample bias. We compute robust standard errors following Hansen and Hodrick (1980) GMM.

Using the augmented regression method and the auto-correlation robust standard errors, we plot the estimated β_θ^j and its 90% confidence

interval in Chart 17. The solid black line depicts β_θ^j, while the dashed black lines represent the 90% confidence interval. The x-axis is the forecast horizon j. Also, we estimate β_θ^0 and plot it in the chart, to capture the contemporaneous effect of the risk premium shock. As is evident in Chart 17, the bias adjusted regression coefficients are indeed of the expected positive sign and are statistically significant. In a month when the risk premium on the foreign currency rises, on average in the same month the dollar appreciates, but in subsequent months the dollar tends to depreciate which is an ex-ante source of expected return identified by the risk premium.

In the context of the Fama regression, researchers usually regress the subsequent change of spot exchange rate on the lagged short term interest rate differential but not on the estimate of the lagged risk premium per se. And we also note that the existence of a risk premium does not in and of itself imply that exchange rate changes are forecastable. Instead, if the exchange rate were unforecastable, fluctuations in the risk premium would be reflected entirely in inflation indexed yield differentials.

Chart 17

Note: when j = 0, we regress $S_t - S_{t-1}$ on $\theta_{t,n}$ and plot the coefficient β_θ^0. When j > 0, β_θ^j in Equation 22 is plotted.
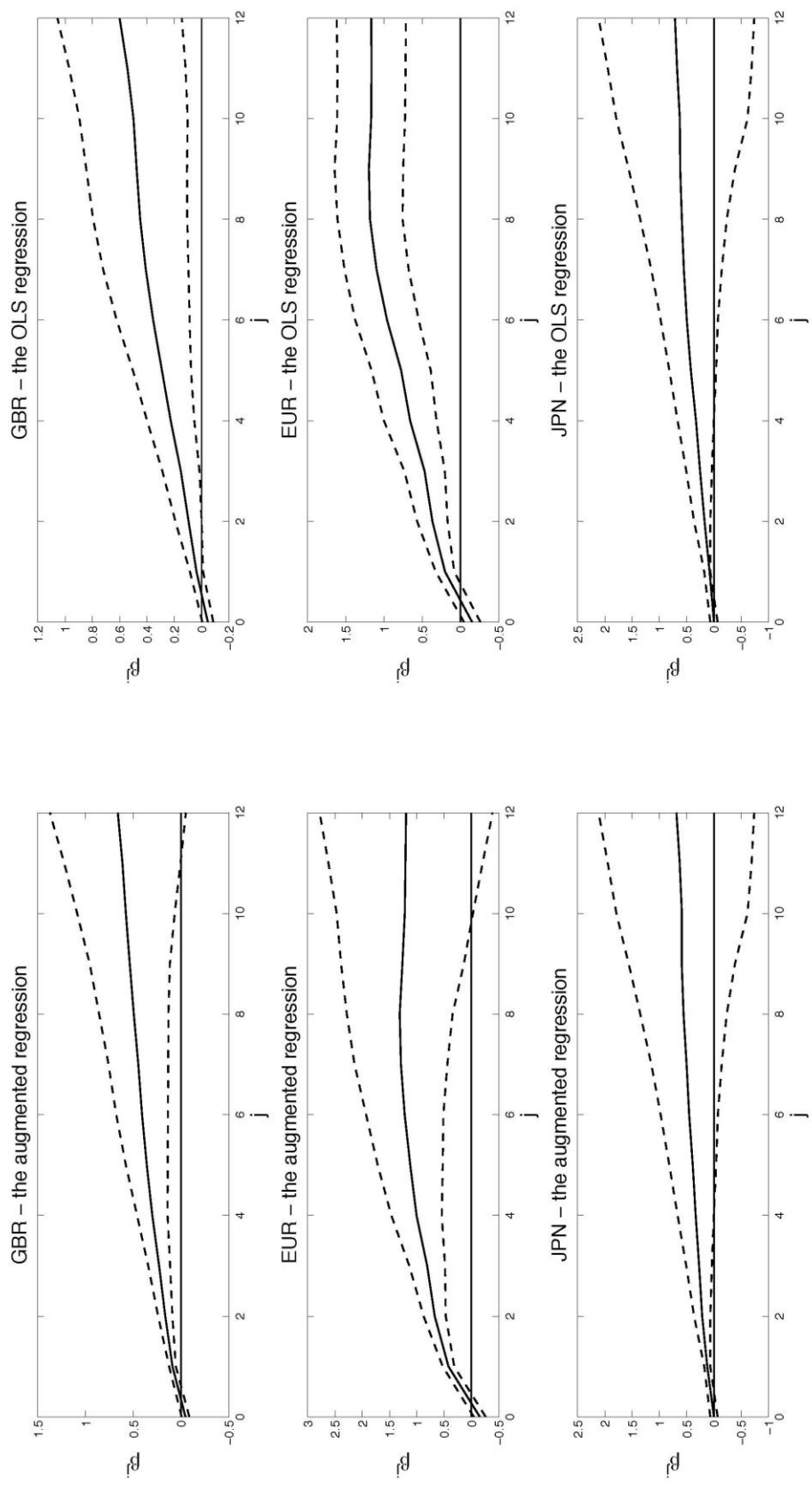
IX. Concluding Remarks

This chapter has derived a novel structural relationship between the nominal exchange rate, national price levels, and observed yields on long maturity inflation - indexed bonds. This relationship can be interpreted as defining the fair value of the exchange rate as well as an empirically observable measure of the risk premium that can open up a wedge between the observed level of the nominal exchange rate and its fair value. We take our theory to the data to study high frequency, real time decompositions of pound, euro, and yen exchange rates into their fair value and risk premium components and find that the relative importance of these two factors varies depending on the sub sample studied. However, sub samples in which, contrary to the Meese-Rogoff (1983) puzzle, 30 to 60 percent of the fluctuations in daily exchange rate changes are explained by contemporaneous changes in fair value are not uncommon. We also find that a striking feature in the data is that, *for all three major exchange rates in our study, on average roughly half of a given rise in the risk premium is reflected in a rise in the inflation indexed bond return differentials in favor the foreign country and the remaining half is reflected in an appreciation of the dollar*. Moreover, we document that our measure of the risk premium , as predicted by theory, contains useful information that can forecast subsequent changes in the exchange rate.

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